Chiral Order
in Frustrated Magnets

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Outline

1. **Frustration and chirality**

2. **Spin-chirality decoupling (I)**
   - Regularly frustrated XY magnets
   - in low dimensions

3. **Chirality-induced novel critical behavior**
   - Stacked-triangular antiferromagnets in 3D

4. **Spin-chirality decoupling (II)**
   - spin glass ordering and the chirality mechanism

5. **Spin-chirality decoupling (III)**
   - chiral glass order in granular high-Tc superconductors
What is frustration?
Various elements compete with each other

- order destabilized
- novel fluctuations enhanced

Frustration induced novel properties
Origin of frustration research — frustrated magnetism

2D antiferromagnetic Ising model

Spin remains disordered
Residual entropy appears (violate the 3rd law?)

triangular lattice
(Wannier 1951)

kagome lattice
(Shoji 1955)
New phenomena!

Frustration

Magnetic ordering suppressed

Weak interaction dominates

The 3rd law

New phenomena!

New paradigm

New concept
Example of geometrical frustration (A)

Three Ising spins coupled antiferromagnetically
Example of geometrical frustration (B)

Three vector spins coupled antiferromagnetically

Spins cant with each other, partly relieving frustration
Chirality (I) --- Vector chirality

**XY spin**

\[ \kappa = \sum S_i \times S_j = \sum \sin(\theta_i - \theta_j) \]

\[ Z_2 \times SO(2) \]

Chirality spin rotation

Right-handed +

Chirality +

Left-handed -

Chirality -

\[ \rightarrow \text{multiferro} \]
Spin super-structure and chirality induced by frustration

Spiral (helical) magnet

[chirality +]  [chirality −]

「right」 「left」

Frustration induced superstructure emergence of the chirality

Electric polarization!
chirality induced ferroelectricity

spin frustration

spin spiral

→ emergence of chirality

→ ferroelectricity

RMnO₃, RMn₂O₅ etc

Chirality controle by electric field

Polarization controle by magnetic field
Spin current mechanism of ferroelectricity
[H. Katsura, N. Nagaosa and A.V. Balatsky, ’05]

Electric polarization is generated by the spin Current (vector chirality)

\[ \vec{P} \propto \vec{e}_{ij} \times (\vec{S}_i \times \vec{S}_j) \]

Inverse Dzyaloshinskii-Moriya interaction

\[ H_{DM} = D_{ij} \cdot \vec{S}_i \times \vec{S}_j \]

Dzyaloshinskii-Moriya interaction
Chirality control by electric fields

- Clockwise spiral
  - Polarization
  - Chirality

- Counterclockwise spiral
  - Polarization
  - Chirality

- Electric field

[T. Arima et al '07]
Chirality (II) --- scalar chirality

Heisenberg spin

\[ \chi = S_i \cdot S_j \times S_k \]

\( Z_2 \times SO(3) \)

right-handed chirality

left-handed chirality

Unconventional anomalous Hall effect
Chirality induced gigantic effective magnetic field and novel transport phenomena
- Berry phase and anomalous Hall effect

Aharonov-Bohm effect
\[ \Phi = B \cdot S \]

phase factor of conduction electrons:

Normal Hall effect due to magnetic fields

Gigantic effective field \(10^4\text{T}\) due to the spin chirality

Chirality induced Hall effect

effective flux \(\Phi\)
\[ = \frac{1}{2} \times \text{chirality } \chi \]

\[ \chi = S_1 \cdot S_2 \times S_3 \]
Locally, chirality is a composite operator of spins, NOT independent of spins

**spin - chirality decoupling**

Beyond a crossover length scale, $L > L^*$, chiral correlations outgrow spin correlations, *i.e.*, 

$$\xi(\text{chiral}) \gg \xi(\text{spin})$$

eventually leading to **two separate transitions**, 

$$T_c(\text{spin}) < T_c(\text{chiral})$$

Fluctuations important!
Spin-chirality decoupling leads to

**Chiral spin liquid (chiral phase)**

- chirality LRO
- spin disordered

possible →

**Ferroelectric spin liquid**

*Ferroelectricity of magnetic origin*
*but without a magnetic LRO*
Spin-chirality decoupling in certain frustrated vector spin systems

Chiral order in regularly frustrated $XY$ magnets in low dimensions
  * 1D triangular-ladder $XY$ antiferromagnet
  * 2D triangular-lattice $XY$ antiferromagnet

Chiral order in 3D Heisenberg spin glasses
  * 3D isotropic Heisenberg SG
  * chirality scenario of experimental spin-glass transitions

Chiral order in ceramic high-$T_c$ superconductors
Monbu-Kagaku-Syo Project

Grant-in-Aid for Scientific Research on Priority Areas

Novel States of Matter Induced by Frustration

Term of Project: 2007-2011 (5 years)

Head: H. Kawamura

Group leaders: S.Maegawa, H.Tsunetsugu, H.Kageyama, T.Arima, K. Hirota, H. Katori

Number of Core Researchers: 36

Budget: 1.2 billion Yen (~ 12 million $)
A01 Fundamental Properties of Frustrated Systems

Novel Order in Geometrically Frustrated Magnet

Frustration and Chirality

Quantum Frustration

A02 Frustration-induced Novel Phenomena and their Applications

Frustration and Quantum Transport

Frustration and Multiferroics

Frustration and Relaxor

Geometrical frustration and functions in spin-charge-lattice coupled system

Organization

http://www.frustration.jp

HK

T. Arima
2. Spin-chirality decoupling (I)

Chiral order in regularly frustrated $XY$ magnets in low dimensions

--- 1D triangular-ladder and 2D triangular-lattice $XY$ antiferromagnet
1D XY AF on the triangular ladder

exact solution [Horiguchi et al ’90]

* Both the spin and the chirality order at \( T_c = 0 \)
* Spin and chirality correlation length exponents are mutually different.

\[ \nu_s = 1 < \nu_k = \infty \]

\( \rightarrow \) two different diverging length scales at the \( T = 0 \) transition, \( i.e., \)

“spin-chirality decoupling” (rigorous !)
Temperature dependence of the spin and the chirality correlation lengths
2D triangular-lattice XY AF

\[ H = J \sum_{ij} S_i \cdot S_j \quad (J > 0) \]

\[ S_i = (S_{ix}, S_{iy}) \]

ordered state: 120 structure with chirality
Specific heat of the 2D triangular XY AF (MC)

[D.H. Lee et al ’84]

divergent specific heat

[S. Miyashita & H. Shiba ’84]

accompanied with the chiral order
Successive spin and chirality transitions are likely to be realized, i.e., the spin-chirality decoupling with

\[ 0 < T_{XY} \text{(spin)} < T_I \text{ (chiral)} \]

\( T = T_I \) : chiral transition
chirality exhibits a LRO while spin decays exponentially.

\( T = T_2 \) : Kosterlitz-Thouless transition of spin (phase)
spin exhibits a quasi-LRO with power-lay decay.

[H.J. Xu et al (’96) (~2%), L. Capriotti et al (’98) (~2%), S. Lee et al (’98) (~2%), D. Loisson et al (~0.3%), Y.Ozeki and N.Ito (’03) (~1%) ]
Length and time scales at the phase transitions of the 2D triangular-lattice $XY$ antiferromagnet
3. **Chirality-induced novel critical behavior**

3D stacked-triangular \textit{XY} and Heisenberg antiferromagnets
3D stacked-triangular-lattice XY AF

Spin and chirality order simultaneously with only one diverging length scale, i.e., no spin-chirality decoupling, whereas, chiral degrees of freedom leads to the possible new chiral universality class

\[ \alpha \sim 0.35, \beta \sim 0.26, \gamma \sim 1.14, \nu \sim 0.55 \]

c.f. \[ \alpha \sim 0, \beta \sim 1/3, \gamma \sim 1.3, \nu \sim 0.7 \]

[H.K. ’85-] symmetry analysis, RG, MC
Instability points in $k$-space (Bragg points)

- **(a)** Ferromagnet
  
- **(b)** Collinear AF
  
- **(c)** Triangular AF
  
- **(d)** Helical magnet
Landau-Ginzburug-Wilson Hamiltonian of chiral magnets for RG analysis

\[ H = (\text{grad} \ a)^2 + (\text{grad} \ b)^2 + r (a^2 + b^2) \]
\[ + u (a^2 + b^2)^2 + \nu \{ (a \cdot b)^2 - a^2 b^2 \} \]

\[ a = (a_1, a_2, a_3, \ldots, a_n), \quad b = (b_1, b_2, b_3, \ldots, b_n) \]

\( n \): number of spin components

\( \nu > 0 \)

spin order at a wavevector \( Q \)

\[ S(r) = a(r) \cos Qr + b(r) \sin Qr \]

\( a \perp b \iff \nu > 0 \)
Continuous transition of new universality vs. first-order transition

Is the new chiral fixed point stable?
For larger $n$, yes!
For physical cases of $n=2$ and 3, still under debate.

Higher-order loop expansion yields a stable fixed point accompanied with exotic oscillating RG flows
FIG. 1. RG flow for the physically important case $D = 3$, $N = 2$. Both $\beta$-functions are represented by the approximants with $\alpha = 2$ and $b = 6$. With this choice, the chiral fixed point coordinates $(\bar{u}, \bar{v})$ are $(1.882, 4.017)$. 

[P. Carabrese et al '02]
Specific heat of the stacked-triangular-lattice $XY$ antiferromagnet $\text{CsMnBr}_3$

$\alpha_{\text{[exp.]}} \sim 0.39 \, (5)$; $A_+/A_- \sim 0.32 \, (20)$

$\alpha_{\text{[theory]}} \sim 0.34 \, (6)$; $A_+/A_- \sim 0.36 \, (20)$

[R.Deutschmann et al ‘92]
Order parameter of the stacked-triangular lattice $XY$ antiferromagnet CsMnBr$_3$

FIG. 1. Magnetic Bragg peak intensity as a function of temperature for $Q=\left(\frac{1}{3}, \frac{1}{3}, 1\right)$. The line is the fit described in Sec. III with $\beta=0.21$.

[Y. Ajiro et al '88]

\[ \beta \, [\text{exp.}] = 0.25(1) \]
\[ \beta \, [\text{theory}] = 0.25(2) \]

[T. Mason et al '89]
Critical properties of the chirality

Chirality orders simultaneously with the spin, with a common correlation length

→ “one-length scaling”

\[ \nu_{\text{spin}} = \nu_{\text{chiral}} \sim 0.55(3) \]

Other exponents differ between for the spin and the chirality:

- \( \beta_{\text{chiral}} = 0.45(2) \)
- \( \phi_{\text{chiral}} = 1.22(6) \)
- \( \beta_{\text{spin}} = 0.25(1) \)
- \( \phi_{\text{spin}} = 1.38(8) \)

How to measure the chirality?
Measurements of the vector chirality

Polarized neutron scattering

\[\Delta I(q) = I(\uparrow - \uparrow) - I(\uparrow - \downarrow)\]

\[d\sigma/d\Omega \propto \langle S_\perp(q) \cdot S_\perp(-q) \rangle + i\mathcal{P} \cdot \langle S_\perp(q) \times S_\perp(-q) \rangle\]

\[q : \text{momentum transfer}\]

\[q \sim \text{integral of } I(q) \propto \text{“chirality”}\]
Chirality of the stacked-triangular-lattice XY antiferromagnet CsMnBr$_3$

$\beta_c = 0.42(7)$

$\beta_c = 0.44(7)$

$\phi_c = 1.29(7)$

[V.P. Plakhty et al ‘00]
In multiferroic materials, chiral critical properties might be measurable by the standard polarization measurements!

\[ P \propto \kappa = S_i \times S_j \]

Use multiferroic properties as a probe of the chirality.
4. **Spin-chirality decoupling (II)**

Chiral order in 3D Heisenberg spin glasses and chirality scenario of experimental SG ordering
Experimental finding of spin glass (SG)

[Canella and Mydosh, 1972]
Canonical SG CuMn, AuFe, etc.

Heisenberg system with weak random magnetic anisotropy

Experimentally, a thermodynamic SG transition and a SG ordered state has been established.

The true nature of the SG transition and the SG order state?

→ still at issue
Numerical results on the 3D EA SG model

* **3D Ising SG**

The existence of a finite-temperature SG transition established in zero field.

* **3D isotropic Heisenberg SG**

Earlier studies suggested no finite-$T$ transition

[Olive, Young, Sherrington, ’86, F. Matsubara et al ‘91]

The possibility of a finite-$T$ transition in the *chiral* sector was suggested [H.K., ’92] --- chiral glass state

**spin-chirality decoupling** $T_{SG} < T_{CG}$

Chirality scenario of experimental SG transition

[H.K. ’92]
Chirality scenario of SG transition [H.K. ’92～]

* Isotropic Heisenberg SG in 3D exhibits a spin-chirality decoupling, with the chiral-glass ordered phase not accompanying the standard SG order.

* Chirality is a hidden order parameter of real SG transitions. Experimental SG transition is a “disguized” chiral-glass transition: The spin is mixed into the chirality via the random magnetic anisotropy.

Scalar chirality

\[ \chi_{ijk} = S_i \cdot (S_j \times S_k) \]
Chiral-glass state

Chiral glass

paramagnetic
Spin-chirality decoupling in the 3D isotropic Heisenberg SG
Recent controversy on the 3D Heisenberg SG

Due to the progress in the computer ability and simulation technique, significant numerical study now becomes possible for the 3D Heisenberg SG.

Consensus in recent numerical studies:
- The 3D Heisenberg SG exhibits a finite- \( T \) transition.
- However, its nature has still been largely controversial.
- Spin & chirality are decoupled or not?

* Yes, decoupling occurs \((T_{SG} < T_{CG})\)
  - H.K. ’98, K.Hukushima & H.K. ’00 ‘05

* No decoupling \((T_{SG} = T_{CG})\)
  - F.Matsubara, T.Shirakura et al,
  - B.W.Lee & A.P.Young ’03; ’07
  - I.Campos et al ’06 (comment: I.Campbell & H.K. ‘07)

Chirality scenario has been contested!
Model

3D Edwards-Anderson Heisenberg SG model

\[ H = - \sum_{ij} J_{ij} S_i \cdot S_j \]

\( S_i = (S_{ix}, S_{iy}, S_{iz}) \) : classical Heisenberg spin

\( J_{ij} \) : Nearest-neighbor random Gaussian coupling with zero mean and variance \( J^2 \)

Simple cubic lattice of size \( N = L^3 \)

Periodic boundary conditions applied

→ new MC simulation

[D.X. Viet and H.K. to appear in PRL]
Spin and chirality correlation length ratio $\xi/L$

--- a quantity most intensively studied

The transition temperature can be estimated from the size and temperature dependence of the dimensionless ratio $\xi/L$

$T>0$ transition $\quad T=0$ transition $\quad$ subtle case

$\xi/L$ $\xi/L$ $\xi/L$

$L$ larger $\quad L$ larger $\quad L$ larger

$T_g$ $\quad T$ $\quad T$ $\quad T$
Correlation length ratios $\xi/L$ [D.X. Viet and H.K., 2008]

$T_{CG}=0.145\pm0.004$

$T_{SG}=0.120\pm0.006$
Binder ratio: $g$

$$g = \frac{1}{2} \left\{ 3 - 2 \frac{\langle q^4 \rangle}{\langle q^2 \rangle^2} \right\}$$

Dimensionless quantity representing the ratio between the fourth and the second moments of the overlap.

**Standard $T>0$ transition**

**$T=0$ transition**

**Special $T>0$ transition**
Binder ratio of the 3D Gaussian Heisenberg SG

[D.X. Viet and H.K., ‘08]

Growing negative dip \( \rightarrow \) consistent with 1-step RSB

\[ T_{SG} \leq 0.12 \]
Glass order parameters $q^{(2)}$ [D.X. Viet and H.K., 2008]

**Chirality**

$T_{CG} \sim 0.148 \pm 0.005$

**Spin**

$T_{SG} \leq 0.13 < T_{CG}$
Critical properties of the chiral-glass transition of the 3D Heisenberg SG

Finite-size scaling plot of the chiral-glass order parameter $q_{CG}^{(2)}$

chiral-glass exponents

$\nu_{CG} = 1.4(2)$
$\eta_{CG} = 0.6(2)$

differ from the 3D Ising values!

$\nu_{CG} \sim 2.5$
$\eta_{CG} \sim -0.35$
Our MC Results on the 3D isotropic Heisenberg SG

1. The spin and the chirality are decoupled, i.e., the chiral-glass order occurs at least 15% higher than the spin-glass order

2. The chiral-glass criticality is different from that of the 3D Ising spin glass, characterized by the exponents:
   \[ \nu_{CG} = 1.4 \pm 0.2, \quad \eta_{CG} = 0.6 \pm 0.2 \]
Chirality hypothesis [H.K. 1992]

Isotropic (ideal) system

→ “spin-chirality decoupling”

Real system is weakly anisotropic!
Spin is “recoupled” to the chirality due to the weak random magnetic anisotropy $D$.

$\mathbb{Z}_2$ [chiral] $\times$ $\text{SO}(3)$ [spin-rotation]

The chiral-glass transition now appears as the SG transition.

“spin-chirality recoupling”

```
+-----------------------------+
| T                          |
+-----------------------------+
| CG                        |
+-----------------------------+
| SG                        |
+-----------------------------+
| para phase                 |
+-----------------------------+
| SG(CG) phase               |
+-----------------------------+
```

$D$
Prediction from the chirality hypothesis for the SG transition of canonical SG

1. SG transition temperature $T=T_g$ is of $O(J)$ depending on the anisotropy $D$ as

$$T_g(D) \sim T_{CG}(0) + cD + \ldots .$$

Only one fixed point (chiral-glass fixed point) governs the SG order both for zero and nonzero $D$.

2. SG critical exponents of canonical SG differ from the 3D Ising exponents, and are $\beta \sim 1$, $\gamma \sim 2$, $\delta \sim 3$ etc. There is no crossover in the standard sense.
Anisotropy \((d)\) dependence of the transition temperature of canonical SG

\[ T_c(d) \sim T_c(d=0)(1+cd^{0.8}) \]

AuFePt & AgMnAu

[A. Fert et al '88]
## Critical properties of canonical SG

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\nu$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CuMn &amp; AgMn</td>
<td>1.0±0.1</td>
<td>2.2±0.1</td>
<td>$\approx 1.4$</td>
<td>$\approx 0.4$</td>
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<tr>
<td>[de Courtenary et al.\textsuperscript{42}]</td>
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<tr>
<td>AgMn</td>
<td>1.0±0.1</td>
<td>2.2±0.2</td>
<td>$\approx 1.4$</td>
<td>$\approx 0.4$</td>
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<tr>
<td>[Bouchiat\textsuperscript{43}]</td>
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<tr>
<td>AgMn</td>
<td>0.9±0.2</td>
<td>2.1±0.1</td>
<td>$\approx 1.3$</td>
<td>$\approx 0.4$</td>
</tr>
<tr>
<td>[Levy et al.\textsuperscript{41}]</td>
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<tr>
<td>CuAlMn</td>
<td>$\approx 1.0$</td>
<td>$\approx 1.9$</td>
<td>$\approx 1.3$</td>
<td>$\approx 0.5$</td>
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<td>[Simpson\textsuperscript{44}]</td>
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<tr>
<td>PdMn</td>
<td>0.9±0.15</td>
<td>2.0±0.2</td>
<td>$\approx 1.3$</td>
<td>$\approx 0.4$</td>
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<tr>
<td>[Coles and Williams\textsuperscript{45}]</td>
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<tr>
<td>CdCr$_2$In$_2$S$_4$</td>
<td>0.75±0.10</td>
<td>2.3±0.4</td>
<td>$\approx 1.3$</td>
<td>$\approx 0.2$</td>
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<tr>
<td>[Vincent et al.\textsuperscript{46}]</td>
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<tr>
<td>$\pm J$</td>
<td>$\approx 0.55$</td>
<td>$\approx 4.0$</td>
<td>1.7±0.3</td>
<td>$-0.35±0.05$</td>
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<tr>
<td>[Kawashima and Young\textsuperscript{31}]</td>
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<tr>
<td>ガウシアン</td>
<td>$\approx 0.64$</td>
<td>$\approx 4.7$</td>
<td>2.0±0.15</td>
<td>$-0.36±0.06$</td>
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<tr>
<td>[Marinari et al.\textsuperscript{32}]</td>
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<tr>
<td>Fe$<em>{0.5}$Mn$</em>{0.5}$TiO$_3$</td>
<td>$\approx 0.54$</td>
<td>4.0±0.3</td>
<td>$\approx 1.7$</td>
<td>$\approx -0.35$</td>
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<tr>
<td>[Gunnarsson et al.\textsuperscript{47}]</td>
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3D Ising theory

3D Ising Exp.

Chiral

$\beta \sim 1$

$\gamma \sim 2$

$\nu \sim 1.2$

$\eta \sim 0.8$
Direct test of the chirality scenario

→ needs to measure the chirality directly

Use an anomalous Hall effect as a probe of chiral order!

Measurements of linear and nonlinear chiral susceptibilities, $X_\chi$ & $X_{\chi_{nl}}$, becomes possible via measurements of Hall coefficient $R_s$.

[G. Tatara & H.K. ’02, H.K. ’03]

$$R_s = \frac{\rho_{xy}}{M} = -A\rho - B\rho^2 - CD [X_\chi + X_{\chi_{nl}} (DM)^2 + ...]$$
* **Approach from the strong coupling**
  [Ye et al, ‘99; Ogushi et al, ’00]

* **Approach from the weak coupling**
  [G. Tatara and H.K., ’02]

Perturbation calculation of the Hall conductivity based on the \textit{s-d} model and the linear response

\[
\sigma_{xy}^{(3)} = \frac{N}{\pi V} \left( \frac{e}{m} \right)^2 (2\pi \nu J)^3 \tau^2 \chi_0 = (4\pi)^2 \sigma_0 J^3 \nu^2 \tau \chi_0.
\]

\[
\chi_0 = \frac{1}{6N} \sum_{x_i} S_{x_i} \cdot (S_{x_2} \times S_{x_3})
\times \left[ \frac{(a \times b)_z}{ab} I'(a)I'(b)I(c) + \frac{(b \times c)_z}{bc} I(a)I'(b)I'(c) + \frac{(c \times a)_z}{ca} I(a)I(b)I'(c) \right].
\]

\[
I(r) = \frac{\sin k_F r}{k_F r} e^{-r/2\ell}
\]

→ The third-order term with respect to the exchange interaction \( J \) gives a finite contribution to \( \rho_{xy} \) which is proportional to the uniform chirality of the system \( \chi_0 \).
○ To get finite $\rho_{xy}(\text{chiral})$, one needs a finite total chirality $\chi_0 > 0$

○ In SG, however, $\chi_0$ is canceled out to be zero in the bulk so that no net $\rho_{xy}(\text{chiral})$ is expected to be realized?

○ In fact, if the system possesses a uniform net magnetization $M$, a uniform component of the total chirality is induced due to the spin-orbit interaction $\lambda$.

[Ye et al, ‘99]

From weak-coupling approach, one has

\[
\langle H_{so} \rangle = D' M \chi_0 = -H_\chi \chi_0
\]

\[
D' = D\lambda (J/\varepsilon_F)^2 (J\tau)^2
\]

which breaks the chiral symmetry and serves as a conjugate field to the chirality.

\[
H_\chi = - D' M \quad \text{(chiral field)}
\]
Anomalous Hall effect as a chiral order in spin glasses

[H.K., PRL (2003)]

Linear and nonlinear chiral susceptibilities

\[ X_\chi = \left(\frac{d\chi_0}{dH}\right)_{H\chi=0}, \]
\[ X_{\chi_{nl}} = \left(\frac{1}{6}\frac{d^3\chi_0}{dH^3}\right)_{H\chi=0} \]

\[ \chi_0 = -X_\chi (DM) - X_{\chi_{nl}} (DM)^3 + ... \]

\[ \rho_{xy}[\text{chiral}] = C_{\chi_0} \]
\[ = -CDM \left[ X_\chi + X_{\chi_{nl}} (DM)^2 + ... \right] \]

Anomalous Hall coefficient \( R_s \)

\[ R_s = \frac{\rho_{xy}}{M} \]
\[ = -A\rho - B\rho^2 - CD \left[ X_\chi + X_{\chi_{nl}} (DM)^2 + ... \right] \]
For canonical spin glasses,

(i) Linear part of $R_s$ exhibits a cusp at $T = T_g$.

(ii) Nonlinear part of $R_s$ exhibits a divergence with an exponent $\sim 2$ at $T = T_g$.

(iii) Chiral part of $R_s$ exhibits a following scaling form

$$R_s[\text{chiral}] \sim |t|^{\beta_{\chi}} \, G(M^2/|t|^{\beta_{\chi} + \gamma_{\chi}})$$

$$\beta_{\chi} \sim 1, \, \gamma_{\chi} \sim 2$$
Anomalous Hall resistivity $\rho_{xy}$ and Hall coefficient $R_S$ of canonical SG; AuMn

Kageyama et al (‘03) FeAl (RSG)
T. Taniguchi et al (‘04) AuFe
P. Pureur et al (‘04) AuMn;
T. Taniguchi et al (‘06) AuMn

[T. Taniguchi et al ’04]

FIG. 1: Simultaneous measurement of the Hall resistivity $\rho_{xy}$ and magnetization $M$ for AuFe 8at.%Fe.

FIG. 2: Temperature dependence of $\rho_{xy}/M$ in the fields indicated. The arrows mark $T_g(H)$. 
Critical property of the chirality

Chiral susceptibility from $R_s$

[T. Taniguchi et al ’06]

$\delta \chi = 3.3$

$\delta_x = 1.7$
What is the nature of the chiral-glass ordered state?

Is there an RSB?
If so, what type?
SG ordered state might possess a complex phase-space structure

Possible multi-valley structure in SG

RSB (replica-symmetry breaking)
Replica symmetry breaking (RSB)

Ergodicity is broken in the phase space into many “pure components” unrelated to global symmetry of the Hamiltonian

[ex.] mean-field SG model (SK model)

Overlap distribution function: $P(q)$

overlap: $q$

$q = \frac{1}{N} \sum_i S^{(a)}_i S^{(b)}_i$ \hspace{0.5cm} (a,b replica index)

$P(q') = [\langle \delta(q'-q) \rangle]$  

$P(q)$ is an indicator of RSB.
Overlap distribution function $P(q)$

- **No RSB**: (a) No RSB
- **full-step RSB**: (b) full-step RSB (*e.g.*, SK model)
- **1-step RSB**: (c) 1-step RSB
- **Combination of (b) & (c)**: (d) Combination of (b) & (c)

**Droplet picture**
Fisher & Huse

**Hierarchical RSB**
Parisi
Overlap distribution of the isotropic model

chirality

spin \[ T = 0.133 < T_{CG} \]

\[ P(q_{\chi}) \]:
central peak & \( \pm q_{EA} \) peak
→ 1-step-like RSB?

\[ \pm J; \ [K.Hukushima & HK '05] \]
Spin glass in off-equilibrium

In equilibrium, FDT holds

\[ R(t_1, t_2); \text{ response function} \]
\[ C(t_1, t_2); \text{ correlation function} \quad T; \text{ heat bath temperature} \]

\[ R(t_1, t_2) = \frac{1}{k_B T} \left[ \frac{dC(t_1, t_2)}{dt_1} \right] \]

In off-equilibrium, FDT does not hold, but there is an off-equilibrium counterpart

\[ R(t_1, t_2) = \frac{X(t_1, t_2)}{k_B T} \left[ \frac{dC(t_1, t_2)}{dt_1} \right] \]

In the limit of \( t_1, t_2 \to \infty \),

\[ \frac{1}{k_B T_{\text{eff}}} \]

\[ X(t_1, t_2) \to X(C(t_1, t_2)) \]

[L.F. Kugliandolo and J. Kurchan '93]

\[ P(q) = \frac{dX(q)}{dq} \quad P(q); \text{ overlap distribution function} \]
Susceptibility $\chi(t_1, t_2)$ vs. Correlation $C(t_1, t_2)$

- (a) No RSB
- (b) full-step RSB
- (c) 1-step RSB
- (d) combination of (b)+(c)

FDT line
Off-equilibrium MC simulation of the isotropic 3D Gaussian Heisenberg SG

Chiral autocorrelations \( C_\chi(t,t_w) \) at \( T=0.05 \)

\( t_w \): waiting time

\( q_{EA} \) plateau

quasi-equilibrium regime

aging regime

\( \log t \)

\( \log t/t_w \)

superaging

Chiral EA parameter \( q_{EA} \)

\( T_{CG} \sim 0.15 \)

\( \beta \sim 1 \)
Off-equilibrium simulation of the weakly anisotropic 3D $\pm J$ Heisenberg SG

$\chi - C$ plot for the spin

$T_g \sim 0.21$

$D = 0.01$

$\frac{T}{T_g} \sim \frac{2T}{T_g}$ irrespective of $T$ !!
Experiment on Heisenberg-like SG

\( CdCr_{1.7}In_{0.3}S_4 \) [D. Herisson and M. Ocio, '02]

\[ T_{eff} \sim 1.9 \, T_g \] (measured at \( T = 0.8 \, T_g \))

1-step-like?
SG (chiral-glass) ordered state of canonical SG exhibits a one-step-like RSB

Analogy to molecular glasses?
Dynamical equations describing structural glass are similar to MF SG models exhibiting a 1-step RSB

[T.E. Kirkpatrick, D. Thirumalai, P.G. Wolynes ’87]

1. **Discontinuous 1-step RSB** (discontinuous $q_{EA}$ at $T=T_g$)
   - $p>2$ spin MF SG, $p>4$ state MF Potts SG
   - dynamical $T_D$ and static $T_g$ ($T_D > T_g$)

   $\rightarrow$ **structural glass**

2. **Continuous 1-step RSB** (continuous $q_{EA}$ at $T=T_g$)
   - $2<p<4$ state MF Potts SG
   - No dynamical $T_D$

   $\rightarrow$ **Heisenberg-like SG**
5. Spin-chirality decoupling (III)

Chiral order in ceramic high-$T_c$ superconductors

Complete isotropy of the gauge space provides a unique opportunity to test the spin(phase)-chirality decoupling
Chirality is hard to measure in magnets. But, in superconductors, the superconducting order parameter is given by

$$\exp(i\theta) = \cos\theta + i\sin\theta$$

where the chirality

$$\sin(\theta_i - \theta_j) \quad (= Si \times Sj)$$

just corresponds to the supercurrent.

Josephson current $I = J\sin(\theta_1 - \theta_2)$
Cuprate high-$T_c$ superconductors: $d$-wave

$\pi$ junction is possible (phase reversal; $J < 0$)

Loop containing odd number of $\pi$ junctions
odd ring ($\pi$ ring)

$\rightarrow$ frustration in “phase” of superconductor
$\rightarrow$ spontaneous loop current

Clockwise current $\rightarrow$ chirality $-\$
Counter-clockwise current $\rightarrow$ chirality $+$

granular (ceramic) superconductors
Cuprate granular (ceramic) superconductors

Random Josephson network consisting of both $\pi$ and 0 junctions → analogy to $XY$ SG

Chiral-glass state

★ spontaneous generation of Random flux in zero field (random freezing of chirality)

★ negative divergence of the nonlinear susceptibility at $T_{CG}$ ($\gamma \sim 4.4$)

★ aging and rejuvenation-memory effect
AC susceptibilities of YBa$_2$Cu$_4$O$_8$ ceramic superconductors

[M. Hagiwara et al, ‘05]

Nonlinear susceptibility near $T_c$

Critical phenomena of nonlinear susceptibility

Intra-grain transition

Intra-grain transition

slope $= 4.4$
Ideal system to detect the possible spin(phase)-chirality decoupling

Gauge space is completely isotropic!

\[ T[\text{chirality}] > T[\text{phase}]? \]

Linear resistivity \( \rho \) should remain nonzero, i.e., an ohmic behavior is expected in the chiral-glass phase, if there occurs the spin(phase)–chirality decoupling

*Experimental work now in progress!*
Summary

* Frustarion causes many interesting phenomena. It might provide us a new paradigm in condensed matter physics.

* Chirality, both the scalar and the vector ones, causes various intriguing phenomena, including multiferroic phenomena.

* A novel “spin-chirality decoupling” phenomenon occurs in certain frustrated systems, including spin glasses and even granular superconductors.