Testing Copernican Principle with kSZ effect

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Note

All of results in this talk are just preliminary. This work is currently in progress.

Any questions and comments are welcome anytime in this talk. But, please don’t expect that I can answer all of them.
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Introduction

-Inhomogeneous universe models-
Spherical Inhomogeneous Universe

We are in the center of the isotropic but inhomogeneous universe. [Zehavi et.al.(1998), Tomita(2000)]

It is quite unnatural!
It completely violates “Copernican principle”!

But, this kind of universe models gather much attention

Why is it?
Why Inhomogeneous Universe?

Dark energy would not be needed

Actually, we succeeded to construct the Lemaître-Tolman-Bondi model whose distance-redshift relation is equivalent to that of $\Lambda$CDM model [arXiv:0807.0932]
LTB Universe

\[ ds^2 = -dt^2 + \frac{\left( \partial_r R(t, r) \right)^2}{1 - k(r) r^2} \, dr^2 + R^2(t, r) \, d\Omega^2 \]

\[ R(t, r) = \left( 6M(r) \right)^{1/3} (t - t_B(r))^{2/3} S(x) \]

\[ x = k(r) r^2 \left( \frac{t - t_B(r)}{6M(r)} \right)^{2/3} \]

Our solution

\[ t_B = 0, \quad M(r) = \frac{4\pi \rho_0 r^3}{3}, \]

\[ k(r) = \frac{0.545745}{(0.211472 + \sqrt{0.026176 + r})} - 2.22881/(0.807782 + \sqrt{0.026176 + r})^2 \]
Fitting $k(r)$
Why Inhomogeneous Universe?

• Dark energy would not be needed

• It has not been completely excluded with current observational data

• It might be possible to exclude or put restrictions on these models in near future. Namely, we are in the stage of observationally confirming the *Copernican principle*. 
Introduction
-Kinematic Sunyaev-Zel’dovich effect-
Sunyaev-Zel’dovich Effect

Change of CMB spectrum due to scattering by a cluster

Thermal Sunyaev-Zel’dovich effect

• Scattering by thermal electrons

Kinematic Sunyaev-Zel’dovich effect

• Scattering by a cluster with drift velocity
  
  drift velocity: the component along the observer’s line-of-site relative to the CMBR
**kSZ Effect**

**drift velocity:** velocity of the cluster relative to CMB

The observer at the cluster sees dipole anisotropy of the CMB temperature

This anisotropy causes the spectrum variation for us
Homogeneous Universe Case

There is the CMB rest frame

Distribution function of photons \( f(\nu) \)

\[
f(\nu) = \frac{1}{\exp(h\nu/k_B T) - 1}
\]

If the cluster has drift velocity, in the cluster rest frame

\[
f(\nu; \theta) = \frac{1}{\exp(h\nu/k_B T(\theta)) - 1}
\]

\[
T(\theta) = \frac{T_0}{\gamma(1 - \beta \cos \theta)}
\]

\[
\gamma = \frac{1}{\sqrt{1 - \beta^2}}
\]

\[
\beta = \frac{\nu}{c}
\]
Current Observational Data

\[ T(\theta) = \frac{T_0}{\gamma(1 - \beta \cos \theta)} \]

\[ \gamma = \frac{1}{\sqrt{1 - \beta^2}} \quad \beta = \frac{v}{c} \]
kSZ Effect in the LTB universe
“current observations of only nine clusters with large error bars already rule out LTB models with void size greater than $\sim 1.5 \text{ Gpc}$”
Problem
J. Garcia-Berillid and T. Haugbolle [JCAP09(2008)016]

• Their own parametrized LTB model which is asymptotically homogeneous universe has been used.

• They assumed that their models are equivalent to corresponding homogeneous universe models in the asymptotic region.

Without this assumption, results can change
Inhomogeneous Universe Case
Inhomogeneous Universe Case

We concentrate on only co-moving clusters

Distribution function of photons

\[ f(\nu; \theta, r_{cl}) = \frac{1}{\exp(h\nu/k_B T(\theta, r_{cl})) - 1} \]

\( \theta \) dependence of the temperature comes from the inhomogeneity of the universe
Last Scattering Surface

Number of interaction per unit time for a photon

$$\Gamma_\gamma = cn_e\sigma_T$$

Ionization rate

$$X_e := \frac{n_e}{n_p} = X_e(\eta, T)$$

Baryon to photon ratio

$$\eta = \eta(t, r)$$

$$\Gamma_\gamma = cn_\gamma(T)\eta(t, r)X_e(\eta(t, r))\sigma_T$$

Decoupling criterion

$$\Gamma_\gamma(\eta(t_{\text{dec}}(r), r), T_{\text{dec}}(r)) = H \sim \sqrt{8\pi\rho/3}$$

$$\rho = \alpha(t_{\text{dec}}(r), r)\rho_\gamma(T_{\text{dec}}(r))$$

4-functional degrees of freedom, 2-equations

$$\alpha(t_{\text{dec}}(r), r), \eta(t_{\text{dec}}(r), r), T_{\text{dec}}(r)$$
How to Fix the Freedoms

We focus on only the dipole component of $T(\theta)$

- We put $T_{\text{dec}}(r)$ by hands. $T_{\text{dec}} = \text{const.}$

- We consider the fictitious reference cluster at $z = z_{\text{cl}}$ and assume that the observer at this cluster observes the temperature distribution given by

$$T(\theta) = \frac{T_{\text{cl}}}{\gamma_{\text{cl}} (1 - \beta_{\text{cl}} \cos \theta)}$$

- We solve null geodesic equation from $z_{\text{cl}}$ to the $z_{\text{dec}}(\theta)$ given by

$$1 + z_{\text{dec}}(\theta) = \frac{T_{\text{cl}}(\theta)}{T_{\text{dec}}} (1 + z_{\text{cl}})$$

- We have $t_{\text{dec}} = t_{\text{dec}}(r)$
We have $t_{\text{dec}} = t_{\text{dec}}(r)$ for $r_{\text{min}} < r < r_{\text{LSS}}$
LSS for Other Clusters

For other cluster at $z < z_{\text{cl}}$, LSS is in $r_{\text{min}} < r$

We can calculate the temperature anisotropy for redshift $z < z_{\text{cl}}$
Functional Degrees of Freedom

4-functional degrees of freedom

\[ \alpha(t_{\text{dec}}(r), r), \eta(t_{\text{dec}}(r), r), T_{\text{dec}}(r) \]

We already fixed 2-degrees of freedom \( T_{\text{dec}} \) and \( t_{\text{dec}} \)

We have two equations

\[ \Gamma^\gamma(\eta(t_{\text{dec}}(r), r), T_{\text{dec}}(r)) = H \sim \sqrt{8\pi\rho/3} \]

\[ \rho = \alpha(t_{\text{dec}}(r), r)\rho^\gamma(T_{\text{dec}}(r)) \]

In general, the other two functions are not constant, i.e., inhomogeneous
Inhomogeneity

Inhomogeneity of $\eta$: iso-curvature perturbation of baryon
Inhomogeneity of $\alpha$: iso-curvature perturbation of DM

These are strongly constrained by WMAP
in the case of homogeneous universe

However, longitudinal components of these inhomogeneity
can exist in inhomogeneous universe model.
They have no observational constraint.
Demonstration
Setup

• Back ground universe model is that given in previous our work

• $z_{\text{LSS}}$ from the observer is fixed 1100, and $T_{\text{dec}}$ is fixed as a constant $T_{\text{dec}} = T_0 \times 1101$

• The effective velocity of the reference cluster is given by $v_{\text{cl}}$

• After the calculation of temperature anisotropy which each cluster at $z < z_{\text{cl}}$ observes, dipole components have been extracted by using method of least squares
Results
Discussion

• The current observational data seems too poor to constraint inhomogeneous universe models

• What about future observations?

• How large is the effect of different function of \( T_{\text{dec}}(r) \)?

• Other problems…?