B Physics in the Warped Extra Dimensions

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Introduction to Extra-dimensions.
All fundamental interactions except gravity can be described by gauge theory

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

i.e. Standard Model (SM)

In SM, there exists two fundamental scales,

Electroweak breaking scale $M_W$ and Planck scale $M_{pl} = 10^{19}$ GeV.

The huge difference between two scales is still a mystery.
Einstein spent his late years to find a theory which unites both gravity and electromagnetic force.

In 1921, Theodor Kaluza suggested that 

*Gravity and Electromagnetism can be unified by extending General Relativity to a five-dimensional Spacetime.*

In 1926, Oskar Klein proposed that 

*The 4th spatial extra dimension is curled up in a circle of very small radius.*

Kaluza-Klein idea was rejected by Einstein.
Mass correction contains a loop diagram with momentum integral.

If we introduce a large but finite $\Lambda_{\text{cut}}$, the correction becomes

$$I(p) \propto \log \Lambda_{\text{cut}}.$$ 

All SM parameters except Higgs mass are scaled at most $\log \Lambda_{\text{cut}}$, which can be handled easily by renormalization.

$\Lambda = M_{\text{pl}}$ is a natural cut-off.
Higgs mass correction is from $\phi^4$ interactions,

\[ M_{\text{Higgs}}^2 \rightarrow M_0^2 + c\Lambda_{\text{cut}}^2 \]

Higgs mass correction is quadratic divergent to cutoff scale.

If $\Lambda = M_{pl}$, the bare mass $M_0$ should be also $M_{pl}$ order and the difference of $M_0^2$ and correction should be smaller than TeV.

The quantum correction should be fine tuned $M_{\text{Higgs}}^2/M_{pl}^2 < 10^{-32}$.

We need an upper bound of $\Lambda_{\text{cut}}$ to be around 1 TeV.
Supersymmetry (SUSY)

SUSY is a symmetry between boson and fermion.

Each particle in SM has a superpartner with the same quantum number but different spin.

Each loop-correction by scalar loop is cancelled by higgsino loop.

(soft) SUSY breaking generates natural cut-off at $\sim$TeV.
A simple idea was brought by


ADD suggest that the hierarchy problem can be solved by

existence of extra spatial dimensions.

Quantum Gravity scale (and thus the cut-off) is lowered to TeV.
Flat Extra dimensions

If our Universe has a thickness $R$, we will find extra dimensions in a small scale $r < R$.

Let the number of extra dimensions $n$.

Then the gravity become stronger in small range.

\[
G \frac{1}{r}, \quad (r > R)
\]
\[
G_{(4+n)} \frac{1}{r^{n+1}}, \quad (r < R)
\]
The 4D gravity can be weakened by extra-dimensions.
Our \((3 + 1)D\) universe
Flat extra dimension $S^1$ (Arkani-Hamed et.al.)
All SM fields should be confined on the 3+1D brane except gravity.
This can be related with the string theory with brane solutions
The object of this model is to solve the hierarchy problem.

To explicit, to keep the Higgs mass 1-loop contribution TeV which should be the QG scale.

\[ \text{QG scale} = \text{TeV} \]

The Quantum gravity effect, like micro Black Hole, graviton production, are expected at the TeV scale.
Graviton wave function in the extra-dimensions is sine functions.

In 3-D, extra-dimensional momentum appears as finite graviton mass. For extra-dimension size $L$, the $n$-th graviton mass is $m_n \sim n\pi/L$.

The graviton mass is very light (a few eV to a few MeV) depending on number of extra-dimensions.

Existence of massive Kaluza-Kelin(KK) graviton is cosmologically dangerous.
Randall-Sundrum model (RS1)

Randall and Sundrum PRL (1999)

RS1 is model with one warped extra-dimension.

\[ ds^2 = e^{-2\sigma(y)}(dt^2 - dx^2) - dy^2, \]

\[ \sigma(y) = k|y| \] is the warp factor

\[ k \sim M_{pl} \] is the curvature of warped space.
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\[ k \sim M_{pl} \] is the curvature of warped space.

The 5th direction \( y \) is bounded and the bulk is AdS\(_5\).

All SM fields are confined on TeV brane (IR boundary)

Gravity resides in the bulk.
The conformal coordinate of $z \equiv e^{\sigma}/k$ is useful.

\[ ds_5^2 = \frac{1}{(kz)^2} \left( dt^2 - d\vec{x}^2 - dz^2 \right). \]
The conformal coordinate of $z \equiv e^\sigma / k$ is useful.

$$ds_5^2 = \frac{1}{(kz)^2} \left( dt^2 - d\vec{x}^2 - dz^2 \right).$$

$y$ is confined in $0 \leq y \leq L$

$z$ is also bounded in $1/k \leq z \leq 1/T$.

$T \equiv e^{-kL} k \equiv \epsilon k$.

If we live on $y = L$ then QG scale $M_{pl}$ scaled to $T = e^{-kL} M_{pl}$.

With $kL \approx 35$, the natural cut-off becomes $O(1)$ TeV.
Where do we live in RS1?

UV(IR) brane is actually a UV(IR) cut-off (boundary).

There is no reason for SM to be confined.

If the energy level is close to TeV,

SM fields behave more tamed in the bulk.

But, how can we develop bulk SM?
Bulk fields in RS1

- Bulk scalar field was introduced at early stage of RS model.
  

- Then Bulk gauge field was introduced by
  

- And bulk fermion was followed soon after.
  
  SC, Hisano, Nakano, Okada and Yamaguchi, (1999),
Graviton in the bulk

In RS1 model, only graviton is the bulk field,

$$\mathcal{L} = -\frac{1}{M^{3/2}} T_{\mu\nu} h_{\mu\nu}(x, z)$$

$T_{\mu\nu}$ is an energy momentum tensor, $h_{\mu\nu}$ is graviton

$$h_{\mu\nu}(x, z) = \sum_{n=0}^{\infty} h_{\mu\nu}^{(n)}(x)f^{(n)}(z)$$

Mode function can be written in Bessel functions

$$f^{(n)}(z) = N_n[J_2(m_nz) + \alpha_n Y_2(m_nz)]$$

1st KK mode graviton mass $m_1 \sim \text{TeV}$.

Zero mass graviton mode function localize at $z_{\text{UV}}$ and weakly couple to matter at $z_{\text{IR}}$. 
Bulk gauge bosons

The action for a 5D $U(1)$ gauge field

$$S_{\text{gauge}} = \int d^4x dz \sqrt{G} \left[ -\frac{1}{4} g^{MP} g^{NQ} F_{MN} F_{PQ} + \frac{1}{2} M^2 g^{MN} A_M A_N \right],$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$, $M, N = \{0, 1, 2, 3, 4\}$.
Bulk gauge bosons

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The general mass term $M^2(z)$,

$$M^2(z) = a_{UV} k^2 \delta(z - z_{UV}) + a_{IR} k^2 \delta(z - z_{IR}) + b^2 k^2,$$

the dimensionless $b$ and $a_{UV}$ ($a_{IR}$) are bulk mass and localized mass on the UV (IR) brane.
The extra-dimension dependence can be separated from the wave function. The KK expansion of the dimension $3/2$ field $A^M(x, z)$ is

$$A_\nu(x, z) = \sqrt{k} \sum_n A^{(n)}_\nu(x) f^{(n)}_A(z)$$

With the equation of motion for mode function

$$-z \partial_z \left( \frac{1}{z} \partial_z f^{(n)}_A(z) \right) + \frac{M^2(z)}{k^2 z^2} f^{(n)}_A(z) = m^{(n)}_A f^{(n)}_A(z),$$

The mode function can be obtained in Bessel function form,

$$f^{(n)}_A(z) = \frac{z}{N^{(n)}_A} \left[ J_\nu(m^{(n)}_A z) + \beta^{(n)}_A Y_\nu(m^{(n)}_A z) \right],$$

where $\nu = \sqrt{1 + b^2}$, $b = 0$ if the gauge symmetry is conserved in the bulk.
For conserved gauge symmetry, there exists a $z$ independent zero mode. i.e. there is a massless gauge boson with a constant gauge coupling in any 3+1D slice of bulk.
For conserved gauge symmetry, there exists a z independent zero mode. i.e. there is a massless gauge boson with a constant gauge coupling in any 3+1D slice of bulk.

SM gauge fields can reside in the bulk but if fermions are on the TeV scale KK gauge mode contributions are too strong and conflicts with electroweak precision data.

It requires that SM fermion should also in the bulk.
The bulk fermion Lagrangian

\[ S_{\text{fermion}} = \int d^4x dy \left[ \bar{\Psi} e^\sigma i \gamma^\mu \partial_\mu \Psi - \frac{1}{2} \bar{\Psi} \gamma_5 \partial_y \Psi + \frac{1}{2} (\partial_y \bar{\Psi}) \gamma_5 \Psi + m_D \bar{\Psi} \Psi \right] \]

where \( m_D \equiv ck \) is Planck scale bulk fermion mass.
Fermions in RS1 Bulk

The bulk fermion Lagrangian

\[ S_{\text{fermion}} = \int d^4x dy \left[ \bar{\Psi} e^{\sigma i} \gamma^\mu \partial_\mu \Psi - \frac{1}{2} \bar{\Psi} \gamma_5 \partial_y \Psi + \frac{1}{2} (\partial_y \bar{\Psi}) \gamma_5 \Psi + m_D \bar{\Psi} \Psi \right] \]

where \( m_D \equiv ck \) is Planck scale bulk fermion mass.

In odd dimension, fermions cannot be chiral, i.e. there should exist both left and right handed fermions.

How can we make SM chiral fermions?

And how can we localize matter at \( z_{\text{IR}} \)?
Bulk Lagrangian should respect the $\mathbb{Z}_2$ parity

$$\gamma_5 \Psi(x, -y) = \pm \Psi(x, y)$$

The bulk fermion can be divided into two chiral components,

$$\hat{\Psi} = \hat{\Psi}_L + \hat{\Psi}_R$$
Bulk Lagrangian should respect the $\mathbb{Z}_2$ parity

$$\gamma_5 \psi(x, -y) = \pm \psi(x, y)$$

The bulk fermion can be divided into two chiral components,

$$\hat{\psi} = \hat{\psi}_L + \hat{\psi}_R$$

which can be expanded to KK modes

$$\hat{\psi}(x, y)_{L(R)} = \sqrt{k} \sum_n \psi_{L(R)}^{(n)}(x)f_{L(R)}^{(n)}(y).$$
Parity of RS1 Warped extra dimension $S^1/\mathbb{Z}_2$
Parity of RS1 Warped extra dimension $S^1/\mathbb{Z}_2$
Localization of chiral fermion in higher dimension

In field theory and Lattice model of odd $n$ spatial dimensions, if a fermion has odd parity mass at $n - 1$ dimensional domain wall only one chirality of fermion mode can be localize at the domain wall. (Massless) Chiral fermion is localized at $n - 1$ dimension domain wall.

Jakiw and Levi (1975), ’t Hooft (1976), Kaplan (1992)
Localization of chiral fermion in higher dimension

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Bulk RS1 model fermion automatically contains chiral fermions in 3+1D.
The general properties of bulk fermions

- The 5-th D parity should be conserved.
- Bulk fermion with odd parity cannot have a zero mode.
- The zero mode is chiral, only $\psi_L^{(0)}$ or $\psi_R^{(0)}$ exists (thus massless).
- Chiral fermion is automatically localized on IR boundary.
- All bulk gauge boson have (massless) zero mode.
- (Massless) SM fields can be induced from bulk fields.
Experimental problems

Energy scale is warped down at IR boundary \((y = y_0)\).

\[ M_{\text{pl}} e^{-k y_0} \sim \text{TeV} \]

The QG scale is about TeV.

Bulk field generates KK modes with about TeV scale mass gap.

t and \( b \) quark KK modes modifies \( W, Z \) boson mass loop corrections.

Quark KK mode contribution to \( M_W / M_Z \) conflicts with experimental data
In 2003, Agashe et al. suggested that if there is

\[ SU(2)_L \times SU(2)_R \times U(1)_{B-L} \text{ gauge symmetry on } S^1/\mathbb{Z}_2 \times \mathbb{Z}'_2. \]
Custodial isospin

In 2003, Agashe et.al. suggested that if there is

\[ SU(2)_L \times SU(2)_R \times U(1)_{B-L} \text{ gauge symmetry on } S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'. \]

There exist Ads5/CFT conformal dual global \( SU(2) \) custodial isospin.

Which protect \( M_W/M_Z \) ratio from KK mode contributions.

Other than custodial isospin,

this model has another important feature in it.
Other than custodial isospin,
this model has another important feature in it.

Two independent parities on each boundary (UV, IR).

\((\pm, \pm)\) and \((\pm, \mp)\)

For bulk fermions, this gives symmetry

\[\gamma_5 \psi(x, -y) = \pm \psi(x, y), \quad \gamma_5 \psi(x, L - y) = \pm \psi(x, L + y)\]
Parities of Agashe et.al. Warped extra dimension $S^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$
Parities of Agashe et.al. Warped extra dimension $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$
Gauge boson KK mode mass on $S^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$

$$m_A^{(n)}/T$$

- $a_{UV} \to \infty$
- $a_{IR} \to \infty$
- $a_{IR,UV} \to \infty$
Only \((+++)\) parity bulk field can have zero mode.

The gauge symmetry in 4D is broken for all other parities.

Thus \(W_R\) fields are massive without bulk mass term.
Only $(++)$ parity bulk field can have zero mode.

The gauge symmetry in 4D is broken for all other parities.

Thus $W_R$ fields are massive without bulk mass term.

The $(+-)$ field contains a much lighter KK mode.

If the boundary mass term turns on and start to increase,

each mode mass transform to the other parity mass spectra.
The boundary mass modifies the KK modes masses

\[ a_{\text{UV}} \rightarrow \infty \]

\[ a_{\text{IR}} \rightarrow \infty \]
KK mass of bulk gauge boson

\[
\begin{align*}
    m_{A^{(++)}}^{(1)} &\approx 2.45 \ T, & m_{A^{(++)}}^{(2)} &\approx 5.57 \ T, & m_{A^{(++)}}^{(3)} &\approx 8.70 \ T, \\
    m_{A^{(+-)}}^{(1)} &\approx 0.24 \ T, & m_{A^{(+-)}}^{(2)} &\approx 3.88 \ T, & m_{A^{(+-)}}^{(3)} &\approx 7.06 \ T, \\
    m_{A^{(--)}}^{(1)} &\approx 2.40 \ T, & m_{A^{(--)}}^{(2)} &\approx 5.52 \ T, & m_{A^{(--)}}^{(3)} &\approx 8.65 \ T, \\
    m_{A^{(--)}}^{(1)} &\approx 3.83 \ T, & m_{A^{(--)}}^{(2)} &\approx 7.02 \ T, & m_{A^{(--)}}^{(3)} &\approx 10.17 \ T.
\end{align*}
\]

If \( T = ke^{-kL} \sim 0.5 \text{TeV} \), \( m_{A^{(+-)}}^{(1)} \) can be about 100 GeV.
The KK mass of bulk gauge boson is given by:

\[ m_{A^{++}}^{(1)} \approx 2.45 \, T, \quad m_{A^{++}}^{(2)} \approx 5.57 \, T, \quad m_{A^{++}}^{(3)} \approx 8.70 \, T, \]
\[ m_{A^{+-}}^{(1)} \approx 0.24 \, T, \quad m_{A^{+-}}^{(2)} \approx 3.88 \, T, \quad m_{A^{+-}}^{(3)} \approx 7.06 \, T, \]
\[ m_{A^{+-}}^{(1)} \approx 2.40 \, T, \quad m_{A^{+-}}^{(2)} \approx 5.52 \, T, \quad m_{A^{+-}}^{(3)} \approx 8.65 \, T, \]
\[ m_{A^{--}}^{(1)} \approx 3.83 \, T, \quad m_{A^{--}}^{(2)} \approx 7.02 \, T, \quad m_{A^{--}}^{(3)} \approx 10.17 \, T. \]

If \( T = ke^{-kL} \sim 0.5 \text{TeV} \), \( m_{A^{++}}^{(1)} \) can be about 100 GeV.

(Even without Higgs!)
Bulk fermion can have four different $\mathbb{Z}_2 \times \mathbb{Z}_2'$ parity sets,

$$\hat{\Psi}_i(x, y) = \sqrt{k} \sum_n [\psi_{iL}^{(n)}(x)f_{iL}^{(n)}(y) + \psi_{iR}^{(n)}(x)f_{iR}^{(n)}(y)]$$
Bulk fermion can have four different $\mathbb{Z}_2 \times \mathbb{Z}_2'$ parity sets,

$$\hat{\Psi}_i(x, y) = \sqrt{k} \sum_n [\psi^{(n)}_{iL}(x)f^{(n)}_{iL}(y) + \psi^{(n)}_{iR}(x)f^{(n)}_{iR}(y)]$$

$i = 1, 2$ represent the parallel conditions ($\pm\pm$)

$f_{iL}$ has ($\pm\pm$) parity and $f_{iR}$ has ($\mp\mp$),

$i = 3, 4$ represent the crossed conditions, where ($\pm\mp$)

$f_{iL}$ has ($\pm\mp$) parity and $f_{iR}$ has ($\mp\pm$).
The fermion mode functions $f_i(y)$ can be transformed into $z$ coordinate easily. $z = e^{\sigma(y)/k}$

The fermion mode functions can be written in Bessel functions as bosons.

$$f_{iL}^{(n)}(z) = \frac{\sqrt{z}}{N_i^{(n)}} \left[ J_{c_i + \frac{1}{2}}(m_i^{(n)} z) + \beta_i^{(n)} Y_{c_i + \frac{1}{2}}(m_i^{(n)} z) \right],$$

$$f_{iR}^{(n)}(z) = \frac{\sqrt{z}}{N_i^{(n)}} \left[ J_{c_i - \frac{1}{2}}(m_i^{(n)} z) + \beta_i^{(n)} Y_{c_i - \frac{1}{2}}(m_i^{(n)} z) \right].$$

$c_i = m_{Di}/k \sim O(1)$ is the dimensionless bulk fermion mass.
Figure: KK mass spectra of bulk fermion without Higgs in unit of $T$
Only \((++\) parity bulk mode function can be a zero mode.

The gauge field with other parity is massive in 4D.

\((+-)\) parity mode has a light gauge boson without Higgs.
Only $(++)$ parity bulk mode function can be a zero mode.

The gauge field with other parity is massive in 4D.

$(+-)$ parity mode has a light gauge boson without Higgs.

The fermions with other than $(++)$ parity become massive in 4D.

$(+-)$ and $(-+)$ mode can have very light KK modes.

Higgs should be confined on IR boundary

**Bulk Higgs scalar brings back hierarchy problem again.**
In bulk SM, there are too many unknowns like bulk field masses.

How do we construct a phenomenological model from bulk SM?
Flavor Physics in Bulk SM

In bulk SM, there are too many unknowns like bulk field masses.

How do we construct a phenomenological model from bulk SM?

Model buildings,

Make least assumptions

and generate as many predictions.
It is not generally possible to deduce all of 5D parameters from the observed fermion mass spectrum and mixing angles.

Thus we introduce two simple assumptions to construct a bulk SM.
Basic formula

It is not generally possible to deduce all of 5D parameters from the observed fermion mass spectrum and mixing angles.

Thus we introduce two simple assumptions to construct a bulk SM.

1. All mass hierarchy are generated from bulk mass structure:

All 5D SM parameters are considered $\mathcal{O}(1)$. 
Basic formula

It is not generally possible to deduce all of 5D parameters from the observed fermion mass spectrum and mixing angles.

Thus we introduce two simple assumptions to construct a bulk SM.

1. All mass hierarchy are generated from bulk mass structure:

   All 5D SM parameters are considered $\mathcal{O}(1)$.

2. There should not be a order changing cancellation during the matrix algebra between mixing and mass matrices.
The SM fermion mass and mixing is decided by both bulk fermion masses and Yukawa coupling with Higgs.

We assume "(almost) universal Yukawa coupling".
Universal Yukawa coupling

The SM fermion mass and mixing is decided by both bulk fermion masses and Yukawa coupling with Higgs. We assume “(almost) universal Yukawa coupling”. There is no hierarchy within 5D Yukawa coupling matrix.

\[
\lambda_5^f \simeq \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{pmatrix}.
\]
If Higgs get the vev $\nu = 174\text{GeV}$ on IR boundary, fermion mass matrix is generated by overlap between left and right handed mode functions.

\[
(M_f)_{ij} = \nu \lambda_{5ij}^f \frac{k}{T} f_R^{(0)}(z, c_{Ri}) f_L^{(0)}(z, c_{Lj}) \bigg|_{z=T^{-1}} \\
\equiv \nu \lambda_{5ij}^f F_R(c_{U_i}) F_L(c_{Q_j}),
\]

where

\[
F_L(c_i) = \epsilon^{c_i-1/2} \sqrt{\frac{2c_i-1}{1-\epsilon^{2c_i-1}}}, \quad F_R(c_i) = \epsilon^{-c_i-1/2} \sqrt{\frac{2c_i+1}{\epsilon^{-2c_i-1}-1}},
\]

Note that left-handed and right handed SM fermion comes from different bulk fermions.
$F_L(c)$ as a function of bulk mass $c$. 

\[ \frac{F_L(c)}{F_L(0.5)} \]
If we increase \((-c_i, F_L(R)(c_i)\) decrease slowly until \((-c_i = 1/2).\)

For \((-c_i > 1/2,\) it decrease fast in power of \(\epsilon (-c_i).\)

Without a large hierarchy in bulk fermion mass \(c_i,\)

All SM fermion masses can be generated.
If we increase \((-c_i, F_{L(R)}(c_i))\) decrease slowly until \((-c_i = 1/2)\).

For \((-c_i > 1/2\), it decrease fast in power of \(\epsilon^{-c_i}\).

Without a large hierarchy in bulk fermion mass \(c_i\),

All SM fermion masses can be generated.

If \((-c_i < 0,\) the 1st KK mode can be very light. \(m^{(1)} \ll \text{TeV}\).

Model with larger symmetry might contains light neutral and stable KK mode, i.e. WIMPS.
Bulk fermions

Since a bulk fermion can contain only one \((++)\) parity zero mode, left and right SM fermions should come from different bulk fields.

\[ Q_i \text{ contains left handed quark} \]

\[ U_i, D_i \text{ contains right handed quark} \]

\[ L_i \text{ contains lepton doublet} \]

\[ E_i \text{ contains lepton singlet} \]

\[ N_i \text{ contains right handed neutrino} \]
Each bulk field contains SM fields which has $(++)$ parity.

\[ Q_i = \begin{pmatrix} u_{iL}^{(++)} \\ d_{iL}^{(++)} \end{pmatrix}, \quad U_i = \begin{pmatrix} u_{iR}^{(++)} \\ D_{iR}^{(--)} \end{pmatrix}, \quad D_i = \begin{pmatrix} U_{iR}^{(--)} \\ d_{iR}^{(++)} \end{pmatrix}, \]

\[ L_i = \begin{pmatrix} \nu_{iL}^{(++)} \\ e_{iL}^{(++)} \end{pmatrix}, \quad N_i = \begin{pmatrix} \nu_{iR}^{(++)} \\ E_{iR}^{(--)} \end{pmatrix}, \quad E_i = \begin{pmatrix} N_{iR}^{(--)} \\ e_{iR}^{(++)} \end{pmatrix}. \]
Fermion mixings

The mass eigenstates of the SM fermions defined by

$$\chi_{fL} = U_{fL}^{\dagger} \psi_{fL}^{(0)}, \quad \chi_{fR} = U_{fR}^{\dagger} \psi_{fR}^{(0)}.$$  

The observed mixing matrix is a multiplication of two independent mixing matrices,

$$V_{\text{CKM}} = U_{uL}^{\dagger} U_{dL} \text{ and } U_{\text{MNS}} = U_{eL}^{\dagger} U_{\nu L}.$$
Minor differences in $\lambda_{5ij}$ are to be absorbed into mixing matrices.

The top quark mass scale is naturally explained by $\nu \simeq 174$ GeV.

Other small SM fermion masses are generated by controlling $c$’s.

This leads to

$$\left( M_f^T M_f \right)_{ij} = \lambda_5^2 \nu^2 F_L(c_{fLi}) F_L(c_{fLj}) \sum_k F_R(c_{Fk})^2.$$  

where $\lambda_5$ is order one.
Quark mass and mixing

For simplified Wolfenstein parameterization and $\lambda \simeq 0.22$.

\[ V_{\text{CKM}} \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \]

We parameterize

\[ (U_{qL})_{ij} = \kappa_{ij} V_{ij}^{\text{CKM}}, \]

where $|\kappa_{ij}| \in [\sqrt{\lambda}, 1/\sqrt{\lambda}]$. 
From the quark masses

\[ M_{u}^{\text{diag}} = \text{diag}(m_u, m_c, m_t) \simeq v_W \text{diag}(\lambda^8, \lambda^{3.5}, 1), \]
\[ M_{d}^{\text{diag}} = \text{diag}(m_d, m_s, m_b) \simeq v_W \text{diag}(\lambda^7, \lambda^5, \lambda^{2.5}). \]

and various experimental constraints (e.g. \( Z \rightarrow b\bar{b} \)),

the bulk quark masses can be determined for \( T \sim \text{TeV} \).

(SC, Kim, Yamaguchi)
From the quark masses

\[ M_u^{\text{diag}} = \text{diag}(m_u, m_c, m_t) \approx v_W \text{ diag}(\lambda^8, \lambda^{3.5}, 1), \]
\[ M_d^{\text{diag}} = \text{diag}(m_d, m_s, m_b) \approx v_W \text{ diag}(\lambda^7, \lambda^5, \lambda^{2.5}). \]

and various experimental constraints (e.g. \(Z \to b\bar{b}\)),
the bulk quark masses can be determined for \(T \sim \text{TeV}\).

(SC, Kim, Yamaguchi)

\[ c_{Q1} \approx 0.61, \quad c_{Q2} \approx 0.56, \quad c_{Q3} \approx 0.3 \pm 0.03. \]
\[ c_{U1} \approx -0.70, \quad c_{U2} \approx -0.52, \quad 0 \lesssim c_{U3} \lesssim 0.2, \]
\[ c_{D1} \approx -0.66, \quad c_{D2} \approx -0.61, \quad c_{D3} \approx -0.56. \]
Lepton mass matrix

\[ U_{\text{MNS}} = U_e^\dagger U_\nu, \]

where

\[ M_\nu^\dagger M_\nu = U_\nu (M_\nu^{\text{diag}})^2 U_\nu^\dagger, \quad M_e^\dagger M_e = U_e (M_e^{\text{diag}})^2 U_e^\dagger. \]

MNS matrix can be approximated as

\[ |U_{\text{MNS}}| \sim \begin{pmatrix} 1 & 1 & \lambda^m \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \]

where the experimental constraint on \( U_{e3} \) gives \( m > 1.3 \).
In this model neutrino mass should have Normal Hierarchy (NH).

\[ M_\nu^T M_\nu \propto \begin{pmatrix} \lambda^{2n} & \lambda^n & \lambda^n \\ \lambda^n & 1 & 1 \\ \lambda^n & 1 & 1 \end{pmatrix} \text{ (NH)}. \]

Also we parameterize lepton masses

\[ M_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3) = v_W \text{ diag}(\lambda^{20.5}, \lambda^{20.5}, \lambda^{19}), \]

\[ M_e^{\text{diag}} = \text{diag}(m_e, m_\mu, m_\tau) = v_W \text{ diag}(\lambda^{8.5}, \lambda^5, \lambda^3). \]
Maximal mixing between 2 and 3 flavors suggests the mixing

\[ U_f \approx \begin{pmatrix}
  1 & \lambda^a_f & \lambda^n \\
  \lambda^b_f & 1 & 1 \\
  \lambda^c_f & 1 & 1
\end{pmatrix}, \]

with \( f = e, \nu. \)

\[ \lambda^a_\nu \sim \lambda^b_\nu + \lambda^c_\nu \sim 1, \quad \lambda^b_e + \lambda^c_e \lesssim \lambda^m, \quad \lambda^n \lesssim \lambda^m. \]

where \( 1.3 \lesssim m \lesssim n \lesssim 1.5. \)

In this model, it is predicted

\[ U_{e3} \approx \lambda^m \approx 0.10 - 0.14. \]
If we start from flavor diagonal 5D gauge interactions, we have FCNC mediated by KK gauge bosons in 4D.

$W_L^{\mu(n)}$, $W_R^{\mu(n)}$ and $B_X^{\mu(n)}$ are the corresponding gauge fields of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. 
Their 5D gauge couplings \((g_5^L, g_5^R \text{ and } g_5^X)\) are related with the 4D effective couplings through

\[
\begin{align*}
g &= g_{4L} = \frac{g_5^L}{\sqrt{kL}}, \\
\tilde{g} &= g_{4R} = \frac{g_5^R}{\sqrt{kL}} \sim g', \\
g_X &= g_{4X} = \frac{g_{4Y} g_{4R}}{\sqrt{g_{4R}^2 - g_{4Y}^2}} \sim g'.
\end{align*}
\]
The 4D gauge interactions with KK gauge modes are

\[
\mathcal{L}_{4D} \supset g_{4D}^a \sum_{n=1}^{\infty} \left( \hat{g}_L^{(n)}(c_i) \bar{\psi}_{iL} \gamma^\mu \psi_{iL}^{(0)} + \hat{g}_R^{(n)}(c_i) \bar{\psi}_{iR} \gamma^\mu \psi_{iR}^{(0)} \right) A_{\mu}^{a(n)},
\]

where \(T^a = (T_L, T_R, Y_X)\) for \(A^a = (W_{3L}, W_{3R}, B_X)\), \(Y_X = (B - L)/2\), \(g_{4D}^a = g_5^a/\sqrt{kL}\)

Effective coupling can be achieved from integration of mode function overlap.

\[
\hat{g}_L^{(n)}(c_f_i) = \sqrt{kL} \int dz k \left[ f_{L}^{(0)} (z, c_f_i) \right]^2 f_{A}^{(n)} (z) \equiv \hat{g}^{(n)}(c_f_i),
\]
\[
\hat{g}_R^{(n)}(c_f_i) = \sqrt{kL} \int dz k \left[ f_{R}^{(0)} (z, c_f_i) \right]^2 f_{A}^{(n)} (z) = \hat{g}^{(n)}(-c_f_i).
\]
$\hat{g}^{(1)}(c)$ saturates around -0.2 if $c > 0.55$. 
Flavor violation mediated by the neutral KK gauge bosons are

\[ \mathcal{L}_{4D} \supset -\frac{1}{2} \sum_{i,j,n} \left[ g \left( K_{Qij}^{(n)} \bar{d}_i \gamma^\mu d_j + K_{Lij}^{(n)} \bar{e}_i \gamma^\mu e_j \right) W_{3L\mu}^{(n)} 
+ \tilde{g} \left( K_{Dij}^{(n)} \bar{d}_i \gamma^\mu d_j + K_{Eij}^{(n)} \bar{e}_i \gamma^\mu e_j \right) W_{3R\mu}^{(n)} 
- g_X \left( K_{Qij}^{(n)} \bar{d}_i \gamma^\mu d_j - K_{Lij}^{(n)} \bar{e}_i \gamma^\mu e_j \right) B_{X\mu}^{(n)} \right], \]

where \( i, j \) are the generation indices \( (i, j = 1, 2, 3) \).
\[ K_{Qij}^{(n)} = \sum_{k=1}^{3} (U_{dL}^\dagger)_{ik} \hat{g}^{(n)}(c_{Q_k}) (U_{dL})_{kj}, \]

\[ K_{Dij}^{(n)} = \sum_{k=1}^{3} (U_{dR}^\dagger)_{ik} \hat{g}^{(n)}(-c_{D_k}) (U_{dR})_{kj}, \]

\[ K_{Lij}^{(n)} = \sum_{k=1}^{3} (U_{eL}^\dagger)_{ik} \hat{g}^{(n)}(c_{L_k}) (U_{eL})_{kj}, \]

\[ K_{Eij}^{(n)} = \sum_{k=1}^{3} (U_{eR}^\dagger)_{ik} \hat{g}^{(n)}(-c_{E_k}) (U_{eR})_{kj}. \]
Lepton flavor violation

If we focus the KK mass within detectable range $2 – 3 \text{ TeV}$, The Lepton flavor violation (LFV) processes give strong bounds.

\[ \frac{\Gamma(\mu \rightarrow 3e)}{\Gamma(\mu \rightarrow e\nu_{\mu}\bar{\nu}_{e})} \simeq (K_{L11}^{(1)}K_{L12}^{(1)})^2 \left( \frac{m_Z}{M_A^{(1)}} \right)^4 \lesssim 1.0 \times 10^{-12}, \]

\[ \frac{\Gamma(\tau \rightarrow 3\mu)}{\Gamma(\tau \rightarrow \mu\nu_{\tau}\bar{\nu}_{\mu})} \simeq (K_{L22}^{(1)}K_{L23}^{(1)})^2 \left( \frac{m_Z}{M_A^{(1)}} \right)^4 \lesssim 10^{-6}. \]
If KK gauge boson is experimentally testable, i.e. $M_A^{(1)} \simeq 2\text{-}3$ TeV, we can specify its mixing matrix

$$U_{eL} \simeq \begin{pmatrix} 1 & \delta & \delta \\ < \delta^2 & 1 & 1 \\ \delta & 1 & 1 \end{pmatrix},$$

where $\delta = \lambda^m \simeq 0.1,$
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where $\delta = \lambda^m \simeq 0.1$, and bulk lepton masses (SC, Kim, Song)

$$c_{L1} \simeq 0.59, \quad c_{L2} \simeq 0.5, \quad c_{L3} \simeq 0.5,$$

$$c_{E1} \simeq -0.74, \quad c_{E2} \simeq -0.65, \quad c_{E3} \simeq -0.55,$$

$$c_{N2} \simeq -1.2, \quad c_{N3} \simeq -1.1.$$
$B$ mixing and $B$ rare decays are well measured and estimated.

In many cases, SM corrections are suppressed to higher order.

It is a good test for the new physics effect.

Also there will be more experimental data, soon.
The total transition amplitude for $b \to s l_i^+ l_i^-$ can be written as

$$\mathcal{M} = \mathcal{M}_{\text{SM}} + \mathcal{M}_{\text{new}}.$$ 

where

$$\mathcal{M}_{\text{new}} = \frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{ts}^* \left[ C_{LL}(\bar{s}_L \gamma^\mu b_L)(\bar{l}_L \gamma^\mu l_L) + C_{LR}(\bar{s}_L \gamma^\mu b_L)(\bar{l}_R \gamma^\mu l_R) \right.$$ 

$$\left. + C_{RL}(\bar{s}_R \gamma^\mu b_R)(\bar{l}_L \gamma^\mu l_L) + C_{RR}(\bar{s}_R \gamma^\mu b_R)(\bar{l}_R \gamma^\mu l_R) \right].$$
The RS contributions can be written as

\[
\mathcal{M}_{\text{RS}} \simeq \sum_{n=1}^{\infty} \frac{1}{4M_A^{(n)2}} \left[ \left( g^2 K_{Q23}^{(n)} K_{Lii}^{(n)} - g_X^2 K_{Q23}^{(n)} K_{Lii}^{(n)} \right) (\bar{s}_L \gamma^\mu b_L) (\bar{t}_i L \gamma^\mu l_i L) 
- g_X^2 K_{Q23}^{(n)} K_{Eii}^{(n)} (\bar{s}_L \gamma^\mu b_L) (\bar{t}_i R \gamma^\mu l_i R) 
- g_X^2 K_{D23}^{(n)} K_{Lii}^{(n)} (\bar{s}_R \gamma^\mu b_R) (\bar{t}_i L \gamma^\mu l_i L) 
+ \left( \tilde{g}^2 K_{D23}^{(n)} K_{Eii}^{(n)} - g_X^2 K_{D23}^{(n)} K_{Eii}^{(n)} \right) (\bar{s}_R \gamma^\mu b_R) (\bar{t}_i R \gamma^\mu l_i R) \right].
\]
new physics parameters \( C_{XX'} \) (\( X, X' = L, R \)) are

\[
C_{LL} \approx \left( \frac{\tilde{G}}{M_A} \right)^2 (g^2 - g_X^2) \kappa^2 \hat{g}(c_{Q3}) K_{Lij},
\]

\[
C_{LR} \approx \left( \frac{\tilde{G}}{M_A} \right)^2 g_X^2 \kappa^2 \hat{g}(c_{Q3}) K_{Eij},
\]

\[
C_{RL} \approx 2 \left( \frac{\tilde{G}}{M_A} \right)^2 g_X^2 \kappa_D^2 \hat{g}(c_{D3}) K_{Lij},
\]

\[
C_{RR} \approx 2 \left( \frac{\tilde{G}}{M_A} \right)^2 (\tilde{g}^2 - g_X^2) \kappa D^2 \hat{g}(c_{D3}) K_{Eij},
\]

where \( \tilde{G} = (\pi/2\sqrt{2}G_F\alpha)^{1/2} \approx 3.5 \) TeV.
**Table:** The values of $C_{LL}$, $C_{RL}$, $C_{LR}$, and $C_{RR}$ for $b \rightarrow s l_i^{+}l_j^{-}$. We set $M_A^{(1)} = 2$ TeV, $\kappa Q$, $D = 1$, and $\delta = 0.15$. 

<table>
<thead>
<tr>
<th></th>
<th>$e^+e^-$</th>
<th>$e^+\mu^-$</th>
<th>$e^+\tau^-$</th>
<th>$\mu^+\mu^-$</th>
<th>$\mu^+\tau^-$</th>
<th>$\tau^+\tau^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{LL}$</td>
<td>$-0.3$</td>
<td>$\pm 7 \times 10^{-3}$</td>
<td>$\pm 0.05$</td>
<td>$-7 \times 10^{-3}$</td>
<td>$\pm 6 \times 10^{-3}$</td>
<td>$-4 \times 10^{-5}$</td>
</tr>
<tr>
<td>$C_{RL}$</td>
<td>$0.02$</td>
<td>$\pm 5 \times 10^{-4}$</td>
<td>$\pm 4 \times 10^{-3}$</td>
<td>$6 \times 10^{-4}$</td>
<td>$\pm 5 \times 10^{-4}$</td>
<td>$3 \times 10^{-6}$</td>
</tr>
<tr>
<td>$C_{LR}$</td>
<td>$-0.1$</td>
<td>$\pm 0.01$</td>
<td>$\pm 10^{-3}$</td>
<td>$-0.1$</td>
<td>$\pm 0.01$</td>
<td>$-0.1$</td>
</tr>
<tr>
<td>$C_{RR}$</td>
<td>$0.03$</td>
<td>$\pm 3 \times 10^{-3}$</td>
<td>$\pm 2 \times 10^{-4}$</td>
<td>$0.03$</td>
<td>$\pm 3 \times 10^{-3}$</td>
<td>$0.02$</td>
</tr>
</tbody>
</table>

Only $B \rightarrow K^* e^+ e^-$ has significant contribution from new physics.
BR distribution is most sensitive to $C_{LL}$.

Since $C_{LL}$ for $B \rightarrow K^* e^+ e^-$ is negative, we have reduced result compared to the SM.

The reduction can be maximally about 20% but the theoretical uncertainty of the form factors are known to be about 15%.

It would be quite challenging for experiments to probe this new physics effect from the BR distribution.
Figure: $d\text{BR}/dq^2$ as a function of $q^2$ for $B \rightarrow K^*e^+e^-$. The blue line is for the bulk RS model with $\kappa = \sqrt{2}$ and $M_A^{(1)} = 2$ TeV.
The zero value position of the forward-backward asymmetry is sensitive to new physics. ($A_{FB}(\hat{s}_0) = 0$.)

The forward-backward asymmetry $A_{FB}(\hat{s})$ is defined by

$$\frac{d}{d\hat{s}} A_{FB}(\hat{s}) = \frac{\int_0^1 dz \frac{d\Gamma}{d\hat{s} dz} - \int_{-1}^0 \frac{d\Gamma}{d\hat{s} dz}}{\int_0^1 dz \frac{d\Gamma}{d\hat{s} dz} + \int_{-1}^0 \frac{d\Gamma}{d\hat{s} dz}},$$

$\hat{s} = q^2 / m_B^2$, $z = \cos \theta$, and $\theta$ is the angle between $K^*$ and $l^-$. 
Figure: $dA_{FB}/d\hat{s}$ as a function of $\hat{s}$ for $B \to K^* e^+ e^-$. The blue line is the new physics result with $\kappa = \sqrt{2}$ and $M_A^{(1)} = 2$ TeV.
One of the most unique FCNC feature is that;

only the $B \rightarrow K^* e^+ e^-$ decay has sizable new physics effect while others have quite negligible effects.

Therefore, we consider the ratio of differential decay rate of $B \rightarrow K^* e^+ e^-$ to that of $B \rightarrow K^* \mu^+ \mu^-$. 
Figure: $\frac{d\Gamma}{d\hat{s}}(B \rightarrow K^* e^+ e^-) / \frac{d\Gamma}{d\hat{s}}(B \rightarrow K^* \mu^+ \mu^-)$ as a function of $\hat{s}$ in the SM and the custodial bulk RS with $\kappa = 1, \sqrt{2}$. We set $M_A^{(1)} = 2$ TeV.
Figure: \( \frac{\Gamma(B \to K^{*} e^+ e^-)}{\Gamma(B \to K^{*} \mu^+ \mu^-)} \) as a function of \( M_A^{(1)} \) in the SM and the custodial bulk RS with \( \kappa = 1, \sqrt{2} \). Here \( \Gamma \) is partially integrated \( d\Gamma/d\hat{s} \) for \( \hat{s} \in [0.1, \hat{m}_c^2] \).
The best chance to observe the custodial bulk RS model effect is through \( b \rightarrow s e^+ e^- \) due to the suppressed couplings of \( \mu^+ - \mu^- - A^{(n)} \) and \( \tau^+ - \tau^- - A^{(n)} \).

\( C_{LL} \) is dominant, and \( C_{LR} \) is the second dominant.

Two other decays of \( b \rightarrow s\mu^+ \mu^- \) and \( b \rightarrow s\tau^+ \tau^- \) have dominant vertex of \( C_{LR} \). Unfortunately, their magnitudes are too small for experiments to probe in near future.

Other non-diagonal decay modes of \( b \rightarrow sl_i^+ l_j^- (i \neq j) \) are quite suppressed in this model.
$B_0^q - \bar{B}_0^q$ mixing data

\[
\begin{align*}
\Delta M_d^{\exp} & = (0.507 \pm 0.004) \text{ ps}^{-1}, \\
\Delta M_s^{\exp} & = \left[ 17.33^{+0.42}_{-0.21}(\text{stat}) \pm 0.07(\text{syst}) \right] \text{ ps}^{-1}.
\end{align*}
\]

There is no tree level contribution to $B_0^q - \bar{B}_0^q$ in SM.

Can be a good test for the RS bulk SM.
The KK gluon contribution to $B^0_q - \bar{B}^0_q$ is tree level while SM contribution is from box diagram.

(SC, Song and Kim 07)
For the $B^0_q - \bar{B}^0_q$ transition amplitude $M^q_{b\bar{b}}$ defined by

$$\langle B^0_q | H_{\Delta B=2}^\text{eff} | \bar{B}^0_q \rangle = 2M_{B_q}M^q_{b\bar{b}},$$

where

$$\Delta M_q = 2 |M^q_{b\bar{b}}|.$$  

“mixing-induced” CP violation by phase

$$\phi_q = \arg (M^q_{b\bar{b}}).$$
$B^0_q - \bar{B}^0_q$ mixing from the SM box diagrams and the RS KK gluons:

\[
M_{bb}^q = M_{bb}^{q,\text{SM}} \left(1 + \frac{M_{bb}^{q,\text{RS}}}{M_{bb}^{q,\text{SM}}} \right).
\]

We parameterize the new physics effect by

\[
r_q e^{i\sigma_q} = \frac{M_{bb}^{q,\text{RS}}}{M_{bb}^{q,\text{SM}}},
\]

where $r_q \geq 0$ and $\sigma_q$ is real.
The $r_q$ and $\sigma_q$ are constrained by the experimental result for $\Delta M_q$ and the theoretical calculation of the $\Delta M_q^{SM}$, of which the ratio is defined by $\rho_q$:

$$
\rho_q \equiv \left| \frac{\Delta M_q}{\Delta M_q^{SM}} \right| = \frac{M_{q_{SM}}^{bb} + M_{q_{RS}}^{bb}}{M_{q_{SM}}^{bb}} = \sqrt{1 + 2r_q \cos \sigma_q + r_q^2}.
$$
$\phi_d$ can be divided into the SM phase and that from New Physics (NP) contributions.

$$\phi_q = \phi_q^{\text{SM}} + \phi_q^{\text{NP}} = \phi_q^{\text{SM}} + \text{arg}(1 + r_q e^{i\sigma_q}).$$

$\phi_q^{\text{NP}}$ is determined by $r_q$ and $\sigma_q$.

$$\sin \phi_q^{\text{NP}} = \frac{r_q \sin \sigma_q}{\sqrt{1 + 2 r_q \cos \sigma_q + r_q^2}},$$

$$\cos \phi_q^{\text{NP}} = \frac{1 + r_q \cos \sigma_q}{\sqrt{1 + 2 r_q \cos \sigma_q + r_q^2}}.$$
SM amplitude is

\[ M_{b\bar{b}}^{q,\text{SM}} = \frac{G_F^2 m_W^2}{12\pi^2} M_{B_q} \hat{\eta}^B \hat{\mathcal{B}}_{B_q} f_{B_q}^2 (V^*_{tq} V_{tb})^2 S_0(x_t), \]

where \( x_t = \frac{m_{top}^2}{m_W^2} \), and \( S_0 \) is an “Inami–Lim” function.

CKM components are

\[ |V^*_{td} V_{tb}| = (8.6 \pm 1.3) \times 10^{-3}, \quad |V^*_{ts} V_{tb}| = (41.3 \pm 0.7) \times 10^{-3}. \]

Short-distance QCD correction \( \hat{\eta}^B = 0.552 \)

\( \hat{\mathcal{B}}_{B_{d,s}} f_{B_{d,s}}^2 \) is determined from lattice simulation by JLQCD collaboration.
New physics effect becomes

\[
    r_q e^{i\sigma_q} \equiv \frac{M_{q_{bb}}^{q_{RS}}}{M_{q_{bb}}^{q_{SM}}} = \frac{16\pi^2}{N_C} \frac{8g_s^2}{g^4S_0(x_t)} m_W^2 \kappa_{33}^2 \kappa_{q3}^2 \sum_{n=1} \left( \frac{\hat{g}^{(n)}(c_{Q3})}{m_A^{(n)}} \right)^2.
\]

\(\mathcal{O}(1)\) ambiguity and phases in mixings are absorbed in \(\sqrt{\lambda} \lesssim \kappa_{ij} \lesssim \frac{1}{\sqrt{\lambda}}\).

\[
    (U_{qL})_{ij} = \kappa_{ij} V_{ij}^{\text{CKM}},
\]
$\phi_d$ associated with $B^0_d - \bar{B}^0_d$ is well measured,

$$(\sin \phi_d)^{cc\bar{s}} = \sin(2\beta + \phi_{d}^{NP}) = 0.687 \pm 0.032,$$

where $\phi_{d}^{SM} = 2\beta$.

$\phi_{d}^{NP}$ is estimated by (Ball and Fleischer, 2006)

$$\phi_{d}^{NP}|_{incl} = -(10.1 \pm 4.6)^{\circ}, \quad \phi_{d}^{NP}|_{excl} = -(2.5 \pm 8.0)^{\circ}.$$

New Physics parameters are constrained by the CP phases.
Figure: Allowed parameter space of \((\sigma_d, M_{KK})\). Red lines satisfy the observed \(\rho_d\) and blue lines for \(\phi_d^{NP}|_{\text{incl}}\), with 1\(\sigma\) uncertainty.
Figure: Allowed parameter space of $(\sigma_s, M_{KK})$ from the observed $\Delta M_s$. 
Results

For the choice of optimistic parameters for bulk SM,

\[ c_{Q3} = 0.32, \quad \kappa = 1/\sqrt{2} \]

\( B^0_d - \bar{B}^0_d \) bounds allow KK mode mass as low as 3 TeV.

\( B_s \) mixing gives a weaker bound than \( B_d \) mixing.
1. SM field can be resides on $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$ bulk space.
Summary of flavor physics study

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5. $B^0_d - \bar{B}^0_d$ constrains 1st KK mode $M_A^{(1)} \gtrsim 3$ TeV.
Even though the Bulk SM model is phenomenologically rich and interesting,

Electroweak precision data (e.g. UTfit 2008) suggests that minimal KK mode mass should be a few TeV or higher.

This might be too high to get at LHC.

However, the possibility of TeV scale QG model is even more exciting since it opens the possibility of opening entirely new physics to explore.