Recent Development in Neutrino Masses and Mixing

APCTP Focus Program
Recent Developments in Neutrino Physics and Astroparticle Physics
(In memory of Prof. B. W. Lee)

June 15–27, 2009

Sin Kyu Kang (SNUT)
B.W. Lee’s contribution to neutrino physics

• Papers on neutrinos:

• The Process $\muon - \text{neutrino} + p \rightarrow \muon - \text{neutrino} + p + \pi^0$ in Weinberg's model of weak interactions. 

• Chiral-symmetry-breaking effects on neutrino scattering. 
  B.W. Lee, J.E. Mandula (Caltech, Kellogg Lab). 

• Neutrino Physics – Theoretical Considerations. 

• Cosmological Lower Bound on Heavy Neutrino Masses. 
  Benjamin W. Lee (Fermilab), Steven Weinberg 
• **Summary Talk: Status of Accelerator Neutrino Physics.**
  Benjamin W. Lee (Fermilab).
  Published in Neutrino 76:0704

• **Estimates of Charm Production in Exclusive Neutrino Reactions.**
  R.E. Shrock, Benjamin W. Lee (Fermilab).

• **Contributions of Vector – Meson – Dominance to Charmed Meson Production in Inelastic Neutrino and anti-neutrino Interactions.**
  Martin B. Einhorn, B.W. Lee (Fermilab).

**Statistics:**

- Papers (Spires search) published in journals: 76
- Total citation: 12138
- Citation per paper: 160
- # of papers cited >1000: 4 (cf. S. Weinberg: 4 till 1977)
What is Neutrino?

- Pauli (1930) proposed neutrino to solve a prob.

In $\beta$-decay

Weakly interacting massless neutral fermion

The first discovery of neutrino: Cowan & Reines 1956
• Neutrinos in the Standard Model

• How neutrinos describe in the SM?

\[ \overline{L} \partial L \]

\[ (D_\mu = \partial_\mu - igG_\mu) \]

• Weak interaction exchanging
The President's View
(excerpted from remarks at the MIT commencement, June 6, 1998)

[W]e must help you to ensure that America continues to lead the revolution in science and technology……..

Just yesterday in Japan, physicists announced a discovery that tiny neutrinos have mass. …..but it may change our most fundamental theories from the nature of the smallest subatomic particles to how the universe itself works, and indeed how it expands.
Neutrinos (meaning: "Small neutral ones") are elementary particles that often travel close to the speed of light, lack an electric charge, are able to pass through ordinary matter almost undisturbed and are thus extremely difficult to detect. Neutrinos have a minuscule, but nonzero mass. They are usually denoted by the Greek letter \( \nu \).
Outline of my talk

• Current status of neutrino physics

• Particle physics implications

• What we know and don’t know
  - new phase of neutrino physics

• Conclusion
Current Status of Neutrino Physics

Discoveries in the last decade

- Atmospheric $\nu_\mu$ are converted to $\nu_\tau$ (SK 98)
- Solar $\nu_e$ are converted to either $\nu_\mu$ or $\nu_\tau$ (SNO 02)
- Reactor anti-$\nu_e$ disappear /reappear (KamLAND 04)
- Accelerator $\nu_\mu$ disappear (K2K 04, MINOS 06)
- Only the LMA solution left for solar neutrino problem
Neutrino oscillations happen

\[ P = \left| \left( -\sin \theta e^{-iE_1 t} \langle \nu_1 | + \cos \theta e^{-iE_2 t} \langle \nu_2 | \right) \left( \cos \theta | \nu_1 \rangle + \sin \theta | \nu_2 \rangle \right) \right|^2 \]

\[ P = \sin^2 (2\theta) \sin^2 \left( \frac{1.27 \Delta m^2 L}{E} \right) \]
Neutrino Oscillations in Matter

Mikheyev-Smirnov-Wolfenstein (MSW) effect

\[ i \frac{d}{dt} \begin{bmatrix} \nu_e \\ \nu_x \end{bmatrix} = H \begin{bmatrix} \nu_e \\ \nu_x \end{bmatrix} \]

In matter:

\[ H = \begin{bmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{bmatrix} \]

\[ \nu_x \rightarrow \nu_x \]

\[ e^- + Z^0 \rightarrow e^- \]

\[ \nu_e \rightarrow e^- \]
\[ P_{\nu_e \rightarrow \nu_\mu}(x) = \sin^2 2\theta_M \sin^2 \left( \frac{\Delta m^2_M x}{4E} \right) \]

\[ \tan 2\theta_M = \frac{\tan 2\theta}{1 - \frac{A_{CC}}{\Delta m^2 \cos 2\theta}} \]

\[ \Delta m^2_M = \left[ (\Delta m^2 \cos 2\theta - A_{CC})^2 + (\Delta m^2 \sin 2\theta)^2 \right]^{1/2} \]

\[ \Delta m^2 = 7 \times 10^{-6} \text{ eV}^2, \quad \theta = 10^{-3} \]
Neutrino Mass and Mixing Parameters

- 7 physical parameters in $\nu$ oscillations

$$U_{\text{MNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 0 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric | Reactor | Solar | Majorana

- Oscillation phase $\delta$
- Majorana phases $\alpha_1, \alpha_2$

3 masses + 3 angles + 1(3) phase(s) = 7(9) new parameters for SM

- Neutrino oscillation experiments sensitive to

$$\Delta m^2_{21} = m_2^2 - m_1^2, \quad \Delta m^2_{32} = m_3^2 - m_2^2$$

3 mixing angles & 1 CP phase ($\delta$)
Catalogue of main oscillation experiments

- Solar neutrino experiments
  - Homestake, SAGE, GALLEX, SuperK, SNO, Borexino

- Atmospheric neutrino experiments
  - SuperKamiokande

- Reactor anti-neutrino experiments
  - KamLAND, CHOOZ, RENO

- Long Baseline experiments
  - K2K, MINOS, OPERA

<table>
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<tr>
<th>Experiments</th>
<th>Parameters of leading effects</th>
<th>Parameters of sub-leading effects</th>
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<tr>
<td>Solar neutrinos,</td>
<td>$\Delta m^2_{12}$, $\theta_{12}$</td>
<td>$\theta_{13}$</td>
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<tr>
<td>KamLAND</td>
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<td>Atmospheric neutrinos</td>
<td>$\Delta m^2_{23}$, $\theta_{23}$</td>
<td>$\Delta m^2_{12}$, $\theta_{12}$, $\theta_{13}$, $\delta$</td>
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<tr>
<td>K2K</td>
<td>$\Delta m^2_{23}$, $\theta_{23}$</td>
<td>$\theta_{13}$</td>
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<td>CHOOZ</td>
<td>$\Delta m^2_{23}$, $\theta_{13}$</td>
<td>strongly suppressed</td>
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<tr>
<td>MINOS</td>
<td>$\Delta m^2_{23}$, $\theta_{23}$</td>
<td>$\theta_{13}$</td>
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</table>
Sun burns!!!

Electron neutrinos are produced

Adiabatic conversion in matter of the Sun

Oscillations in matter of the Earth
We observed solar neutrino flux deficit.
Evidence for Solar $\nu_e$ oscillation from SNO

- **Charged Current**: $\nu_e$

- **Neutral Current**: $\nu_e + \nu_\mu + \nu_\tau$

$\Phi_{\text{NC}}^{\text{SNO}} = 5.21 \pm 0.27 \pm 0.38 \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$

$\Phi_{\text{CC}}^{\text{SNO}} = 1.59^{+0.08+0.06}_{-0.07-0.08} \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$

7.6$\sigma$ difference

$\Rightarrow \nu_\mu, \nu_\tau$ are coming from the Sun!
SNO (II, III) experiments

\[ \frac{\phi_{\text{CC}}^{\text{SNO}}}{\phi_{\text{NC}}^{\text{SNO}}} = 0.340 \pm 0.038 \]  

2005

7σ evidence for a non-zero \( \nu_{\mu, \tau} \) flux

\[ \frac{\phi_{\text{CC}}^{\text{SNO}}}{\phi_{\text{NC}}^{\text{SNO}}} = 0.301 \pm 0.033 \]  

2008

\( \theta_{12} \) shifts to a lower value
Constraints on solar parameters from solar $\nu$ data

2008 Best-fit

\[ \Delta m_{21}^2 = 6.2 \times 10^{-5} \text{ eV}^2 \]
\[ \sin^2 \theta_{12} = 0.286 \]

1$\sigma$ intervals

\[ \Delta m_{21}^2 = (3.5 - 7.5) \times 10^{-5} \text{ eV}^2 \]
\[ \sin^2 \theta_{12} = 0.28 - 0.33 \]

Large Mixing Angle (LMA) solution as preferred solution
KamLAND Experiment

First terrestrial $\nu$ exp. relevant to solar $\nu$ problem

$\bar{\nu}_e + p \rightarrow n + e^+$

To probe LMA, need $L \sim 100\text{km}$, 1kt, need low $E_\nu$, high $\Phi_\nu$

First result (2002)
No oscillation hypothesis
Excluded at 99.95%
KamLAND experiments (2008)

Almost two oscillations observed !!!

No distortions excluded at > 5 \sigma

Testing solar L/E dep.
Impact of KamLAND on $\Delta m^2_{21}$ and $\theta_{12}$

Impact of KamLAND on $\Delta m^2_{21}$ and $\theta_{12}$

- $\Delta m^2_{21}$ constrained to within 8% ($3\sigma$) by KamLAND
- $\sin^2 \theta_{12}$ constrained to within 19% by mainly solar data

08 Best-fit to solar+KamLAND

$\Delta m^2_{21} = 7.7 \times 10^{-5} \text{ eV}^2$
$\sin^2 \theta_{12} = 0.31$

3σ range solar+KamLAND

$\Delta m^2_{21} = (7.1 - 8.3) \times 10^{-5} \text{ eV}^2$
$\sin^2 \theta_{12} = (0.25 - 0.37)$

Maltoni et al. (2008), Fogli et al. (2008), Bandyopadhyay et al. (2008)
First results from Borexino (2007)

- Low background liquid scintillator detector.
- Measurement of $^7$Be neutrinos via $\nu$-e scattering.
- First real time spectral measurement of sub-MeV solar $\nu$.
- Observed rate: $47 \pm 7_{\text{stat}} \pm 12_{\text{syst}}/(\text{day}.100 \text{ ton})$

  cf. expected rate without oscillation: $75 \pm 4/(\text{day}.100 \text{ ton})$

$$P_{ee} = 0.62 \pm 0.18$$
Measurement of Solar $^8$B $\nu_e$ flux with 246 days of Borexino (2008)

confirming MSW-LMA solution of solar neutrino anomaly

$P_{ee} = 1 - \frac{1}{2} \sin^2 2\theta$

$P_{ee} = \sin^2 \theta$

non-oscillation pred.

oscillation fit
Atmospheric Neutrinos

Primary cosmic ray p, He, ...

Isotropic flux of cosmic rays

atmosphere

neutrino direction 10000 km

Zenith

θ

25 km

Ratio of $\nu_\mu / \nu_e \sim 2$
(for $E_V < \text{few GeV}$)

Up-Down Symmetric Flux
(for $E_V > \text{few GeV}$)
Superkamiokande (1998-2006)

deficit of mu-like events
Interpretation of SK data:

- $\nu_\mu \leftrightarrow \nu_\tau$ oscillations

$1.5 \times 10^{-3}$ eV$^2 < \Delta m^2 < 3.4 \times 10^{-3}$ eV$^2$

$\sin^2 2\theta > 0.92$ at 90% CL

Best Fit:

- $\sin^2 2\theta = 1.02$
- $\Delta m^2 = 2.1 \times 10^{-3}$ eV$^2$
- $\chi^2 = 174.9/177$ dof
- $\chi^2 = 465/179$ dof for no osc
L/E distribution of events in SuperK

2004 SK

oscillation „dip”

L/E distribution of events in SuperK
Long Baseline (LBL) experiments

- Beam with well known properties
- Distance > 100km
- Tune L/E to study interesting $\Delta m^2$

<table>
<thead>
<tr>
<th>Expt</th>
<th>Year</th>
<th>$E_\nu$</th>
<th>L</th>
<th>Detector</th>
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<tr>
<td>K2K</td>
<td>1999 - 2004</td>
<td>1 GeV</td>
<td>250 Km</td>
<td>Water Cerenkov</td>
</tr>
<tr>
<td>MINOS</td>
<td>2005-</td>
<td>3 GeV</td>
<td>732 km</td>
<td>Iron Scintillator</td>
</tr>
<tr>
<td>OPERA</td>
<td>2008-</td>
<td>17 GeV</td>
<td>732</td>
<td>Emulsion</td>
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Physics Goals

- **K2K**: confirming the atmospheric $\nu$ oscillation
- **MINOS**: precise determination of $|\Delta m^2_{\text{atm}}|$ & $\theta_{23}$
- **OPERA**: observe $\tau$-appearance using $\nu_\mu - \nu_\tau$ channel
K2K (1999-2006)


\[ \Delta m^2 (10^{-3} \text{eV}^2) \]

\[ \sin^2(2\theta) \]

\[ E_{\text{rec}}^{\text{GeV}} \]
MINOS (2005-)  

- MINOS (Main Injector Neutrino Oscillation Search) - a long-baseline neutrino oscillation experiment:

  - Neutrino beam provided by 120 GeV protons from the Fermilab Main Injector.

  - A Near detector at Fermilab to measure the beam composition and energy spectrum

  - A Far detector deep underground in the Soudan Mine, Minnesota, to search for evidence of oscillations
MINOS

Clear dip in E spectrum
(arXiv: 0806.2237)
Atmospheric parameters from MINOS

Best fit (2008): $|\Delta m^2_{\text{atm}}| = 2.38 \times 10^{-3} \text{ eV}^2$, $\sin^2 2\theta_{23} = 1.0$
Impact of MINOS on $\Delta m^2_{atm}$ and $\theta_{23}$

Best fit (Atm+MINOS) :

$$|\Delta m^2_{atm}| = 2.4 \times 10^{-3} \text{ eV}^2$$

$$\sin^2 \theta_{23} = 0.5$$

$$3 \sigma \ (Atm+MINOS) :$$

$$|\Delta m^2_{atm}| = (2.1 - 2.8) \times 10^{-3} \text{ eV}^2$$

$$\sin^2 \theta_{23} = 0.36 - 0.67$$

No information on $\text{sign}(\Delta m^2_{atm})$ and octant of $\theta_{23}$

Maltoni et al. (2008), Fogli et al. (2008)
Current Status of $\theta_{13}$

**CHOOZ Exp.**
(reactor exp. in France)

- No evidence of oscillation
- Best fit: $\theta_{13} = 0$
- Limit on $\theta_{13}$ depends on $|\Delta m^2_{\text{atm}}|$
Global analysis of $3 \nu$ oscillation with SK+K2K+CHOOZ
(Fogli, Lisi, Morrone, Palazzo, hep-ph/0506083)

But, Hint for $\theta_{13} > 0$

$\star$ Non-zero $\theta_{13}$ preferred at $\sim 0.9\sigma$
But SK gets no preference for non-zero $\theta_{13}$
Non-zero $\theta_{13}$ helps to solve the small tension between $\theta_{12}$ and $\Delta m^2$ preferred by solar and KamLAND.

Mismatch between Solar & KamLAND bets-fit $\sin^2 \theta_{12}$, Solar best fit lower than KamLAND. Non-zero $\theta_{13}$ reconcile the mismatch.
Best-fit to global data

\[ \sin^2 \theta_{13} = 0.016(0.01) \] at \( 1.6\sigma (< 1\sigma) \)

Fogli et al., arXiv:0806.2649
Maltoni, Schwetz, arXiv:0812.3161

\( 1(3)\sigma \) range global data

\[ \sin^2 \theta_{13} < 0.046(0.056) \]
Very recently,
**MiniBooNE**

- Motivated by LSND result for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation at $\Delta m^2 \sim 1 \text{ eV}^2$

*Diagram of detector setup*

- $E_{LSND} \sim 30 \text{ MeV}$, $E_{MB} \sim 500 \text{ MeV}$
- $L_{LSND} \sim 30 \text{ m}$, $L_{MB} \sim 500 \text{ m}$
- $L/E \sim 1 \text{ MeV/m}$ – same as LSND by increasing both $L$ and $E$
- This changes signature of signal and major backgrounds in the detector
- Channel $\bar{\nu}_\mu - \bar{\nu}_e$ (LSND), $\nu_\mu - \nu_e$ (MiniBoone)
First results from MinibooNE

- With two largely independent analyses, observe no excess of events for reconstructed $E_\nu > 475$ MeV.

- The data are consistent with no oscillations within a two neutrino appearance-only oscillation model.

- However, an excess of events observed for $E_\nu < 475$ MeV. This low energy excess can not be explained by two neutrino oscillation

- in the framework of (3+2) five-neutrino oscillation, MinibooNE is in agreement with LSND: Maltoni & Schwetz (2007)
New results from Miniboone

Data sample increased from $5.58 \times 10^{20}$ POT to $6.46 \times 10^{20}$

$475 \text{ MeV} < E < 3 \text{ GeV}$

Low energy anomaly!
What Is Known, Putting It All Together

The mixing angles

\[ \theta_{12}^\circ = 33.7^{+1.3}_{-1.3} \left( ^{+4.3}_{-3.5} \right) \]
\[ \theta_{23}^\circ = 43.1^{+4.3}_{-3.8} \left( ^{+9.8}_{-8.8} \right) \]
\[ \theta_{13}^\circ = 7.6 < +6 \left( < +11.5 \right) \]

- two large mixings
- small shift from maximal mixing
- tiny \( \theta_{13} \)

Mixing matrix:

\[ U_{\text{MNS}} = \begin{pmatrix}
0.77 - 0.86 & 0.50 - 0.63 & 0.00 - 0.22 \\
0.22 - 0.56 & 0.44 - 0.73 & 0.57 - 0.80 \\
0.21 - 0.55 & 0.40 - 0.71 & 0.59 - 0.82 \\
\end{pmatrix} \]

mass splittings

Global fit:

\[
\begin{align*}
\Delta m_{31}^2 &= 2.4 \pm 0.12 \ (0.6) \times 10^{-3} \ eV^2 \\
\Delta m_{21}^2 &= 7.90 \pm 0.27 \left( + 1.10 \right) \times 10^{-5} \ eV^2 \\
\end{align*}
\]
Present status of $\nu$ oscillation parameters

\[ U_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

- Atmospheric
- Reactor
- Solar
- Majorana

Oscillation phase $\delta$
Majorana phases $\alpha_1, \alpha_2$

3 masses + 3 angles + 1(3) phase(s) = 7(9) new parameters for SM

**neutrino mass hierarchy**

$|m_2/m_3| > 0.18$

the weakest mass hierarchy

Solar neutrino data: $\Delta m^2_{21} > 0$

But sign of $|\Delta m^2_{31}|$ not known
Absolute Neutrino Masses

- From Tritium $\beta$-decay

$$m_{\nu_e} = \left( \sum |U_{ei}|^2 m_i^2 \right)^{\frac{1}{2}}$$
$$= \left[ c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2 \right]^{\frac{1}{2}}$$

- Neutrinoless double $\beta$-decay

$$m_{ee} = \left| \sum U_{ei}^2 m_i \right|$$
$$= \left| c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3} \right|$$

- Cosmology

$$m_{\text{cosmo}} = \Sigma m_i$$
Tritium $\beta$-decay

- Bound from Mainz and Troisk Exp.
  \[ m_\beta < 1.8 \text{ eV (2}\sigma) \]

- Improvement in the bound from Katrin exp. down to \( m_\beta \sim 0.3 \text{ eV} \)

Hierarchy sensitivity is below the reach of Katrin
Neutrinoless double $\beta$-decay

$(A, Z) \rightarrow (A, Z+2)+2e^-$

Lepton number violating interaction

Majorana neutrinos

Present bound $m_{\beta\beta} < 0.35 \text{ eV} + \text{theor. uncert. (3 times)(90CL)}$

Several proposed experiments to reach the sensitivity $0.01 \text{ eV}$
\[
\langle m \rangle = |m_1|U_{e1}|^2 + m_2|U_{e2}|^2 e^{i\phi_1} + m_3|U_{e3}|^2 e^{i\phi_2}.
\]
Particle Physics Implications
Interpretation of lepton mixing

Neutrino data prefer Tri-Bimaximal mixing

\[ U_{tbm} = \begin{pmatrix} 2/3 & 1/3 & 0 \\ -1/6 & 1/3 & 1/2 \\ 1/6 & -1/3 & 1/2 \end{pmatrix} \]

\( \nu_2 \) is tri-maximally mixed

\( \nu_3 \) is bi-maximally mixed

\[ \nu_2 = \nu_e + \nu_\mu + \nu_\tau \]

\[ \nu_3 = \nu_\mu + \nu_\tau \]
What is the origin of the tri-bimaximal mixing?

Is it related with mass eigenstates?

No analogy in the Quark sector?

- maximal 2-3 mixing
- zero 1-3 mixing
- no CP-violation

$U_{tbm} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1/2} & \sqrt{1/2} \\ 0 & -\sqrt{1/2} & \sqrt{1/2} \end{pmatrix} \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}$

with $\sin^2 \theta_{12} = 1/3$
How can we obtain $U_{\text{tbm}}$?

- We can obtain $U_{\text{tbm}}$ by diagonalizing light neutrino mass matrix (in a basis where charged lepton diagonal)

\[
M_v = \begin{pmatrix}
x & y & y \\
y & z & w \\
y & w & z
\end{pmatrix}
\]

\[
U_v = \begin{pmatrix}
-c & s & 0 \\
\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & -\frac{1}{\sqrt{2}}
\end{pmatrix}
\]

\[
\theta_{13} = 0 \\
\theta_{23} = \theta_{\text{atm}} = \pi/4
\]

\[
\sin^2 2\theta_{12} = \sin^2 \theta = \frac{8y^2}{(x-w-z)^2 + 8y^2}
\]
Require further relation:

\[ M_{11} = M_{22} + M_{23} - M_{13} \]

\[ x = z + w - y \]

\[ \sin^2 2\theta = \frac{8}{9} \]

\[ \sin^2 \theta = \frac{1}{3} \]

A forms of neutrino mass matrix diagonalized by \( U_{\text{tbm}} \):

\[
M_\nu = \begin{pmatrix}
x & y & y \\
y & x + v & x - v \\
y & x - v & x + v
\end{pmatrix}
\]

\[ m_1 = x - y \]

\[ m_2 = x + 2y \]

\[ m_3 = x - y + 2v \]
Models with lepton flavor symmetry

Applies to the effective LH Majorana (or Dirac) neutrino mass matrix in the flavor basis.

- $\mu-\tau$ interchange symmetry:

$$
M_{\nu} = \begin{pmatrix}
  x & y & y \\
  y & z & w \\
  y & w & z
\end{pmatrix}
$$

If $x,y < z,w$ normal hierarchy.

If $x=z=w=0$, the lepton number $L_e - L_\mu - L_\tau$ is conserved and an inverted hierarchy results but with 2 massless neutrinos unless the symmetry is broken.
\( S_3 \) Lepton Flavor Symmetry

- Permutation group of 3 flavors applies to both rows and columns.
- \( S_3 \) symmetry breaking required to obtain suitable diagonal charged lepton mass matrix and approximate tri-bimaximal mixing matrix.

\[ M_\nu = \begin{pmatrix} x & y & y \\ y & x & y \\ y & y & x \end{pmatrix} \]

\( z = w = y \)

\( A_4 \) Flavor Symmetry

Isomorphic to group of tetrahedron rotation
It contains $1, 1', 1''$ and 3 irrep. (smallest non-Abelian)

$$3 \times 3 = 1 + 1' + 1'' + 3 + 3$$

For two 3 rep. $\chi, \varphi$

$$1 \sim (\chi \varphi) = (\chi_1 \varphi_1 + \chi_2 \varphi_3 + \chi_3 \varphi_2),$$
$$1' \sim (\chi \varphi)' = (\chi_3 \varphi_3 + \chi_1 \varphi_2 + \chi_2 \varphi_1),$$
$$1'' \sim (\chi \varphi)'' = (\chi_2 \varphi_2 + \chi_1 \varphi_3 + \chi_3 \varphi_1)$$

Typically, we assign

$$l^c_1 \sim 1, \quad l^c_2 \sim 1'', \quad (L_1, L_2, L_3) \sim 3, \quad l^c_3 \sim 1'$$

A4 is well fit for 3 families
A model based on A4

\[ w_l = y_e e^c(\phi_T l) + y_\mu \mu^c(\phi_T l)' + y_\tau \tau^c(\phi_T l)'' + (x_a \xi + \tilde{x}_a \tilde{\xi})(ll) + x_b (\phi_s ll) + h.c. \]

Here, \( y_e e^c(\phi_T l) \equiv y_e e^c(\phi_T l) h_d / \Lambda, \) \( x_a \xi(ll) \equiv x_a \xi(lh_u lh_u) / \Lambda^2 \)

\[ \phi_s, \phi_T, \xi, \tilde{\xi} \sim (3,3,1,1) \]

with the vacuum alignment

\[ m_l = v_T \frac{v_d}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \]

\[ m = \begin{pmatrix} x & y & y \\ y & x + v & y - v \\ y & y - v & x + v \end{pmatrix} \]

many versions of A4 models exist by now!
Though TB mixing pattern is consistent with current data, no convincing reason for exact TB mixing

expect deviations (Rodejohann, SKK, Xing)

TB is exact at LO, and NLO corrections are included from higher dimensional operators

→ each mixing angle receives corrections of order

\[ O(\lambda_C^2) \]

We expect \( \theta_{13} \sim O(\lambda_C^2) \) measurable in next run of exp.
Quark-Lepton Complementarity

QLC- relations (empirical):

\[ \theta_{12} + \theta_{12}^q \sim \pi/4 \]

\[ \theta_{23} + \theta_{23}^q \sim \pi/4 \]

\[ \theta_{12} + \theta_C = 46.5^\circ \pm 1.3^\circ \]

\[ \theta_{23} + V_{cb} = 43.9^\circ \pm 5.1/3.6^\circ \]

H. Minakata, A.Smirnov, Raidal (04)

Qualitatively,

- 2-3 leptonic mixing is close to maximal because 2-3 quark mixing is small
- 1-2 leptonic mixing deviates from maximal substantially because 1-2 quark mixing is relatively large
``Lepton mixing = bi-maximal mixing - quark mixing''

Quark-lepton symmetry

Existence of structure which produces bi-maximal mixing

Hint for GUT?

Mixing matrix weakly depends on mass eigenvalues

- We need precision experiments to identify or exclude QLC

- Quantum corrections to QLC may be important $\rightarrow$ RG evolution
  (SKK, Kim, Lee; Smirnov et al)
Neutrino Masses: Theoretical Ideas

- No fundamental reason why neutrinos must be massless
  - But why are they much lighter than other particles?

- Modified Higgs sector to accommodate neutrino mass

- Extra Dimensions
  - Neutrinos live outside of 3 + 1 space

- Grand Unified Theories
  - Dirac and Majorana Mass
    - See-saw Mechanism
  - Many of these models have at least one EW singlet \( \nu \)
  - Right-handed partner of the left-handed \( \nu \)
  - Mass uncertain from light (< 1 eV) to heavy (>10^{16} eV)
  - Would be “sterile” - Doesn't couple to standard W and Z
What we know
What we don't know
• The first phase of studies of neutrino mass and mixing is essentially over.

- What we learnt
  - Neutrinos are massive particles
  - Neutrino mix a lot
    discovery of two large mixing angles
  - Very different from quarks

- The first evidence for
  demise of the minimal standard model
What we don’t know

• What is the sign of $\Delta m^2_{32}$
• Neutrino mass spectrum:

![Diagram showing normal and inverted hierarchies of neutrino mass eigenstates]

• Are 3 flavor oscillations enough?
• How small is $\sin^2 2\theta_{13}$?
• Is the CP phase non-zero?
• Is $\theta_{23}$ maximal? If not, what is the octant?
• Are Neutrinos Majorana?
New phase just started

the main objectives:

- Precision measurements of already known parameters
- Determination of the absolute scale of neutrino mass;
- Subdominant structures of mixing matrix;
- Identification of the mass hierarchy
- CP violation in lepton sector
future neutrino oscillations

\[ U_{\text{MNS}} = \begin{bmatrix}
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23} \\
0 & s_{13}e^{-i\delta} & 0
\end{bmatrix} \]

improving

Reactors:
\[ c_{13} & 0 & s_{13}e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{i\delta} & 0 & c_{13} \]

improving

Solar:
\[ c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1 \]

Majorana:
\[ e^{i\alpha_1/2} & 0 & 0 \\
0 & e^{i\alpha_2/2} & 0 \\
0 & 0 & 1 \]

Oscillation phase \( \delta \)
Majorana phases \( \alpha_1, \alpha_2 \)

3 masses + 3 angles + 1(3) phase(s) = 7(9) new parameters for SM

Aims:  
\( \Rightarrow \) improved precision of the leading 2x2 oscillations  
\( \Rightarrow \) detection of generic 3-neutrino effects: \( \theta_{13}, \) CP violation

\( \Rightarrow \) precision neutrino physics
Three Generations of Experiments Needed

- **0. Only three or more?**  \( \Rightarrow \) MiniBoone + Cosmology
- **I. Precision measurements for Solar & Atm. Sector**
  - \( \Delta m_{12}^2, \theta_{12} \) \( \Rightarrow \) Borexino
  - \( |\Delta m_{23}^2|, \theta_{23} \) \( \Rightarrow \) OPERA
- **II. Connection between both Sectors**  \( \Rightarrow \)
  - \( \theta_{13}, \text{Sign} (\Delta m_{23}^2) \) \( \Rightarrow \) Double CHOOZ, RENO, T2K, NOVA, INO, …
- **III. CP-Violating Interference**  \( \Rightarrow \)
  - \( \delta, \alpha_{2,3} \)  
    - Super-Beams?  
    - Beta Beams?  
    - Neutrino Factory?
What is precision neutrino physics good for?

- unique flavour information
- tests models / ideas about flavour
- lessen: elimination of SMA
Determination of $\theta_{13}$

the key parameter for next generation of neutrino oscillation experiments.

Why is it so important?

- Relatively large $\theta_{13}$ opens the possibility to observe generic 3-flavor effects including CP violation and mass hierarchy.

Why is it so small?

- $\theta_{13} \ll 1$ hint for some flavor symmetry
the required facility
- $\sin^2 2\theta_{13} > 0.01 \Rightarrow$ conventional neutrino beam
- $\sin^2 2\theta_{13} < 0.01 \Rightarrow \mu$ storage ring, $\beta$ beam

Two paths to determine $\theta_{13}$
- Long-baseline accelerator: T2K, NO$\nu$A
- Reactor neutrino exp.: $2 \times$ CHOOZ, RENO, Daya Bay

Sensitivity to $\theta_{13}$

Second Generation Experiments for $U(e3)$
Survey of predictions for $\theta_{13}$

Albright, M.C.Chen
Determination of neutrino mass hierarchy is a crucial prerequisite to understand the origin of lepton mass and mixing and to establish their relationship to the analogous properties in quark sector.
To resolve the hierarchy we need:

- Long baseline neutrino experiments

(or any other smart idea?)
- Tritium beta decay: (Strumia, Vissani’05)

- If Tritium beta decay could exclude then, Normal Hierarchy

\[ m_{\nu_e} \geq \frac{1}{30} \text{eV} \]
From Neutrinoless Double Beta Decay

\[ m_{ee} = |m_1 U_{e1}^2 + m_2 U_{e2}^2 e^{i\alpha_{21}} + m_3 U_{e3}^2 e^{i\alpha_{31}}| \]

Majorana phases in \( U_{\text{MNS}} \)

- **Normal Hierarchy**

\[ m_{ee} \leq |\sqrt{\Delta m_{\text{sol}}^2 (1-|U_{e3}|^2)} \sin^2 \theta_{\text{sol}} + \sqrt{\Delta m_{\text{atm}}^2} |U_{e3}|^2 e^{i(\alpha_{31}-\alpha_{21})}| \]

\( \text{few} \times 10^{-4} \leq m_{ee} \leq 6 \times 10^{-3} \text{(eV)} \)

- **Inverted Hierarchy**

\[ \sqrt{\Delta m_{\text{atm}}^2 (1-|U_{e1}|^2)} |\cos 2\theta_{\text{sol}}| \leq m_{ee} \leq \sqrt{\Delta m_{\text{atm}}^2 (1-|U_{e1}|^2)} \]

\[ 0.01 \leq m_{ee} \leq 0.05 \text{(eV)} \]
Phase I: Distinguish between IH & NH

Phase II:

Phase III:

Feruglio Strumia Vissani

90% CL (1 dof)

\[ |m_{ee}| \text{ in eV} \]

\[ \Delta m^2_{23} < 0 \]

\[ \Delta m^2_{23} > 0 \]

Lightest neutrino \( (m_1) \) in eV

Distinguish between IH & NH
Third Generation Experiments: CP Violation

- CP violation may be probable!

\[
A_{\text{CP}} \equiv [P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)] \propto \pm 8J \left(\frac{\Delta m_{21}^2 L}{2E}\right) \sin^2 \left(\frac{\Delta m_{31}^2 L}{2E}\right),
\]

\[
J = \frac{1}{8} \cos \theta_{13} \sin 2\theta_{23} \sin 2\theta_{13} \sin 2\theta_{12} \sin \delta
\]

- $|Ue3|$ gives the strength of $P(\nu_e \rightarrow \nu_\mu)$

- $\delta$ gives the interference pattern: CP odd term is odd in E/L

- CP violation accessible in suppressed appearance experiments only if

\[
\Delta m_{12}^2, s_{12} \text{ large enough (LMA)}
\]

\[
\theta_{13} \text{ large enough}
\]
Measuring by $\nu_\mu \Rightarrow \nu_e$ appearance channel suffers from degeneracies

- **Intrinsic ($\delta, \theta_{13}$)-degeneracy**: (Burguet-Castell et al, 2001)
  (also: Barger, Marfatia, Whisnant, 2001)
- **$\text{sgn}(\Delta m^2_{13})$-degeneracy**: (Minakata, Nunokawa, 2001)
- **$(\theta_{23}, \pi/2-\theta_{23})$-degeneracy**: (Fogli, Lisi, 1996)
Altogether → 8-fold degeneracy
Breaking of degeneracies:

- Combining information from detectors at different baselines
- Using additional oscillation channels ($\mu \rightarrow e$)
- Spectral information (wideband beam)
- Adding information on $\theta_{13}$ from a reactor experiment
- Adding information from atmospheric neutrino experiments

<table>
<thead>
<tr>
<th>Variable Measured</th>
<th>LBL $\nu_\mu \rightarrow \nu_\mu$</th>
<th>LBL $\nu_\mu \rightarrow \nu_e$</th>
<th>Reactor $\bar{\nu}_e \rightarrow \bar{\nu}_e$</th>
<th>Comments</th>
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<tbody>
<tr>
<td>$</td>
<td>\Delta m_{32}^2</td>
<td>$</td>
<td>Y</td>
<td>n</td>
</tr>
<tr>
<td>$\sin^2 2\theta_{23}$</td>
<td>Y</td>
<td>Y</td>
<td>n</td>
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<tr>
<td>$\sin^2 \theta_{13}$</td>
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<td>Y</td>
<td>n</td>
<td>direct measurement</td>
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<tr>
<td>$\sin^2 \theta_{23} \sin^2 \theta_{13}$</td>
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<td>Y</td>
<td>n</td>
<td>combination of $\theta_{23}$ and $\theta_{13}$ via matter effects</td>
</tr>
<tr>
<td>$\text{sign}(\Delta m_{32}^2)$</td>
<td>n</td>
<td>Y</td>
<td>n</td>
<td>CP violation extremely difficult</td>
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<tr>
<td>$\cos \theta_{23} \sin \delta_{CP}$</td>
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<td>n</td>
<td>n</td>
<td></td>
</tr>
<tr>
<td>$\cos \theta_{23} \cos \delta_{CP}$</td>
<td>n</td>
<td>?</td>
<td>n</td>
<td></td>
</tr>
</tbody>
</table>
Lifting neutrino parameter degeneracy at T2KK  (Kajita et al., 06)
Roles of Neutrinos in Particle Physics and Cosmology

- Probing new physics beyond SM
  - origin of tiny neutrino mass and mixing
    (Seesaw mechanism, radiative generation, SUSY...)
- Unification of matter, forces and flavor
  - GUT, Flavor symmetries...

- Playing a role in our existence
  - Leptogenesis

- Playing a role in forming galaxies
- Playing a role in birth of the Universe
- Shedding light on dark energy

Particle Phys.  
Cosmology
Conclusion

• Revolutions in neutrino physics

• The solar & atmospheric neutrino problems solved!

• Small but finite neutrino mass:
  - Probes physics beyond the standard model
  - New insights into the origin of flavor
  - Interesting interplay between neutrinos and cosmos

• A lot more to learn in the next few years
  (mass spectrum, hierarchy, $\theta_{13}$, CP violation ....)