GR Simulations of Neutron Star-Neutron Star & Black Hole-Neutron Star Binaries

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Introduction: Why NS-NS/BH-NS simulations are important?

1. One of the most promising sources of gravitational waves
2. Huge laboratory of nuclear physics → Information for high-density nuclear matter could be extracted
3. High-energy phenomena: Possible progenitor of short GRBs

Numerical relativity is probably the unique approach.
Three motivations

1. One of the most promising sources of gravitational waves
2. Huge laboratory of nuclear physics
   → Information for high-density nuclear matter could be extracted
3. High-energy phenomena:
   Possible progenitor of short GRBs
BH/NS-NS merger = GW source

LIGO

VIRGO

LCGT: Funded!
Sensitivity (standard)

\[ h = h(f) \]

Frequency (Hz)

2016 ~

AdvLIGO, VIRGO, LCGT

NS-NS@100Mpc

\[ \sim 1-10 \text{ per year} \]
Before merger ($10 < f < \sim \text{kHz}$)

- Post-Newtonian
- EOB

- Chirp

after merger ($f > \sim \text{kHz}$)

- GR gravity
- Hydro-interaction
- Neutrino emission
- Magnetic field

Numerical relativity
Three motivations

1. One of the most promising sources of gravitational waves

2. Huge laboratory of nuclear physics → Information for high-density nuclear matter could be extracted

3. High-energy phenomena: Possible progenitor of short GRBs
Neutron-star composition is still unknown

Components: Neutron + ????

Radius: 10—15 km?

$\rho_0 = 2.8 \times 10^{14} \text{ g/cm}^3$

$\rho \sim 10^{15} \text{ g/cm}^3$: Too high
1.97 ± 0.04 \( M_{\odot} \)

Still many possibilities
Gravitational waves by numerical relativity

- GWs in the late inspiral and merger phases could constrain EOS of NS.

Blue: $R=15.2$ km, Red: $R=11.6$ km

Many templates are necessary
Three motivations

1. One of the most promising sources of gravitational waves
2. Huge laboratory of nuclear physics → Information for high-density nuclear matter could be extracted
3. High-energy phenomena: Possible progenitor of short GRBs
\( \gamma \)-ray bursts (GRBs)

- High-energy transient phenomena of very short duration 10 ms—1000 s
- Emit mostly \( \gamma \)-rays with huge total energy \( E \sim 10^{48} - 10^{52} \) ergs
- Central engine = BH + hot torus

Should Verify!
Evolution of NS-NS (on which I focus today)

Formation (after two supernovae)
Evolve as a result of GW emission

Last 1 hour at $r \sim 1000$ km
$f_{GW} \sim 7$ Hz

Merger starts when
$r \sim 30$ km ($f_{GW} \sim 1$ kHz)

If mass is high enough, a black hole is formed

Otherwise, “hypermasive NS”

Large EOS-dependence

Case I

Case II

Case I

Case II

~ $10^6$ km

~ $10^8$ yrs

~ $10^3$ km

~ 1 h
## Galactic compact NS-NS observed

<table>
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<tr>
<th>PSR</th>
<th>( P(\text{day}) )</th>
<th>( e )</th>
<th>( M(M_{\odot}) )</th>
<th>( M_1 )</th>
<th>( M_2 )</th>
<th>( T_{\text{GW}} )</th>
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<td>0.617</td>
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<td>1.37</td>
<td>3.0</td>
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5+1(GC) NS-NS, which will merge in Hubble time (13.7 Gyr), has been found.
Detection rate by population synthesis

LIGO

advLIGO

dot: NSNS
solid: BHNS
dashed: BHBH

NSNS
• $10^{-2\pm1}$ per year for LIGO
• $10^{1.6\pm1}$ per year for advLIGO

Latest study
⇒ Kim san’s talk

V. Kalogera et al. 07
Numerical simulation for BH/NS-BH/NS

\[ G_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu} \]

\[ \begin{cases} 
\nabla \mu T^\mu_\nu = 0 \\
\nabla_\mu (\rho u^\mu) = 0 
\end{cases} \]

\[ \begin{cases} 
\nabla_\mu F^{\mu\nu} = -4\pi j^\nu \\
\nabla [\mu F^\nu_\lambda] = 0 \\
\text{Radiation .....} 
\end{cases} \]

- We have to solve many equations including Einstein’s equation
- Now, it is possible to numerically solve: Accurate simulations are feasible (except for radiation transfer)
Current status

1. Einstein’s evolution equations solver
2. GR Hydrodynamic equations solver
3. Gauge conditions (coordinate conditions)
4. Realistic initial conditions
5. Gravitational wave extraction techniques
6. Apparent horizon (Event horizon) finder
7. Special techniques for handling BHs
8. Physics: EOS, neutrino processes, B-field, radiation transfer
   New frontier
9. Powerful supercomputers or AMR
Latest subjects in numerical relativity

• Derive gravitational waveforms in the late inspiral and merger phases, in particular, to clarify dependence on EOS of NS. (related to motivations I and II)
• Clarify the outcome formed the merger; to determine density, temperature, neutrino luminosity, magnetic field strength, etc. (related motivation III)

In this lecture, I will focus on
• how to model NS matter
• current status of numerical simulations
Thermal condition of NS 1

- **Late inspiral phase** = NS is cold; NS of age > $10^7$ yrs $\rightarrow$ long-term $\nu + \gamma$ cooling $\rightarrow T < 10^5\ \text{K} \sim 10\ \text{eV} \ll E_F \sim 100\ \text{MeV}$ $\rightarrow$ NS should be modeled by cold EOS (EOS is still unknown; need systematic survey)

- **Merger phase**: Shock heating rapidly increases temperature to $kT \sim 0.1-0.2\ E_F \sim 10\ \text{MeV}$ $\rightarrow$ *Finite temperature effects, lepton fraction $Y_e$, neutrino thermal pressure, neutrino cooling, etc could play an important role* (need many runs for many EOS, too, but there are only a few EOSs; need more)
Cooling curve models of NS surface

Surface temperature

For $M = 1.35M_{\text{sun}}$

With superfluid

via emission of neutrinos

Lai & Harding 06
Two approaches will be suitable

1. *Inspiral + Early merger with cold EOS*  
   →  Detailed calculation of gravitational waveforms for late inspiral + early merger phases to clarify their dependence on NS’s cold EOSs  ←  relatively easy, low costs

2. *Merger  → Hypermassive NS/ BH + disk with Finite temperature EOS + neutrino cooling*  (and B-fields): This is required in particular for the study for GRBs  ←  expensive, need details of microphysics; Frontier
I Cold EOSs by piecewise polytrope

- Too many EOSs by nuclear theory: For systematic studies, analytic EOS with few parameters is preferable.
- Piecewise polytrope: \( P = K_i \rho^{\Gamma_i} \) for \( \rho_i < \rho < \rho_{i+1} \)
- Piecewise polytrope with 3 or 4 parameters is good for most of EOSs; \( \Delta P/P < \sim 1\% \)

(Read et al. (UWM) PRD79, 2009)
Piecewise polytrope by UWM

Log $P$ (cgs)

4 parameters

$P_1$

$\Gamma_1$

$\Gamma_0 \sim 4/3$

Fixed for crust

$\Gamma_2$

$\Gamma_3$

Necessary only for massive NS

EOS of NS is stiff:
$\Gamma_1 = 2.7 - 3.0$

Log $\rho$ (g/cm$^3$)

14.7

15.0

We approximately know
Nuclear-theory based cold EOS
Piecewise polytrope with 2 pieces for low mass

\[ P = \begin{cases} \kappa_1 \rho^{\Gamma_1} & (\rho < \rho_1) \\ \kappa_2 \rho^{\Gamma_2} & (\rho > \rho_1) \end{cases} \]

Read+ (2009)

Match well

crust
Some of latest numerical results

I here focus on

• Gravitational waveforms & their dependence of EOS

• Status for simulations with microphysics

• Latest progress in MHD simulations will be presented by Luca Baiotti
Typical NR formulation

- Einstein’s eqs solver: BSSN formalism with 4th-order finite difference in time & space
- Hydro: Shock capturing scheme; 3rd-order in space (no shock); 1st-order (around shock)
- Gauge: Dynamical gauge (puncture gauge)
- Quasiequilibrium initial condition (LORENE library) with a large orbital separation (MW ∼ 0.02)
- Gravitational waves are extracted by NP formalism
- Black hole is determined by finding apparent horizon
- Adaptive mesh refinement (ours=SACRA)
A result of long-term evolution

9.5 orbits
Gravitational waveform

1.35-1.35 \( M_{\text{sun}} \)

\( R=11.6 \text{ km} \)

Blue: Enhanced post-Newtonian (TT4)

Red: Numerical-relativity

\( h r / M_0 \)

\( t_{\text{ret}} / M_0 \)
For all, $1.35-1.35M_{\text{sun}}$

- $R=11.6 \text{ km}$
- $R=11.0 \text{ km}$
- $R=10.7 \text{ km}$

26.6 ms
For all, $1.35 - 1.35 M_{\text{sun}}$

For $R=11.6\,\text{km}$

For $R=15.2\,\text{km}$

Short inspiral phase

$26.6\,\text{ms}$
Fourier spectrum

For all, $1.35 - 1.35 M_{\text{sun}}$

Appreciable dependence on EOS in high frequency

$h_{\text{eff}} \sim f^{-1/6}$

$f h_{f} (r=100 \text{ Mpc})$

$1e-21$

$1e-22$

$R=15.2 \text{ km}$

Blue=11.0 km

Pink=10.7 km

Cyan=10.3 km

$11.6 \text{ km}$

QNM of BH

PP. Newton

advLIGO

advLIGO/10

$f (\text{Hz})$
Assume BH: $10M_{\text{sun}}$、NS: $1.4M_{\text{sun}}$

Spectrum

@100 Mpc

Frequency $f$ (Hz)

Initial LIGO

Cut-off depends on EOS

HMNS form

~ 2-4 kHz

BH form

~ 6 kHz

advLIGO

$h$

$h_{\text{rms}}$

$h_{\text{SB}}$
Developing a template for extracting EOS is urgent

\[ \frac{\omega^2}{\dot{\omega}} \]

Baiotti et al. (2011)
NS-NS II: With non-cold EOS

• Study with finite-temperature-EOS + neutrino physics

• The purpose is to clarify the detail in the merger: evolution of hypermassive neutron stars & BH-torus, and possible relation with short GRBs

• Our current implementation (by Sekiguchi): Shen’s (tabulated) EOS + electron, positron, neutrinos; Neutrino cooling by a GR leakage scheme

• Magnetic field → Luca’s talk
Neutrino leakage in GR

$$\nabla_\mu T^\mu_\nu_{\text{hydro}} = \dot{Q} u^\nu; \quad \dot{Q} = \text{cooling function}$$

$$T^\mu_\nu_{\text{hydro}} = \rho \left(1 + \varepsilon + P/\rho\right) u^\mu u^\nu + P g^{\mu_\nu}$$

$$\nabla_\mu T^\mu_\nu_{\text{rad}} = -\dot{Q} u^\nu$$

Assume $$T^\mu_\nu_{\text{rad}} = E n^\mu n^\nu + F^\mu n^\nu + F^\nu n^\mu + P^\mu_\nu$$

$$\nabla_\mu \left(T^\mu_\nu_{\text{hydro}} + T^\mu_\nu_{\text{rad}}\right) = 0$$

We have to evolve electron fraction:

$$u^t \frac{dY_e}{dt} = -Q_{Y_e}; \quad Y_e \equiv \frac{n_e}{n_{\text{baryon}}} \quad \text{or} \quad \nabla_\mu \left(\rho u^\mu Y_e\right) = -\rho Q_{Y_e}$$

Need extension for radiation transfer:
but a mile-stone development
Neutrino emission & absorption

Emission

\[
\begin{align*}
\text{electron capture: } & \quad e^- + p \rightarrow n + \nu_e \\
\text{positron capture: } & \quad e^+ + n \rightarrow p + \nu_e \\
\text{pair annihilation: } & \quad e^- + e^+ \rightarrow \nu + \bar{\nu} \\
\text{Plasmon decay: } & \quad \gamma \rightarrow \nu + \bar{\nu}
\end{align*}
\]

Absorption

\[
\begin{align*}
\text{Absorption: } & \quad \begin{cases}
n + \nu_e \rightarrow e^- + p \\
p + \nu_e \rightarrow e^+ + n
\end{cases}
\end{align*}
\]

Scattering

\[
\begin{align*}
\text{Scattering: } & \quad \begin{cases}
n + \nu \rightarrow n + \nu \\
p + \nu \rightarrow p + \nu \\
N + \nu \rightarrow N + \nu
\end{cases}
\end{align*}
\]
NS-NS merger with finite-temp EOS + neutrino

Long lived hypermassive neutron star is formed: HMNS is supported by thermal pressure.

Shen’s EOS
1.5—1.5 $M_{\odot}$

Sekiguchi et al.
PRL in press
NS-NS merger with finite-temp EOS + neutrino

$x$-$z$ plane; only after the merger

High neutrino luminosity in the polar surface
→ May be favorable for pair annihilation near the polar region
Related quantities

**Red:** $M = 1.35 - 1.35$

**Green:** $M = 1.5 - 1.5$

**Blue:** $M = 1.6 - 1.6$

**HMNS:** $T \sim 20 - 30$ MeV

**BHdisk:** $T \sim 10$ MeV

**HMNS:** $L \sim 3 - 5 \times 10^{53}$ erg/s

**BHdisk:** $L \sim 1 \times 10^{53}$ erg/s

Graph showing $L_{\nu}$, $T_{\max}$, and $t - t_{\text{merge}}$ over time for different colors and labels.
Gravitational waveforms

- **L**: Spheroidal HMNS is formed
- **M**: Spheroidal HMNS is formed
- **H**: BH is formed

Graph showing gravitational waveforms over time, with different lines representing distinct events and outcomes in the scenario of gravitational wave interactions.
NS-NS merger with hyperon (x-y plane)

Collapse is triggered by hyperon production: By softening of equation of state

Shen’s + hyperon EOS 1.35—1.35 $M_{\odot}$

Sekiguchi et al. in submission
NS-NS merger with hyperon (x-z plane)

- Disk mass ~ 0.1 $M_{\odot}$
- High mass & high luminosity disk

only after the BH formation is shown
Gravitational waveforms

Appearance of hyperon is reflected
Summary of NSNS

• Inspiral and merger of NS-NS is being explored by several groups in detail in numerical relativity

• NS-NS merger is an excellent field for exploring the nature of nuclear EOS → Appropriate modeling of gravitational waveform is the next step

• NS-NS merger often produce HMNS which may be progenitor of short GRB → More detailed physical modeling, e.g., neutrino radiation transfer is needed
Merger simulations of BH-NS

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Coalescence of BH-NS binaries

1. Promising source of gravitational waves
2. Experimental field for high-density matter
3. Possible source of central engine of gamma-ray bursts

as in NS-NS binaries
Evolution of BH-NS (mass = 4.2:1.4)

At formation

Evolve as a result of GW emission

Last 1 hour at $r \sim 3500 \text{ km}$ ($f_{GW} \sim 1 \text{ Hz}$)

Merger starts at $r \sim 5GMc^{-2}$ ($f_{GW} \sim 1 \text{ kHz}$)

NS is swallowed by BH for small $R_{NS}$ or $M_{BH} \gg M_{NS}$

Case I

Case II

NS is disrupted

Large EOS-dependence
Two kinds of the final fate

1. No tidal disruption
2. NS is tidally disrupted before merger

The fate is determined by 4 parameters

- NS mass $\sim 1.3-1.4 \, M_{\text{sun}}$
- NS radius $\sim 10-15 \, \text{km}$?
- BH mass: no information
- BH spin: no information

A wide range has to be surveyed
Condition for tidal disruption

- **BH tidal force > NS self-gravity**

\[
\frac{M_{\text{BH}}}{r^2} \frac{2(\alpha R)}{r} > \frac{M_{\text{NS}}}{(\alpha R)^2} \Rightarrow 2\alpha^3 \left(\frac{M_{\text{BH}}}{r}\right)^3 > \left(\frac{M_{\text{NS}}}{R}\right)^3 \left(\frac{M_{\text{BH}}}{M_{\text{NS}}}ight)^2
\]

\[
\frac{\zeta}{6} \geq \left(\frac{M_{\text{BH}}}{r}\right) \Rightarrow \frac{2\alpha^3 \zeta^3}{216} \geq \left(\frac{M_{\text{NS}}}{R}\right)^3 \left(\frac{M_{\text{BH}}}{M_{\text{NS}}}ight)^2
\]

\[
\left(\frac{M_{\text{BH}}}{M_{\text{NS}}}\right) \leq 2.0 \zeta^{3/2} \left(\frac{\alpha}{1.5}\right)^{3/2} \left(\frac{R}{5M_{\text{NS}}}ight)^{3/2}
\]

Low-mass BH or Large NS radius is necessary
Condition for tidal disruption

Hereafter

Mass ratio \( Q = \frac{M_{\text{BH}}}{M_{\text{NS}}} \), Compactness \( C = \frac{M_{\text{NS}}}{R} \)

\[ Q \leq 2.0\zeta^{3/2} \left( \frac{\alpha}{1.5} \right)^{3/2} \left( \frac{C}{0.2} \right)^{-3/2} \]

No spin BH(\( \zeta = 1 \)) \( Q \leq 2 \left( \frac{C}{0.2} \right)^{-3/2} \)

Max spin BH(\( \zeta = 6 \)) \( Q \leq 30 \left( \frac{C}{0.2} \right)^{-3/2} \)

Approximate guideline
Gravitational-wave frequency at the onset of merger

\[ f = \frac{1}{\pi} \sqrt{\frac{GM}{r^3}} = \frac{c^3}{6^{3/2} \pi GM} \left( \frac{6GM}{rc^2} \right)^{3/2} \]

\[
\begin{cases}
440 \text{ Hz} \left( \frac{M}{10M_{\odot}} \right)^{-1} \left( \frac{6GM}{rc^2} \right)^{3/2} & 1.4M_{\odot} + 8.6M_{\odot} \\
880 \text{ Hz} \left( \frac{M}{5M_{\odot}} \right)^{-1} \left( \frac{6GM}{rc^2} \right)^{3/2} & 1.4M_{\odot} + 3.6M_{\odot}
\end{cases}
\]

\[ \text{cf. } \frac{GM}{c^3} = 4.926 \mu s \]

The frequency may be lower than for NS-NS: Good.
Groups working

- Albert-Einstein Institute
- Cornell-Caltech-CITA-Washington
- Illinois
- Kyoto
- Louisiana-New-york-Perimeter...

Similar results have been published in the past 3 years.
Moving puncture framework

Brandt-Bruegmann 97, AEI, UIUC, Kyoto employ

• Einstein-Rosen bridge + NS

\[ dl^2 \sim \psi^4 \left( dx^2 + dy^2 + dz^2 \right); \quad \psi = 1 + \frac{m_{BH}}{2 | \vec{r} - \vec{r}_{BH} |} + \phi \]

• No singularity

• BSSN formalism + dynamical gauge
Excision framework

- Generalized Harmonic gauge approach
- Excision at apparent horizon

Two approaches have given similar results (e.g., Shibata-Taniguchi, Living Review 2011)
A Black-hole Spin=0

• I focus on the dependence of EOS and BH spin
• Piecewise polytrope is employed for EOS
• Based on Kyutoku, Shibata, Taniguchi, PRD 82, 044049 (2010)
Piecewise polytrope EOS

- \( P = K_i \rho^{\Gamma_i} \) for \( \rho_i < \rho < \rho_{i+1} \) (Cold EOS) + approximate thermal correction:
  \[
P_{\text{corr}} = (\Gamma-1) \rho \varepsilon_{\text{th}}
\]
- Piecewise polytrope with 3 or 4 parameters is a good approximation for most of cold EOSs; \( \Delta P/P < \sim 1\% \)
  (Read et al. (UWM) PRD79, 2009)
- 2-parameter PWP is OK for low-mass NS: We here present results by 2-parameter case
BH-NS: tidal disruption case

- Example: Black-hole neutron-star binary (Animation by K. Kyutoku)
- $M_{BH} = 2M_{NS} = 2.4M_{\text{sun}}$, $R = 15.2$ km (EOS 2H)
- Neutron star with large radius is tidally disrupted before swallowed by black hole $\Rightarrow$ BH + disk is the results
\[ M_{\text{BH}} = 2M_{\text{NS}} = 2.4M_{\text{sun}}, \quad R = 15.2 \text{ km} \quad (\text{EOS 2H}) \]
BH-NS: no tidal disruption case

- Example: Black-hole neutron-star binary (Animation by K. Kyutoku)
- $M_{BH}=3M_{NS}=4.05M_{\text{sun}}, \ R=11.0 \ \text{km}$ (EOS B)
- Neutron star with small radius is not tidally disrupted before swallowed by black hole
- $M_{\text{BH}} = 3M_{\text{NS}} = 4.05M_{\text{sun}}$, $R = 11.0$ km (EOS B)
\[ M_{\text{BH}} = 2.7M_{\text{sun}}, \quad M_{\text{NS}} = 1.35M_{\text{sun}} \]

- BH ringdown
- \( R = 15.2 \text{ km} \)
- sudden shutdown
- \( R = 11.6 \text{ km} \)

Green = Tayloy T4
Imprint of EOS in tidal disruption

• Large NS Radius $\rightarrow$ tidal disruption at a distant orbit & a low frequency

• Small NS Radius $\rightarrow$ tidal disruption at a high frequency

Assume the same mass
BH-NS with piecewise polytrope

For all, $1.35-2.7M_{\text{sun}}$ Clear dependence on NS radius

Larger radius of NS
3 Types of spectrum

- No TD
- Weak TD
- TD during inspiral

Amp

$D=100\text{Mpc}$

frequency $f$ (Hz)
$f_{\text{cut}} \propto C^\beta$

$\beta \sim 3.9 \pm 1.5$
Disk mass: $Q=2$, various EOSs

Smaller radius
B with black-hole Spin

- Spin-Orbit interaction turns on
- Assume that BH spin and orbital angular momentum are parallel

→ Angular velocity, $\Omega$, is *smaller* than that for $S=0$ for a given orbital separation
→ Tidal force depends weakly on separation
→ Tidal disruption occurs for *lower* $\Omega$
→ $dE_{GW}/dt$ is smaller for smaller $\Omega$, and lifetime of binary is longer
→ Effective GW amplitude is higher
Effect of the BH spin II

- The BH spin changes the ISCO radius
  - For Schwarzschild BH, $r_{ISCO} = 6M_{BH}$
  - For extreme Kerr, $r_{ISCO} = M_{BH}$

for prograde orbits:
  - due to spin-orbit coupling
  (prograde = parallel, aligned)

- For prograde orbits, tidal disruption is enhanced even for a heavy BH or large mass ratio $Q$

- For a retrograde (anti-parallel) case, tidal disruption is suppressed.
$M_{BH} = 4.05M_{\text{sun}}, \ M_{NS} = 1.35M_{\text{sun}}, \ R = 11.6 \ \text{km}$

Dash = Tayloy T4

UIUC originally pointed out
GW spectrum for $Q=2$, $M_{NS}=1.35M_{\text{sun}}$

Spin increases
GW spectrum for $Q=3, a=0.75$
GW spectrum for $Q=4$, $a=0.75$, $M_{NS}=1.35M_{\text{sun}}$

$f_{\text{cut}}$ may be detectable in near future
Enhanced disk mass

• Example: Black-hole neutron-star binary
  HB_q5_M135.a75.gif
  Animation by K. Kyutoku
  \( M_{\text{BH}} = 5M_{\text{NS}} = 6.75M_{\text{sun}} \), \( R = 11.6 \text{ km} \) (EOS HB), \( a = 0.75 \)
  • Wide disk with mass \( \sim 0.1 M_{\text{sun}} \)
• $M_{\text{BH}} = 5M_{\text{NS}} = 6.75M_{\text{sun}}$, $R=12.3 \text{ km}$ (EOS H), $a=0.75$
Disk mass: $Q=3$, $M_{NS}=1.35M_{\text{sun}}$, $R=11.6\text{km}$

HB $Q=3$

UIUC, CCCW group originally pointed out
Disk mass with high spin

$Q=5 \ a=0.75$
Disk mass vs compactness

\[ a = 0.75 \]
Summary

• Gravitational waves in the merger phase reflect the radius (EOS) of neutron star
• Appreciable dependence on EOS is found for frequency 1—3 kHz (high frequency)
• Advanced LIGO (+ special design for high-frequency GWs) could constrain EOS for data with high spin and $Q \sim 4,5$
• Mass of disk is $> 0.1 M_{\odot}$ for spin $a=0.75$ even for $Q \sim 5$ with realistic compactness of NS, $C \sim 0.14—0.18$
Summary for 2 days

• Many simulations are ongoing for NS-NS and BH-NS: Current primary interest is to clarify gravitational waveforms.

• In ~3 years, a variety of theoretical waveforms will be derived.

• First implementation of finite-temperature EOS + neutrino physics + cooling is done by Sekiguchi for astrophys. simulation → Simulation with (approximate) neutrino transfer will be one of challenges for PFlopes machine in the next ~ 3-5 years.
Merger to NS

$1.4M_{\text{sun}}$  $1.4M_{\text{sun}}$

$0.00034482 \text{ ms}$

$\log(\text{g/cm}^3)$
How mass and spin of BH and NS are determined

• These could be determined by GWs in the inspiral phase

\[
\frac{t}{P} = \frac{5}{512\pi} \left( \frac{GM}{c^2 r} \right)^{-5/2} \left( \frac{M}{\mu} \right) \approx 1.1 \left( \frac{6GMc^{-2}}{r} \right)^{-5/2} \left( \frac{M}{4\mu} \right)
\]

\[\Rightarrow t > P\]

Adiabatic
LCGT was funded last year!
3 km-3 km
In operation from ~2017

Super Kamioka around here
Inside of Kamioka place

Japan sea/East sea
Gravitational waveform for black hole formation case

Kiuchi et al.

Qualitatively universal

$Dh_+/M_0$

$\text{Ring down}$

Inspiral

Merger
Universal spectrum for BH formation

\[ h_{\text{eff}} \sim f^n \]

Qualitatively depends on disk mass & mass ratio

No disk

CTRL QNM

Kiuchi et al.
GR leakage works well

- Neutrino luminosities consistent with result by 1D-GR radiation hydro (Liebendoerfer et al. 04)
  - Collapse of 15 $M_{\text{sun}}$ model by WHW02
  - Besides convection induced modulation in luminosities
- Neutrino luminosities in BNS merger will be estimated

Results by Sekiguchi (2009)
Numerical relativity

Relativity, Mathematical relativity

Numerical computation

Physics Astrophysics
Adaptive Mesh Refinement

\[ L > \lambda >> GM/c^2 \]

\[ l \sim 4GM/c^2 \]

- For each box \( \sim 121^2 \times 61 \) grids (+12 for margin) \( \rightarrow \) NS diameter is covered by \( \sim 100 \) grids.
- Number of levels \( \sim 7 \) (3 coarse*1 + 4 fine*2)
- Number of variables \( \sim 150 \) in our SACRA code
  \( \rightarrow \) Memory \( \sim 15 \) GBytes

Computation is feasible by PC of \( \sim \$200,000 \)!
Binary simulation @ home (@office)

- Core i7x, 3.33 GHz, 4 cores, 12 or 24 GB memory
  Better than supercomputer of 10 yrs ago
- 121*121*61*10 AMR domains ➔ 15 GB memory
- About 40 days for ~7 orbits by SACRA code
- Parameter parallel by ~30 machines now