Supersymmetric AdS_6 solutions of type IIB supergravity

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with Hyojoong Kim and Nakwoo Kim

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Duality and Novel Geometry in M-theory

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Minwoo Suh

Department of Physics

Sogang University

Plan

Motivation: The AdS_6/CFT_5 correspondence

Method: The method of Killing spinors

Supersymmetric AdS_6 solutions of type IIB supergravity

Summary and outlook

Motivation: The AdS_6/CFT_5 correspondence

• Perturbatively five-dimensional gauge theories are non-renormalizable.

• [Seiberg, 1996], [Intriligtor, Morrison, Seiberg, 1996]

$\mathcal{N} = 1$ supersymmetric $Sp(N)$ gauge theory in five dimensions
with $N_f < 8$ fundamental and 1 antisymmetric hypermultiplets

$\downarrow$ infinite gauge coupling limit

Superconformal fixed point theories
with $SO(2N_f) \times U(1)$ global symmetry enhanced to E_{N_f+1}

- [Ferrara, Kehagias, Partouche, Zaffaroni, 1998]

String theory realization: N D4-branes probing a $O8^-$ plane with N_f coincident D8-branes.

- [Brandhuber, Oz, 1999]

Near horizon limit of D4-D8 system in massive type IIA supergravity:

$$\begin{aligned} ds^2 =& \frac{W^2 L^2}{4}\left[9 ds^2_{AdS_6} + 4 ds^2_{S^4}\right] , \\ F_4 =& 5 L^4 W^{-2} \sin^3\theta d\theta \wedge vol_{S^3} , \\ e^{-\Phi} =& \frac{3L}{2W^2} , \qquad W = (m\cos\theta)^{-1/6} . \end{aligned}$$

The only known supersymmetric AdS_6 solution till 2012.

- [Passias, 2012]

The Brandhuber-Oz solution is the only supersymmetric AdS_6 solution of massive type IIA supergravity.

• [Lozano, O Colgain, Rodriguez-Gomez, Sfetsos, 2012]

Non-Abelian T-dual of the Brandhuber-Oz solution
in type IIB supergravity.

The field theory dual is unknown so far.

• [Hee-Cheol Kim, Sung-Soo Kim, Kimyeong Lee, 2012]

$SO(2N_f) \times U(1)$ global symmetry enhancement to E_{N_f+1}
via 5d superconformal index.

• [Apruzzi, Fazzi, Passias, Rosa, Tomasiello, 2014]

No supersymmetric AdS_6 solutions of type IIA and $d = 11$
supergravity.

The most general conditions for supersymmetric AdS_6 solutions
of type IIB supergravity via the pure spinor method
from generalized geometry.

• Summary so far:

Type IIA and $d = 11$ supergravity: No supersymmetric AdS_6 solution

Massive type IIA supergravity: The unique Brandhuber-Oz solution

$\updownarrow$

5d fixed point theories

Type IIB supergravity: The non-Abelian T-dual of the Brandhuber-Oz

and unknown but possible solutions

$\updownarrow$

Dual field theories unknown

• Our goal:

To find the most general conditions for supersymmetric AdS_6 solutions of type IIB supergravity via the Killing spinor method

Method: The method of Killing spinors

- A method of finding conditions for supersymmetric solutions of supergravity:

[Gauntlett, Pakis, 2002], "The geometry of D = 11 Killing spinors",

[Gauntlett, Gutowski, Hull, Pakis, Reall, 2002], "All supersymmetric solutions of minimal supergravity in five dimensions",

[Lin, Lunin, Maldacena, 2004], "Bubbling AdS space and 1/2 BPS geometries",

[Gauntlett, Martelli, Sparks, Waldram, 2004], "Supersymmetric AdS_5 solutions of M-theory",
[Gauntlett, Martelli, Sparks, Waldram, 2004], "A new infinite class of Sasak-Einstein manifolds",

[Gauntlett, Martelli, Sparks, Waldram, 2005], "Supersymmetric AdS_5 solutions of type IIB supergravity",

[Nakwoo Kim, 2005], "AdS_3 solutions of IIB supergravity from D3-branes".

- There is also the method of pure spinors via generalized geometry.

Supersymmetric AdS_6 solutions of type IIB supergravity

- The ansatz

$$ds^2 = e^{2U} ds^2_{AdS_6} + ds^2_4, \qquad \tau = C_{(0)} + i e^{-\Phi},$$
$$F_{(5)} = 0, \qquad G_{(3)} = G_{mnp} e^{mnp}.$$

- Two Majorana-Weyl spinors

$$\varepsilon = \psi_+ \otimes \xi_{1-} + \psi_+^c \otimes \xi_{2-}^c + \psi_- \otimes \xi_{1+} + \psi_-^c \otimes \xi_{2+}^c .$$

- Supersymmetry variation of gravitino and dilatino

$$\delta\psi_M = D_M \varepsilon - \frac{1}{96}\left(\Gamma_M{}^{NPQ} G_{NPQ} - 9\,\Gamma^{NP} G_{MNP}\right)\varepsilon^c + \frac{i}{1920}\Gamma^{NPQRS} F_{NPQRS}\Gamma_M \varepsilon,$$
$$\delta\lambda = i\,\Gamma^M P_M \varepsilon^c + \frac{i}{24}\Gamma^{MNP} G_{MNP}\varepsilon .$$

- Supersymmtry conditions

$$
\begin{aligned}
2D_m\xi_{1\pm} - \frac{1}{48}(G_{npq}\gamma_m{}^{npq} - 9G_{mpq}\gamma^{pq})\xi_{2\pm} &= 0\,,\\
2\bar{D}_m\xi_{2\pm} - \frac{1}{48}(G^*_{npq}\gamma_m{}^{npq} - 9G^*_{mpq}\gamma^{pq})\xi_{1\pm} &= 0\,,\\
\partial_n U\gamma^n\xi_{1\pm} + ime^{-U}\xi_{1\mp} - \frac{1}{48}G_{npq}\gamma^{npq}\xi_{2\pm} &= 0\,,\\
\partial_n U\gamma^n\xi_{2\pm} + ime^{-U}\xi_{2\mp} - \frac{1}{48}G^*_{npq}\gamma^{npq}\xi_{1\pm} &= 0\,,\\
\frac{1}{2}P_n\gamma^n\xi_{2\pm} + \frac{1}{48}G_{npq}\gamma^{npq}\xi_{1\pm} &= 0\,,\\
\frac{1}{2}P^*_n\gamma^n\xi_{1\pm} + \frac{1}{48}G^*_{npq}\gamma^{npq}\xi_{2\pm} &= 0\,.
\end{aligned}
$$

- Spinor bilinear equations (e.g. vector bilinears)

$$
\begin{aligned}
&2\nabla^{[l}(\bar{\xi}_{1+}\gamma^{m]}\xi_{1-}+\bar{\xi}_{2+}\gamma^{m]}\xi_{2-})+6\partial^{[l}U(\bar{\xi}_{1+}\gamma^{m]}\xi_{1-}+\bar{\xi}_{2+}\gamma^{m]}\xi_{2-})\\
&-3ime^{-U}(\bar{\xi}_{1+}\gamma^{lm}\xi_{1+}+\bar{\xi}_{1-}\gamma^{lm}\xi_{1-}+\bar{\xi}_{2+}\gamma^{lm}\xi_{2+}+\bar{\xi}_{2-}\gamma^{lm}\xi_{2-})=0\,,
\end{aligned}
$$

$$
\begin{aligned}
&2\nabla^{[l}(\bar{\xi}_{1+}\gamma^{m]}\xi_{1-}-\bar{\xi}_{2+}\gamma^{m]}\xi_{2-})+4\partial^{[l}U(\bar{\xi}_{1+}\gamma^{m]}\xi_{1-}-\bar{\xi}_{2+}\gamma^{m]}\xi_{2-})\\
&-ime^{-U}(\bar{\xi}_{1+}\gamma^{lm}\xi_{1+}+\bar{\xi}_{1-}\gamma^{lm}\xi_{1-}-\bar{\xi}_{2+}\gamma^{lm}\xi_{2+}-\bar{\xi}_{2-}\gamma^{lm}\xi_{2-})=0\,,
\end{aligned}
$$

$$
\begin{aligned}
&D^{[l}(\bar{\xi}_{2+}\gamma^{m]}\xi_{1-})+4\partial^{[l}U\bar{\xi}_{2+}\gamma^{m]}\xi_{1-}-P^{[l}(\bar{\xi}_{1+}\gamma^{m]}\xi_{2-})\\
&-2ime^{-U}(\bar{\xi}_{2+}\gamma^{lm}\xi_{1+}+\bar{\xi}_{2-}\gamma^{lm}\xi_{1-})=0\,,
\end{aligned}
$$

$$
\begin{aligned}
&\overline{D}^{[l}(\bar{\xi}_{1+}\gamma^{m]}\xi_{2-})+4\partial^{[l}U\bar{\xi}_{1+}\gamma^{m]}\xi_{2-}-P^{*[l}(\bar{\xi}_{2+}\gamma^{m]}\xi_{1-})\\
&-2ime^{-U}(\bar{\xi}_{1+}\gamma^{lm}\xi_{2+}+\bar{\xi}_{1-}\gamma^{lm}\xi_{2-})=0\,.
\end{aligned}
$$

• Three Killing vectors

$$K_1^m = Re\left(\overline{\xi}_{1+}\gamma^m\xi_{1-} + \overline{\xi}_{2+}\gamma^m\xi_{2-}\right),$$

$$K_2^m = Im\left(\overline{\xi^c_{1}}_{+}\gamma^m\xi_{2-} + \overline{\xi^c_{2}}_{+}\gamma^m\xi_{1-}\right),$$

$$K_3^m = Re\left(\overline{\xi^c_{1}}_{+}\gamma^m\xi_{2-} + \overline{\xi^c_{2}}_{+}\gamma^m\xi_{1-}\right),$$

satisfying an $SU(2)$ algebra

$$[K_i, K_j]^m = \varepsilon_{ijk} K_k^m .$$

This is the $SU(2)$ R-symmetry in the dual 5d gauge theories.

- Two vectors orthogonal to all three Killing vectors

$$L_1^m = Re\left(\bar{\xi}_{1+}\gamma^m\xi_{2-} - \bar{\xi}_{2+}\gamma^m\xi_{1-}\right),$$
$$L_2^m = Im\left(\bar{\xi}_{1+}\gamma^m\xi_{2-} + \bar{\xi}_{2+}\gamma^m\xi_{1-}\right).$$

- Defining coordinates, (y, z)

$$\nabla_m\left[e^{U+\Phi/2}(\bar{\xi}_{1+}\xi_{2+} + \bar{\xi}_{2+}\xi_{1+})\right] - m\,e^{\Phi/2}\,Im\left(\bar{\xi}_{1+}\gamma^m\xi_{2-} + \bar{\xi}_{2+}\gamma^m\xi_{1-}\right) = 0$$

$$\nabla_m\left[\frac{1}{i}e^{U-\Phi/2}(\bar{\xi}_{1+}\xi_{2+} - \bar{\xi}_{2+}\xi_{1+})\right] - e^{U+\Phi/2}(\bar{\xi}_{1+}\xi_{2+} + \bar{\xi}_{2+}\xi_{1+})\partial_m C_{(0)}$$
$$+ m\,e^{-\Phi/2}\,Re\left(\bar{\xi}_{1+}\gamma^m\xi_{2-} - \bar{\xi}_{2+}\gamma^m\xi_{1-}\right) = 0.$$

$$y = 3m\,e^{U+\Phi/2}\left(\bar{\xi}_{1+}\xi_{2+} + \bar{\xi}_{2+}\xi_{1+}\right),$$
$$z = -\,i3m\,e^{U-\Phi/2}\left(\bar{\xi}_{1+}\xi_{2+} - \bar{\xi}_{2+}\xi_{1+}\right).$$

- Defining coordinates, (x, y)

$$z = e^{-2U-\Phi}\sqrt{e^{8U+\Phi} - e^{\Phi}x^2 - e^{4U}y^2}.$$

- The metric

$$
\begin{aligned}
ds^2 =& e^{2U} ds^2_{AdS_6} + ds^2_4 , \\
ds^2_4 =& \frac{1}{9m^2} \left[e^{-6U} x^2 \, ds^2_{S^2} + \frac{e^{-2U}}{e^{8U+\Phi} - e^{\Phi} x^2 - e^{4U} y^2} \right. \\
& \left. \times \left[(e^{4U+\Phi} - y^2) \, dx^2 + 9 \, (e^{8U} - x^2) \, dy^2 + 6 \, x \, y \, dx \, dy \right] \right] .
\end{aligned}
$$

- The one-form flux

$$
\begin{aligned}
dC_{(0)} = & \frac{e^{-2U-\Phi}}{\sqrt{e^{8U+\Phi} - e^{\Phi} x^2 - e^{4U} y^2}} \left[2(e^{8U+\Phi} + e^{\Phi} x^2) \frac{dU}{y} \right. \\
& \left. - \frac{1}{2} (e^{8U+\Phi} - e^{\Phi} x^2 - 2e^{4U} y^2) \frac{d\Phi}{y} - \frac{2}{3} e^{\Phi} \frac{x dx}{y} \right] .
\end{aligned}
$$

- The three-form flux

$$Re(G) = \frac{1}{27m^2}\,\frac{e^{-6U-\Phi/2}x}{\sqrt{e^{8U+\Phi}-e^{\Phi}x^2-e^{4U}y^2}}\Big[(e^{8U+\Phi}-3e^{\Phi}x^2-2e^{4U}y^2)dU$$

$$-\frac{1}{4}(e^{8U+\Phi}-e^{\Phi}x^2)d\Phi+e^{\Phi}xdx+2e^{4U}ydy\Big]\wedge vol_{s^2}\,,$$

$$Im(G) = -\frac{1}{27m^2}\,\frac{x}{y}\,e^{-8U-\Phi/2}\Big[(e^{8U+\Phi}+e^{\Phi}x^2-2e^{4U}y^2)dU$$

$$-\frac{1}{4}(e^{8U+\Phi}-e^{\Phi}x^2)d\Phi-\frac{1}{3}e^{\Phi}xdx+2e^{4U}ydy\Big]\wedge vol_{s^2}\,.$$

• Two PDEs: The most general conditions for supersymmetric AdS_6 solutions of type IIB supergravity

$$
\begin{aligned}
4\,e^{\Phi}\,x\,(e^{8U+\Phi}+e^{\Phi}x^2)\,dU\wedge dx \\
-\,e^{\Phi}\,x\,(e^{8U+\Phi}-e^{\Phi}x^2-2e^{4U}y^2)\,d\Phi\wedge dx \\
-12\,e^{4U}\,y\,(e^{8U+\Phi}-3e^{\Phi}x^2-2e^{4U}y^2)\,dU\wedge dy \\
+3\,e^{4U}\,y\,(e^{8U+\Phi}-e^{\Phi}x^2)\,d\Phi\wedge dy \\
-4\,e^{4U+\Phi}\,x\,y\,dx\wedge dy = 0\,,
\end{aligned}
$$

$$
\begin{aligned}
8\,e^{\Phi}\,x\,y\,dU\wedge dx - 12\,(e^{8U+\Phi}+e^{\Phi}x^2-2e^{4U}y^2)\,dU\wedge dy \\
+2\,e^{\Phi}\,x\,y\,d\Phi\wedge dx + 3(e^{8U+\Phi}-e^{\Phi}x^2)\,d\Phi\wedge dy + 4\,e^{\Phi}\,x\,dx\wedge dy = 0\,.
\end{aligned}
$$

• To the non-Abelian T-dual of the Brandhuber-Oz solutions in
[Lozano, O Colgain, Rodriguez-Gomez, Sfetsos, 2012]

$$\begin{aligned} x &\to \frac{c_1^4}{c_2}\frac{4}{L^2}\, m^{1/3}\, r \sin^2\theta\,, \\ y &\to c_1^2 \cos^{2/3}\theta\,, \\ e^{2A} &\to c_1^2 \cos^{-1/3}\theta\,, \\ e^{2\Phi} &\to c_2^2\, \frac{\cos^{-2/3}\theta}{\sin^6\theta}\, \frac{1}{1+\frac{16}{L^4}m^{2/3}r^2\cos^{2/3}\theta\sin^{-4}\theta}\,. \end{aligned}$$

• To the general conditions of supersymmetric AdS_6 solutions in
[Apruzzi, Fazzi, Passias, Rosa, Tomasiello, 2014]

$$U \to A-\phi/4\,, \qquad \Phi \to \phi\,, \qquad x \to p\,, \qquad y \to q\,,$$

$$dC_{(0)} \to F_1\,, \qquad Re(G) \to -e^{-\Phi/2}H\,, \qquad Im(G) \to e^{\Phi/2}F_3\,.$$

Summary

• Via the method of Killing spinors, we have reproduced the two PDEs as general conditions for supersymmetric AdS_6 solutions of type IIB supergravity.

Outlook

• It would be interesting to search for new supersymmetric AdS_6 solutions by solving the two PDEs.

• Dual field theories for non-Abelian T-dual of the Brandhuber-Oz solution and possible new AdS_6 solutions are still beyond our reach.

• Study of the four-dimensional effective action will provide us with a better understanding of the $AdS_6 \times_w M_4$ geometry. (Continued in Hyojoong Kim's talk in this conference.)

Thank you for your attention.