AdS_6 solutions of type IIB supergravity and their hidden symmetry

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with Nakwoo Kim and Minwoo Suh

Duality and Novel Geometry in M-theory
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Search for AdS_6 solutions in IIB supergravity

Metric ansatz : $$ds^2 = e^{2U} ds^2_{\mathrm{AdS}_6} + ds^2_{M_4}.$$

To preserve the symmetry of AdS_6, we set

- warp factor U, axion C and dilaton ϕ : functions on M_4,
- F_5 =0,
- complex G_3 on M_4 : They can be dualized to scalar fields f and g.

Looking for supersymmetric solutions using Killing spinor equations
$\longrightarrow$ Minwoo's talk.

Dimensional reduction on AdS_6

4 dimensional effective action from the equations of motion

$$\mathcal{L} = \sqrt{g_4}\Big[R - 24(\partial U)^2 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}e^{2\phi}(\partial C)^2 + \frac{1}{2}e^{-12U-\phi}(\partial f)^2 + \frac{1}{8}e^{-12U+\phi}(\partial g + 2C\partial f)^2 - 30e^{-8U}\Big].$$

- We obtain a non-linear sigma model of 5 scalar fields coupled to gravity.
- There is a non-trivial scalar potential.
- The signs of the kinetic terms for dualized scalars are reversed.

D=5 target space

$$ds_5^2 = 48dU^2 + d\phi^2 + e^{2\phi}dC^2 - \frac{1}{4}e^{-12U+\phi}(dg + 2Cdf)^2 - e^{-12U-\phi}df^2.$$

- We have found that there are 8 Killing vectors.
- They generate $sl(3,\mathbb{R})$ algebra.

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- We have found that there are 8 Killing vectors.
- They generate $sl(3, \mathbb{R})$ algebra.
- The five-dimensional target space parametrize the coset $SL(3, \mathbb{R})/SO(2, 1)$.

Scalar kinetic terms : $SL(3,\mathbb{R})/SO(2,1)$

Construct a coset representative in Borel gauge

$$\mathcal{V} = e^{\frac{1}{\sqrt{2}}\phi H_1} e^{-2\sqrt{6}U H_2} e^{C E_{\alpha_1}} e^{f E_{\alpha_2}} e^{\frac{1}{2} g E_{\alpha_3}}.$$

Introduce a Lie algebra-valued 1-form as

$$\mathcal{V}_i^{\ m} \partial_\mu (\mathcal{V}^{-1})_m^{\ \ k} \eta_{kj} = P_{\mu(ij)} + Q_{\mu[ij]},$$

with $\eta_{ij} = \mathrm{diag}(1,1,-1)$.

The scalar kinetic terms in the Lagrangian become $\mathcal{L}_{\mathrm{kin}} = -\mathrm{Tr}(P_\mu P^\mu)$.

Scalar potential

The scalar potential $V = 30\, e^{-8U}$

- breaks the $SL(3,\mathbb{R})$ global symmetry into a nontrivial subalgebra,
- is invariant under the action of 5 Killing vectors, which form a certain algebra $A_{5,40} \cong sl(2,\mathbb{R}) \ltimes \mathbb{R}^2$.

  [Patera, Sharp, Winternitz, Zassenhaus 76]

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Goal : Write the scalar potential in terms of the coset representative.

Study the integrability conditions with the Killing spinor equations.

Killing spinor equations

Killing spinor equations can be written as

$$\begin{pmatrix} \delta\psi_{\mu+} \\ \delta\psi_{\mu-} \end{pmatrix} = \begin{pmatrix} \nabla_\mu + \frac{1}{4}Q_{\mu ij}\Gamma^{ij} & S\gamma_\mu \\ S\gamma_\mu & \nabla_\mu + \frac{1}{4}Q_{\mu ij}\bar{\Gamma}^{ij}_\mu \end{pmatrix} \begin{pmatrix} \xi_+ \\ \xi_- \end{pmatrix},$$

$$\begin{pmatrix} \delta\lambda_{i+} \\ \delta\lambda_{i-} \end{pmatrix} = \begin{pmatrix} M_{ij}\bar{\Gamma}^j & P_{\mu ij}\gamma^\mu\bar{\Gamma}^j \\ P_{\mu ij}\gamma^\mu\Gamma^j & M_{ij}\Gamma^j \end{pmatrix} \begin{pmatrix} \xi_+ \\ \xi_- \end{pmatrix},$$

where

$$S = \frac{3i}{2L}e^{-4U}, \qquad M_{ij} = \frac{2i}{L}e^{-4U}\mathrm{diag}(1,1,2).$$

Integrability conditions

The gravitino-gravitino integrability condition

$$\gamma_\mu{}^{\nu\rho}\,[\mathcal{D}_\nu,\mathcal{D}_\rho]\begin{pmatrix}\xi_+\\ \xi_-\end{pmatrix}-\frac{1}{2}\begin{pmatrix}-M^i{}_j\bar{\Gamma}^j & P_\nu{}^i{}_j\,\gamma^\nu\Gamma^j\\ *** & ***\end{pmatrix}\gamma_\mu\begin{pmatrix}\delta\lambda_{i+}\\ \delta\lambda_{i-}\end{pmatrix}$$

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$$\left(R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}-\left(P_{\mu ij}P_\nu{}^{ij}-\frac{1}{2}g_{\mu\nu}P_{\rho ij}P^{\rho ij}\right)+g_{\mu\nu}\left(-12S^2+\frac{1}{2}M_{ij}M^{ij}\right)\right)\gamma^\nu\xi_+$$
$$-4\left(\partial_\nu S+\frac{1}{4}M_{ij}P_\nu{}^{ij}\right)\gamma^\nu{}_\mu\xi_-$$
$$-\left(\left(SQ_{\nu ij}+\frac{1}{2}\left(P_\nu\eta M\right)_{ij}\right)-K_i{}^l\left(SQ_{\nu lk}+\frac{1}{2}\left(P_\nu\eta M\right)_{lk}\right)K^k{}_j\right)\Gamma^{ij}\gamma^\nu{}_\mu\xi_-$$
$$-\frac{1}{2}\left(\left(P_\mu\eta M\right)_{ij}+\left(KP_\mu\eta MK\right)_{ij}\right)\Gamma^{ij}\xi_-,$$

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Einstein equation

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Equations for S and M_{ij}

Integrability conditions

The integrability conditions with the Killing spinor equations

$$\gamma_\mu{}^{\nu\rho}\,[\mathcal{D}_\nu, \mathcal{D}_\rho]\begin{pmatrix} \xi_+ \\ \xi_- \end{pmatrix}, \qquad \gamma^\mu D_\mu \delta\lambda_{i-},$$

- the Einstein equation and the scalar equations of motion part
- the remaining part, which should be canceled by specific S and M_{ij}.

The scalar potential is

$$V = M_{ij}M^{ij} - 24S^2,$$

where

$$S = \frac{3}{4}\mathcal{V}_{11}\,\mathcal{V}_{22}, \qquad M_{ij} = \mathcal{V}_{11}\,\mathcal{V}_{22}\,\mathrm{diag}(1,1,2).$$

Summary

Via dimensional reduction on AdS_6, the problem of finding AdS_6 solutions of type IIB supergravity is reduced to a four-dimensional non-linear sigma model, i.e. a gravity theory coupled to 5 scalar fields with a non-trivial scalar potential.

$$\boxed{\mathcal{L} = \sqrt{g_4}\left[R - \mathrm{Tr}\left(P_\mu P^\mu\right) - \left(M_{ij}M^{ij} - 24S^2\right)\right].}$$

- The scalar kinetic terms parameterize $SL(3,\mathbb{R})/SO(2,1)$.
- The scalar potential breaks the global symmetry to a certain subalgebra.
- The Killing spinor equations are re-written in terms of coset NLSM.

$SL(n,\mathbb{R})/SO(p,q)$ generalization

We have generalized this construction to $SL(n,\mathbb{R})/SO(p,q)$.

$$S = \frac{n}{4}\left(\prod_{i'=1}^{n-1}\mathcal{V}_{i'i'}\right), \qquad M_{ij} = \left(\prod_{i'=1}^{n-1}\mathcal{V}_{i'i'}\right)\tilde{M}_i{}^k\eta_{kj}.$$

$$\begin{aligned}
\eta &= \mathrm{diag}(\underbrace{1,\cdots,1}_{p},\underbrace{-1\cdots,-1}_{q}),\\
K_i{}^j &= \mathrm{diag}(\underbrace{1,\cdots,1}_{n-1},-1),\\
\tilde{M}_i{}^j &= \mathrm{diag}(\underbrace{1,\cdots,1}_{n-1},-(n-1)).
\end{aligned}$$

We can construct or classify the geometries admitting Killing spinor equations without supersymmetry.

Action of G

The coset representative $\mathcal{V}$ transforms as

$$\mathcal{V} \longrightarrow K\,\mathcal{V}\,G,$$

under global G and local K transformations.

Our case : $G/K = SL(3,\mathbb{R})/SO(2,1)$

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The scalar potential is invariant under the action of h_1, e_1, e_2, e_3 and f_1.

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$$\begin{aligned} e^{\phi} &\to e^{\phi} + 2aCe^{\phi}, \\ C &\to C + a\left(e^{-2\phi} - C^2\right), \\ f &\to f - \frac{a}{2}g. \end{aligned}$$

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This corresponds to $SL(2,\mathbb{R})$ transformation with $\begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}$ in type IIB supergravity.

$h_1,\ e_1,\ f_1$ generate $SL(2,\mathbb{R})$ symmetry and e_2, e_3 generate trivial gauge symmetries. This is consistent with the algebra $A_{5,40} \cong sl(2,\mathbb{R}) \ltimes \mathbb{R}^2$ generated by 5 Killing vectors.

- The two known AdS_6 solutions can be re-written in terms of coset representative $\mathcal{V}$ to show the symmetry manifestly.
- By applying group transformations on $\mathcal{V}$ of this "seed solution", we could not obtain a new solution. The transformed solution can be obtained by $SL(2,\mathbb{R})$ symmetry of type IIB supergravity directly.

AdS_6 in M-theory

The similar structure appears in AdS_6 solutions in M-theory.

There are two scalars : the warp factor U and a dualized scalar f.

We obtain a five-dimensional effective Lagrangian as

$$\mathcal{L} = \sqrt{g_5}\left(R - 18\,(\partial U)^2 + 18\,e^{-12U}\,(\partial f)^2 - 30\,e^{-6U}\right).$$

The target space parameterize the coset $SL(2,\mathbb{R})/SO(1,1)$.

the Killing spinor equations, the integrability conditions, $\cdots$.

Concluding remarks

- Searching for new supersymmetric AdS_6 solutions by solving the two coupled PDEs directly.
- Constructing the dual field theories for IIB AdS_6 solutions.

Thank you!!!