

Non-Relativistic Strings and Non-geometric Backgrounds in DFT

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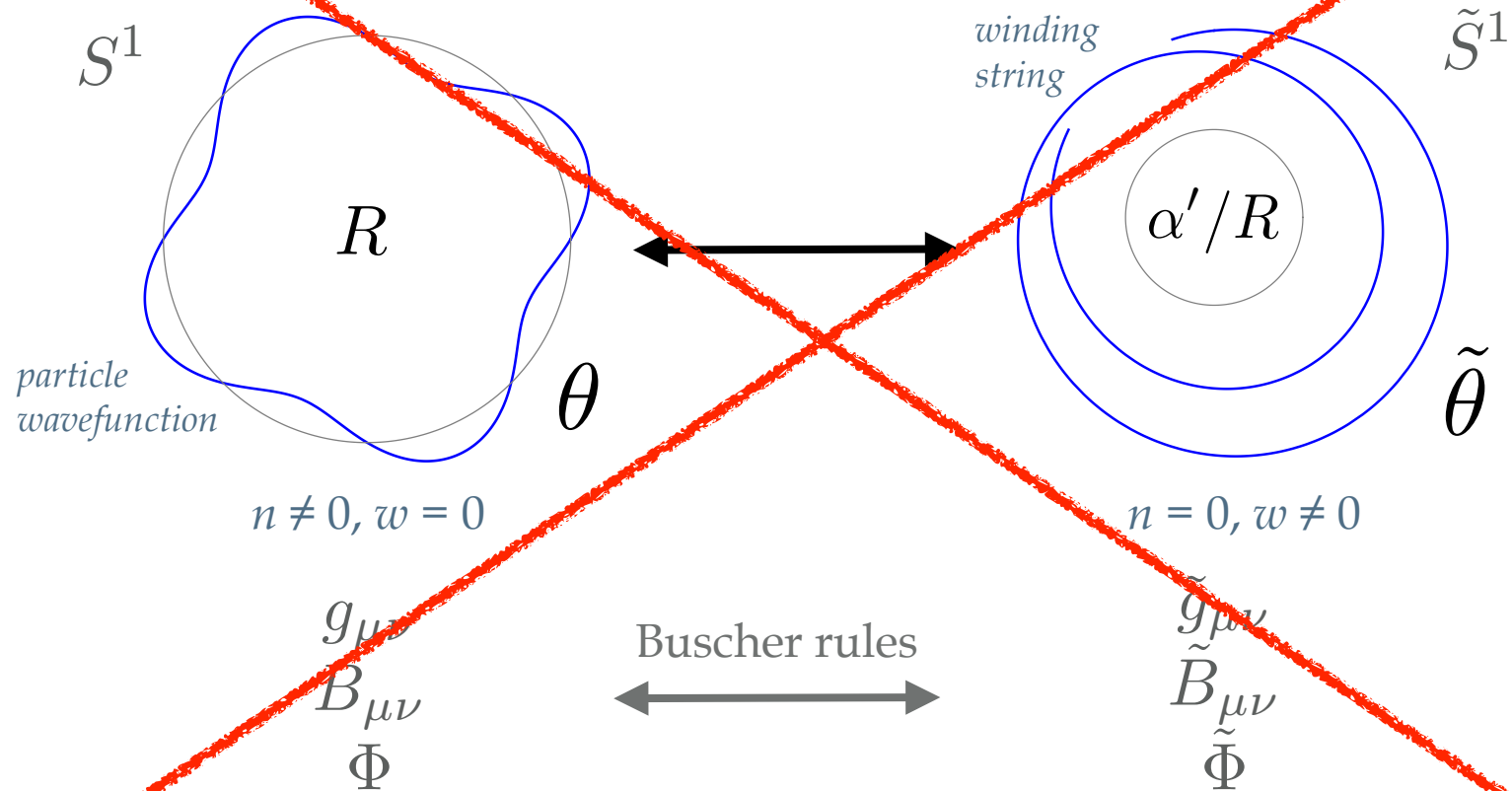
APCTP: Duality and Novel Geometry in M-theory

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Based on [arXiv:1508.01121](https://arxiv.org/abs/1508.01121)
with S.M. Ko, R. Meyer, and J.-H. Park

What is Double Field Theory?

Idea: Make T-duality manifest ($n \leftrightarrow w$)



Generalization to $T^D : O(D, D; \mathbb{Z})$ duality group

DFT: Quick Review

Doubled spacetime: $X^M = (\tilde{x}_\mu, x^\mu)$

Canonical inner product $J_{MN} = \begin{pmatrix} 0 & \delta_n^m \\ \delta_m^n & 0 \end{pmatrix}$ used to raise/lower indices

Coordinate patches glued using **generalized diffeomorphisms**.

infinitesimally: $\delta_\xi Y^M = \xi^N \partial_N Y^M - Y^N \partial_N \xi^M + Y^N \partial^M \xi_N$

Too many coordinates
 \Rightarrow impose "section condition": $\begin{cases} \partial_M \partial^M \Phi(X) = 0 \\ \partial_M \Phi_1(X) \partial^M \Phi_2(X) = 0 \end{cases}$

Always choose
 $\frac{\partial}{\partial \tilde{x}_\mu} = 0$

Geometry: DFT dilaton $e^{-2d} = \sqrt{-g} e^{-2\Phi}$ and **generalized metric**

$$\mathcal{H}_{MN} = \mathcal{H}_{NM} = \begin{pmatrix} g^{-1} & -g^{-1} B \\ B g^{-1} & g - B g^{-1} B \end{pmatrix} \in O(D, D)$$

T-duality group $O(D, D)$ acts by $\tilde{\mathcal{H}}_{MN} = \mathcal{O}_M^P \mathcal{O}_N^Q \mathcal{H}_{PQ}$, $\mathcal{O}_N^M \in O(D, D)$

DFT: Geometry and Non-Geometry

Non-geometry (e.g. "T-folds")

Locally geometric, but transition functions mix x^μ and \tilde{x}^μ .

Example on torus: Going once around one circle, glue others using T-duality.

Also R fluxes, ...

Hellerman, McGreevy, Williams;
Hull, Dabholkar
Shelton, Taylor, Wecht

Standard geometry

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}$$

Transition functions don't mix x^μ and \tilde{x}^μ non-trivially.

Locally non-Riemannian spaces

Generalized geometries (with a sector) of the form

$$\mathcal{H}_{MN} = \begin{pmatrix} 0 & N_\mu^\nu \\ (N^T)^\mu_\nu & S_{\mu\nu} \end{pmatrix}$$

exist and are non-trivial $O(D,D)$ elements!

Lee, Park

Locally Non-geometric Background

Fundamental string soliton background:

K.-H. Lee & J.-H. Park
arXiv:1307.8377

$$ds^2 = f^{-1}(-dt^2 + (dx^1)^2) + (dx^2)^2 + \dots + (dx^9)^2,$$

$$B = (f^{-1} - \hat{c})dt \wedge dx^1, \quad e^{-2\phi} = f e^{-2\phi_0}, \quad \begin{array}{l} x^1 \text{ compact,} \\ 0 < \hat{c} < 2 \end{array}$$

$$f = 1 + \frac{Q}{r^6}, \quad r^2 = \sum_{a=2}^9 (x^a)^2,$$

Apply T-duality

$$\mathcal{O}_A{}^B = \begin{pmatrix} 0 & 0 & \eta^{\alpha\beta} & 0 \\ 0 & \delta^i{}_j & 0 & 0 \\ \eta_{\alpha\beta} & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_i{}^j \end{pmatrix} \Rightarrow$$

$$ds^2 = \frac{1}{\hat{c}(2-\hat{c}f)} (-dt^2 + (dx^1)^2) + (dx^2)^2 + \dots + (dx^9)^2$$

$$B = -\frac{1-\hat{c}f}{\hat{c}(2-\hat{c}f)} dt \wedge dx^1,$$

$$e^{-2\phi} = e^{-2\phi_0} \hat{c}(2-\hat{c}f).$$

and take $\hat{c} \rightarrow 0$.

Obtain generalized metric:

$$\mathcal{H}_{MN} = \begin{pmatrix} 0 & 0 & \mathcal{E}^\alpha{}_\beta & 0 \\ 0 & \delta^{ij} & 0 & 0 \\ -\mathcal{E}_\alpha{}^\beta & 0 & f\eta_{\alpha\beta} & 0 \\ 0 & 0 & 0 & \delta_{ij} \end{pmatrix}$$

inverse metric
component
vanishes(!)

Non-relativistic Strings

J. Gomis and H. Ooguri
hep-th/0009181

Introduce $x^\alpha = (t, x^1)$ and wrap x^1 on radius R circle.

Use metric $ds^2 = c^2(dx^\alpha)^2 + (dx^i)^2$ and constant B field $B_{01} = B$.

Dispersion relation:

$$\frac{1}{c^2} \left(E + \frac{wRB}{\alpha'} \right)^2 = k^2 + c^2 \left(\frac{wR}{\alpha'} \right)^2 + \frac{1}{c^2} \left(\frac{n}{R} \right)^2 + \frac{2}{\alpha'} (N + \tilde{N} - 2)$$

Note that $c \rightarrow \infty$ limit doesn't make sense.

But if we set $B = c^2 - \mu^2$ (with μ a constant) then limit exists:

$$E = \mu \frac{wR}{\alpha'} + \frac{\alpha' k^2}{2wR} + \frac{N + \tilde{N} - 2}{wR}$$

(Also take string coupling to infinity, $g_s^2 = c^2 g_0^2$)

Resulting theory is non-relativistic: spectrum is invariant under Galilean transformations, ...

Non-relativistic Strings

J. Gomis and H. Ooguri
hep-th/0009181

Need a worldsheet theory.

Action in original background ($\gamma = x^1 + t$, $\bar{\gamma} = x^1 - t$, $G = c^2$)

$$S = \frac{1}{4\pi\alpha'} \int d^2z \left((G - B)\partial\gamma\bar{\partial}\bar{\gamma} + (G + B)\partial\bar{\gamma}\bar{\partial}\gamma + 2\partial X^i\bar{\partial}X^i \right)$$

Non-relativistic limit is singular. (!?)

Try Lagrange multipliers: write

$$S = \frac{1}{2\pi\alpha'} \int d^2z \left(\beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} - \frac{2}{G+B}\beta\bar{\beta} + \frac{1}{2}(G-B)\partial\gamma\bar{\partial}\bar{\gamma} + \partial X^i\bar{\partial}X^j \right)$$

Limit now exists:

$$S = \frac{1}{2\pi\alpha'} \int d^2z \left(\beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} + \frac{\mu}{2}\partial\gamma\bar{\partial}\bar{\gamma} + \partial X^i\bar{\partial}X^i \right)$$

Sigma model has $c = D$, is unitary, etc... but also is Galilean invariant.

GO Limit of Double Field Theory

Start with same configuration: $ds^2 = G(dx^\alpha)^2 + (dx^i)^2$, $B_{01} = G - \mu$

Doubled geometry:
$$\mathcal{H}_{AB} = \begin{pmatrix} \frac{1}{G}\eta^{\alpha\beta} & \frac{G-\mu}{G}\mathcal{E}^\alpha_\beta \\ -\frac{G-\mu}{G}\mathcal{E}_\alpha^\beta & 2\mu\eta_{\alpha\beta} \end{pmatrix}$$

Limit $G \rightarrow \infty$ exists!

Takes locally non-geometric form:

$$\mathcal{H}_{AB} = \begin{pmatrix} 0 & \sigma_\alpha^\beta \\ \sigma^\alpha_\beta & 2\mu\eta_{\alpha\beta} \end{pmatrix}$$

Dilaton in GO string: $g_s^2 = Gg_0^2$

DFT dilaton: $e^{-2d} = g_s^{-2}G = g_0^{-2}$

GO limit gives a good DFT background.

GO vs. DFT: Sigma Models

K.-H. Lee & J.-H. Park
arXiv:1307.8377

DFT sigma model: $\mathcal{L} = -\frac{1}{2}\sqrt{-h}h^{ab}D_a X^M D_b X^N \mathcal{H}_{MN}(X) - \epsilon^{ab}D_a X^M \mathcal{A}_{bM}$

Worldsheet gauge field: $A^M = (A_\mu, \tilde{A}^\mu)$

Generalized 1-form: $DX^M = dX^M - \mathcal{A}^M$

Section condition: $\tilde{\partial}^\mu = 0 \implies \tilde{A}^\mu = 0$

Non-Riemannian background: (t, x^1) sector in conformal gauge

$$\mathcal{L}_{0,1} = -2\partial_- \gamma A_+ + 2\partial_+ \bar{\gamma} \bar{A}_- - f(\partial_+ \gamma \partial_- \bar{\gamma} + \partial_+ \bar{\gamma} \partial_- \gamma)$$

Identify: $A_+ = \beta \quad A_- = -\bar{\beta} \quad f = 2\mu$

Wick rotation \Leftrightarrow Gomis-Ooguri string!

Gomis-Ooguri β fields are the DFT gauge field
 \Rightarrow DFT is the natural geometric framework in which to
understand the GO string

Symmetries

Galilean-invariant spectrum:
$$E = \mu \frac{wR}{\alpha'} + \frac{\alpha' k^2}{2wR} + \frac{N + \tilde{N} - 2}{wR}$$

⇒ should realize Bargmann algebra (central extension of Galilean algebra)

Realized in DFT by doubled diffeomorphisms:

$$H = -\partial_t \qquad Q = -\partial_1$$

$$P_i = -\partial_i \qquad \text{Mass number: } N = -\tilde{\partial}^1$$

$$M_{ij} = -(x^i \partial_j - x^j \partial_i) \qquad \text{Boosts: } B_i = -t \partial_i - x^i \tilde{\partial}^1$$

Closes onto Bargmann algebra under C-bracket (up to trivial generators):

$$[\xi, \eta]_C = \xi^A (\partial_A \eta^B) \partial_B - \eta^A (\partial_A \xi^B) \partial_B - \frac{1}{2} \xi_A (\partial^B \eta^A) \partial_B + \frac{1}{2} \eta_A (\partial^B \xi^A) \partial_B$$

Schrödinger Backgrounds

Split spacetime as: $x^\mu = (x^\alpha, x^m)$ $x^i = (x^m, u)$

DFT coordinates: $X^A = (\tilde{x}_\alpha, x^\alpha)$ $X^I = (\tilde{x}_i, x^i)$

$$\mathcal{H}_{AB} = \begin{pmatrix} 0 & \sigma^\alpha_\beta(u) \\ \sigma_\alpha^\beta(u) & \mathcal{H}_{\alpha\beta} \end{pmatrix}, \quad \mathcal{H}_{IJ} = \begin{pmatrix} u^2 \delta^{ij} & 0 \\ 0 & u^{-2} \delta_{ij} \end{pmatrix}, \quad \mathcal{H}_{AI} = 0,$$

$$\mathcal{H}_{\alpha\beta} = \begin{pmatrix} -\frac{1}{u^{2z}} & 0 \\ 0 & u^{4-2z} \end{pmatrix}, \quad \sigma^\alpha_\beta(u) = (\sigma_\beta^\alpha(u))^T = \begin{pmatrix} 0 & -u^2 \\ -\frac{1}{u^2} & 0 \end{pmatrix}.$$

Generalized isometries:

Schrödinger conformal algebra

$$H = -\partial_t,$$

$$D = -zt\partial_t - x^m\partial_m - u\partial_u - (z-2)x^1\partial_1$$

$$P_m = -\partial_m,$$

$$B_m = -t\partial_m - x^m\tilde{\partial}^1,$$

$$N = -\tilde{\partial}^1,$$

$$M_{mn} = -(x^m\partial_n - x^n\partial_m).$$

$$C = -t^2\partial_t - tx^m\partial_m - tu\partial_u - \frac{1}{2}(x^2 + u^2)\tilde{\partial}^1 \quad (z=2 \text{ only})$$

Applications to
non-relativistic
holography?

Fluctuations

O. Hohm, C. Hull, B. Zwiebach [arXiv:1006.4823]

I. Jeon, K. Lee, J. H. Park [arXiv:1105.6294]

Work in the semi-covariant formalism.

DFT action:
$$\mathcal{L} = \frac{1}{8}e^{-2d} [(P^{AC} P^{BD} - \bar{P}^{AC} \bar{P}^{BD})S_{ABCD} - 2\Lambda]$$

$$S_{ABCD} := \frac{1}{2}(R_{ABCD} + R_{CDAB} - \Gamma^E{}_{AB}\Gamma_{ECD}) \quad \bar{P}_{AB} = \frac{1}{2}(\mathcal{J}_{AB} - \mathcal{H}_{AB})$$

$$\nabla_P P_{MN} = \nabla_P \bar{P}_{MN} = \nabla_M d = 0 \quad P_{AB} = \frac{1}{2}(\mathcal{J}_{AB} + \mathcal{H}_{AB})$$

Quadratic action:

S.M. Ko, R. Meyer, CMT, J.H. Park [arXiv:1508.01121]

$$\mathcal{L}_{\text{eff.}} := e^{-2d} \left[\frac{1}{2}(P - \bar{P})^{AB} \partial_A \delta d \partial_B \delta d - \frac{1}{2} \partial_A \delta d \nabla_B \delta P^{AB} + \frac{1}{8} \delta P^{AB} (\bar{\Delta}_A{}^C P_B{}^D - \Delta_A{}^C \bar{P}_B{}^D) \delta P_{CD} \right]$$

$$(P^{AB} - \bar{P}^{AB}) \nabla_A \partial_B \delta d - \frac{1}{2} \nabla_A \nabla_B \delta P^{AB} \equiv 0,$$

Linearized EOM:

$$P_A{}^C \bar{P}_B{}^D \nabla_C \partial_D \delta d + \frac{1}{4} (P_A{}^C \bar{\Delta}_B{}^D - \Delta_A{}^C \bar{P}_B{}^D) \delta P_{CD} \equiv 0$$

with

$$\Delta_A{}^B := P_A{}^B P^{CD} \nabla_C \nabla_D - 2P_A{}^D P^{BC} (\nabla_C \nabla_D - S_{CD})$$

$$\bar{\Delta}_A{}^B := \bar{P}_A{}^B \bar{P}^{CD} \nabla_C \nabla_D - 2\bar{P}_A{}^D \bar{P}^{BC} (\nabla_C \nabla_D - S_{CD})$$

New Quadratic Differential Operator

Two more operators: $\mathfrak{D}_A{}^B = \Delta_A{}^B - P_A{}^B P^{CD} S_{CD}$, $\bar{\mathfrak{D}}_A{}^B = \bar{\Delta}_A{}^B - \bar{P}_A{}^B \bar{P}^{CD} S_{CD}$

Covariant \Leftrightarrow commutes with generalized Lie derivative. Use:

$$(\delta_X - \hat{\mathcal{L}}_X)\Gamma_{CAB} = 2[(\mathcal{P} + \bar{\mathcal{P}})_{CAB}{}^{FDE} - \delta_C^F \delta_A^D \delta_B^E] \partial_F \partial_{[D} X_{E]}$$

$$(\delta_X - \hat{\mathcal{L}}_X)\nabla_C T_{A_1 \dots A_n} = \sum_{i=1}^n 2(\mathcal{P} + \bar{\mathcal{P}})_{CA_i}{}^{BFDE} \partial_F \partial_{[D} X_{E]} T_{A_1 \dots A_{i-1} B A_{i+1} \dots A_n}$$

$$(\delta_X - \hat{\mathcal{L}}_X)S_{ABCD} = 2\nabla_{[A} ((\mathcal{P} + \bar{\mathcal{P}})_{B][CD]}{}^{EFG} \partial_E \partial_{[F} X_{G]}) + 2\nabla_{[C} ((\mathcal{P} + \bar{\mathcal{P}})_{D][AB]}{}^{EFG} \partial_E \partial_{[F} X_{G]}) .$$

Following contractions give new quadratic tensorial differential operators:

$$\mathfrak{D}_A{}^C \bar{P}_{B_1}{}^{D_1} \dots \bar{P}_{B_n}{}^{D_n} T_{CD_1 \dots D_n}, \quad \bar{\mathfrak{D}}_A{}^C P_{B_1}{}^{D_1} \dots P_{B_n}{}^{D_n} T_{CD_1 \dots D_n}$$

$$\Delta_A{}^C \bar{P}_{B_1}{}^{D_1} \dots \bar{P}_{B_n}{}^{D_n} T_{CD_1 \dots D_n}, \quad \bar{\Delta}_A{}^C P_{B_1}{}^{D_1} \dots P_{B_n}{}^{D_n} T_{CD_1 \dots D_n}$$

Fluctuations in GO Background

Expand linearly in fluctuations:

$$\mathcal{H}_{AB} \mapsto \mathcal{H}_{AB} + h_{AB}, \quad d \mapsto d + \psi$$

$$\mathcal{H}_{MN} = \begin{pmatrix} 0 & 0 & \mathcal{E}^\alpha_\beta & 0 \\ 0 & g^{ij} & 0 & 0 \\ -\mathcal{E}_\alpha^\beta & 0 & f\eta_{\alpha\beta} & 0 \\ 0 & 0 & 0 & g_{ij} \end{pmatrix}$$

Background

Constraint $\mathcal{H}_{AC} J^{CD} \mathcal{H}_{DB} = J_{AB}$ means:

$$h^{\alpha\beta} = -\sigma_\gamma^\alpha h^{\gamma\delta} \sigma_\delta^\beta, \quad h^\alpha_\beta = -\sigma_\gamma^\alpha h^{\gamma\delta} \sigma_\delta^\beta - f\sigma_\gamma^\alpha h^{\gamma\delta} \eta_{\delta\beta},$$

$$h^{\alpha i} = -\sigma_\beta^\alpha h^\beta_j g^{ji}, \quad h_{\alpha\beta} = -\sigma_\alpha^\gamma h_{\gamma\delta} \sigma_\beta^\delta - f\sigma_\alpha^\gamma h_{\gamma\delta} \eta_{\delta\beta} - f\eta_{\alpha\gamma} h^\gamma_\delta \sigma_\beta^\delta - f^2 \eta_{\alpha\gamma} h^{\gamma\delta} \eta_{\delta\beta}, \quad (\sigma_\beta^\alpha = \mathcal{E}^\alpha_\beta)$$

$$h_i^j = b_{ik} g^{kj}, \quad h_{\alpha i} = -f\eta_{\alpha\gamma} h^{\gamma j} g_{ji} - \sigma_\alpha^\beta h_\beta^j g_{ji},$$

$$h^i_j = -g^{ik} b_{kj}, \quad h^{ij} = -g^{im} h_{mn} g^{nj},$$

Gauge-fixing: gauge parameter $\xi^M = (\tilde{\lambda}_\mu, \lambda^\mu)$

Use λ^α to fix $h^\alpha_\beta = -\frac{1}{2} f \hat{h} \sigma_\beta^\alpha$

Use $\tilde{\lambda}_\alpha$ to fix $h_{\alpha\beta} = 0$.

Use $\tilde{\lambda}_i$ and λ^i to fix $h_{\alpha i} = 0$.

Expand in plane waves:

$$h_{AB}(x) = h_{AB} e^{ip_+ x^+ + ip_- x^- + ik_i x^i}$$

Fluctuations in GO Background

Transverse/
longitudinal
decomposition

$$h_{ij} = h_{ij}^{\perp} + k_i \zeta_j^{\perp} + k_j \zeta_i^{\perp} + (k_i k_j - \frac{1}{D-2} k^2 g_{ij}) \rho + \frac{1}{D-2} h g_{ij},$$

$$b_{ij} = b_{ij}^{\perp} + k_i \chi_j^{\perp} - k_j \chi_i^{\perp},$$

$$k^i h_{ij}^{\perp} = k^i b_{ij}^{\perp} = k^i \zeta_i^{\perp} = k^i h_i^{\perp \alpha} = 0$$

$$h_i^{\alpha} = h_i^{\perp \alpha} + k_i \phi^{\alpha},$$

Result: **No normalizable fluctuating modes!**

$$\mathcal{E}_{\psi} = 2p_+ p_- \hat{h} + 2k^2 (p_- \phi^- - p_+ \phi^+) + \frac{1}{D-2} k^2 [h - (D-3)k^2 \rho + 4(D-2)\psi],$$

$$\mathcal{E}^{-+} = k^2 \hat{h},$$

$$\mathcal{E}^{-+} = 2p_+ (k^2 \phi^- + p_+ \hat{h}),$$

$$\mathcal{E}_{-+} = f k^2 (p_- \phi^- - p_+ \phi^+ + \frac{1}{4} f \hat{h}) + 8p_+ p_- \psi,$$

$$\mathcal{E}_{-+} = 2p_- (k^2 \phi^+ - p_- \hat{h}),$$

$$\mathcal{E}_{-i} = p_- k^m (h_{mi} - b_{mi}) + 2p_-^2 h_i^- + \frac{f}{2} k^2 h_i^{\perp +} + 4p_- k_i \psi,$$

$$\mathcal{E}_{-i} = -k^2 h_i^{\perp -} + p_+ k_i \hat{h},$$

$$\mathcal{E}_{i+} = p_+ k^m (h_{mi} + b_{mi}) - 2p_+^2 h_i^+ - \frac{f}{2} k^2 h_i^{\perp -} + 4p_+ k_i \psi,$$

$$\mathcal{E}_{i+} = -k^2 h_i^{\perp +} - p_- k_i \hat{h},$$

$$\mathcal{E}_{ij} = \frac{1}{2} k_i \left[2p_- h_j^- + g^{mn} k_m (h_{nj} - b_{nj}) \right]$$

$$- \frac{1}{2} k_j \left[2p_+ h_i^+ - g^{mn} k_m (h_{ni} + b_{ni}) \right] - k^2 (h_{ij} - b_{ij}) + 2k_i k_j \psi.$$

Complete
equations
of motion

T-Dual Fluctuations

Apply T-duality transformation

$$(\mathcal{O}_A^B) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Resulting configuration is geometric:

$$ds^2 = -f dt^2 + 2 dt dx + (dx^i)^2$$

– light-cone compactification.

Choose gauge: $h_{0\mu} = b_{0\mu} = 0$

Equation of motion reduce to

$$\mathcal{E}_\psi = p^\mu p^\nu h_{\mu\nu} - 4p^2 \psi = 0,$$

$$\mathcal{E}_{(\mu\nu)} = p^\lambda p_{(\mu} h_{\nu)\lambda} - p^2 h_{\mu\nu} + 2p_\mu p_\nu \psi = 0,$$

$$\mathcal{E}_{[\mu\nu]} = p^\lambda p_{[\mu} h_{\nu]\lambda} + p^2 b_{\mu\nu} = 0.$$

Gives (D-1)(D-3) modes with dispersion

$$E = \frac{1}{2} f p_\theta + \frac{\vec{k}^2}{2p_\theta}$$

in accordance with expectations.

Reproduces the $N = \tilde{N} = 1$ sector of Gomis-Ooguri string with $wR \mapsto \alpha' p_\theta$.

Summary

- ▶ Double Field Theory: Manifestly T-duality invariant description of geometric, globally non-geometric, and locally non-geometric backgrounds
- ▶ Gomis-Ooguri string: worldsheet theory for a Galilean-invariant target spacetime
- ▶ Gomis-Ooguri limit gives non-singular configuration in DFT which is “locally non-Riemannian”
- ▶ Gomis-Ooguri worldsheet theory a special case of DFT sigma model
- ▶ Fluctuation spectrum in GO background and its dual match

Outlook

- ▶ Quantum properties of non-relativistic sigma model in curved spacetime
- ▶ Applications to holography?
- ▶ Insights on action formulation of Newtonian gravity

Thank you!