Double Sigma Model

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Nucl.Phys. B898 (2015) 30-52

February 2, 2016

Reference

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Action

$$S_{\text{bulk}} = \frac{1}{2}\int d^2\sigma\ \left(\partial_1 X^A \mathcal{H}_{AB}\partial_1 X^B - \partial_1 X^A \eta_{AB}\partial_0 X^B\right).$$

The notations are $\alpha = 0, 1$ (We use the Greek indices to indicate the worldsheet coordinates.), $A = 0, 1, \cdots, 2D - 1$ (We define the double target indices from A to K.), and

Using the strong constraints $\tilde{\partial}^m=0$ ($\partial_m \equiv \frac{\partial}{\partial x^m}$, $\tilde{\partial}^m \equiv \frac{\partial}{\partial \tilde{x}_m}$ and $\partial_A \equiv \begin{pmatrix} \tilde{\partial}^m \\ \partial_m \end{pmatrix}$.) and a self-duality relation

$$\mathcal{H}^m{}_B \partial_1 X^B - \eta^m{}_B \partial_0 X^B = 0$$

to guarantee classical equivalence with the ordinary sigma model. The ordinary sigma model is

$$\frac{1}{2}\int d^2\sigma \left(\partial_\alpha X^m g_{mn} \partial^\alpha X^n - \epsilon^{\alpha\beta}\partial_\alpha X^m B_{mn} \partial_\beta X^n\right).$$

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and the boundary condition on σ^0-direction (The Dirichlet boundary condition) is

$$\delta X^m = 0.$$

Low-Energy Effective Action

$$
\begin{aligned}
S_T &= S_1 + \alpha S_2 \\
&= \int dx\ d\tilde{x}\ \Bigg[e^{-d} \Big(-\det(\mathcal{H}_{mn}) \Big)^{\frac{1}{4}} \\
&\quad + \alpha e^{-2d} \Big(\frac{1}{8} \mathcal{H}^{AB} \partial_A \mathcal{H}^{CD} \partial_B \mathcal{H}_{CD} - \frac{1}{2} \mathcal{H}^{AB} \partial_B \mathcal{H}^{CD} \partial_D \mathcal{H}_{AC} \\
&\quad - 2\partial_A d \partial_B \mathcal{H}^{AB} + 4\mathcal{H}^{AB} \partial_A d \partial_B d \Big) \Bigg],
\end{aligned}
$$

where α is an arbitrary constant and

$$
e^{-d} \equiv \Big(-\det g \Big)^{\frac{1}{4}} e^{-\phi}.
$$

Quantum Equivalence with the Strong Constraints

When we perform Gaussian integration, the result of the integration on the exponent is equivalent to using

$$\partial_1 \tilde{X}_p = g_{pn}\partial_0 X^n + B_{pn}\partial_1 X^n .$$

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$$\int DX^A$$

becomes

$$\int DX^m \sqrt{\det g} \equiv \int D'X^m$$

when we integrate out the dual coordinates. We obtain the diffemorphism invariant measure ($D'X^m$) with shift symmetry.

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Gauge symmetry implies that we need a projector.

- We can use the projector to get closed algebra although it is not associative.

$$A * B = B * A, \qquad A * (B + C) = A * B + A * C,$$
$$\int A * (B * C) = \int (A * B) * C, \qquad \int A \cdot (B * C) = \int A \cdot B \cdot C.$$

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We only need to modify gauge transformation from the projector, but the action still remains the same form. All things are consistent with the string field theory.

Conclusion and Discussion

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- We show equivalence between standard and double sigma models with a boundary condition.
- Weakly constrained double field theory can be found from the non-associative algebra (projector).
- Without using the strong constraints, we have global symmetry structures to avoid the non-gauge invariant entanglement entropy in closed string theory.