Curved $\mathrm{SL}(5)$ exceptional field theory

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Scalar potential

- "Fluctuations" (scalar degrees of freedom)
- Introduce generalised vielbeine $\tilde{\mathbb{E}}_a{}^{\bar{a}}$, $\bar{a}$ is $\mathrm{SO}(5)$ index
- Generalised metric $\mathcal{M}_{ab} = \tilde{\mathbb{E}}_a{}^{\bar{a}}\tilde{\mathbb{E}}_b{}^{\bar{b}}\delta_{\bar{a}\bar{b}}$ ($\mathrm{SO}(5)$ structure)
- Define connection for $\mathcal{L}$

$$\tilde{\nabla}_{ab} V^c = \nabla_{ab} V^c + \tilde{\Gamma}_{ab,d}{}^c V^d - w\gamma_{ab} V^c \,, \tag{1}$$

$$\tilde{\Gamma}_{ab,c}{}^d = -\tilde{\mathbb{E}}^{\bar{c}}{}_c \nabla_{ab} \tilde{\mathbb{E}}_{\bar{c}}{}^d \,, \qquad \gamma_{ab} = \frac{5}{7} D_{ab} \ln e \,, \tag{2}$$

- $e_\mu{}^{\bar{\mu}}$ is external vielbein
- $\tilde{\Gamma}$ is not $\mathrm{SO}(5)$ invariant (we will restore this in the potential)

Generalised torsion

- Generalised Torsion:

$$\left(\mathcal{L}^{\tilde{\nabla}}_{\Lambda} - \mathcal{L}_{\Lambda}\right) V^a = \frac{1}{2} T_{bc,d}{}^a \Lambda^{bc} V^d - \frac{1}{2} w T_{bc} \Lambda^{bc} V^a \,, \tag{3}$$

- Covariant under $\mathcal{L}$!
- Irreps:

$$T_{ab,c}{}^d = \frac{1}{2}\delta^d_{[a}\tilde{S}_{b]c} + \frac{1}{2}\epsilon_{abcef}\tilde{Z}^{ef,d} - \frac{1}{27}\left(25\,\delta^d_{[a}T_{b]c} + 5\,\delta^d_c T_{ab}\right) \,. \tag{4}$$

- Explicitly,

$$\begin{aligned}
\tilde{S}_{ab} &= T_{c(a,b)}{}^c = 4\tilde{\Gamma}_{c(a,b)}{}^c \,, \\
\tilde{Z}^{ab,c} &= \frac{1}{3!}\epsilon^{abdef} T_{de,f}{}^c = \frac{1}{2}\epsilon^{abdef}\tilde{\Gamma}_{de,f}{}^c - \frac{1}{2}\epsilon^{abcde}\tilde{\Gamma}_{[fd,e]}{}^f \,, \\
T_{ab} &= -\frac{5}{3} T_{c[a,b]}{}^c = \frac{6}{5}\gamma_{ab} + \tilde{\Gamma}_{e[a,b]}{}^e \,,
\end{aligned} \tag{5}$$

- The torsion is not invariant under $\mathrm{SO}(5)$!

SO(5)-invariance

- Construct independent generalised diffeomorphism scalar densities

$$A = \tilde{S}_{ab}\tilde{S}_{cd}\mathcal{M}^{ac}\mathcal{M}^{bd}\,, \qquad B = \left(\tilde{S}_{ab}\mathcal{M}^{ab}\right)^2\,,$$
$$C = \mathcal{M}^{ac}\mathcal{M}^{bd}T_{ab}T_{cd}\,, \qquad D = \mathcal{M}_{ab}\mathcal{M}_{cd}\mathcal{M}_{ef}\tilde{Z}^{ac,e}\tilde{Z}^{bd,f}\,,$$
$$E = \mathcal{M}_{ab}\mathcal{M}_{cd}\mathcal{M}_{ef}\tilde{Z}^{ac,b}\tilde{Z}^{de,f}\,, \quad F = \mathcal{M}^{ac}\mathcal{M}^{bd}\tilde{\nabla}_{ab}T_{cd}\,. \tag{6}$$

- Combine them to form $\mathrm{SO}(5)$ invariant (up to constraints):

$$\begin{aligned} V_1 = & -\frac{1}{16}\mathcal{M}^{ac}\mathcal{M}^{bd}\tilde{S}_{ab}\tilde{S}_{cd} + \frac{1}{32}\mathcal{M}^{ac}\mathcal{M}^{bd}\tilde{S}_{ac}\tilde{S}_{bd} - \frac{5}{12}\mathcal{M}^{ac}\mathcal{M}^{bd}T_{ab}T_{cd} \\ & - \frac{1}{2}\mathcal{M}_{ab}\mathcal{M}_{cd}\mathcal{M}_{ef}\tilde{Z}^{ac,e}\tilde{Z}^{bd,f} + \frac{1}{2}\mathcal{M}_{ab}\mathcal{M}_{cd}\mathcal{M}_{ef}\tilde{Z}^{ac,b}\tilde{Z}^{ed,f} - \mathcal{M}^{ac}\mathcal{M}^{bd}\tilde{\nabla}_{ab}T_{cd}\,. \end{aligned} \tag{7}$$

- Up to constraints we can write this as

$$
\begin{aligned}
V_1 = & -\frac{1}{2}\nabla_{ab}\mathcal{M}^{ac}\nabla_{cd}\mathcal{M}^{bd} + \frac{1}{8}\mathcal{M}^{ac}\mathcal{M}^{bd}\nabla_{ab}\mathcal{M}^{ef}\nabla_{cd}\mathcal{M}_{ef} + \frac{1}{2}\mathcal{M}^{ac}\mathcal{M}^{bd}\nabla_{ab}\mathcal{M}^{ef}\nabla_{ec}\mathcal{M}_{df} \\
& - \mathcal{M}^{ac}\nabla_{ab}\nabla_{cd}\mathcal{M}^{bd} + \mathcal{M}^{ac}\mathcal{M}^{bd}\left(\frac{1}{2}\omega_{ae,c}{}^{e}\omega_{bf,d}{}^{f} - \frac{1}{2}\omega_{ae,c}{}^{f}\omega_{bf,d}{}^{e} + \frac{1}{2}\omega_{ae,b}{}^{e}\omega_{df,c}{}^{f}\right. \\
& \left. -\frac{1}{2}\omega_{ae,b}{}^{f}\omega_{df,c}{}^{e} - \omega_{ae,d}{}^{f}\omega_{fc,b}{}^{e} - \omega_{ae,f}{}^{e}\omega_{cd,b}{}^{f}\right) .
\end{aligned} \tag{8}
$$

- ω^2 terms are background fluxes and appear also in DFT_{WZW}.
- Section condition term is

$$
\Delta_0 = \frac{1}{2}\mathcal{M}^{ac}\mathcal{M}^{bd}\left(-\mathring{\Gamma}_{ae,b}{}^{e}\mathring{\Gamma}_{df,c}{}^{f} - 2\mathring{\Gamma}_{ab,c}{}^{e}\mathring{\Gamma}_{ef,d}{}^{f} + \mathring{\Gamma}_{ae,c}{}^{e}\mathring{\Gamma}_{bf,d}{}^{f} + \mathring{\Gamma}_{af,b}{}^{e}\mathring{\Gamma}_{de,c}{}^{f} - \mathring{\Gamma}_{af,c}{}^{e}\mathring{\Gamma}_{be,d}{}^{f}\right) \tag{9}
$$

where

$$
\mathring{\Gamma}_{ab,c}{}^{d} = -\tilde{\mathbb{E}}_{c}{}^{\bar{c}}D_{ab}\tilde{\mathbb{E}}_{\bar{c}}{}^{d} . \tag{10}
$$

Local flatness

- Can construct the full "curved" $\mathrm{SL}(5)$ EFT action this way.
- Everything is manifestly coordinate-invariant.
- Similar to gauged EFT but "twists" $E_{ab} \in \mathrm{GL}(10)$.
- When M_{10} locally flat $\Rightarrow \mathrm{SL}(5)$ EFT.
- Then coordinate patches are patched with $\mathrm{SL}(5)$!
- Do we need local flatness? Background-independence? (Hohm, Marques) Applying the section condition? SUSY?...

Conclusions and outlook

- Conditions for local flatness. Better connections?
- What 4-manifolds (3-manifolds) can be embedded into locally flat M_{10}?
- Section condition?
- Global sections as reduced structure group? Non-geometric spaces?
- Scherk-Schwarz (probably need local flatness!)
- Non-abelian T-duality?

Tensor Hierarchy

Module(w)	Representations	Gauge field	Field strength
$\mathcal{A}(1/5)$	$\mathbf{10}$	$\mathcal{A}^{ab}$	$\mathcal{F}^{ab}$
$\mathcal{B}(2/5)$	$\overline{\mathbf{5}}$	$\mathcal{B}_a$	$\mathcal{H}_a$
$\mathcal{C}(3/5)$	$\mathbf{5}$	$\mathcal{C}^a$	$\mathcal{J}^a$
$\mathcal{D}(4/5)$	$\overline{\mathbf{10}}$	$\mathcal{D}_{ab}$	$\mathcal{K}_{ab}$

$\bullet$-product (wedge product)

We define a $\bullet$ product between these spaces.

$\bullet$	$\mathcal{A}(1/5)$	$\mathcal{B}(2/5)$	$\mathcal{C}(3/5)$	$\mathcal{D}(4/5)$
$\mathcal{A}(1/5)$	$\mathcal{B}(2/5)$	$\mathcal{C}(3/5)$	$\mathcal{D}(4/5)$	$\mathcal{S}(1)$
$\mathcal{B}(2/5)$	$\mathcal{C}(3/5)$	$\mathcal{D}(4/5)$	$\mathcal{S}(1)$	
$\mathcal{C}(3/5)$	$\mathcal{D}(4/5)$	$\mathcal{S}(1)$		
$\mathcal{D}(4/5)$	$\mathcal{S}(1)$			

$$
\begin{aligned}
(\mathcal{A}_1 \bullet \mathcal{A}_2)_a &= \frac{1}{4}\epsilon_{abcde}\mathcal{A}_1^{bc}\mathcal{A}_2^{de}\,, \\
(\mathcal{A} \bullet \mathcal{B})^a &= \mathcal{A}^{ab}\mathcal{B}_b\,, \\
(\mathcal{A} \bullet \mathcal{C})_{ab} &= \frac{1}{4}\epsilon_{abcde}\mathcal{A}^{cd}\mathcal{C}^e\,, \\
\mathcal{A} \bullet \mathcal{D} &= \frac{1}{2}\mathcal{A}^{ab}\mathcal{D}_{ab}\,, \\
(\mathcal{B}_1 \bullet \mathcal{B}_2)_{ab} &= \mathcal{B}_{2[a}\mathcal{B}_{1]b}\,, \\
\mathcal{B} \bullet \mathcal{C} &= \mathcal{B}_a\mathcal{C}^a\,,
\end{aligned}
\tag{11}
$$

"Exterior derivative"

Want nilpotent derivative

$$\mathcal{A}(1/5) \xleftarrow{\hat{\partial}} \mathcal{B}(2/5) \xleftarrow{\hat{\partial}} \mathcal{C}(3/5) \xleftarrow{\hat{\partial}} \mathcal{D}(4/5)\,. \tag{12}$$

Use

$$\hat{\partial}\mathcal{B}^{ab} = \frac{1}{2}\epsilon^{abcde}\nabla_{cd}\mathcal{B}_e\,, \qquad \hat{\partial}\mathcal{C}_a = \nabla_{ba}\mathcal{C}^b\,, \qquad \hat{\partial}\mathcal{D}^a = \frac{1}{2}\epsilon^{abcde}\nabla_{bc}\mathcal{D}_{de}\,, \tag{13}$$

Constraints $\Rightarrow$ Nilpotency, covariant under $\mathcal{L}$.

- and $\hat{\partial}$ satisfy for all $\Lambda \in \mathcal{A}(1/5)$ and $\mathcal{T} \in \mathcal{B}(2/5)$ or $\mathcal{C}(3/5)$,

$$\mathcal{L}_\Lambda \mathcal{T} = \Lambda \bullet \left(\hat{\partial}\mathcal{T}\right) + \hat{\partial}\left(\Lambda \bullet \mathcal{T}\right)\,. \tag{14}$$