The $\mathrm{SL}(2) \times \mathbb{R}^+$ exceptional field theory and F-theory

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Duality and Novel Geometry in M-theory, APCTP

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Based on 1512.06115 with David Berman, Emanuel Malek and Felix Rudolph

Motivations

- Understand dualities geometrically $\Rightarrow$ DFT/EFT.
- What problems can we apply these to?
- Compare to F-theory: geometric approach to S-duality, becomes a powerful tool for compactifications
- Can we understand the F-theory approach in terms of EFT? (Conceptually, technically.)

F-theory or not F-theory?

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High Energy Physics - Theory

M-theory and F-theory from a Duality Manifest Action

Chris D. A. Blair, Emanuel Malek, Jeong-Hyuck Park

(Submitted on 20 Nov 2013 (this version), **latest version 6 Feb 2014** *(v3))*

We revisit the SL(5) U-duality manifest action constructed by Berman and Perry in an extended spacetime. Upon choosing a four-dimensional solution to the section condition constraint, the theory reduces to a four-dimensional truncation of eleven-dimensional supergravity. In this paper, we show that the theory contains more than this M-theory reduction. The section condition also admits an SL(5) inequivalent three-dimensional solution, upon which the action directly reduces to a three-dimensional truncation of type IIB supergravity. The extended space then geometrises the S-duality of the theory and thus can be seen to incorporate F-theory as well as M-theory. We also discuss the reduction to IIB^* supergravity.

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Outline

Goal: To explicitly connect F-theory and exceptional field theory (EFT)

- Features of F-theory
- Features of EFT
- Construction of $\mathrm{SL}(2) \times \mathbb{R}^+$ EFT: 12-dimensional theory with M-theory and IIB sections
- Connection to F-theory and outlook

IIB in 10-d

- Massless fields are

$$\text{metric } g_{\mu\nu} \quad \text{2-forms } \begin{matrix} B_2 \\ C_2 \end{matrix} \quad \text{4-form } C_4$$

$$\text{scalars } \tau = C_0 + ie^{-\phi}$$

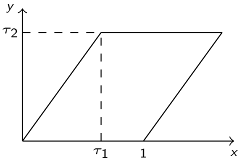
- $\mathrm{SL}(2)$ invariant

$$\tau \to \frac{a\tau + b}{c\tau + d} \qquad \begin{pmatrix} B_2 \\ C_2 \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} B_2 \\ C_2 \end{pmatrix}$$

where $ad - bc = 1$.

Geometrical origin

- Shape of a torus is parametrised by a complex structure τ, identified up to $\mathrm{SL}(2,\mathbb{Z})$



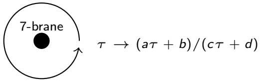
$(x, y) \sim (x + 1, y) \sim (x + \tau_1, y + \tau_2)$, $\tau = \tau_1 + i\tau_2$

- Introduce an auxiliary torus $T^2 \to$ 10-d IIB whose complex structure τ varies in spacetime.

- 12-d theory? Not really - no volume for torus, no 12-d SUGRA, field reduction doesn't make sense.

F-theory

- F-theory is a framework for analysing these fibrations. Vafa 1996
- In particular, 7-brane backgrounds. $z \in \mathbb{C}$ transverse coordinate. $\tau = \tau(z)$ describes fibration of torus (elliptic fibration).
- At D7-brane position, $\tau \sim \frac{1}{2\pi i} \log(z - z_{D_7})$. So $\tau \to i\infty$ at brane position. Paths around the brane have an $\mathrm{SL}(2)$ monodromy $\tau \to \tau + 1$.



- More general monodromies allowed $\Rightarrow$ non-perturbative (recall $\tau = C_0 + i/g_s$).
- Coincident branes $\Rightarrow$ enhanced gauge symmetries.

M-theory/F-theory duality

- M-theory on S^1_A gives IIA theory. M-theory on $S^1_A \times S^1_B$ plus T-duality on S^1_B gives IIB on $\tilde{S}^1_B$. $\tilde{R}_B = \alpha'/R_A$.
- M-theory on T^2 in limit of vanishing torus area gives IIB in 10-dimensions. No area: only complex structure τ, becomes exactly τ of IIB.

F-theory

- In F-theory, we **geometrise** duality. Guided by scalars:

$$\tau \in \frac{\mathrm{SL}(2)}{\mathrm{SO}(2)} \qquad \text{duality: } \mathrm{SL}(2)$$

  Identify/interpret τ as moduli of auxiliary space: torus of vanishing area.

- This is **S**-duality, what about **U**-duality?

U-duality

- Reduce maximal SUGRA on a D-torus.
- Enhanced symmetry G. Scalars $\mathcal{M} \in G/H$ coset transform non-linearly, $\mathcal{M} \to P^T \mathcal{M} P$, $P \in G$. Forms in linear reps of G.

10-D	D	G	H
9	1	$\mathrm{SL}(2) \times \mathbb{R}^+$	$\mathrm{SO}(2)$
8	2	$\mathrm{SL}(3) \times \mathrm{SL}(2)$	$\mathrm{SO}(3) \times \mathrm{SO}(2)$
7	3	$\mathrm{SL}(5)$	$\mathrm{SO}(5)$
6	4	$\mathrm{SO}(5,5)$	$\mathrm{SO}(5) \times \mathrm{SO}(5)$
5	5	E_6	$\mathrm{USp}(8)$
4	6	E_7	$\mathrm{SU}(8)$
3	7	E_8	$\mathrm{SO}(16)$

U-manifolds?

- Associate scalars $\mathcal{M} \in G/H$ to geometric object? Kumar, Vafa 1996 .
- A limited number of possible geometrisations. In general, no obvous geometric interpretations.
- More recently: not geometry but **generalised geometry**. Make symmetry G manifest on an **extended space**: double field theory (T-duality) and exceptional field theory (U-duality).

  Full EFT construction Hohm, Samtleben 2013 earlier work Hillmann 2009; Berman, Perry 2010 and many more Builds on DFT Siegel 1993; Hull, Hohm, Zwiebach 2009 and many more generalised geometry Hitchin; Gualtieri 2003; Coimbra, Strickland-Constable, Waldram 2011 and many more

Exceptional field theory: general features

- All fields, and coordinates $\in$ reps of $G \Rightarrow$ manifest duality symmetry.
- Extra coordinates needed
- Treats metric + forms together $\Rightarrow$ novel generalised diffeomorphism symmetry.
- Consistency $\Rightarrow$ section condition, reduces coordinate dependence.
- After imposing this condition, $\rightarrow$ 11-dimensional SUGRA or 10-dimensional IIB SUGRA. CB, Malek, Park 2013; Hohm, Samtleben 2013

Exceptional field theory: constructive explanation

- Take 10-d (or 11-d) SUGRA. Split coordinates

$$x^{\hat{\mu}} \to (x^{\mu}, y^{i}) \quad \hat{\mu} = 1, \ldots, 10 \quad \mu = 1, \ldots, 10 - D \quad i = 1, \ldots, D$$

- Extended coordinates

$$Y^{M} = (y^{i}, \tilde{y}_{i}, \tilde{z}_{i}, \tilde{z}_{[ijk]}, \ldots)$$

in a representation of G.

- Dual coordinates $\sim$ brane wrappings.

Exceptional field theory: constructive explanation

- Classify dofs under splitting $\mathrm{SO}(1, 10-D) \times \mathrm{SO}(D)$, (as if KK reduction) e.g. $g_{\mu\nu} \to g_{\mu\nu}, A_{\mu i}, g_{ij}$.
- Repackage into G representations:
  - Metric $g_{\mu\nu}$
  - Vector field $A_\mu{}^M$
  - Forms $B_{\mu\nu}{}^{(MN)}, C_{\mu\nu\rho}{}^{M\cdots}, \ldots$ (after dualisation)
  - Scalars $\mathcal{M}_{MN} \in G/H$

Exceptional field theory: constructive explanation

- Repackage symmetries: diffeomorphisms plus gauge

$$\delta_\Lambda V^i = \Lambda^j \partial_j V^i - V^j \partial_j \Lambda^i$$

$$\delta_\lambda C_{i_1 \dots i_p} = p \partial_{[i_1} \lambda_{i_2 \dots i_p]}$$

- Gives generalised diffeomorphisms

$$\delta_\Lambda V^M = \Lambda^N \partial_N V^M - V^N \partial_N \Lambda^M + Y^{MN}{}_{PQ} \partial_N \Lambda^P V^Q$$

  Y-tensor invariant under G Berman, Cederwall, Kleinschmidt, Thompson 2012

- These plus gauge transformations of form fields $\Rightarrow$ tensor hierarchy.

- Also have external diffeomorphisms (covariantised under generalised diffeomorphisms) $\partial_\mu \to D_\mu \equiv \partial_\mu - \delta_{A_\mu}$.

Exceptional field theory: consistency of symmetries

- Normal diffeomorphisms form an algebra under the Lie bracket.
- Generalised diffeomorphisms: closure if

  $$Y^{MN}{}_{PQ}\partial_M\mathcal{O}_1\partial_N\mathcal{O}_2 = 0 \quad Y^{MN}{}_{PQ}\partial_M\partial_N\mathcal{O} = 0$$

  known as **section condition**.
- Two inequivalent solutions: restricting coordinate dependence to
  - at most $D+1$ of the Y^M coordinates, total coordinates $10 - D + D + 1 = 11$ **M-theory section**
  - at most D of the Y^M coordinates, total coordinates $10 - D + D = 10$ **IIB section**

Exceptional field theory: action

- The (bosonic) action is entirely fixed by the (bosonic) symmetries:

$$S = \int dx dY \sqrt{|g|} \left(R(g) + \mathcal{L}_{kin} + \frac{1}{\sqrt{|g|}} \mathcal{L}_{top} + V(M, g) \right)$$

- Ricci scalar $R(g) \sim (D_\mu g)^2$
- Kinetic and gauge field terms $\mathcal{L}_{kin} \sim (D_\mu \mathcal{M})^2 + \mathcal{F}^2$
- Chern-Simons $\mathcal{L}_{top}$
- Scalar potential $V(M, g) \sim (\partial_M \mathcal{M})^2 + (\partial_M g)^2$.

- Invariant under local G by construction, global G manifest.

- Input section condition choice: find equivalent to 11-dimensional SUGRA/10-dimensional IIB SUGRA, no reduction.

Exceptional field theory: backgrounds

- Recall D7 brane, $\mathrm{SL}(2)$ monodromy $\tau \to \tau + 1$
- Non-geometric branes with general U-duality monodromy, globally defined up to duality. e.g. 5^2_2 de Boer, Shigemori 2012

$$
\begin{aligned}
ds^2\Big|_{\text{transverse}} &= dr^2 + r^2 d\theta^2 + \frac{H(r)}{H(r)^2 + h^2\theta^2}(dx^2 + dy^2) \\
B_2 &= -\frac{h\theta}{H(r)^2 + h^2\theta^2} dx \wedge dy
\end{aligned}
$$

- $(g, B_2)(\theta = 2\pi) =$ (T-)duality transformation of $(g, B_2)(\theta = 0)$.
- Natural description in DFT/EFT.

Cases

- The full EFT has been constructed for
  - E_8, E_7, E_6 Hohm, Samtleben 2013
  - $\mathrm{SO}(5,5)$ Abzalov, Bakhmatov, Musaev 2015
  - $\mathrm{SL}(5)$ Musaev 2015
  - $\mathrm{SL}(3) \times \mathrm{SL}(2)$ Hohm, Wang 2015
  - $+$ SUSY E_7, E_6 Godazgar, Godazgar, Hohm, Nicolai, Samtleben 2014; Musaev, Samtleben 2014
- So what about $\mathrm{SL}(2) \times \mathbb{R}^+$? Duality group for 11-dim SUGRA on T^2 / 10-dim SUGRA on S^1 / F-theory.
- Our goal: to clarify EFT vs F-theory relationship Berman, CB, Malek, Rudolph 2015
  (different approach: Linch, Siegel 2015)

$\mathrm{SL}(2) \times \mathbb{R}^+$ EFT

- Coordinates x^μ, $\mu = 0, \ldots, 8$ and $Y^M = (y^\alpha, y^s)$ in $\mathbf{2}_2 \oplus \mathbf{1}_{-1}$ of $\mathrm{SL}(2) \times \mathbb{R}^+$.

- Y-tensor Wang 2015
$$Y^{\alpha s}{}_{\beta s} = \delta^\alpha_\beta$$
so section condition is
$$\partial_\alpha \otimes \partial_s = 0$$

- M-theory section $\partial_s = 0$, coordinate dependence on (x^μ, y^α)
- IIB section $\partial_\alpha = 0$, coordinate dependence on (x^μ, y^s).

- Generalised diffeomorphisms
$$\delta_\Lambda V^\alpha = \Lambda^M \partial_M V^\alpha - V^\gamma \partial_\gamma \Lambda^\alpha + \partial_s \Lambda^s V^\alpha$$
$$\delta_\Lambda V^s = \Lambda^M \partial_M V^s + V^s \partial_\gamma \Lambda^\gamma - \partial_s \Lambda^s V^s$$

Digression - phase space?

- Recall string phase space (X, P) closely related to double coordinates $(X, \tilde{X})$.
- From M-theory point of view: 2 physical coordinates y^α plus 1 dual coordinate $y_{\alpha\beta} \sim$ membrane winding.
- Hamiltonian mechanics $\{X, P\} \rightarrow$ Nambu mechanics $\{X, P, Q\}$ with 3-bracket.
- Suggests $y^\alpha, y_{\alpha\beta}$ as a Nambu triplet for M2.
- (Note: here only S-duality, no transformations between winding and momentum.)

$\mathrm{SL}(2) \times \mathbb{R}^+$ EFT field content

- External metric $g_{\mu\nu}$
- Coset valued generalised metric $\mathcal{M}_{MN} \in \mathrm{SL}(2) \times \mathbb{R}^+/\mathrm{SO}(2)$

$$\mathcal{H}_{\alpha\beta} \in \mathrm{SL}(2)/\mathrm{SO}(2) \Rightarrow \mathcal{H}_{\alpha\beta} = \frac{1}{\mathrm{Im}\,\tau} \begin{pmatrix} |\tau|^2 & \mathrm{Re}\,\tau \\ \mathrm{Re}\,\tau & 1 \end{pmatrix}$$

$$\mathcal{M}_{ss} \in \mathbb{R}^+$$

$\mathrm{SL}(2) \times \mathbb{R}^+$ EFT field content

- Tensor hierarchy of gauge potentials Cederwall, Edlund, Karlsson 2013; Wang 2015; Berman, CB, Malek, Rudolph 2015

Representation	Gauge potential	Field strength
$\mathbf{2}_1 \oplus \mathbf{1}_{-1}$	${A_\mu}^M$	${\mathcal{F}_{\mu\nu}}^M$
$\mathbf{2}_0$	${B_{\mu\nu}}^{\alpha s}$	${\mathcal{H}_{\mu\nu\rho}}^{\alpha s}$
$\mathbf{1}_1$	${C_{\mu\nu\rho}}^{[\alpha\beta]s}$	${\mathcal{J}_{\mu\nu\rho\sigma}}^{[\alpha\beta]s}$
$\mathbf{1}_0$	${D_{\mu\nu\rho\sigma}}^{[\alpha\beta]ss}$	${\mathcal{K}_{\mu\nu\rho\sigma\lambda}}^{[\alpha\beta]ss}$
$\mathbf{2}_1$	${E_{\mu\nu\rho\sigma\kappa}}^{\gamma[\alpha\beta]ss}$	${\mathcal{L}_{\mu\nu\rho\sigma\kappa\lambda}}^{\gamma[\alpha\beta]ss}$
$\mathbf{2}_0 \oplus \mathbf{1}_2$	${F_{\mu\nu\rho\sigma\kappa\lambda}}^M$	

- Interrelated gauge transformations, field strengths, Bianchi identities, starting point:

$$\delta_\Lambda {A_\mu}^M = \partial_\mu \Lambda^M - \delta_{A_\mu} \Lambda^M$$

$\mathrm{SL}(2) \times \mathbb{R}^+$ EFT action

- The (bosonic) action is entirely fixed by the (bosonic) symmetries:

$$S = \int d^9x d^3Y \sqrt{|g|}\left(R(g) + \mathcal{L}_{kin} + V\right) + S_{top}$$

- One finds

$$\begin{aligned}
\mathcal{L}_{kin} = &-\frac{7}{32} g^{\mu\nu} D_\mu \ln \mathcal{M}_{ss} D_\nu \ln \mathcal{M}_{ss} + \frac{1}{4} g^{\mu\nu} D_\mu \mathcal{H}_{\alpha\beta} D_\nu \mathcal{H}^{\alpha\beta} \\
&- \frac{1}{2\cdot 2!} \mathcal{M}_{MN} \mathcal{F}_{\mu\nu}{}^{M} \mathcal{F}^{\mu\nu N} - \frac{1}{2\cdot 3!} \mathcal{M}_{\alpha\beta} \mathcal{M}_{ss} \mathcal{H}_{\mu\nu\rho}{}^{\alpha s} \mathcal{H}^{\mu\nu\rho\beta s} \\
&- \frac{1}{2\cdot 2!4!} \mathcal{M}_{ss} \mathcal{M}_{\alpha\gamma} \mathcal{M}_{\beta\delta} \mathcal{J}_{\mu\nu\rho\sigma}{}^{[\alpha\beta]s} \mathcal{J}^{\mu\nu\rho\sigma[\gamma\delta]s} \,.
\end{aligned}$$

$$\begin{aligned}
S_{top} = \frac{1}{5!\cdot 48} \int d^{10}x\, d^3Y\, \varepsilon^{\mu_1\ldots\mu_{10}} \frac{1}{4} \epsilon_{\alpha\beta}\epsilon_{\gamma\delta} \Big[&\frac{1}{5} \partial_s \mathcal{K}_{\mu_1\ldots\mu_5}{}^{\alpha\beta ss} \mathcal{K}_{\mu_6\ldots\mu_{10}}{}^{\gamma\delta ss} \\
&- \frac{5}{2} \mathcal{F}_{\mu_1\mu_2}{}^{s} \mathcal{J}_{\mu_3\ldots\mu_6}{}^{\alpha\beta s} \mathcal{J}_{\mu_7\ldots\mu_{10}}{}^{\gamma\delta} \\
&+ \frac{10}{3} 2 \mathcal{H}_{\mu_1\ldots\mu_3}{}^{\alpha s} \mathcal{H}_{\mu_4\ldots\mu_6}{}^{\beta s} \mathcal{J}_{\mu_7\ldots\mu_{10}}{}^{\gamma\delta} \Big] \,.
\end{aligned}$$

Action continued

- And

$$V = \frac{1}{4}\mathcal{M}^{ss}\left(\partial_s \mathcal{H}^{\alpha\beta}\partial_s \mathcal{H}_{\alpha\beta} + \partial_s g^{\mu\nu}\partial_s g_{\mu\nu} + \partial_s \ln|g|\partial_s \ln|g|\right)$$
$$+ \frac{9}{32}\mathcal{M}^{ss}\partial_s \ln \mathcal{M}_{ss}\partial_s \ln \mathcal{M}_{ss} - \frac{1}{2}\mathcal{M}^{ss}\partial_s \ln \mathcal{M}_{ss}\partial_s \ln|g|$$
$$+ \mathcal{M}_{ss}^{3/4}\Bigg[\frac{1}{4}\mathcal{H}^{\alpha\beta}\partial_\alpha \mathcal{H}^{\gamma\delta}\partial_\beta \mathcal{H}_{\gamma\delta} + \frac{1}{2}\mathcal{H}^{\alpha\beta}\partial_\alpha \mathcal{H}^{\gamma\delta}\partial_\gamma \mathcal{H}_{\delta\beta}$$
$$+ \partial_\alpha \mathcal{H}^{\alpha\beta}\partial_\beta \ln\left(|g|^{1/2}\mathcal{M}_{ss}^{3/4}\right)$$
$$+ \frac{1}{4}\mathcal{H}^{\alpha\beta}\Big(\partial_\alpha g^{\mu\nu}\partial_\beta g_{\mu\nu} + \partial_\alpha \ln|g|\partial_\beta \ln|g|$$
$$+ \frac{1}{4}\partial_\alpha \ln \mathcal{M}_{ss}\partial_\beta \ln \mathcal{M}_{ss} + \frac{1}{2}\partial_\alpha \ln g \partial_\beta \ln \mathcal{M}_{ss}\Big)\Bigg]$$

$\mathrm{SL}(2) \times \mathbb{R}^+$ EFT reduction to SUGRA

- M-theory section $\partial_s = 0$, coordinates (x^μ, y^α).
- IIB section $\partial_\alpha = 0$, coordinates (x^μ, y^s).
- Relate EFT fields to SUGRA fields as follows (schematically):

EFT field	M-theory	IIB
$\mathcal{H}_{\alpha\beta}$	$g^{-1/2} g_{\alpha\beta}$	$\mathcal{H}_{\alpha\beta}$
$\mathcal{M}_{ss}$	$g^{-6/7}$	$(g_{ss})^{8/7}$
${A_\mu}^\alpha$	${A_\mu}^\alpha$	$B_{\mu s}, C_{\mu s}$
${A_\mu}^s$	$C_{\mu\alpha\beta}$	${A_\nu}^s$
${B_{\mu\nu}}^{\alpha s}$	$C_{\mu\nu\alpha}$	$B_{\mu\nu},\ C_{\mu\nu}$
${C_{\mu\nu\rho}}^{\alpha\beta s}$	$C_{\mu\nu\rho}$	$C_{\mu\nu\rho s}$
${D_{\mu\nu\rho\sigma}}^{\alpha\beta ss}$	dual	$C_{\mu\nu\rho\sigma}$

- Theory is then equivalent to 11-dim SUGRA/IIB SUGRA.

Relationship to F-theory

- Both $\mathrm{SL}(2) \times \mathbb{R}^+$ EFT and F-theory: a 12-dimensional perspective on IIB

- EFT: Extended space has local $\mathrm{SL}(2) \times \mathbb{R}^+$ symmetry via generalised diffeomorphisms - not conventional geometry. "Scalars" in generalised metric.
- F-theory: Auxiliary torus not truly geometric, zero volume. Scalars in torus moduli.

- EFT on IIB section: two isometries in 12-d ($\partial_\alpha = 0$), interpret as torus fibration. Key point: no volume modulus as generalised metric in coset.

Relationship to F-theory: M-theory/F-theory duality

- Explicit realisation of M-theory/F-theory duality. Direct mapping between fields via EFT definitions.

- Consider just extended directions,

$$\text{“}ds^2\text{”} = (\mathcal{M}_{ss})^{-3/4}\mathcal{H}_{\alpha\beta}dy^\alpha dy^\beta + \mathcal{M}_{ss}(dy^s)^2$$

  Limits $\mathcal{M}_{ss} \to 0 \Rightarrow$ M-theory directions large, $\mathcal{M}_{ss} \to \infty$ IIB direction large. $\mathcal{M}_{ss} = v^{9/7}$ gives

$$ds^2_M = v\mathcal{H}_{\alpha\beta}dy^\alpha dy^\beta$$

  and

$$ds^2_{IIB} = v^{-3/2}(dy^s)^2$$

  usual relation $R_{IIB} \sim v^{-3/4}$.

Relationship to F-theory: solutions

- e.g. 7-branes

$$
\begin{aligned}
ds^2_{(9)} &= -dt^2 + d\vec{x}^2_{(6)} + \tau_2 |f|^2 dz d\bar{z} \\
\text{“} ds^2_{ext} \text{''} &= \frac{1}{\tau_2} \left[|\tau|^2 (dy^1)^2 + 2\tau_1 dy^1 dy^2 + (dy^2)^2 \right] + (dy^s)^2
\end{aligned} \tag{1}
$$

$$\tau = j^{-1}(P(z)/Q(z))$$

where $P(z)$ and $Q(z)$ are polynomials in z. Roots of $Q(z) \to$ brane locations. Near brane, solution like "smeared monopole" in extended space.

- In spacetime sections

$$ds^2_{IIB} = -dt^2 + d\vec{x}^2_{(6)} + (dy^s)^2 + \tau_2 |f|^2 dz d\bar{z}$$

and

$$ds^2_M = -dt^2 + d\vec{x}^2_{(6)} + \tau_2 |f|^2 dz d\bar{z} + \tau_2 (dy^1)^2 + \frac{1}{\tau_2} \left(dy^2 + \tau_1 dy^1 \right)^2 \ .$$

Conclusions and outlook

- Have constructed the $\mathrm{SL}(2) \times \mathbb{R}^+$ EFT and argued it is an action for F-theory.
- Supersymmetrisation and compactification - find permitted backgrounds, derive effective actions directly.
- Embedding and/or extension to higher rank duality groups?
- Scherk-Schwarz F-theory? Connections to massive supergravity?
- Global issues - simple setting to understand finite transformations? Global choice of section - exchange M-theory and F-theory sections?

Thanks for listening!