Supersymmetric gauged Double Field Theory: Systematic derivation by virtue of *Twist*

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Duality and Novel Geometry in M-theory

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Based on arXiv1505.01301, with Wonyoung Cho, J.J. Fernandez-Melgarejo, Jeong-Hyuck Park

Intruduction

- A characteristic of Double Field Theory is the section condition :

  The $\mathbf{O}(D,D)$ d'Alembert operator is trivial, acting on arbitrary fields or gauge parameters as well as their products:

$$\partial_A\partial^A = \mathcal{J}^{AB}\partial_A\partial_B = 2\frac{\partial^2}{\partial\tilde{x}_\mu\partial x^\mu} \sim 0$$

  i.e.

$$\partial_A\partial^A\Phi \sim 0\,, \qquad \text{and} \qquad \partial_A\Phi_1\partial^A\Phi_2 \sim 0$$

- DFT action is (locally) equivalent to the effective action:

$$S_{\text{DFT}} \Longrightarrow S_{\text{eff.}} = \int \mathrm{d}x^D\sqrt{-g}e^{-2\phi}\left(R_g + 4(\partial\phi)^2 - \tfrac{1}{12}H^2\right).$$

- Section condition(strong constraint) seems necessary to write a complete theory, because of action invariance and closedness of symmetry algebra .

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Introduction

- 'generalized Lie derivative' [Siegel, Courant, Grana ...]

$$\hat{\mathcal{L}}_X T_{\omega A} := X^B \partial_B T_{\omega A} + \omega \partial_B X^B T_{\omega A} + \partial_A X^B T_{\omega B} - \partial^B X_A T_{\omega B}.$$

- Commutator of the generalized Lie derivatives is closed, up to the section condition , by using **c**-bracket,

$$[\hat{\mathcal{L}}_X, \hat{\mathcal{L}}_Y] \sim \hat{\mathcal{L}}_{[X,Y]_{\mathbf{C}}} ,$$

where $[X, Y]_{\mathbf{C}}$ denotes C-bracket

$$[X, Y]^A_{\mathbf{C}} := X^B \partial_B Y^A - Y^B \partial_B X^A + \tfrac{1}{2} Y^B \partial^A X_B - \tfrac{1}{2} X^B \partial^A Y_B ,$$

- Understanding the level matching condition(weak constraint) in DFT is still on-going [K. Lee's talk] .
- However, "relaxing" the section condition to some extent has been understood. [Geissbühler; Aldazabal, Baron, Marqués, Nunez; Grana, Marqués]
  The section condition is sufficient but not the necessary condition for the algebra closure and action invariance.
- The relaxation of the section condition is allowed when doing Sherk-Schwarz reduction in DFT and it gets low dimensional gauged DFT
- A variety of the known gauged supergravities in lower dimensions can be reproduced, i.e. DFT provides the higher dimensional origin of a various gauged supereravities. (Electric gauging. *cf*. Magnetic gauging is from EFT [Berman, Musaev, C. Thompson;...])
  - A systematic classification of all possible deformations as for gaugings is allowed by embedding tensor method [de Wit, Samtleben]
  - Only some of them can be obtained by a Sherk-Schwarz dimensional reduction of 11 or 10-dimensional supergravities.

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  - Only some of them can be obtained by a Sherk-Schwarz dimensional reduction of 11 or 10-dimensional supergravities.

- This is an indication that DFT goes beyond the ordinary supergravity or generalized geometry .
- Particularly, one needs explicitly section-condition-breaking terms, which depend on both of x and $\tilde{x}$. Geissbühler realized necessity of introducing such term,

$$\Delta\mathcal{L} = -\frac{1}{6}\hat{F}_{\hat{A}\hat{B}\hat{D}}\hat{F}^{\hat{A}\hat{B}\hat{C}} ,$$

to reproduce the complete classification of $N = 4$, $D = 4$ gauged SUGRAs.

- ‘Geometric’ understanding of DFT
  - Flux formulatin [Hohm, Kwak]
  - Semi-covariant formulation [IJ, Lee, Park]
    - Direct analogy of the Riemann geometry using Christofel connection,
    - Fully covariant with respect to all the symmetries in DFT,
    - Maximal and half maximal supersymmetric DFT is realized in full order of fermions, where Maximal supersymmetric DFT unifies the type II supergravities [IJ, Lee, Park, Suh]

- ‘Geometric’ understanding of gauged DFT
  - in flux formulation (for bosonic DFT) [Geissbuhler; Marques, Aldazabal, Nunez; Berman, Bair, Malek, Perry]
  - in semi-covariant formulation, understood by torsionful deformation of gauged DFT, where half-maximal supersymmetry was realized.[Berman, Lee]

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Goal

- To have **systematic understanding** of the low dimensional gauged SDFT in the semi-covariant formulation

    - We twist the semi-covariant formulation of the ungaged SDFT **without any ambiguity**.

        - By the formulation, all the symmetries in DFT are fully covariant.

        - Torsionful deformation of the gauged DFT is derived from twisting.

        - Definition of curvature includes the section condition breaking term

- To realize the maximal as well as half maximal supersymmetric gauged DFT in full order of fermions

    - Constraint on the structure constant for the maximal SDFT.

    - **Two half maximal gauged SDFTs.**

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Contents

2. Semi-covariant formulation of DFT/SDFT

[1011.1324, 1105.6294, 1109.2035, 1112.0069, 1206.3478, 1210.5078]

Symmetries of SDFT

Semi covariant formulation manifests all the bosonic symmetries.

- **O**(D, D) **T-duality:**
- **DFT-diffeomorphism (generalized Lie derivative)**
  - **Diffeomorphism**
  - B**-field gauge symmetry**
- **A pair of local Lorentz symmetries, Spin**$(1, D-1)_L \times$ **Spin**$(D-1, 1)_R$
- $D = 10$ **Local SUSY**

Field contents of $D=10$ Maximal SDFT

- **Bosons**
  - **NS-NS sector** $\left\{\begin{array}{ll}\textbf{DFT-dilaton:} & d \\ \textbf{Double-vielbeins:} & V_{Ap}\,, \quad \bar{V}_{A\bar{p}}\end{array}\right.$
  - **R-R potential:** $\mathcal{C}^{\alpha}{}_{\bar{\alpha}}$

- **Fermions**
  - **DFT-dilatinos:** $\rho^{\alpha}\,, \quad \rho'^{\bar{\alpha}}$
  - **Gravitinos:** $\psi^{\alpha}_{\bar{p}}\,, \quad \psi'^{\bar{\alpha}}_{p}$

Index	Representation	Metric (raising/lowering indices)
$A, B, \cdots$	$\mathbf{O}(D,D)$ vector	$\mathcal{J}_{AB}$
$p, q, \cdots$	$\mathbf{Spin}(1,D{-}1)_L$ vector	$\eta_{pq} = \mathbf{diag}(-++\cdots+)$
$\alpha, \beta, \cdots$	$\mathbf{Spin}(1,D{-}1)_L$ spinor	$C_{+\alpha\beta}\,, \quad (\gamma^{p})^{T} = C_{+}\gamma^{p}C_{+}^{-1}$
$\bar{p}, \bar{q}, \cdots$	$\mathbf{Spin}(D{-}1,1)_R$ vector	$\bar{\eta}_{\bar{p}\bar{q}} = \mathbf{diag}(+--\cdots-)$
$\bar{\alpha}, \bar{\beta}, \cdots$	$\mathbf{Spin}(D{-}1,1)_R$ spinor	$\bar{C}_{+\bar{\alpha}\bar{\beta}}\,, \quad (\bar{\gamma}^{\bar{p}})^{T} = \bar{C}_{+}\bar{\gamma}^{\bar{p}}\bar{C}_{+}^{-1}$

Semi-covariant formulation

- The DFT-vielbeins satisfy the **four defining properties:**

$$V_{Ap}V^{A}{}_{q} = \eta_{pq}\,, \qquad \bar{V}_{A\bar{p}}\bar{V}^{A}{}_{\bar{q}} = \bar{\eta}_{\bar{p}\bar{q}}\,, \qquad V_{Ap}\bar{V}^{A}{}_{\bar{q}} = 0\,, \qquad V_{Ap}V_{B}{}^{p} + \bar{V}_{A\bar{p}}\bar{V}_{B}{}^{\bar{p}} = \mathcal{J}_{AB}\,.$$

- They generate **a pair of two-index projectors,**

$$P_{AB} := V_{A}{}^{p}V_{Bp}\,, \qquad \bar{P}_{AB} := \bar{V}_{A}{}^{\bar{p}}\bar{V}_{B\bar{p}}\,,$$

$P_{AB}, \bar{P}_{AB}$ are projection matrices('left and right'),

$$P_{A}{}^{B}P_{B}{}^{C} = P_{A}{}^{C}\,, \qquad \bar{P}_{A}{}^{B}\bar{P}_{B}{}^{C} = \bar{P}_{A}{}^{C}\,, \qquad P_{A}{}^{B}\bar{P}_{B}{}^{C} = 0$$

which are related to $\mathcal{H}$ and $\mathcal{J}$,

$$P_{AB} + \bar{P}_{AB} = \mathcal{J}_{AB}\,, \qquad P_{AB} - \bar{P}_{AB} = \mathcal{H}_{AB}$$

- We further define a pair of six-index projectors,

$$\mathcal{P}_{CAB}{}^{DEF} := P_{C}{}^{D}P_{[A}{}^{[E}P_{B]}{}^{F]} + \tfrac{2}{D-1}P_{C[A}P_{B]}{}^{[E}P^{F]D}\,, \qquad \mathcal{P}_{CAB}{}^{DEF}\mathcal{P}_{DEF}{}^{GHI} = \mathcal{P}_{CAB}{}^{GHI}\,,$$
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which satisfy the following properties, symmetric and traceless,

$$\mathcal{P}_{CABDEF} = \mathcal{P}_{DEFCAB} = \mathcal{P}_{C[AB]D[EF]}\,, \qquad \bar{\mathcal{P}}_{CABDEF} = \bar{\mathcal{P}}_{DEFCAB} = \bar{\mathcal{P}}_{C[AB]D[EF]}\,,$$
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Semi-covariant derivatives

- We introduce **master 'semi-covariant' derivative**

$$\mathcal{D}_A = \partial_A + \Gamma_A + \Phi_A + \bar{\Phi}_A\,.$$

- It is also useful to set

$$\nabla_A = \partial_A + \Gamma_A\,,$$

- The 'semi-covariant' derivative for the DFT-diffeomorphism is

$$\nabla_C T_{\omega A_1 A_2 \cdots A_n} := \partial_C T_{\omega A_1 A_2 \cdots A_n} - \omega \Gamma^B{}_{BC} T_{\omega A_1 A_2 \cdots A_n} + \sum_{i=1}^{n} \Gamma_{CA_i}{}^B T_{\omega A_1 \cdots A_{i-1} B A_{i+1} \cdots A_n}\,.$$

- Spin connections

$$\Phi_{Apq} = V^B{}_p \nabla_A V_{Bq}\,, \qquad \bar{\Phi}_{A\bar{p}\bar{q}} = \bar{V}^B{}_{\bar{p}} \nabla_A \bar{V}_{B\bar{q}}\,,$$

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$$\mathcal{D}_A V_{Bp} = 0\,, \qquad \mathcal{D}_A \bar{V}_{B\bar{p}} = 0\,. \qquad (cf.\ \mathcal{D}_\mu e_\nu{}^a = 0)$$

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- **compatibility for the whole NS-NS sector**

$$\mathcal{D}_A d = 0\,, \qquad \mathcal{D}_A V_{Bp} = 0\,, \qquad \mathcal{D}_A \bar{V}_{B\bar{p}} = 0\,. \qquad (cf.\, \mathcal{D}_\mu e_\nu{}^a = 0)$$

together with

$$\mathcal{D}_A \eta_{pq} = \mathcal{D}_A \bar{\eta}_{\bar{p}\bar{q}} = \mathcal{D}_A (\gamma^p)^\alpha{}_\beta = \mathcal{D}_A (\bar{\gamma}^{\bar{p}})^{\bar{\alpha}}{}_{\bar{\beta}} = \mathcal{D}_A C_{+\alpha\beta} = \mathcal{D}_A \bar{C}_{+\bar{\alpha}\bar{\beta}} = 0\,.$$

It follows that

$$\nabla_A d = 0\,, \qquad \nabla_A P_{BC} = 0\,, \qquad \nabla_A \bar{P}_{BC} = 0\,, \qquad (cf. \nabla_\mu g_{\nu\lambda} = 0)$$

- Spin connections

$$\Phi_{Apq} = V^B{}_p \nabla_A V_{Bq}\,, \qquad \bar{\Phi}_{A\bar{p}\bar{q}} = \bar{V}^B{}_{\bar{p}} \nabla_A \bar{V}_{B\bar{q}}\,,$$

- Torsion free conection is uniquely determined in terms of basic geometrical variables, [IJ, Lee, Park '11]

$$\Gamma^0_{CAB} = \quad 2\left(P\partial_C P\bar{P}\right)_{[AB]} + 2\left(\bar{P}_{[A}{}^{D}\bar{P}_{B]}{}^{E} - P_{[A}{}^{D}P_{B]}{}^{E}\right)\partial_D P_{EC}$$

$$-\tfrac{4}{D-1}\left(\bar{P}_{C[A}\bar{P}_{B]}{}^{D} + P_{C[A}P_{B]}{}^{D}\right)\left(\partial_D d + (P\partial^E P\bar{P})_{[ED]}\right) ,$$

from

$$\nabla_A d = 0\,, \qquad \nabla_A P_{BC} = 0\,, \qquad \nabla_A \bar{P}_{BC} = 0\,, \qquad (cf.\nabla_\mu g_{\nu\lambda} = 0)$$

$$\Gamma^0_{[ABC]} = 0\,, \qquad (\Leftrightarrow \quad \hat{\mathcal{L}}^{\partial}_X = \hat{\mathcal{L}}^{\nabla}_X)$$

$$\mathcal{P}_{CAB}{}^{DEF}\Gamma^0_{DEF} = 0\,, \qquad \bar{\mathcal{P}}_{CAB}{}^{DEF}\Gamma^0_{DEF} = 0\,.$$

Semi-covariant formulation

- Under $\delta_X \mathcal{H}_{AB} = \hat{\mathcal{L}}_X \mathcal{H}_{AB}$, $\delta_X d = \hat{\mathcal{L}}_X d$, namely DFT-difeomorphism (= diffeomorphism + B-field gauge symmetry), the variation of $\nabla_C T_A$ contains an anomalous non-covariant part,

$$(\delta_X - \hat{\mathcal{L}}_X)\nabla_C T_A \sim 2(\mathcal{P}+\bar{\mathcal{P}})_{CA}{}^{BFDE}\partial_F\partial_{[D}X_{E]}T_B \,.$$

- However, the anomalous term are controlled by the rank six projectors , so they can be projected out by combining the projection matrices P_{AB} and $\bar{P}_{AB}$.

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Projection-aided covariant derivatives

"semi-covariant derivative" :

combined with the projections , we can get various covariant quantities:

Examples:

- **For $\mathbf{O}(D,D)$ tensors:**

$$P_C{}^D\bar{P}_A{}^B\nabla_D T_B\,,\qquad \bar{P}_C{}^D P_A{}^B\nabla_D T_B\,,$$

$$P^{AB}\nabla_A T_B\,,\qquad \bar{P}^{AB}\nabla_A T_B\,,\qquad \text{Divergences}\,,$$

$$P^{AB}\bar{P}_C{}^D\nabla_A\nabla_B T_D\,,\qquad \bar{P}^{AB}P_C{}^D\nabla_A\nabla_B T_D\,.\qquad \text{Laplacians}$$

- Rule: need opposite chirality or contraction

Projection-aided covariant derivatives

- **For Spin$(1, D-1)_L \times$ Spin$(D-1, 1)_R$ tensors:**

$$\mathcal{D}_p T_{\bar{q}} \,, \qquad \mathcal{D}_{\bar{p}} T_q \,,$$
$$\mathcal{D}^p T_p \,, \qquad \mathcal{D}^{\bar{p}} T_{\bar{p}} \,,$$
$$\mathcal{D}_p \mathcal{D}^p T_{\bar{q}} \,, \qquad \mathcal{D}_{\bar{p}} \mathcal{D}^{\bar{p}} T_q \,,$$

where we set

$$\mathcal{D}_p := V^A{}_p \mathcal{D}_A \,, \qquad \mathcal{D}_{\bar{p}} := \bar{V}^A{}_{\bar{p}} \mathcal{D}_A \,.$$

These are the pull-back of the previous results using the double-vielbeins.

Projection-aided covariant derivatives

- **Dirac operators for fermions,** ρ^{α}, $\psi_{\bar{p}}^{\alpha}$, $\rho'^{\bar{\alpha}}$, $\psi_{p}'^{\bar{\alpha}}$ **:** [IJ, Lee, Park '11]

$$\gamma^{p}\mathcal{D}_{p}\rho = \gamma^{A}\mathcal{D}_{A}\rho\,, \qquad \gamma^{p}\mathcal{D}_{p}\psi_{\bar{p}} = \gamma^{A}\mathcal{D}_{A}\psi_{\bar{p}}\,,$$

$$\mathcal{D}_{\bar{p}}\rho\,, \qquad \mathcal{D}_{\bar{p}}\psi^{\bar{p}} = \mathcal{D}_{A}\psi^{A}\,,$$

$$\bar{\gamma}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho' = \bar{\gamma}^{A}\mathcal{D}_{A}\rho'\,, \qquad \bar{\gamma}^{\bar{p}}\mathcal{D}_{\bar{p}}\psi'_{p} = \bar{\gamma}^{A}\mathcal{D}_{A}\psi'_{p}\,,$$

$$\mathcal{D}_{p}\rho'\,, \qquad \mathcal{D}_{p}\psi'^{p} = \mathcal{D}_{A}\psi'^{A}\,,$$

Projection-aided covariant derivatives

- **For Spin$(1, D-1)_L \times$ Spin$(D-1, 1)_R$ bi-fundamental spinors, $\mathcal{C}^{\alpha}{}_{\bar{\beta}}$:**
  [IJ, Lee, Park '12]

$$\gamma^A \mathcal{D}_A \mathcal{C}\,, \qquad\qquad \mathcal{D}_A \mathcal{C} \bar{\gamma}^A\,.$$

- Further define

$$\mathcal{D}_+ \mathcal{C} := \gamma^A \mathcal{D}_A \mathcal{C} + \gamma^{(D+1)} \mathcal{D}_A \mathcal{C} \bar{\gamma}^A\,,$$

$$\mathcal{D}_- \mathcal{C} := \gamma^A \mathcal{D}_A \mathcal{C} - \gamma^{(D+1)} \mathcal{D}_A \mathcal{C} \bar{\gamma}^A\,.$$

- Especially for the torsionless case, the corresponding operators are **nilpotent** up to the section condition

$$(\mathcal{D}^0_+)^2 \mathcal{C} \sim 0\,, \qquad\qquad (\mathcal{D}^0_-)^2 \mathcal{C} \sim 0\,,$$

- **The field strength of the R-R potential, $\mathcal{C}^{\alpha}{}_{\bar{\alpha}}$, is then defined by**

$$\mathcal{F} := \mathcal{D}^0_+ \mathcal{C}\,.$$

Projection-aided covariant derivatives

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Curvatures I [1105.6294]

- From the usual DFT-diffeomorhphism field strength, we define *semi-covariant four-index curvature*

$$S_{ABCD} := \tfrac{1}{2}\left(R_{ABCD} + R_{CDAB} - \Gamma^{E}{}_{AB}\Gamma_{ECD}\right) .$$

- It satisfies
  - just like the Riemann curvature,

$$S_{ABCD} = \tfrac{1}{2}\big(S_{[AB][CD]} + S_{[CD][AB]}\big) ,$$

$$S_{A[BCD]} = 0 \quad : \quad \text{Bianchi identity} ,$$

  - and with projectors,

$$(P^{AB}P^{CD} + \bar{P}^{AB}\bar{P}^{CD})S_{ACBD} \sim 0 ,$$

$$P_{I}{}^{A}P_{J}{}^{B}\bar{P}_{K}{}^{C}\bar{P}_{L}{}^{D}S_{ABCD} \sim 0 ,$$

$$P_{I}{}^{A}\bar{P}_{J}{}^{B}P_{K}{}^{C}\bar{P}_{L}{}^{D}S_{ABCD} \sim 0 , \quad etc.$$

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Curvatures I [1105.6294]

- This is still Not covariant tensor, but contracting with projection operators, we can obtain covariant quatities.

  - Rank two-tensor:

$$P_I{}^A \bar{P}_J{}^B S_{AB} \,, \qquad \text{where } S_{AB} := S^C{}_{ACB}$$

  - Scalar curvature: defines the Lagrangian for NS-NS sector

$$(P^{AB}P^{CD} - \bar{P}^{AB}\bar{P}^{CD})S_{ACBD}$$

Curvatures II

- Alternative way to define the curvature is using the field strength for the local Lorentz group, *c.f.* YM-gauge field strength

$$\mathcal{F}_{ABpq} := \nabla_A \Phi_{Bpq} - \nabla_B \Phi_{Apq} + \Phi_{Ap}{}^r \Phi_{Brq} - \Phi_{Bp}{}^r \Phi_{Arq} \,,$$

$$\bar{\mathcal{F}}_{AB\bar{p}\bar{q}} := \nabla_A \bar{\Phi}_{B\bar{p}\bar{q}} - \nabla_B \bar{\Phi}_{A\bar{p}\bar{q}} + \bar{\Phi}_{A\bar{p}}{}^{\bar{r}} \bar{\Phi}_{B\bar{r}\bar{q}} - \bar{\Phi}_{B\bar{p}}{}^{\bar{r}} \bar{\Phi}_{A\bar{r}\bar{q}} \,,$$

We define *Semi-covariant four-index curvature of the spin connections* ,

$$\mathcal{G}_{ABCD} := \tfrac{1}{2}\left[(\mathcal{F} + \bar{\mathcal{F}})_{ABCD} + (\mathcal{F} + \bar{\mathcal{F}})_{CDAB} + (\Phi + \bar{\Phi})^E{}_{AB}(\Phi + \bar{\Phi})_{ECD}\right] \,,$$

where

$$\mathcal{F}_{ABCD} = \mathcal{F}_{ABpq} V_C{}^p V_D{}^q \,, \qquad \bar{\mathcal{F}}_{ABCD} = \bar{\mathcal{F}}_{AB\bar{p}\bar{q}} \bar{V}_C{}^{\bar{p}} \bar{V}_D{}^{\bar{q}} \,.$$

Curvatures II

- These two four-index curvatures are closely related to each other,

$$\mathcal{G}_{ABCD} = S_{ABCD} + \tfrac{1}{2}(V_A{}^p \partial_E V_{Bp} + \bar{V}_A{}^{\bar{p}} \partial_E \bar{V}_{B\bar{p}})(V_C{}^q \partial^E V_{Dq} + \bar{V}_C{}^{\bar{q}} \partial^E \bar{V}_{D\bar{q}}),$$

  such that upon the section condition they are equivalent.

- Later, $\mathcal{G}_{ABCD}$ will be the proper curvature when we relax the section condition.

- The later terms will correspond to what Geissbuler introduced in order to reproduce the the low dimensional gauged SUGRA.

- The curvature cannot be written in terms of generalized metric only, but should be written in terms of vielbein.

3. Twisting the semi-covariant formulation

- Twisting ansatz (Scherk-Schwarz reduction ansatz)
- Twistability conidition ("relaxing" the section condition) by closure of the algebra.
- Obtain low dimensional gauged SDFT with maximal and half maximal supersymmetry

U-twisting ansatz

- For the twisting we use the two twisting datas:
  a scalar $\lambda(x)$ and $U(x)_A{}^{\dot{A}} \in \mathbf{O}(D,D)$,

  $$U\dot{\mathcal{J}}U^t = \mathcal{J}\,, \qquad \dot{\mathcal{J}}_{\dot{M}\dot{N}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

  using which we set the ansatz for *U-twist*

  $$T_{A_1\cdots A_n} = e^{-2\omega\lambda} U_{A_1}{}^{\dot{A}_1} \cdots U_{A_n}{}^{\dot{A}_n} \dot{T}_{\dot{A}_1\cdots\dot{A}_n}\,.$$

- The $\lambda(x)$ and $U(x)_A{}^{\dot{A}}$ do not satisfy the section condition, but shall be require to satisfy the consistency conditions, i.e. *twistability condition* .
- The twisted field is denoted by dot with dotted indices,
- and $U(x)_A{}^{\dot{A}}$ carries one undotted index and other dotted index, such that the additional $\mathbf{O}(D,D)$ metric, $\dot{\mathcal{J}}_{\dot{M}\dot{N}}$ is introduced.
- While the twisted metric $\dot{\mathcal{J}}_{\dot{M}\dot{N}}$ may coincide numerically with the untwisted metric $\mathcal{J}_{MN}$, we deliberately distinguish them as the two different kinds of indices will never be contracted.

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- If we assume the U matrix to be in a block diagonal form,

$$U = \begin{pmatrix} 1 & 0 \\ 0 & u \end{pmatrix},$$

  and split all the internal coordinate dependency into the U matrix, then this twisting ansatz is nothing but the usual Sherk-Schwarz reduction ansatz.

- But all the forthcoming analyses do not necessarily demand this ansatz, so we will use above general twisting ansatz.

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- *The only field variables to be twisted* are

$$e^{-2d} = e^{-2\lambda}e^{-2\dot{d}}\,, \qquad V_{Ap} = U_A{}^{\dot{A}}\dot{V}_{\dot{A}p}\,, \qquad \bar{V}_{A\bar{p}} = U_A{}^{\dot{A}}\dot{\bar{V}}_{\dot{A}\bar{p}}\,.$$

Other fields (fermions and the R-R potential) are weightless and $\mathbf{O}(D,D)$ singlet.

- The twist of the $\mathcal{N}=1$ or the $\mathcal{N}=2$, $D=10$ SDFT simply amounts to inserting the above expressions for the dilaton and the vielbeins into the untwisted Lagrangian.

U-twisting ansatz

- The derivatives of the untwisted fields then assume a generic form,

$$\partial_C T_{A_1\cdots A_n} = e^{-2\omega\lambda} U_C{}^{\dot{C}} U_{A_1}{}^{\dot{A}_1} \cdots U_{A_n}{}^{\dot{A}_n} \dot{D}_{\dot{C}} \dot{T}_{\dot{A}_1\cdots\dot{A}_n} ,$$

- U-derivative, $\dot{D}_{\dot{C}}$, is defined to act on a twisted field by

$$\dot{D}_{\dot{C}} \dot{T}_{\dot{A}_1\cdots\dot{A}_n} := \dot{\partial}_{\dot{C}} \dot{T}_{\dot{A}_1\cdots\dot{A}_n} - 2\omega\dot{\partial}_{\dot{C}}\lambda\, \dot{T}_{\dot{A}_1\cdots\dot{A}_n} + \sum_{i=1}^{n} \Omega_{\dot{C}\dot{A}_i}{}^{\dot{B}} \dot{T}_{\dot{A}_1\cdots\dot{B}\cdots\dot{A}_n} .$$

  With the pull-back of the naked derivative,

$$\dot{\partial}_{\dot{C}} = U^{-1}{}_{\dot{C}}{}^{C} \partial_C ,$$

  and a pure gauge “connection",

$$\Omega_{\dot{C}\dot{A}}{}^{\dot{B}} := \left(U^{-1}\dot{\partial}_{\dot{C}} U\right)_{\dot{A}}{}^{\dot{B}} ,$$

- The U-derivatives are all commutative,

$$[D_A\, , D_B] = 0\, , \qquad [D_A\, , \dot{D}_{\dot{B}}] = 0\, , \qquad [\dot{D}_{\dot{A}}\, , \dot{D}_{\dot{B}}] = 0\, .$$

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U-twisting ansatz

- Those replacement leads to twisted SDFT Lagrangian,

$$\mathcal{L}^{\mathcal{N}=1}_{D=10}(\mathcal{J}_{AB},\partial_A,d,V_{Ap},\bar{V}_{A\bar{p}},\rho,\psi_{\bar{p}}) = e^{-2\lambda}\dot{\mathcal{L}}^{\text{Half-maximal}}_{\text{Twisted SDFT}}(\dot{\mathcal{J}}_{\dot{A}\dot{B}},\dot{D}_{\dot{A}},\dot{d},\dot{V}_{\dot{A}p},\dot{\bar{V}}_{\dot{A}\bar{p}},\rho,\psi_{\bar{p}})\,,$$

$$\mathcal{L}^{\mathcal{N}=2}_{D=10}(\mathcal{J}_{AB},\partial_A,d,V_{Ap},\bar{V}_{A\bar{p}},\mathcal{C},\rho,\psi_{\bar{p}},\rho',\psi'_{p})$$

$$= e^{-2\lambda}\dot{\mathcal{L}}^{\text{Maximal}}_{\text{Twisted SDFT}}(\dot{\mathcal{J}}_{\dot{A}\dot{B}},\dot{D}_{\dot{A}},\dot{d},\dot{V}_{\dot{A}p},\dot{\bar{V}}_{\dot{A}\bar{p}},\mathcal{C},\rho,\psi_{\bar{p}},\rho',\psi'_{p})\,.$$

- The section condition

$$\dot{D}_{\dot{A}}\dot{D}^{\dot{A}} \sim 0\,.$$

- If we impose this, it is nothing but the field redefinition of the untwisted SDFT.

  We want to find an alternative conditions i.e. *Twistability condition* by imposing the closure of the algebra.

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Twistability condition

- Define key quantities out of the twisting data are *c.f.* *[Grana, Marques]*

$$f_{\dot{A}} := \Omega^{\dot{B}}{}_{\dot{B}\dot{A}} - 2\dot{\partial}_{\dot{A}}\lambda = \partial_C U^C{}_{\dot{A}} - 2\dot{\partial}_{\dot{A}}\lambda\,,$$

and the 'structure constant',

$$f_{\dot{A}\dot{B}\dot{C}} := \Omega_{\dot{A}\dot{B}\dot{C}} + \Omega_{\dot{B}\dot{C}\dot{A}} + \Omega_{\dot{C}\dot{A}\dot{B}} = f_{[\dot{A}\dot{B}\dot{C}]}\,.$$

Twistability condition

- Consider the diffeomorphism, which also be twisted and generated by the *U-twisted generalized Lie derivative*,

$$\dot{\mathcal{L}}_{\dot{X}}\dot{T}_{\dot{A}_1\cdots\dot{A}_n} := \dot{X}^{\dot{B}}\dot{D}_{\dot{B}}\dot{T}_{\dot{A}_1\cdots\dot{A}_n} + \omega\dot{D}_{\dot{B}}\dot{X}^{\dot{B}}\dot{T}_{\dot{A}_1\cdots\dot{A}_n} + \sum_{i=1}^{n}(\dot{D}_{\dot{A}_i}\dot{X}_{\dot{B}} - \dot{D}_{\dot{B}}\dot{X}_{\dot{A}_i})\dot{T}_{\dot{A}_1\cdots\dot{A}_{i-1}}{}^{\dot{B}}{}_{\dot{A}_{i+1}\cdots\dot{A}_n}\,.$$

- Closure of the diffeomorphism

$$\left([\dot{\mathcal{L}}_{\dot{X}}, \dot{\mathcal{L}}_{\dot{Y}}] - \dot{\mathcal{L}}_{[\dot{X},\dot{Y}]_{\dot{C}}}\right)\dot{T}_{\dot{A}_1\cdots\dot{A}_n}$$

$$= \tfrac{1}{2}(\dot{X}^{\dot{N}}\dot{D}^{\dot{M}}\dot{Y}_{\dot{N}} - \dot{Y}^{\dot{N}}\dot{D}^{\dot{M}}\dot{X}_{\dot{N}})\dot{D}_{\dot{M}}\dot{T}_{\dot{A}_1\cdots\dot{A}_n} + \tfrac{1}{2}\omega(\dot{X}^{\dot{N}}\dot{D}_{\dot{M}}\dot{D}^{\dot{M}}\dot{Y}_{\dot{N}} - \dot{Y}^{\dot{N}}\dot{D}_{\dot{M}}\dot{D}^{\dot{M}}\dot{X}_{\dot{N}})\dot{T}_{\dot{A}_1\cdots\dot{A}_n}$$

$$+ \textstyle\sum_{i=1}^{n}(\dot{D}_{\dot{M}}\dot{Y}_{\dot{A}_i}\dot{D}^{\dot{M}}\dot{X}_{\dot{B}} - \dot{D}_{\dot{M}}\dot{X}_{\dot{A}_i}\dot{D}^{\dot{M}}\dot{Y}_{\dot{B}})\dot{T}_{\dot{A}_1\cdots\dot{A}_{i-1}}{}^{\dot{B}}{}_{\dot{A}_{i+1}\cdots\dot{A}_n}\,,$$

where $[\dot{X}, \dot{Y}]_{\dot{C}}$ denotes the U-twisted C-bracket,

$$[\dot{X}, \dot{Y}]_{\dot{C}}^{\dot{A}} := \dot{X}^{\dot{B}}\dot{D}_{\dot{B}}\dot{Y}^{\dot{A}} - \dot{Y}^{\dot{B}}\dot{D}_{\dot{B}}\dot{X}^{\dot{A}} + \tfrac{1}{2}\dot{Y}^{\dot{B}}\dot{D}^{\dot{A}}\dot{X}_{\dot{B}} - \tfrac{1}{2}\dot{X}^{\dot{B}}\dot{D}^{\dot{A}}\dot{Y}_{\dot{B}}\,.$$

Twistability condition

- Closure of the diffeomorphism

$$\left([\dot{\mathcal{L}}_{\dot{X}}, \dot{\mathcal{L}}_{\dot{Y}}] - \dot{\mathcal{L}}_{[\dot{X},\dot{Y}]_{\mathrm{C}}}\right) \dot{T}_{\dot{A}_1\cdots\dot{A}_n}$$

$$= \left(\tfrac{1}{2}\dot{X}^{\dot{N}}\dot{\partial}^{\dot{M}}\dot{Y}_{\dot{N}} - \tfrac{1}{2}\dot{Y}^{\dot{N}}\dot{\partial}^{\dot{M}}\dot{X}_{\dot{N}} + \Omega^{\dot{M}}{}_{\dot{N}\dot{G}}\dot{X}^{\dot{N}}\dot{Y}^{\dot{G}}\right)\dot{\partial}_{\dot{M}}\dot{T}_{\dot{A}_1\cdots\dot{A}_n}$$

$$+\tfrac{1}{2}\omega\Big[\ \dot{X}^{\dot{N}}\dot{\partial}_{\dot{M}}\dot{\partial}^{\dot{M}}\dot{Y}_{\dot{N}} - \dot{Y}^{\dot{N}}\dot{\partial}_{\dot{M}}\dot{\partial}^{\dot{M}}\dot{X}_{\dot{N}} + 2\dot{X}^{\dot{N}}\Omega^{\dot{M}}{}_{\dot{N}\dot{G}}\dot{\partial}_{\dot{M}}\dot{Y}^{\dot{G}} - 2\dot{Y}^{\dot{N}}\Omega^{\dot{M}}{}_{\dot{N}\dot{G}}\dot{\partial}_{\dot{M}}\dot{X}^{\dot{G}}$$

$$+2\dot{X}^{\dot{N}}\dot{Y}^{\dot{G}}\left(\dot{\partial}^{\dot{M}}f_{\dot{M}\dot{N}\dot{G}} + f^{\dot{M}}f_{\dot{M}\dot{N}\dot{G}} + 2\dot{\partial}_{[\dot{N}}f_{\dot{G}]}\right) + f_{\dot{M}}\left(\dot{X}^{\dot{N}}\dot{\partial}^{\dot{M}}\dot{Y}_{\dot{N}} - \dot{Y}^{\dot{N}}\dot{\partial}^{\dot{M}}\dot{X}_{\dot{N}}\right)\Big]\dot{T}_{\dot{A}_1\cdots\dot{A}_n}$$

$$+\textstyle\sum_{i=1}^{n}\Big[\ \dot{\partial}_{\dot{M}}\dot{Y}_{\dot{A}_i}\dot{\partial}^{\dot{M}}\dot{X}_{\dot{B}} - \dot{\partial}_{\dot{M}}\dot{X}_{\dot{A}_i}\dot{\partial}^{\dot{M}}\dot{Y}_{\dot{B}} - \tfrac{1}{2}\Omega_{\dot{M}\dot{A}_i\dot{B}}\left(\dot{X}^{\dot{N}}\dot{\partial}^{\dot{M}}\dot{Y}_{\dot{N}} - \dot{Y}^{\dot{N}}\dot{\partial}^{\dot{M}}\dot{X}_{\dot{N}}\right)$$

$$+3\Omega_{\dot{M}[\dot{A}_i\dot{B}}\dot{X}^{\dot{N}}\dot{\partial}^{\dot{M}}\dot{Y}_{\dot{N}]} - 3\Omega_{\dot{M}[\dot{A}_i\dot{B}}\dot{Y}^{\dot{N}}\dot{\partial}^{\dot{M}}\dot{X}_{\dot{N}]}$$

$$+\dot{X}^{\dot{N}}\dot{Y}^{\dot{G}}\left(\dot{\partial}_{\dot{A}_i}f_{\dot{B}\dot{N}\dot{G}} - 3f_{\dot{M}[\dot{B}\dot{N}}f^{\dot{M}}{}_{\dot{G}]\dot{A}_i} - 3\dot{\partial}_{[\dot{B}}f_{\dot{N}\dot{G}]\dot{A}_i}\right)\Big]\dot{T}_{\dot{A}_1\cdots\dot{A}_{i-1}}{}^{\dot{B}}{}_{\dot{A}_{i+1}\cdots\dot{A}_n}\,.$$

Twistability condition

Sufficient conditions for the closure

1. The section condition for all the dotted twisted fields,
$$\dot{\partial}_{\dot{M}}\dot{\partial}^{\dot{M}} \equiv 0\,.$$
2. The orthogonality between the connection and the derivatives of the dotted twisted fields,
$$\Omega^{\dot{M}}{}_{\dot{F}\dot{G}}\dot{\partial}_{\dot{M}} \equiv 0\,.$$
3. The Jacobi identity for $f_{\dot{A}\dot{B}\dot{C}} = f_{[\dot{A}\dot{B}\dot{C}]}$,
$$f_{[\dot{A}\dot{B}}{}^{\dot{E}}f_{\dot{C}]\dot{D}\dot{E}} \equiv 0\,.$$
4. The constancy of the structure constant, $f_{\dot{A}\dot{B}\dot{C}}$,
$$\dot{\partial}_{\dot{E}}f_{\dot{A}\dot{B}\dot{C}} \equiv 0\,.$$
5. The triviality of $f_{\dot{A}}$,
$$f_{\dot{A}} = \Omega^{\dot{C}}{}_{\dot{C}\dot{A}} - 2\dot{\partial}_{\dot{A}}\lambda = \partial_C U^C{}_{\dot{A}} - 2\dot{\partial}_{\dot{A}}\lambda \equiv 0\,.$$

It might be interesting to investigate the general compatibility condition if any.

Twistability condition

Sufficient conditions for the closure

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For the usual Sherk-Shwarz ansatz, 3-5th conditions are genuine consistency conditions same as [Grana, Marques]

Twistability condition

- The U-twisted generalized Lie derivative reduces, upon the twistability conditions, to

$$\hat{\mathcal{L}}_{\dot{X}}\dot{T}_{\dot{A}_1\cdots\dot{A}_n} \equiv \dot{X}^{\dot{B}}\dot{\partial}_{\dot{B}}\dot{T}_{\dot{A}_1\cdots\dot{A}_n} + \omega\dot{\partial}_{\dot{B}}\dot{X}^{\dot{B}}\dot{T}_{\dot{A}_1\cdots\dot{A}_n} + \sum_{i=1}^{n}\left(2\dot{\partial}_{[\dot{A}_i}\dot{X}_{\dot{B}]} + f_{\dot{A}_i\dot{B}\dot{C}}\dot{X}^{\dot{C}}\right)\dot{T}_{\dot{A}_1\cdots\dot{A}_{i-1}}{}^{\dot{B}}{}_{\dot{A}_{i+1}\cdots\dot{A}_n}$$

This is the gauge transformation of gauged DFT

Twisted semi-covariant formalism

- *U-twisted master semi-covariant derivative* is

$$\dot{\mathcal{D}}_{\dot{A}} = \dot{\nabla}_{\dot{A}} + \dot{\Phi}_{\dot{A}} + \dot{\bar{\Phi}}_{\dot{A}} \,,$$

of which the twisted semi-covariant derivative and the twisted spin connections are given by

$$\dot{\nabla}_{\dot{A}} = \dot{D}_{\dot{A}} + \dot{\Gamma}_{\dot{A}} = \dot{\partial}_{\dot{A}} + \Omega_{\dot{A}} + \dot{\Gamma}_{\dot{A}} \,, \qquad \dot{\Phi}_{\dot{A}pq} = \dot{V}^{\dot{B}}{}_{p} \dot{\nabla}_{\dot{A}} \dot{V}_{\dot{B}q} \,, \qquad \dot{\bar{\Phi}}_{\dot{A}\bar{p}\bar{q}} = \dot{\bar{V}}^{\dot{B}}{}_{\bar{p}} \dot{\nabla}_{\dot{A}} \dot{\bar{V}}_{\dot{B}\bar{q}} \,,$$

- the twisted torsionless connection reads

$$\begin{aligned} \dot{\Gamma}_{\dot{C}\dot{A}\dot{B}} = \; & 2(\dot{P}\dot{D}_{\dot{C}}\dot{P}\dot{\bar{P}})_{[\dot{A}\dot{B}]} + 2(\dot{\bar{P}}_{[\dot{A}}{}^{\dot{D}}\dot{\bar{P}}_{\dot{B}]}{}^{\dot{E}} - \dot{P}_{[\dot{A}}{}^{\dot{D}}\dot{P}_{\dot{B}]}{}^{\dot{E}})\dot{D}_{\dot{D}}\dot{P}_{\dot{E}\dot{C}} \\ & - \tfrac{4}{D-1}(\dot{\bar{P}}_{\dot{C}[\dot{A}}\dot{\bar{P}}_{\dot{B}]}{}^{\dot{D}} + \dot{P}_{\dot{C}[\dot{A}}\dot{P}_{\dot{B}]}{}^{\dot{D}})\left(\dot{D}_{\dot{D}}\dot{d} + (\dot{P}\dot{D}^{\dot{E}}\dot{P}\dot{\bar{P}})_{[\dot{E}\dot{D}]}\right) \,, \end{aligned}$$

- "effective connection" reads explicitly,

$$\begin{aligned} \Omega_{\dot{C}\dot{A}\dot{B}} + \dot{\Gamma}_{\dot{C}\dot{A}\dot{B}} \equiv \; & 2(\dot{P}\dot{\partial}_{\dot{C}}\dot{P}\dot{\bar{P}})_{[\dot{A}\dot{B}]} + 2(\dot{\bar{P}}_{[\dot{A}}{}^{\dot{D}}\dot{\bar{P}}_{\dot{B}]}{}^{\dot{E}} - \dot{P}_{[\dot{A}}{}^{\dot{D}}\dot{P}_{\dot{B}]}{}^{\dot{E}})\dot{\partial}_{\dot{D}}\dot{P}_{\dot{E}\dot{C}} \\ & - \tfrac{4}{D-1}(\dot{\bar{P}}_{\dot{C}[\dot{A}}\dot{\bar{P}}_{\dot{B}]}{}^{\dot{D}} + \dot{P}_{\dot{C}[\dot{A}}\dot{P}_{\dot{B}]}{}^{\dot{D}})\left(\dot{\partial}_{\dot{D}}\dot{d} + (\dot{P}\dot{\partial}^{\dot{E}}\dot{P}\dot{\bar{P}})_{[\dot{E}\dot{D}]}\right) \\ & + (\dot{\bar{P}}_{\dot{C}}{}^{\dot{D}}\dot{P}_{\dot{A}}{}^{\dot{E}}\dot{P}_{\dot{B}}{}^{\dot{F}} + \dot{P}_{\dot{C}}{}^{\dot{D}}\dot{\bar{P}}_{\dot{A}}{}^{\dot{E}}\dot{\bar{P}}_{\dot{B}}{}^{\dot{F}})f_{\dot{D}\dot{E}\dot{F}} \\ & + (\dot{\mathcal{P}} + \dot{\bar{\mathcal{P}}})_{\dot{C}\dot{A}\dot{B}}{}^{\dot{D}\dot{E}\dot{F}}\Omega_{\dot{D}\dot{E}\dot{F}} \,. \end{aligned}$$

The torsionful deformation [Berman, Lee] is derived as the effective torsion.

Twisted semi-covariant formalism

- *U-twisted master semi-covariant derivative* is

$$\dot{\mathcal{D}}_{\dot{A}} = \dot{\nabla}_{\dot{A}} + \dot{\Phi}_{\dot{A}} + \dot{\bar{\Phi}}_{\dot{A}}\,,$$

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- the twisted torsionless connection reads

$$\begin{aligned}\dot{\Gamma}_{\dot{C}\dot{A}\dot{B}} = \;& 2(\dot{P}\dot{D}_{\dot{C}}\dot{P}\dot{\bar{P}})_{[\dot{A}\dot{B}]} + 2(\dot{\bar{P}}_{[\dot{A}}{}^{\dot{D}}\dot{\bar{P}}_{\dot{B}]}{}^{\dot{E}} - \dot{P}_{[\dot{A}}{}^{\dot{D}}\dot{P}_{\dot{B}]}{}^{\dot{E}})\dot{D}_{\dot{D}}\dot{P}_{\dot{E}\dot{C}} \\ & -\tfrac{4}{D-1}(\dot{\bar{P}}_{\dot{C}[\dot{A}}\dot{\bar{P}}_{\dot{B}]}{}^{\dot{D}} + \dot{P}_{\dot{C}[\dot{A}}\dot{P}_{\dot{B}]}{}^{\dot{D}})\left(\dot{D}_{\dot{D}}\dot{d} + (\dot{P}\dot{D}^{\dot{E}}\dot{P}\dot{\bar{P}})_{[\dot{E}\dot{D}]}\right)\,,\end{aligned}$$

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- the twisted torsionless connection reads

$$\begin{aligned}\dot{\Gamma}_{\dot{C}\dot{A}\dot{B}} = \;& 2(\dot{P}\dot{D}_{\dot{C}}\dot{P}\dot{\bar{P}})_{[\dot{A}\dot{B}]} + 2(\dot{\bar{P}}_{[\dot{A}}{}^{\dot{D}}\dot{\bar{P}}_{\dot{B}]}{}^{\dot{E}} - \dot{P}_{[\dot{A}}{}^{\dot{D}}\dot{P}_{\dot{B}]}{}^{\dot{E}})\dot{D}_{\dot{D}}\dot{P}_{\dot{E}\dot{C}} \\ & -\tfrac{4}{D-1}(\dot{\bar{P}}_{\dot{C}[\dot{A}}\dot{\bar{P}}_{\dot{B}]}{}^{\dot{D}} + \dot{P}_{\dot{C}[\dot{A}}\dot{P}_{\dot{B}]}{}^{\dot{D}})\left(\dot{D}_{\dot{D}}\dot{d} + (\dot{P}\dot{D}^{\dot{E}}\dot{P}\dot{\bar{P}})_{[\dot{E}\dot{D}]}\right)\,,\end{aligned}$$

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The torsionful deformation [Berman, Lee] is derived as the effective torsion.

Twisted semi-covariant formalism

- The semi-covariant formulation also works for the twisted semi-covariant derivative.
  Upon all the twistability conditions, we obtain

$$(\delta_{\dot{X}} - \hat{\mathcal{L}}_{\dot{X}})(\dot{\nabla}_{\dot{C}}\dot{T}_{\dot{A}_1\cdots\dot{A}_n}) \equiv \sum_{i=1}^{n} (\mathcal{P}+\bar{\mathcal{P}})_{\dot{C}\dot{A}_i}{}^{\dot{B}}\dot{T}_{\dot{A}_1\cdots\dot{A}_{i-1}\dot{B}\dot{A}_{i+1}\cdots\dot{A}_n}\,,$$

  once again *the anomalies are all controlled by the index-six projection operators. Namely, they are still semi-covariant.*

- We get various covariant derivatives in the same way,

$$\dot{P}_{\dot{C}}{}^{\dot{D}}\dot{\bar{P}}_{\dot{A}_1}{}^{\dot{B}_1}\cdots\dot{\bar{P}}_{\dot{A}_n}{}^{\dot{B}_n}\dot{\nabla}_{\dot{D}}\dot{T}_{\dot{B}_1\cdots\dot{B}_n}\,,\cdots$$

$$\dot{\mathcal{D}}_p T_{\bar{q}_1\cdots\bar{q}_n}\,, \qquad \dot{\mathcal{D}}_p T^p{}_{\bar{q}_1\cdots\bar{q}_n}\,,\cdots$$

$$\gamma^p\dot{\mathcal{D}}_p\rho\,, \qquad \gamma^p\dot{\mathcal{D}}_p\psi_{\bar{p}}\,,\cdots\,,etc.$$

Curvature

- Compare the two possible semi-covariant curvatures upon the twistability conditions,
$$\dot{\mathcal{G}}_{\dot{A}\dot{B}\dot{C}\dot{D}} \equiv \dot{S}_{\dot{A}\dot{B}\dot{C}\dot{D}} + \tfrac{1}{2}\Omega_{\dot{E}\dot{A}\dot{B}}\Omega^{\dot{E}}{}_{\dot{C}\dot{D}}\,.$$
- $\mathcal{G}_{\dot{A}\dot{B}\dot{C}\dot{D}}$ differs from $\dot{S}_{\dot{A}\dot{B}\dot{C}\dot{D}}$ after the twist.
- In the twisted SDFT to be constructed below, we shall employ $\mathcal{G}_{\dot{A}\dot{B}\dot{C}\dot{D}}$ only. It turns out to be semi-covariant, while the other is not.
- The completely covariant index-two (“Ricci") and index-zero (scalar) twisted curvatures are as untwisted cases,
$$\dot{\mathcal{G}}_{pr\bar{q}}{}^{r}\,,\qquad \dot{\mathcal{G}}_{p\bar{r}\bar{q}}{}^{\bar{r}}\,,\qquad \dot{\mathcal{G}}_{pq}{}^{pq}\,,\qquad \dot{\mathcal{G}}_{\bar{p}\bar{q}}{}^{\bar{p}\bar{q}}\,.$$
Their covariance is guarented as they are related to the completely twisted covariant derivatives,
$$\begin{aligned}
&\tfrac{1}{2}[\dot{\mathcal{D}}_p,\dot{\mathcal{D}}_{\bar{q}}]T^p \equiv \dot{\mathcal{G}}_{pr\bar{q}}{}^{r}T^p\,, && \tfrac{1}{2}[\dot{\mathcal{D}}_p,\dot{\mathcal{D}}_{\bar{q}}]T^{\bar{q}} \equiv -\dot{\mathcal{G}}_{p\bar{r}\bar{q}}{}^{\bar{r}}T^{\bar{q}}\,,\\
&[\gamma^p\dot{\mathcal{D}}_p,\dot{\mathcal{D}}_{\bar{q}}]\varepsilon \equiv \dot{\mathcal{G}}_{pr\bar{q}}{}^{r}\gamma^p\varepsilon\,, && [\dot{\mathcal{D}}_p,\bar{\gamma}^{\bar{q}}\dot{\mathcal{D}}_{\bar{q}}]\varepsilon' \equiv -\dot{\mathcal{G}}_{p\bar{r}\bar{q}}{}^{\bar{r}}\gamma^{\bar{q}}\varepsilon'\,,\\
&(\gamma^p\dot{\mathcal{D}}_p)^2\varepsilon + \dot{\mathcal{D}}_{\bar{p}}\dot{\mathcal{D}}^{\bar{p}}\varepsilon \equiv -\tfrac{1}{4}\dot{\mathcal{G}}_{pq}{}^{pq}\varepsilon\,, && (\bar{\gamma}^{\bar{p}}\dot{\mathcal{D}}_{\bar{p}})^2\varepsilon' + \dot{\mathcal{D}}_p\dot{\mathcal{D}}^p\varepsilon' \equiv -\tfrac{1}{4}\dot{\mathcal{G}}_{\bar{p}\bar{q}}{}^{\bar{p}\bar{q}}\varepsilon'\,.
\end{aligned}$$
- Using $\mathcal{G}_{\dot{A}\dot{B}\dot{C}\dot{D}}$, the supersymmetric completion will be possible.

Curvature

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$$\dot{\mathcal{G}}_{\dot{A}\dot{B}\dot{C}\dot{D}} \equiv \dot{S}_{\dot{A}\dot{B}\dot{C}\dot{D}} + \tfrac{1}{2}\Omega_{\dot{E}\dot{A}\dot{B}}\Omega^{\dot{E}}{}_{\dot{C}\dot{D}}\,.$$
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[\gamma^{p}\dot{\mathcal{D}}_p,\dot{\mathcal{D}}_{\bar{q}}]\varepsilon \equiv \dot{\mathcal{G}}_{pr\bar{q}}{}^{r}\gamma^{p}\varepsilon\,, & [\dot{\mathcal{D}}_p,\bar{\gamma}^{\bar{q}}\dot{\mathcal{D}}_{\bar{q}}]\varepsilon' \equiv -\dot{\mathcal{G}}_{p\bar{r}\bar{q}}{}^{\bar{r}}\gamma^{\bar{q}}\varepsilon'\,,\\
(\gamma^{p}\dot{\mathcal{D}}_p)^2\varepsilon + \dot{\mathcal{D}}_{\bar{p}}\dot{\mathcal{D}}^{\bar{p}}\varepsilon \equiv -\tfrac{1}{4}\dot{\mathcal{G}}_{pq}{}^{pq}\varepsilon\,, & (\bar{\gamma}^{\bar{p}}\dot{\mathcal{D}}_{\bar{p}})^2\varepsilon' + \dot{\mathcal{D}}_{p}\dot{\mathcal{D}}^{p}\varepsilon' \equiv -\tfrac{1}{4}\dot{\mathcal{G}}_{\bar{p}\bar{q}}{}^{\bar{p}\bar{q}}\varepsilon'\,.
\end{array}$$
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Curvature

- Compare the two possible semi-covariant curvatures upon the twistability conditions,

$$\dot{\mathcal{G}}_{\dot{A}\dot{B}\dot{C}\dot{D}} \equiv \dot{S}_{\dot{A}\dot{B}\dot{C}\dot{D}} + \tfrac{1}{2}\Omega_{\dot{E}\dot{A}\dot{B}}\Omega^{\dot{E}}{}_{\dot{C}\dot{D}} \,.$$

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\tfrac{1}{2}[\dot{\mathcal{D}}_{p},\dot{\mathcal{D}}_{\bar{q}}]T^{p} \equiv \dot{\mathcal{G}}_{pr\bar{q}}{}^{r}T^{p}\,, & \tfrac{1}{2}[\dot{\mathcal{D}}_{p},\dot{\mathcal{D}}_{\bar{q}}]T^{\bar{q}} \equiv -\dot{\mathcal{G}}_{p\bar{r}\bar{q}}{}^{\bar{r}}T^{\bar{q}}\,,\\
[\gamma^{p}\dot{\mathcal{D}}_{p},\dot{\mathcal{D}}_{\bar{q}}]\varepsilon \equiv \dot{\mathcal{G}}_{pr\bar{q}}{}^{r}\gamma^{p}\varepsilon\,, & [\dot{\mathcal{D}}_{p},\bar{\gamma}^{\bar{q}}\dot{\mathcal{D}}_{\bar{q}}]\varepsilon' \equiv -\dot{\mathcal{G}}_{p\bar{r}\bar{q}}{}^{\bar{r}}\gamma^{\bar{q}}\varepsilon'\,,\\
(\gamma^{p}\dot{\mathcal{D}}_{p})^{2}\varepsilon + \dot{\mathcal{D}}_{\bar{p}}\dot{\mathcal{D}}^{\bar{p}}\varepsilon \equiv -\tfrac{1}{4}\dot{\mathcal{G}}_{pq}{}^{pq}\varepsilon\,, & (\bar{\gamma}^{\bar{p}}\dot{\mathcal{D}}_{\bar{p}})^{2}\varepsilon' + \dot{\mathcal{D}}_{p}\dot{\mathcal{D}}^{p}\varepsilon' \equiv -\tfrac{1}{4}\dot{\mathcal{G}}_{\bar{p}\bar{q}}{}^{\bar{p}\bar{q}}\varepsilon'\,.
\end{array}$$

- Using $\mathcal{G}_{\dot{A}\dot{B}\dot{C}\dot{D}}$, the supersymmetric completion will be possible.

Condition for RR cohomology

- We replace $\dot{S}_{\dot{A}\dot{B}\dot{C}\dot{D}}$ by $\dot{\mathcal{G}}_{\dot{A}\dot{B}\dot{C}\dot{D}}$.
- Almost all the properties of the four-index curvature still hold after the twist, up to the twistability conditions,

$$\dot{\mathcal{G}}_{\dot{A}[\dot{B}\dot{C}\dot{D}]} \equiv 0\,. \quad \text{etc.}$$

The only exception is

$$\dot{\mathcal{G}}_{pq}{}^{pq} + \dot{\mathcal{G}}_{\bar{p}\bar{q}}{}^{\bar{p}\bar{q}} \equiv \tfrac{1}{6} f_{\dot{A}\dot{B}\dot{C}} f^{\dot{A}\dot{B}\dot{C}}\,.$$

- It follows the modification of the Ramond-Ramnond cohomology,

$$(\dot{\mathcal{D}}_{\pm})^2 \mathcal{T} \equiv -\tfrac{1}{24} f_{\dot{A}\dot{B}\dot{C}} f^{\dot{A}\dot{B}\dot{C}} \mathcal{T}\,.$$

We should separately impose

$$f_{\dot{A}\dot{B}\dot{C}} f^{\dot{A}\dot{B}\dot{C}} \equiv 0\,.$$

- To incorporate RR field, we have to impose this extra consistency condition.
- This will also be necessary for the maximal supersymmetry .

Supersymmetric gauged double field they

- The half-maximal supersymmetric gauged double field theory Lagrangian

$$\dot{\mathcal{L}}^{\text{Half}-\text{maximal}}_{\text{Twisted SDFT}} = e^{-2\dot{d}}\Big[\tfrac{1}{4}\dot{\mathcal{G}}_{pq}{}^{pq} + i\tfrac{1}{2}\bar{\rho}\gamma^{p}\dot{\mathcal{D}}_{p}\rho - i\bar{\psi}^{\bar{p}}\dot{\mathcal{D}}_{\bar{p}}\rho - i\tfrac{1}{2}\bar{\psi}^{\bar{p}}\gamma^{q}\dot{\mathcal{D}}_{q}\psi_{\bar{p}}\Big]\,.$$

  The leading order half-maximal twisted supersymmetry transformation rules

$$\delta_{\varepsilon}\dot{d} = -i\tfrac{1}{2}\bar{\varepsilon}\rho\,, \qquad \delta_{\varepsilon}\dot{V}_{Ap} = -i\dot{\bar{V}}_{A}{}^{\bar{q}}\bar{\varepsilon}\gamma_{p}\psi_{\bar{q}}\,, \qquad \delta_{\varepsilon}\dot{\bar{V}}_{A\bar{p}} = +i\dot{V}_{A}{}^{q}\bar{\varepsilon}\gamma_{q}\psi_{\bar{p}}\,,$$

$$\delta_{\varepsilon}\rho = -\gamma^{p}\dot{\mathcal{D}}_{p}\varepsilon\,, \qquad \delta_{\varepsilon}\psi_{\bar{p}} = \dot{\mathcal{D}}_{\bar{p}}\varepsilon\,.$$

- Higher order of fermionic terms are same as the terms in untwisted SDFT.

Supersymmetric gauged double field theory

- From the Z_2 symmetry which exchanges the two spin groups, $\mathbf{Spin}(1,9) \leftrightarrow \mathbf{Spin}(9,1)$, there is a parallel formulation of the half-maximal SDFT,

$$\overline{\dot{\mathcal{L}}^{\text{Half-maximal}}_{\text{Twisted SDFT}}} = e^{-2\dot{d}}\Big[-\tfrac{1}{4}\dot{\mathcal{G}}_{\bar{p}\bar{q}}{}^{\bar{p}\bar{q}} - i\tfrac{1}{2}\bar{\rho}'\bar{\gamma}^{\bar{p}}\dot{\mathcal{D}}_{\bar{p}}\rho' + i\bar{\psi}'^{p}\dot{\mathcal{D}}_{p}\rho' + i\tfrac{1}{2}\bar{\psi}'^{p}\bar{\gamma}^{\bar{q}}\dot{\mathcal{D}}_{\bar{q}}\psi'{}_{p}\Big] .$$

The supersymmetry is realized by

$$\delta_\varepsilon \dot{d} = -i\tfrac{1}{2}\bar{\varepsilon}'\rho' , \qquad \delta_\varepsilon \dot{V}_{\dot{A}p} = +i\bar{\varepsilon}'\bar{\gamma}_{\dot{A}}\psi'_{p} , \qquad \delta_\varepsilon \dot{\bar{V}}_{\dot{A}\bar{p}} = -i\bar{\varepsilon}'\bar{\gamma}_{\bar{p}}\psi'_{\dot{A}} ,$$

$$\delta_\varepsilon \rho' = -\bar{\gamma}^{\bar{p}}\dot{\mathcal{D}}_{\bar{p}}\varepsilon' , \qquad \delta_\varepsilon \psi'_{p} = \dot{\mathcal{D}}_{p}\varepsilon' .$$

Supersymmetric gauged double field theory

- Maximal supersymmetric gauged double field theory Lagrangian,

$$\dot{\mathcal{L}}^{\text{Maximal}}_{\text{Twisted SDFT}} = e^{-2\dot{d}}\Big[\ \tfrac{1}{8}(\dot{\mathcal{G}}_{pq}{}^{pq} - \dot{\mathcal{G}}_{\bar{p}\bar{q}}{}^{\bar{p}\bar{q}}) + \tfrac{1}{2}\mathrm{Tr}(\dot{\mathcal{F}}\bar{\dot{\mathcal{F}}}) - i\bar{\rho}\dot{\mathcal{F}}\rho' + i\bar{\psi}_{\bar{p}}\gamma_q\dot{\mathcal{F}}\bar{\gamma}^{\bar{p}}\psi'^{q}$$
$$+i\tfrac{1}{2}\bar{\rho}\gamma^p\dot{\mathcal{D}}_p\rho - i\bar{\psi}^{\bar{p}}\dot{\mathcal{D}}_{\bar{p}}\rho - i\tfrac{1}{2}\bar{\psi}^{\bar{p}}\gamma^q\dot{\mathcal{D}}_q\psi_{\bar{p}} - i\tfrac{1}{2}\bar{\rho}'\bar{\gamma}^{\bar{p}}\dot{\mathcal{D}}_{\bar{p}}\rho' + i\bar{\psi}'^{p}\dot{\mathcal{D}}_p\rho' + i\tfrac{1}{2}\bar{\psi}'^{p}\bar{\gamma}^{\bar{q}}\dot{\mathcal{D}}_{\bar{q}}\psi'{}_p$$

$$\delta_\varepsilon\dot{d} = -i\tfrac{1}{2}(\bar{\varepsilon}\rho + \bar{\varepsilon}'\rho')\,,$$
$$\delta_\varepsilon\dot{V}_{\dot{A}p} = i\bar{\dot{V}}_{\dot{A}}{}^{\bar{q}}(\bar{\varepsilon}'\bar{\gamma}_{\bar{q}}\psi'_p - \bar{\varepsilon}\gamma_p\psi_{\bar{q}})\,,\quad \delta_\varepsilon\bar{\dot{V}}_{\dot{A}\bar{p}} = i\dot{V}_{\dot{A}}{}^{q}(\bar{\varepsilon}\gamma_q\psi_{\bar{p}} - \bar{\varepsilon}'\bar{\gamma}_{\bar{p}}\psi'_q)\,,$$
$$\delta_\varepsilon\mathcal{C} = i\tfrac{1}{2}(\gamma^p\varepsilon\bar{\psi}'_p - \varepsilon\bar{\rho}' - \psi_{\bar{p}}\bar{\varepsilon}'\bar{\gamma}^{\bar{p}} + \rho\bar{\varepsilon}') + \mathcal{C}\delta_\varepsilon\dot{d} - \tfrac{1}{2}(\bar{\dot{V}}^{\dot{A}}{}_{\bar{q}}\,\delta_\varepsilon\dot{V}_{\dot{A}p})\gamma^{(11)}\gamma^p\mathcal{C}\bar{\gamma}^{\bar{q}}\,,$$
$$\delta_\varepsilon\rho = -\gamma^p\dot{\mathcal{D}}_p\varepsilon\,, \delta_\varepsilon\rho' = -\bar{\gamma}^{\bar{p}}\dot{\mathcal{D}}_{\bar{p}}\varepsilon'\,,$$
$$\delta_\varepsilon\psi_{\bar{p}} = \dot{\mathcal{D}}_{\bar{p}}\varepsilon + \dot{\mathcal{F}}\bar{\gamma}_{\bar{p}}\varepsilon'\,, \delta_\varepsilon\psi'_p = \dot{\mathcal{D}}_p\varepsilon' + \bar{\dot{\mathcal{F}}}\gamma_p\varepsilon\,.$$

Supersymmetric gauged double field theory

Under the supersymmetry transformation

$$\delta_\varepsilon \dot{\mathcal{L}}^{\text{Maximal}}_{\text{Twisted SDFT}} \equiv i\tfrac{1}{48}e^{-2\dot{d}}\left(\bar{\rho}\varepsilon - \bar{\rho}'\varepsilon' + \bar{\varepsilon}\mathcal{C}\rho' + \bar{\varepsilon}\gamma^{p}\mathcal{C}\psi'_{p} + \bar{\rho}\mathcal{C}\varepsilon' + \bar{\psi}_{\bar{p}}\mathcal{C}\bar{\gamma}^{\bar{p}}\varepsilon'\right) \times f_{\dot{A}\dot{B}\dot{C}}f^{\dot{A}\dot{B}\dot{C}}$$

$$+i\tfrac{1}{8}e^{-2d}(\bar{\varepsilon}\gamma_{p}\psi_{\bar{q}} - \bar{\varepsilon}'\bar{\gamma}_{\bar{q}}\psi'_{p})\text{Tr}\left(\gamma^{p}\dot{\mathcal{F}}_{-}\bar{\gamma}^{\bar{q}}\overline{\dot{\mathcal{F}}_{-}}\right) .$$

- Thus, requiring the extra condition

$$f_{\dot{A}\dot{B}\dot{C}}f^{\dot{A}\dot{B}\dot{C}} \equiv 0\,,$$

the action is supersymmetric invariant modulo the self-duality.

Comparison with the untwisted case

- Look at the NS-NS sector of two half-maximal Lagrangian.

$$
\begin{aligned}
+\dot{\mathcal{G}}_{pq}{}^{pq} \quad \equiv \quad & \tfrac{1}{16}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{\mathcal{H}}_{\dot{C}\dot{D}}\dot{\partial}_{\dot{B}}\dot{\mathcal{H}}^{\dot{C}\dot{D}} + \tfrac{1}{4}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}^{\dot{C}}\dot{\mathcal{H}}_{\dot{A}\dot{D}}\dot{\partial}^{\dot{D}}\dot{\mathcal{H}}_{\dot{B}\dot{C}} - \tfrac{1}{2}\dot{\partial}_{\dot{A}}\dot{\partial}_{\dot{B}}\dot{\mathcal{H}}^{\dot{A}\dot{B}} \\
& -2\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{d}\dot{\partial}_{\dot{B}}\dot{d} + 2\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{\partial}_{\dot{B}}\dot{d} + 2\dot{\partial}_{\dot{A}}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{B}}\dot{d} \\
& +\tfrac{1}{8}f_{\dot{A}\dot{B}\dot{C}}f^{\dot{A}\dot{B}}{}_{\dot{D}}\dot{\mathcal{H}}^{\dot{C}\dot{D}} - \tfrac{1}{24}f_{\dot{A}\dot{B}\dot{C}}f_{\dot{D}\dot{E}\dot{F}}\dot{\mathcal{H}}^{\dot{A}\dot{D}}\dot{\mathcal{H}}^{\dot{B}\dot{E}}\dot{\mathcal{H}}^{\dot{C}\dot{F}} - \tfrac{1}{4}f_{\dot{A}\dot{B}\dot{C}}\dot{\mathcal{H}}^{\dot{B}\dot{D}}\dot{\mathcal{H}}^{\dot{C}\dot{E}}\dot{\partial}_{\dot{D}}\dot{\mathcal{H}}_{\dot{E}}{}^{\dot{A}} \\
& +\tfrac{1}{12}f_{\dot{A}\dot{B}\dot{C}}f^{\dot{A}\dot{B}\dot{C}}\,, \\
-\dot{\mathcal{G}}_{\bar{p}\bar{q}}{}^{\bar{p}\bar{q}} \quad \equiv \quad & \tfrac{1}{16}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{\mathcal{H}}_{\dot{C}\dot{D}}\dot{\partial}_{\dot{B}}\dot{\mathcal{H}}^{\dot{C}\dot{D}} + \tfrac{1}{4}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}^{\dot{C}}\dot{\mathcal{H}}_{\dot{A}\dot{D}}\dot{\partial}^{\dot{D}}\dot{\mathcal{H}}_{\dot{B}\dot{C}} - \tfrac{1}{2}\dot{\partial}_{\dot{A}}\dot{\partial}_{\dot{B}}\dot{\mathcal{H}}^{\dot{A}\dot{B}} \\
& -2\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{d}\dot{\partial}_{\dot{B}}\dot{d} + 2\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{\partial}_{\dot{B}}\dot{d} + 2\dot{\partial}_{\dot{A}}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{B}}\dot{d} \\
& +\tfrac{1}{8}f_{\dot{A}\dot{B}\dot{C}}f^{\dot{A}\dot{B}}{}_{\dot{D}}\dot{\mathcal{H}}^{\dot{C}\dot{D}} - \tfrac{1}{24}f_{\dot{A}\dot{B}\dot{C}}f_{\dot{D}\dot{E}\dot{F}}\dot{\mathcal{H}}^{\dot{A}\dot{D}}\dot{\mathcal{H}}^{\dot{B}\dot{E}}\dot{\mathcal{H}}^{\dot{C}\dot{F}} - \tfrac{1}{4}f_{\dot{A}\dot{B}\dot{C}}\dot{\mathcal{H}}^{\dot{B}\dot{D}}\dot{\mathcal{H}}^{\dot{C}\dot{E}}\dot{\partial}_{\dot{D}}\dot{\mathcal{H}}_{\dot{E}}{}^{\dot{A}} \\
& -\tfrac{1}{12}f_{\dot{A}\dot{B}\dot{C}}f^{\dot{A}\dot{B}\dot{C}}\,.
\end{aligned}
$$

- It reproduces the previous result [Geissbuler,Aldazabal,Grana, Marques] , and it matches with the $N = 4$ $D = 4$ gauged SUGRA [Schon, Weidner] .
- The fourth line is the cosmological constant. Each one has different sign of the cosmological constant.

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& -2\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{d}\dot{\partial}_{\dot{B}}\dot{d} + 2\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{\partial}_{\dot{B}}\dot{d} + 2\dot{\partial}_{\dot{A}}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{B}}\dot{d} \\
& +\tfrac{1}{8}f_{\dot{A}\dot{B}\dot{C}}f^{\dot{A}\dot{B}}{}_{\dot{D}}\dot{\mathcal{H}}^{\dot{C}\dot{D}} - \tfrac{1}{24}f_{\dot{A}\dot{B}\dot{C}}f_{\dot{D}\dot{E}\dot{F}}\dot{\mathcal{H}}^{\dot{A}\dot{D}}\dot{\mathcal{H}}^{\dot{B}\dot{E}}\dot{\mathcal{H}}^{\dot{C}\dot{F}} - \tfrac{1}{4}f_{\dot{A}\dot{B}\dot{C}}\dot{\mathcal{H}}^{\dot{B}\dot{D}}\dot{\mathcal{H}}^{\dot{C}\dot{E}}\dot{\partial}_{\dot{D}}\dot{\mathcal{H}}_{\dot{E}}{}^{\dot{A}} \\
& +\tfrac{1}{12}f_{\dot{A}\dot{B}\dot{C}}f^{\dot{A}\dot{B}\dot{C}}\,, \\
-\dot{\mathcal{G}}_{\bar{p}\bar{q}}{}^{\bar{p}\bar{q}} \equiv\; & \tfrac{1}{16}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{\mathcal{H}}_{\dot{C}\dot{D}}\dot{\partial}_{\dot{B}}\dot{\mathcal{H}}^{\dot{C}\dot{D}} + \tfrac{1}{4}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}^{\dot{C}}\dot{\mathcal{H}}_{\dot{A}\dot{D}}\dot{\partial}^{\dot{D}}\dot{\mathcal{H}}_{\dot{B}\dot{C}} - \tfrac{1}{2}\dot{\partial}_{\dot{A}}\dot{\partial}_{\dot{B}}\dot{\mathcal{H}}^{\dot{A}\dot{B}} \\
& -2\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{d}\dot{\partial}_{\dot{B}}\dot{d} + 2\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{\partial}_{\dot{B}}\dot{d} + 2\dot{\partial}_{\dot{A}}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{B}}\dot{d} \\
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$$

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Comparison with the untwisted case

- NS-NS sector of maximal gauged SDFT.

$$
\begin{aligned}
&+\dot{\mathcal{G}}_{pq}{}^{pq} - \dot{\mathcal{G}}_{\bar{p}\bar{q}}{}^{\bar{p}\bar{q}} \\
&\equiv \tfrac{1}{8}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{\mathcal{H}}_{\dot{C}\dot{D}}\dot{\partial}_{\dot{B}}\dot{\mathcal{H}}^{\dot{C}\dot{D}} + \tfrac{1}{2}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}^{\dot{C}}\dot{\mathcal{H}}_{\dot{A}\dot{D}}\dot{\partial}^{\dot{D}}\dot{\mathcal{H}}_{\dot{B}\dot{C}} - \dot{\partial}_{\dot{A}}\dot{\partial}_{\dot{B}}\dot{\mathcal{H}}^{\dot{A}\dot{B}} \\
&\; - 4\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{d}\dot{\partial}_{\dot{B}}\dot{d} + 4\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{\partial}_{\dot{B}}\dot{d} + 4\dot{\partial}_{\dot{A}}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{B}}\dot{d} \\
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\end{aligned}
$$

Comparison with the untwisted case

- For fermions,

$$\gamma^p \dot{\mathcal{D}}_p \rho \equiv \gamma^p \dot{\mathcal{D}}_p \rho\Big|_{\dot{\partial}} + \tfrac{1}{12} f_{pqr}\gamma^{pqr}\rho \,,$$
$$\dot{\mathcal{D}}_{\bar{p}} \rho \equiv \dot{\mathcal{D}}_{\bar{p}} \rho\Big|_{\dot{\partial}} + \tfrac{1}{4} f_{\bar{p}qr}\gamma^{qr}\rho \,,$$
$$\gamma^q \dot{\mathcal{D}}_q \psi_{\bar{p}} \equiv \gamma^q \dot{\mathcal{D}}_q \psi_{\bar{p}}\Big|_{\dot{\partial}} + \tfrac{1}{12} f_{qrs}\gamma^{qrs}\psi_{\bar{p}} + f_{r\bar{p}\bar{q}}\gamma^r \psi^{\bar{q}} \,,$$

  and

$$\bar{\gamma}^{\bar{p}} \dot{\mathcal{D}}_{\bar{p}} \rho' \equiv \bar{\gamma}^{\bar{p}} \dot{\mathcal{D}}_{\bar{p}} \rho'\Big|_{\dot{\partial}} + \tfrac{1}{12} f_{\bar{p}\bar{q}\bar{r}}\bar{\gamma}^{\bar{p}\bar{q}\bar{r}}\rho' \,,$$
$$\dot{\mathcal{D}}_{p} \rho' \equiv \dot{\mathcal{D}}_{p} \rho'\Big|_{\dot{\partial}} + \tfrac{1}{4} f_{p\bar{q}\bar{r}}\bar{\gamma}^{\bar{q}\bar{r}}\rho' \,,$$
$$\bar{\gamma}^{\bar{q}} \dot{\mathcal{D}}_{\bar{q}} {\psi'}_p \equiv \bar{\gamma}^{\bar{q}} \dot{\mathcal{D}}_{\bar{q}} {\psi'}_p\Big|_{\dot{\partial}} + \tfrac{1}{12} f_{\bar{q}\bar{r}\bar{s}}\bar{\gamma}^{\bar{q}\bar{r}\bar{s}}{\psi'}_p + f_{\bar{r}pq}\bar{\gamma}^{\bar{r}}\psi'^{q} \,.$$

  agree with Berman and Lee.

- For R-R as our new result

$$\dot{\mathcal{F}} = \dot{\mathcal{D}}_+ \mathcal{C} \equiv \dot{\mathcal{D}}_+ \mathcal{C}\Big|_{\dot{\partial}} + \tfrac{1}{12} f_{pqr}\gamma^{pqr}\mathcal{C} - \tfrac{1}{4} f_{p\bar{q}\bar{r}}\gamma^p \mathcal{C}\bar{\gamma}^{\bar{q}\bar{r}} - \tfrac{1}{12} f_{\bar{p}\bar{q}\bar{r}}\gamma^{(11)}\mathcal{C}\bar{\gamma}^{\bar{p}\bar{q}\bar{r}} + \tfrac{1}{4} f_{pq\bar{r}}\gamma^{(11)}\gamma^{pq}\mathcal{C}\bar{\gamma}^{\bar{r}} \,,$$

  The nilpotency of this twisted R-R cohomology implies the Bianchi identity for the twisted R-R flux, which is expected to produce the 'tensor hierarchy' [Bergshoeff, *et. al*; Fernandez-Melgarejo *et. al*]

Summary and comments

- We successfully twisted the semi-covariant formulations of the $\mathcal{N} = 2$ and the $\mathcal{N} = 1, D = 10$ SDFT.

- The semi-covariant four index curvature is refined.

$$\mathcal{G}_{ABCD} := \frac{1}{2}\left[(\mathcal{F}+\bar{\mathcal{F}})_{ABCD} + (\mathcal{F}+\bar{\mathcal{F}})_{CDAB} + (\Phi+\bar{\Phi})^{E}{}_{AB}(\Phi+\bar{\Phi})_{ECD}\right] \,,$$

  It cannot be written in terms of only generalized metric, but should be written in terms of double vielbein.

- Imposing the twistablility conditions, it systematically derives the gauged maximal and half-maximal supersymmetric double field theories,

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- Half-maximal gauged SDFTs has two sectors , which have different signs of the cosmological constant!
- For maximal gauged SDFT, we require

$$f_{\dot{A}\dot{B}\dot{C}}f^{\dot{A}\dot{B}\dot{C}} \equiv 0\,,$$

- R-R potential $\mathcal{C}$ is not twisted. Only the field strength $\dot{\mathcal{F}} = \dot{D}_{+}\mathcal{C}$ is influenced by twisting though the twisted nilpotent operator. We expect that this will change when U-duality is twisted in $\mathcal{M}$-theory setup.

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Thank you!

Conclusion

Thank you.