NON-ABELIAN T-DUALITY AS AN O(D,D) TRANSFORMATION

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Aim of our work

- To see if the new solutions of Type II SUGRA obtained by Non-Abelian T-duality (NATD) can be generated by an O(D,D) transformation.

- We have some partial results for an affirmative answer. However, the O(D,D) matrix in question should be non-constant.

- O(D,D) is the T-duality symmetry of string theory on toroidal backgrounds, which becomes a continous, solution generating symmetry in the supergravity limit.
- Recent devolopments in DFT implies that O(D,D) should be more general , not necessarily requiring the existence of a torus in the background geometry.
- We focus on backgrounds with SU(2) isometry group. The NATD of such backgrounds has been studied recently.

- NATD is applied by using the standard tools of Buscher method:
- consider string theory propogating on a target space with a compact symmetry.
- gauge the symmetry and introduce a Lagrange multiplier which constrains the gauge field to be pure gauge.
- integrating out the Lagrange multiplier gives the original action.
- integrating out the gauge field gives the T-dual action.
- Well- understood when the gauge symmetry is Abelian.
- Some global issues for non-abelian gauge symmetry.

- We focus on NATD of backgrounds with SU(2) isometry:
- We claim:

The dual solutions can be generated by the following non-constant O(10,10) matrix:

$$T = \begin{pmatrix} 0_{3\times 3} & 0_{3\times 7} & I_{3\times 3} & 0_{3\times 7} \\ 0_{7\times 3} & 1_{7\times 7} & 0_{7\times 3} & 0_{7\times 7} \\ I_{3\times 3} & 0_{3\times 7} & b_{3\times 3} & 0_{3\times 7} \\ 0_{7\times 3} & 0_{7\times 7} & 0_{7\times 3} & I_{7\times 7} \end{pmatrix}$$

Here b is the 3 x 3 antisymmetric matrix

$$b = \begin{pmatrix} 0 & x_3 & -x_2 \\ -x_3 & 0 & x_1 \\ x_2 & -x_1 & 0 \end{pmatrix}$$

where x_i are the local coordinates on the group manifold

- This is an O(3,3) matrix embedded in O(10,10):

$$T_3 = \begin{pmatrix} 0_3 & I_3 \\ I_3 & b \end{pmatrix}$$

where O(3,3) acts along the SU(2) isometry directions. Note that $[b]_{ij} = f_{ij}{}^{k} x_k = \epsilon_{ij}{}^{k} x_k$. In the limit where the structure constants vanish we have

$$T_3 \rightarrow \begin{pmatrix} 0_3 & I_3 \\ I_3 & 0_3 \end{pmatrix}$$

OUTLINE

- A brief review of O(D,D), particularly its action on the RR fields.
- A case study (in the Abelian case): generating Lunin-Maldacena solutions with O(3,3), embedded in O(10,10).
- The discussion of NATD as an O(10,10) transformation.

O(D,D) action on the NS-NS sector

- An O(D,D) matrix

$$O = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \quad \text{with} \;\; O^T \, JO = J, \qquad J = \begin{pmatrix} 0 & I_D \\ I_D & 0 \end{pmatrix}$$

acts on the NS sector as

$$E = g + B, \quad \text{as} \quad E \rightarrow (AE + B)(CE + D)^{-1}$$

$$\Phi \rightarrow \frac{\Phi}{det(CE + D)^{-1}}$$

$$g \rightarrow ((CE + D)^{-1})^T \; g \; (CE + D)^{-1}$$

This is for a flat D-dim background. For an N-dim curved background with D commuting isometries, O(D,D) has to be embedded in O(N,N).

O(D,D) is generated by the following 3 types of matrices:

$$\Theta \text{ shifts}: \quad T_b = \begin{pmatrix} I & \Theta \\ 0 & I \end{pmatrix}, \quad \text{with} \quad \Theta^T = -\Theta,$$

$$\text{Basis change}: \quad T_A = \begin{pmatrix} A & 0 \\ 0 & (A^T)^{-1} \end{pmatrix}, \quad \text{with} \quad A \in GL(D,R),$$

$$\text{Factorized duality}: \quad T_{D_i} = \begin{pmatrix} I - e_i & e_i \\ e_i & I - e_i \end{pmatrix}, \quad \text{with} \quad (e_i)_{jk} = \delta_{ij}\delta_{ik}.$$

$$\Theta \text{ shifts}: \quad g \to g, \quad B \to B + \Theta$$

$$\text{Basis change}: \quad g \to A^T g A, \quad B \to A^T B A$$

Factorized duality : generalizes $R \leftrightarrow 1/R$ in the ith direction

$$J = \pm T_{D_1} \cdots T_{D_D} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}: \quad \text{generalizes } R \leftrightarrow 1/R \text{ in all directions}$$

R-R SECTOR

- RR potentials fit naturally into a spinor representation of Pin(D,D). [Fukuma-Oota-Tanaka, Hohm- Zwiebach-Kwak]
- If the NS-NS sector transforms under the action of a given O(D,D) matrix T, the RR sector will transform under S, where ρ(S)=T.

$$\rho: \mathrm{Pin}(D,D) \longrightarrow O(D,D), \qquad \rho(S)\, V = S\, V\, S^{-1}$$

- For a given T, the corresponding Pin(D,D) element is found by solving

$$S\Gamma_M S^{-1} = \Gamma_N\, T^N{}_M, \qquad \rho(S) = T$$

where Γ_M are the 2^D x 2^D Dirac matrices satisfying the Clifford algebra C(D,D)

$$\{\Gamma_M, \Gamma_N\} = 2J_{MN}$$

The RR sector transforms under the action of corresponding elements of Pin(D,D):

$$\rho : \ \mathrm{Pin}(D,D) \to O(D,D)\,,$$

$$\rho(S_b) = T_b : \qquad S_b = e^{-\mathbf{B}} = \exp\left(-\frac{1}{2}B_{ij}\psi^{i\dagger}\psi^{j\dagger}\right)$$

$$\rho(S_A) = T_A : \qquad S_A = \det(A)^{1/2}\exp\left(-\psi^{i\dagger}R_i^j\psi_j\right) \quad (A = (A_i^j) = \exp(R_i^j)),$$

$$\rho(S_i) = T_{D_i} : \qquad S_i = \psi^{i\dagger} + \psi_i, \quad i = 1, \cdots, D.$$

$$\{\boldsymbol{\psi}_i, \boldsymbol{\psi}^{j\dagger}\} = \delta_i^{\ j}\,\mathbf{1}, \qquad \{\boldsymbol{\psi}_i, \boldsymbol{\psi}_j\} = 0 = \{\boldsymbol{\psi}^{i\dagger}, \boldsymbol{\psi}^{j\dagger}\} \qquad (i, j = 1, ..., d).$$

The oscillators realize the Clifford algebra by defining

$$\Gamma_M = (\Gamma_1, \cdots, \Gamma_D, \Gamma_{D+1}, \cdots, \Gamma_{2D}) = (\sqrt{2}\psi^{i\dagger}, \sqrt{2}\psi_i), \quad i = 1, \cdots, D$$

A spinor χ in the 2^D Fock space can be identified with

$$\mid \chi >= \sum_{i=0}^{D} C_{i_1 \cdots i_p} \psi^{i_1\dagger} \cdots \psi^{i_p\dagger} \mid 0 >$$

$$\psi_i \mid 0 >= 0$$

The isomorphism (as a linear space) between the Clifford algebra and Exterior algebra gives a correspondence between polyforms and spinor states

$$\omega = \sum_n \Omega_{i_1 \cdots i_n} dx^{i_1} \wedge \cdots \wedge dx^{i_n} \longleftrightarrow \Omega = \sum_n \Omega_{i_1 \cdots i_n} \Psi^{\dagger i_1} \cdots \Psi^{\dagger i_n}$$

Then we define a spinor state corresponding to ω as

$$\omega \longleftrightarrow \ | \omega > \equiv \ \Omega \, | \, 0 >$$

$$\Omega \, | \, \xi > = | \, \omega \wedge \xi >$$

The transformation of the RR forms under T-duality is determined by the action of the Pin(D,D) elements on the spinor state I F > associated with the polyform encoding all the RR fields of type IIA and Type IIB along with the dual forms:

$$| \, D > \longrightarrow \ S \, | \, D > \qquad | \, F > \longrightarrow \ S \, | \, F >$$

The p-form fields in the RR sector transform as a spinor of Pin(D,D) as follows:

- We introduce the polyforms (democratic formulation of sugra)

$$D \equiv \sum_{p+1=0}^{4} D_{p+1}, \qquad F \equiv \sum_{p+2=1}^{5} F_{p+2}.$$

where D_0 , …, D_4 are the gauge potentials of Type IIA and IIB Sugra (combined with the B-fields in a convenient way) , and

$$F = e^{-B} \wedge \, dD$$

we also introduce dual gauge potentials by solving the eom of D_i inclusion of which results in the polyform

$$F = \sum_{p=0}^{10} F_p, \qquad F_p = -(-1)^{\frac{1}{2}p(p+1)} * F_{10-p}$$

$$| D > \longrightarrow \; S \, | D > \qquad | F > \longrightarrow \; S \, | F >$$

Chirality:

- The 2^D dimensional spinor representation of O(D,D) is reducible.
- One can impose a Weyl condition which yields two spinor representations of opposite chirality of dimension 2^{D-1}
- The spinor states they act on can be identified with polyforms consisting of forms of odd and even degree.
- RR fields of Type IIA and Type IIB transform under these chiral representations.
- S_b and S_A are in $Spin^+(D,D)$ and they preserve the chirality of spinor states they act on.
- Odd powers of S_i changes the chirality of the spinor state exchanging Type IIA and Type IIB
- An important chirality changing element of Pin(D,D) is C, where $\rho(C) = J$.

Example: Multi-parameter Lunin-Maldacena Deformations

- Given a CFT and its supergravity dual via the AdS/CFT correspondence, Lunin and Maldacena found the dual sugra solutions corresponding to marginal deformations of the CFT.

- The method works if the original geometry has 3 commuting isometries. The new solutions can be found via T-dualities.

- In the O(D,D) language, this corresponds to generating a new solution by the action of the following SO(3,3) matrix embedded in O(10,10) in an appropriate way:

$$\begin{pmatrix} 1_3 & 0_3 \\ \Gamma & 1_3 \end{pmatrix}, \qquad \begin{pmatrix} 0 & -\gamma_3 & \gamma_2 \\ \gamma_3 & 0 & -\gamma_1 \\ -\gamma_2 & \gamma_1 & 0 \end{pmatrix}$$

This SO(3,3) matrix can be factorized as

$$\begin{pmatrix} 1_3 & 0_3 \\ \Gamma & 1_3 \end{pmatrix} = \begin{pmatrix} 0 & 1_3 \\ 1_3 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1_3 & \Gamma \\ 0 & 1_3 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1_3 \\ 1_3 & 0 \end{pmatrix}$$

Then the corresponding Spin(3,3) matrix is

$$CS_bC = \exp\left(\frac{1}{2}\Gamma_{mn}\psi_m\psi_n\right)$$

$$\rho(CS_bC) = \rho(C)\rho(S_b)\rho(C) = JT_bJ.$$

Acting on a sugra solution with 3 commuting isometries, such as AdS_5 x $T^{1,1}$ supported by self-dual 5-form flux we end up with a new solution describing a deformed geometry supported by 5-form and 3-form fluxes.

$$
\begin{aligned}
F &= (Vol(AdS_5) + *Vol(AdS_5)) \longrightarrow \exp\,(-\frac{1}{2}\hat{B}_{ij}\Psi^{\dagger i}\Psi^{\dagger j})\hat{F}_{\rm new} \\
\hat{F}_{\rm new} &= \exp\,(\frac{1}{2}\hat{B}_{ij}\Psi^{\dagger i}\Psi^{\dagger j})\;[\exp\,(\frac{1}{2}\Gamma_{mn}\Psi_m\;\Psi_n)(F_5)] \\
&= (Vol(AdS_5) + \hat{*}Vol(AdS_5)) + F_3 + \hat{*}F_3.
\end{aligned}
$$

NATD as an O(D,D) Transformation:

- The NS-NS sector

$$ds^2 = G_{\mu\nu}(x)dx^\mu dx^\nu + 2G_{\mu i}(x)dx^\mu L^i + g_{ij}(x)L^i L^j$$

$$B = \frac{1}{2}B_{\mu\nu}(x)dx^\mu \wedge dx^\nu + B_{\mu i}(x)dx^\mu \wedge L^i + \frac{1}{2}b_{ij}L^i \wedge L^j$$

$$Q_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}, \quad Q_{\mu i} = G_{\mu i} + B_{\mu i}, \quad Q_{i\mu} = G_{i\mu} + B_{i\mu}, \quad E_{ij} = g_{ij} + b_{ij}$$

$$Q = \begin{pmatrix} E_{ij} & Q_{i\mu} \\ Q_{\mu i} & Q_{\mu\nu} \end{pmatrix}$$

$$Q \longrightarrow \hat{Q} = (AQ + B)(CQ + D)^{-1}$$

$$T = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 0_{3\times3} & 0_{3\times7} & I_{3\times3} & 0_{3\times7} \\ 0_{7\times3} & 1_{7\times7} & 0_{7\times3} & 0_{7\times7} \\ I_{3\times3} & 0_{3\times7} & b_{3\times3} & 0_{3\times7} \\ 0_{7\times3} & 0_{7\times7} & 0_{7\times3} & I_{7\times7} \end{pmatrix}$$

$$\hat{Q} = \begin{pmatrix} \hat{E}_{ij} & \hat{Q}_{i\mu} \\ \hat{Q}_{\mu i} & \hat{Q}_{\mu\nu} \end{pmatrix} = \begin{pmatrix} \hat{E}_{ij} & -\hat{E}_{ij}Q_{j\mu} \\ Q_{\mu j}\hat{E}_{ji} & Q_{\mu\nu} - Q_{\mu i}\hat{E}_{ij}Q_{j\nu} \end{pmatrix}$$

$$\hat{E} = (0E + I)(IE + b_{ij}(\hat{x}))^{-1} = (E + \epsilon_{ij}{}^{k} z_k)^{-1}.$$

$$\begin{pmatrix} \tilde{x}^i \\ x^i \end{pmatrix} = \begin{pmatrix} z^i \\ x^i \end{pmatrix} \longrightarrow T \begin{pmatrix} z^i \\ x^i \end{pmatrix} = \begin{pmatrix} x^i \\ z^i \end{pmatrix}$$

$$\begin{aligned} ds^2 &= \hat{G}_{\mu\nu} dx^\mu dx^\nu + 2\hat{G}_{\mu i} dx^\mu dz^i + \hat{g}_{ij} dz^i dz^j \\ \hat{B} &= \frac{1}{2}\hat{B}_{\mu\nu} dx^\mu \wedge dx^\nu + \hat{B}_{\mu i} dx^\mu \wedge dz^i + \frac{1}{2}\hat{B}_{ij} dz^i \wedge dz^j \end{aligned}$$

This is the same background obtained by the standard methods of NATD.

- Note that the part of the geometry along SU(2) directions transform under O(3,3):

$$T_3 = \begin{pmatrix} 0_3 & I_3 \\ I_3 & b \end{pmatrix}$$

where that $[b]_{ij} = f_{ij}{}^{k} x_k = \epsilon_{ij}{}^{k} x_k$. In the limit where the structure constants vanish this reduces to 3 consecutive abelian dualities.

- This O(3,3) matrix can be factorized as

$$T = \begin{pmatrix} 0 & I \\ I & b \end{pmatrix} = \begin{pmatrix} 0 & I_D \\ I_D & 0 \end{pmatrix} \begin{pmatrix} I & b \\ 0 & I \end{pmatrix}$$

- The first transformation turns on a non constant B-field. The second transformation inverts the background matrix, taking from IIA to IIB and vice versa.

RAMOND RAMOND SECTOR

- The corresponding Pin(10,10) matrix can be found easily to be

$$\begin{aligned} S &= CS_b \\ &= (\psi^1 - \psi_1)(\psi^2 - \psi_2)(\psi^3 - \psi_3)\exp\left(1/2\; b_{ij}\psi^i\psi^j\right) \\ &= (\psi^1 - \psi_1)(\psi^2 - \psi_2)(\psi^3 - \psi_3)(1 + z^1\psi^2\psi^3 + z^2\psi^3\psi^1 + z^3\psi^1\psi^2) \end{aligned}$$

EXAMPLE: AdS_3 x T^4 x S^3 [Sfetsos,Thompson]

- NATD of this background supported by 3- and 7-form RR fluxes has been studied by Sfetsos and Thompson.

$$F_3 + *F_3, \qquad F_3 = Vol(S^3) + Vol(AdS_3)$$

Acting on the corresponding spinor state with the Pin(10,10) operator above, we obtain a new spinor with the corresponding polyform F_{new} :

$$F_{\text{new}} = e^{-\hat{B}}\hat{F}$$

From this we read the new RR fluxes:

$$\begin{aligned} F_0 &= 1 \\ F_2 &= \frac{r^3}{1+r^2}\,Vol(S^2) \\ F_4 &= -rdr \wedge Vol(AdS_3) + Vol(T^4) \end{aligned}$$

and their Hodge duals *with respect to the new geometry*.

CONCLUSIONS

- We have presented evidence that shows that NATD of backgrounds with SU(2) isometry can be described through the action of a non-constant O(3,3) transformation.
- This T-duality transformation amounts to turning on non-constant B-field and then applying the transformation that exchanges Type IIA and IIB.
- We have looked at several examples, case by case. For a general proof of why the method works (eom, Hodge duality) we find the framework of DFT very useful.