Type II superspace with manifest T-duality

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I. Introduction
II. Local gauge symmetries of SUGRA fields
III. Global type II superalgebra
IV. Superstring action with manifest T-duality
V. Conclusions

I. INTRODUCTION

- Motivation is to understand "superstring theory" —a candidate of quantum gravity — & its characteristic feature "T-duality" in order to explore
  - big bang singularity which is removed by the minimum length from T-duality
  - new aspects of superstring theory
  - alternative way to represent the superstring theory

- Our approach is to construct a theory with manifest T-duality as well as supersymmetry and gauge symmetries.

References

- Basis of the space with manifest T-duality

Momentum winding mode	N=1 Super	Lorentz	R/R
$p_{\rm m}$ $\partial_\sigma X^{\rm m}$	D_μ Ω^μ	S_{mn} Σ^{mn}	$\Upsilon_{\mu\nu'}$ $F^{\mu\nu'}$
`93 Siegel	`93 Siegel	`13 Polacek & Siegel	`15 Kamimura, Siegel & M.H.
$p_{\rm m} - \partial_\sigma X^{\rm m}$	$D_{\mu'}$ $\Omega^{\mu'}$	$S_{m'n'}$ $\Sigma^{m'n'}$	
N=2 Super	`14 Kamimura, Siegel & M.H.		

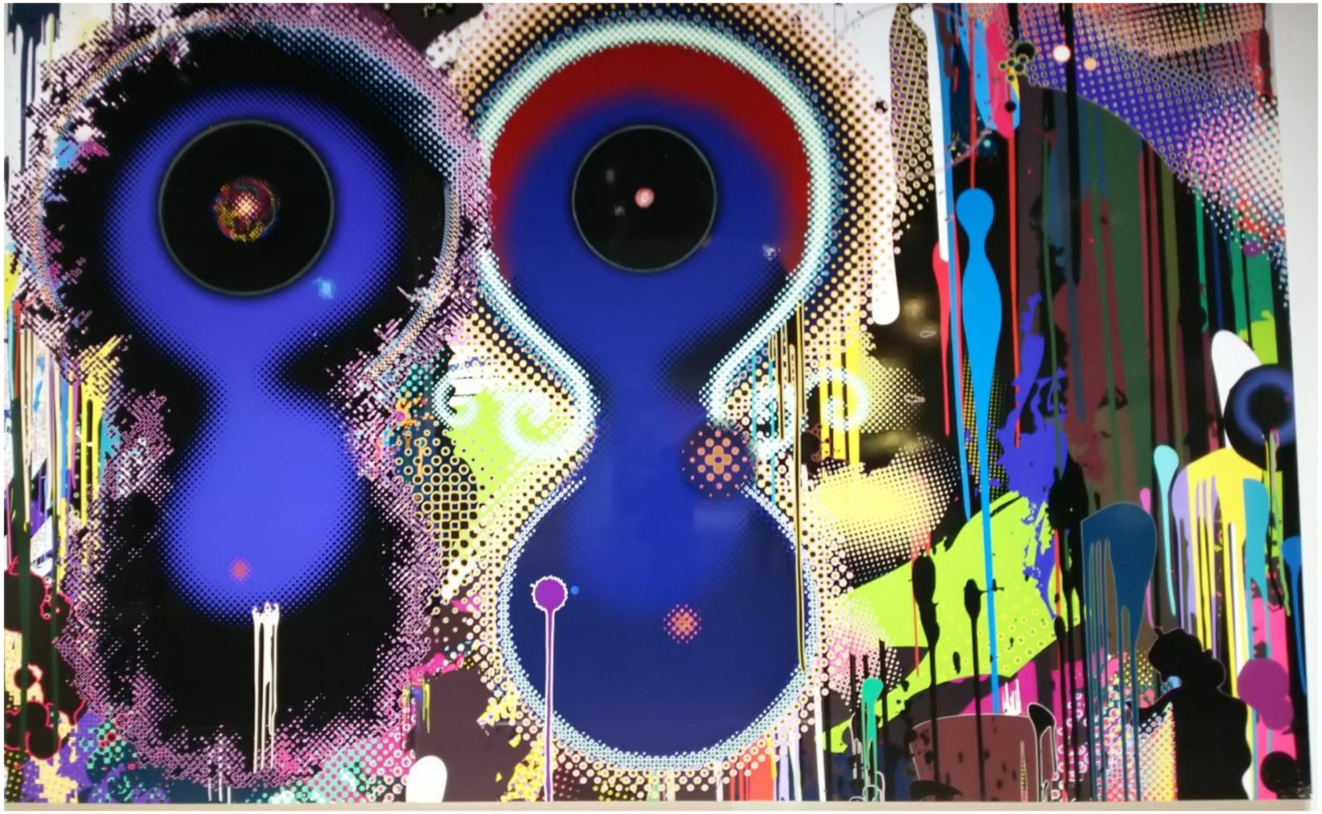
- N=2 SUGRA with RR-gauge field in doubled space

` 99 Fukuma, Oota & Tanaka, `00 Hassan, `05 Grana, Minasian, Petrini and Tomasiello, `11 Coimbra1, Strickland-Constable and Waldram, `11 Hohm, Kwak & Zwiebach, `12 Jeon, Lee, Park and Suh, …more

- Bosonic string action in doubled space

`89 Duff, `90 Tseytlin, `08 Berman, N. Copland & D. Thompson, `13 Lee & Park, '14 De Angelis, G. Gionti S.J., R. Marotta and F. Pezzella, …more

- Superstring action in doubled space `15 Bandos; Nikolic & Sazdovic



Double Face: Two Eleventh Dimensions… 2015   Takashi Murakami

II. LOCAL GAUGE SYMMETRIES OF SUGRA FIELDS

II-1. Gauge symmetry of bosonic gravity fields

- Bosonic gravity fields

$$(G_{\mathrm{mn}},\ B_{\mathrm{mn}})\ \Rightarrow E_{\underline{\mathrm{a}}}{}^{\underline{\mathrm{m}}} = \begin{pmatrix} e_{\mathrm{a}}{}^{\mathrm{m}} & e_{\mathrm{a}}{}^{\mathrm{n}} B_{\mathrm{nm}} \\ 0 & e_{\mathrm{m}}{}^{\mathrm{a}} \end{pmatrix}$$

- Gauge symmetry parameters

$$(\xi^{\mathrm{m}},\ \xi_{\mathrm{m}})\ = \xi^{\underline{\mathrm{m}}}$$

- Covariant derivative current algebra

$$\left(p_{\mathrm{m}} = \frac{\partial}{\partial X^{\mathrm{m}}},\ \partial_\sigma X^{\mathrm{m}}\right)\ = \mathring{\triangleright}_{\underline{\mathrm{m}}}$$

$$[\mathring{\triangleright}_{\underline{\mathrm{m}}}(\sigma_1), \mathring{\triangleright}_{\underline{\mathrm{n}}}(\sigma_2)] = -i\eta_{\underline{\mathrm{mn}}}\partial_\sigma\delta(\sigma_2-\sigma_1)$$

O(d,d) inv. metric

$$\eta_{\underline{\mathrm{mn}}} = \begin{pmatrix} 0 & \delta^{\mathrm{n}}_{\mathrm{m}} \\ \delta^{\mathrm{m}}_{\mathrm{n}} & 0 \end{pmatrix}$$

II-1. Gauge symmetry of bosonic gravity fields

- Manifest T-duality
  - Gravity fields

$$\begin{pmatrix} G^{-1} & G^{-1}B \\ -BG^{-1} & G-BG^{-1}B \end{pmatrix} = E\hat{\eta}E^T \,, \quad \hat{\eta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$E\eta E^T = \eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

  - Under O(d,d) T-duality transf.

$$\mathrm{O(d,d)} \ni A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Fractional transf. $(G+B)_{\mathrm{mn}} \rightarrow \dfrac{a(G+B)+b}{c(G+B)+d}$

Linear transf. $E_{\underline{\mathrm{a}}}{}^{\underline{\mathrm{m}}} \longrightarrow E_{\underline{\mathrm{a}}}{}^{\underline{\mathrm{n}}} A_{\underline{\mathrm{n}}}{}^{\underline{\mathrm{m}}}$

manifest T-dualiy

II-1. Gauge symmetry of bosonic gravity fields

`93 Siegel

- Gauge symmetry transformation

m's are contracted ∴T-duality manifest

$$[E_{\underline{a}}{}^{\underline{m}}\mathring{\triangleright}_{\underline{m}}(\sigma_1), \xi^{\underline{n}}\mathring{\triangleright}_{\underline{n}}(\sigma_2)]$$

$$= -i\delta_\xi E_{\underline{a}}{}^{\underline{m}}\mathring{\triangleright}_{\underline{m}}\ \delta(\sigma_1-\sigma_2) - iE_{\underline{a}}{}^{\underline{m}}\xi^{\underline{n}}(\sigma_2)\eta_{\underline{mn}}\ \partial_\sigma\delta(\sigma_2-\sigma_1)$$

$$\delta_\xi E_{\underline{a}}{}^{\underline{m}} = E_{\underline{a}}{}^{\underline{n}}\partial_{\underline{n}}\xi^{\underline{m}} - \xi^{\underline{n}}\partial_{\underline{n}}E_{\underline{a}}{}^{\underline{m}} + E_{\underline{a}}{}^{\underline{n}}\partial^{\underline{m}}\xi^{\underline{l}}\eta_{\underline{nl}} = [E_{\underline{a}}{}^{\underline{m}}, \xi]_{\rm T}$$

C-bracket

One parameter ambiguity `12 Kimura & M.H.

- Gauge transf. in component with $\partial^{\rm m}E_{\underline{a}}{}^{\underline{n}}(X^{\rm m}, \cancel{X_{\rm n}}) = 0$

by gauge choice

$$\begin{cases} \delta_\xi G_{\rm mn} = \xi^{\rm l}\partial_{\rm l}G_{\rm mn} + \partial_{(\rm m}\xi^{\rm l}G_{\rm l|n)} \\ \delta_\xi B_{\rm mn} = \xi^{\rm l}\partial_{\rm l}B_{\rm mn} + \partial_{[\rm m}\xi^{\rm l}B_{\rm l|n]} + \partial_{[\rm m}\xi_{\rm n]} \end{cases}$$

Treatment of Schwinger term

- Double coordinate & derivative

$$[\mathring{\triangleright}_{\underline{\mathrm{n}}}, \Phi(X^{\underline{\mathrm{m}}})] = -i\partial_{\underline{\mathrm{n}}}\Phi(X^{\underline{\mathrm{m}}})$$

- σ diffeomorphism generator & σ derivative

$$\mathcal{H}_\sigma = p_{\mathrm{m}}\partial_\sigma X^{\mathrm{m}} = \frac{1}{2}\mathring{\triangleright}_{\underline{\mathrm{m}}}\eta^{\underline{\mathrm{mn}}}\mathring{\triangleright}_{\underline{\mathrm{n}}} = 0$$

\`12 Kimura & M.H.

$$\partial_\sigma\Phi(X^{\underline{\mathrm{m}}}(\sigma)) = i[\int \mathcal{H}_\sigma, \Phi(X)] = \eta^{\underline{\mathrm{mn}}}\partial_{\underline{\mathrm{n}}}\Phi(X)\ \mathring{\triangleright}_{\underline{\mathrm{m}}}$$

★ σ-derivative gives cov. derivative!

- Section condition on fields

$$\langle\Phi\mathcal{H}_\sigma\Psi\rangle = 0,\ \text{for}^\forall\Phi,\Psi \ \Rightarrow \partial^{\underline{\mathrm{m}}}\partial_{\underline{\mathrm{m}}}\Psi = 0 = \partial^{\underline{\mathrm{m}}}\Phi\partial_{\underline{\mathrm{m}}}\Psi$$

II-2. Gauge symmetry of type II SUGRA fields

`14 Kamimura, Siegel & M.H.

- Supergravity fields

$$\left(G_{\rm mn}, B_{\rm mn}, \phi, \psi_m{}^\mu, \psi_{m'}{}^{\mu'}, \psi^\mu, \psi^{\mu'}, \boxed{C_{\rm RR};} \omega_a{}^{mn}, F^{\mu\nu}_{\rm NS}, F^{\mu\nu'}_{\rm RR}\right) \in E_{\underline{A}}{}^{\underline{M}}$$

- Gauge symmetry parameters

$$\left(\lambda_{mn}, \ \lambda_{m'n'}, \ \varepsilon^\mu, \ \varepsilon^{\mu'}, \ \xi^m, \ \xi^{m'}; \boxed{\Lambda_{\rm RR}{}^{\mu\nu'}}\right) \in \Lambda^{\underline{M}}$$

- Double coordinates + central extension

indices $\underline{M} = (M, M'; \boxed{\Upsilon, \mathbf{F}})$

Left/right indices

II-2. Gauge symmetry of type II SUGRA fields

- Covariant derivatives $\mathring{\triangleright}_{\underline{M}} = \left(\mathring{\triangleright}_{M},\ \mathring{\triangleright}_{M'};\ \Upsilon_{\mu\nu'},\ \mathbf{F}^{\mu\nu'}\right)$

$$\mathring{\triangleright}_{M} = \left(S_{mn},\ D_{\mu},\ P_{m},\ \Omega^{\mu},\ \Sigma^{mn}\right)$$

$$\mathring{\triangleright}_{M'} = \left(S_{m'n'},\ D_{\mu'},\ P_{m'},\ \Omega^{\mu'},\ \Sigma^{m'n'}\right)$$

Left/right indices

$$P_{m} = (p_{\mathrm{m}} + \partial_\sigma X^{\mathrm{m}})/\sqrt{2}$$

$$P_{m'} = (p_{\mathrm{m}'} - \partial_\sigma X^{\mathrm{m}'})/\sqrt{2}$$

- Current algebras

Nondegenerate group metric

$$\eta_{MN} = \begin{array}{c|ccccc} & S & D & P & \Omega & \Sigma \\ S & & & & & 1 \\ D & & & & -1 & \\ P & & & 1 & & \\ \Omega & & 1 & & & \\ \Sigma & 1 & & & & \end{array}$$

$$[\mathring{\triangleright}_{M}(\sigma_1), \mathring{\triangleright}_{N}(\sigma_2)] = -if_{MN}{}^{K}\,\mathring{\triangleright}_{K}\ \delta(2-1) - i\eta_{MN}\ \partial_\sigma\delta(2-1)$$

$$[\mathring{\triangleright}_{M'}(\sigma_1), \mathring{\triangleright}_{N'}(\sigma_2)] = if_{MN}{}^{K}\,\mathring{\triangleright}_{K'}\ \delta(2-1) + i\eta_{MN}\ \partial_\sigma\delta(2-1)$$

Condition from current algebra

- Nondegenerate group metric: η_{MN}

& Totally antisymmetric structure constant :f_{MNL}

  – from Jacobi identity of current algebra

$$\det \eta_{MN} \neq 0$$

$$f_{MNL} = f_{MN}{}^{K} \eta_{KL} = \frac{1}{3!} f_{[MNL]}$$

- String action
  – Wess-Zumino term is constructed by f_{MNL}

Nondegenerate Super-Poincare algebra

- Covariant derivative: $\mathring{\triangleright}_M = \left(S_{mn},\ D_\mu,\ P_m,\ \Omega^\mu,\ \Sigma_{mn} \right)$
- Algebra

$$
\begin{aligned}
\{D_\mu(1), D_\nu(2)\} &= \underline{2P_m \gamma^m_{\mu\nu}}\ \delta(_{2=1}) \\
[D_\mu(1), P_n(2)] &= \underline{2(\gamma_n \Omega)_\mu}\ \delta(_{2=1}) \\
[S_{mn}(1), \Sigma^{lk}(2)] &= -i\delta^{[k}_{[m} \Sigma_{n]}{}^{l]}\ \delta(_{2=1}) \quad + i\delta^l_{[m} \delta^k_{n]}\ \partial_\sigma(_{2=1}) \\
\{D_\mu(1), \Omega^\nu(2)\} &= -\tfrac{i}{4}\Sigma^{mn}(\gamma_{mn})^\nu{}_\mu\ \delta(_{2=1}) \quad + i\delta^\nu_\mu\ \partial_\sigma(_{2=1}) \\
[P_m(1), P_n(2)] &= i\Sigma_{mn}\ \delta(_{2=1}) \quad + i\eta_{mn}\ \partial_\sigma(_{2=1})
\end{aligned}
$$

totally antisymmetric

$f_{DDP} \Rightarrow f_{DPD} = f_{DP}{}^{\Omega}$

Central extension

- Type II supercovariant derivative: $(D_\mu,\ D_{\mu'})$
- Superalgebra with central extension

$$\left\{\begin{array}{lcl}\{D_\mu, D_\nu\} & = & P_m\gamma^m_{\mu\nu}\\ \{D_\mu, D_{\nu'}\} & = & \Upsilon_{\mu\nu'}\\ \{D_{\mu'}, D_{\nu'}\} & = & -P_{m'}\gamma^{m'}_{\mu'\nu'}\end{array}\right.$$

- Nondegeneracy

$$[\mathsf{F}^{\mu\nu'}(_1), \Upsilon_{\nu\mu'}(_2)]$$
$$= (\delta^\mu_\nu\Sigma^{m'n'}(\gamma_{m'n'})^{\nu'}{}_{\mu'} + \delta^{\nu'}_{\mu'}\Sigma^{mn}(\gamma_{mn})^\mu{}_\nu)\delta(_{2=1}) + \delta^\mu_\nu\delta^{\nu'}_{\mu'}\ \partial_\sigma\delta(_{2=1})$$

II-2. Gauge symmetry of type II SUGRA fields

- Gauge symmetry transformation

$$[E_{\underline{A}}{}^{\underline{M}} \mathring{\triangleright}_{\underline{M}}(\sigma_1), \Lambda^{\underline{N}} \mathring{\triangleright}_{\underline{N}}(\sigma_2)]$$

$$= -i\delta_\Lambda E_{\underline{A}}{}^{\underline{M}} \mathring{\triangleright}_{\underline{M}} \, \delta(_{2-1}) - iE_{\underline{A}}{}^{\underline{M}} \Lambda^{\underline{N}}(\sigma_2)\eta_{\underline{MN}} \, \partial_\sigma \delta(_{2-1})$$

$$\delta_\Lambda E_{\underline{A}}{}^{\underline{M}} = E_{\underline{A}}{}^{\underline{N}} \partial_{\underline{N}} \Lambda^{\underline{M}} - \Lambda^{\underline{N}} \partial_{\underline{N}} E_{\underline{A}}{}^{\underline{M}} + E_{\underline{A}}{}^{\underline{N}} (D^{\underline{M}} \Lambda^{\underline{L}}) \eta_{\underline{NL}}$$
$$+ E_{\underline{A}}{}^{\underline{N}} \Lambda^{\underline{L}} f_{\underline{NL}}{}^{\underline{M}}$$

non-abelian term

$$= [E_{\underline{A}}{}^{\underline{M}}, \Lambda]_{\mathrm{T}}$$

Supersymmetric C-bracket

- Gauge transf. of RR gauge field

$$e_{\mathrm{m}}{}^{\mathrm{a}} E_{\mathrm{a}}{}^{\Upsilon} = C_{\mathrm{RR;m}}{}^{\mu\nu'} \quad , \quad \Lambda^{\Upsilon} = \Lambda_{\mathrm{RR}}{}^{\mu\nu'}$$

$$\delta_{\Lambda^\Upsilon} C_{\mathrm{RR;m}}{}^{\mu\nu'} = D_{\mathrm{m}} \Lambda_{\mathrm{RR}}{}^{\mu\nu'} + (F_{\mathrm{NS}} \wedge \Lambda_{\mathrm{RR}})_{\mathrm{m}}{}^{\mu\nu'}$$

- Local SUSY transf. of gravitino

$$\omega_P{}^S \varepsilon^D f_{SD}{}^{\mu}$$

$$\delta_\varepsilon \psi_{\mathrm{m}}{}^{\mu} = \frac{1}{2}\Big(D_{\mathrm{m}}\varepsilon^{\mu} + \omega_{\mathrm{m}}{}^{nl}(\gamma_{nl}\varepsilon)^{\mu} - E^{\mu}{}_{\underline{\nu}}\partial_{\mathrm{m}}\varepsilon^{\underline{\nu}}$$

$$+F_{\mathrm{NS}}{}^{\mu\nu}(\gamma_{\mathrm{m}}\varepsilon)_{\nu} + F_{\mathrm{RR}}{}^{\mu\nu'}(\gamma_{\mathrm{m}}\varepsilon)_{\nu'}\Big)$$

$$F_{\mathrm{NS};}{}^{\mu D}\varepsilon^D f_{PDD} \qquad F_{\mathrm{RR};}{}^{\mu D'}\varepsilon^{D'} f_{P'D'D'}$$

- SUGRA fields in vielbein field
  - Gravitino $\psi_{\mathrm{m}}{}^{\underline{\mu}} = e_{\mathrm{m}}{}^{\underline{P}} E_{\underline{P}}{}^{\underline{D}},\ E_{\underline{P}}{}^{\underline{D}} = -E^{\underline{D}}{}_{\underline{P}}$
  - NS/NS field strength $(F_{\mathrm{NS}}{}^{\mu\nu},\ F_{\mathrm{NS}}{}^{\mu'\nu'}) = (E^{DD},\ E^{D'D'})$
  - R/R field strength $F_{\mathrm{RR}}{}^{\mu\nu'} = E^{DD'} = E^{D'D}$

- Gauge transf.

$$\delta_\Lambda E_{\underline{A}}{}^{\underline{M}} = E_{\underline{A}}{}^{\underline{N}}\partial_{\underline{N}}\Lambda^{\underline{M}} - \Lambda^{\underline{N}}\partial_{\underline{N}}E_{\underline{A}}{}^{\underline{M}} + E_{\underline{A}}{}^{\underline{N}}(D^{\underline{M}}\Lambda^{\underline{L}})\eta_{\underline{NL}}$$

$$+E_{\underline{A}}{}^{\underline{N}}\Lambda^{\underline{L}} f_{\underline{NL}}{}^{\underline{M}}$$

II-2. Gauge symmetry of type II SUGRA fields

- Local susy transf.

$$\delta_\varepsilon e_{\mathrm{a}}{}^{\mathrm{m}} = -B_{\mathrm{a}\underline{\mu}}\partial^{\mathrm{m}}\varepsilon^{\underline{\mu}} + \psi_{\mathrm{a}}{}^{\mu}(\gamma^{\mathrm{m}}\varepsilon)_\mu + \psi_{\mathrm{a}}{}^{\mu'}(\gamma^{\mathrm{m}}\varepsilon)_{\mu'}$$

$$\delta_\varepsilon B_{\mathrm{mn}} = -B_{[\mathrm{m}|\underline{\mu}}\partial_{|\mathrm{n}]}\varepsilon^{\underline{\mu}} + \psi_{[\mathrm{m}}{}^{\mu}(\gamma_{\mathrm{n}]}\varepsilon)_\mu - \psi_{[\mathrm{m}}{}^{\mu'}(\gamma_{\mathrm{n}]}\varepsilon)_{\mu'}$$

$$\delta_\varepsilon \psi_{\mathrm{m}}{}^{\mu} = \frac{1}{2}\Big(D_{\mathrm{m}}\varepsilon^{\mu} + \omega_{\mathrm{m}}{}^{nl}(\gamma_{nl}\varepsilon)^{\mu} - E^{\mu}{}_{\underline{\nu}}\partial_{\mathrm{m}}\varepsilon^{\underline{\nu}} + F_{\mathrm{NS}}{}^{\mu\nu}(\gamma_{\mathrm{m}}\varepsilon)_\nu + F_{\mathrm{RR}}{}^{\mu\nu'}(\gamma_{\mathrm{m}}\varepsilon)_{\nu'}\Big)$$

Lorentz

NS & R fluxes

$$\delta_\varepsilon \psi_{\mathrm{m}}{}^{\mu'} = \frac{1}{2}\Big(D_{\mathrm{m}}\varepsilon^{\mu'} - \omega_{\mathrm{m}}{}^{n'l'}(\gamma_{n'l'}\varepsilon)^{\mu'} - E^{\mu'}{}_{\underline{\nu}}\partial_{\mathrm{m}}\varepsilon^{\underline{\nu}} + F_{\mathrm{NS}}{}^{\mu'\nu'}(\gamma_{\mathrm{m}}\varepsilon)_{\nu'} + F_{\mathrm{RR}}{}^{\nu\mu'}(\gamma_{\mathrm{m}}\varepsilon)_\nu\Big)$$

$$\delta_\varepsilon C_{\mathrm{RR;m}}{}^{\mu\nu'} = \frac{1}{2}\Big(\psi_{\mathrm{m}}{}^{(\mu}\varepsilon^{\nu')} + (\psi\gamma_{\mathrm{m}})^{(\mu}\varepsilon^{\nu')} - E_{\mathrm{F}}{}^{(\mu}(\gamma_{\mathrm{m}}\varepsilon)^{\nu')}\Big)$$

Treatment of Schwinger term

- σ diffeomorphism generator

$$\mathcal{H}_\sigma = \frac{1}{2}\mathring{\triangleright}_{\underline{M}}\eta^{\underline{MN}}\mathring{\triangleright}_{\underline{N}} = \frac{1}{2}\triangleright_{\underline{A}}\eta^{\underline{AB}}\triangleright_{\underline{B}} = 0$$

- Covariant derivative in curved space

$$\triangleright_{\underline{A}} = E_{\underline{A}}{}^{\underline{M}}\mathring{\triangleright}_{\underline{M}}$$

- Orthonormal vielbein

$$E_{\underline{A}}{}^{\underline{M}}E_{\underline{B}}{}^{\underline{N}}\eta_{\underline{MN}} = \eta_{\underline{AB}}$$

Orthonormality guarantees the σ-diffeo. inv.

- Double coordinate & derivative

$$[\mathring{\triangleright}_{\underline{M}}, \Phi(X^{\underline{N}})] = -iD_{\underline{M}}\Phi(X^{\underline{N}})$$

nonabelian derivative

Ensồ: Black Hole 2015
Takashi Murakami

III. GLOBAL TYPE II SUPERALGEBRA

III-1. Global type II superalgebra

- SUSY theory has

global SUSY $Q_\mu = \int \tilde{\nabla}_\mu$ with $\{Q_\mu, Q_\nu\} = 2\not{P}_{\mu\nu}$

&

local susy $D_\mu = \mathring{\nabla}_\mu$ with $\{D_\mu, D_\nu\} = -2\not{p}_{\mu\nu}$

where SUSY charge is local super-invariant $\{Q_\mu, D_\nu\} = 0$

- Super - Symmetry generator generates left action

$$\delta g = \Lambda g = [\Lambda \int \tilde{\nabla}, g] \;,\; dgg^{-1} = dzL \Rightarrow \tilde{\nabla} = L^{-1}\partial$$

- Super - Covariant derivative generates right action

$$\delta g = g\lambda = [\int \lambda\mathring{\nabla}, g] \;,\; g^{-1}dg = dzR \Rightarrow \mathring{\nabla} = R^{-1}\partial$$

$$[\partial, z] = 1$$

Puzzle: WZW construction for type II?

- WZW construction: $\mathrm{G} \ni g(z(\sigma^+)), g'(z(\sigma^-))$

  $\underline{g} = g(z(\sigma^+))g'(z(\sigma^-))$

  - Symmetry generator for left moving sector

    $\partial_+ \underline{gg}^{-1} = \partial_+ gg^{-1}(\sigma^+) \quad \Rightarrow \quad \tilde{\triangleright}(\sigma^+)\ , ?\tilde{\triangleright}(\sigma^-)?$

  - Covariant derivative for right moving sector

    $\underline{g}^{-1}\partial_- \underline{g} = g'^{-1}\partial_- g'(\sigma^-) \quad \Rightarrow \quad ?\mathring{\triangleright}(\sigma^+)?,\ \mathring{\triangleright}(\sigma^-)$

- Type II SUSY needs super-symmetry generator & super-covariant derivative for both left/right sectors ! HOW??

Answer: Chirality from "Doubling"

`91 Tseytlin (abelian case)
`14 Kamimura, Siegel & M.H.

- Double the group

$$G \to \underline{G} = G \times G' \ni \underline{g} = gg' = g'g$$

$$G \ni g(Z^M(\sigma^+)),\ G' \ni g'(Z^{M'}(\sigma^-))$$

then it gives

– Symmetry generators for both left & right !

$$d\underline{g}\underline{g}^{-1} = dgg^{-1}(\sigma^+) + dg'g'^{-1}(\sigma^+) \quad \Rightarrow \quad \tilde{\nabla}(\sigma^+)\ ,\ \tilde{\nabla}(\sigma^-)$$

– Covariant derivatives for both left & right !

$$\underline{g}^{-1}d\underline{g} = g^{-1}dg(\sigma^+) + g'^{-1}dg'(\sigma^-) \quad \Rightarrow \quad \mathring{\nabla}(\sigma^+)\ ,\ \mathring{\nabla}(\sigma^-)$$

⇒ N=2 global and local SUSY algebras !

Local & Global by "B-field & dilatation"

Covariant derivative $\mathring{\triangleright}_{\underline{M}}$ & Symmetry generator $\tilde{\triangleright}_{\underline{M}}$

$$[\mathring{\triangleright}_{\underline{M}}(\sigma_1), \mathring{\triangleright}_{\underline{N}}(\sigma_2)] = -if_{\underline{MN}}{}^{\underline{K}} \mathring{\triangleright}_{\underline{K}}\, \delta(2{=}1) - i\eta_{\underline{MN}}\, \partial_\sigma \delta(2{=}1)$$

$$[\tilde{\triangleright}_{\underline{M}}(\sigma_1), \tilde{\triangleright}_{\underline{N}}(\sigma_2)] = +if_{\underline{MN}}{}^{\underline{K}} \tilde{\triangleright}_{\underline{K}}\, \delta(2{=}1) + i\eta_{\underline{MN}}\, \partial_\sigma \delta(2{=}1)$$

$$[\mathring{\triangleright}_{\underline{M}}(\sigma_1), \tilde{\triangleright}_{\underline{N}}(\sigma_2)] = 0$$

i.e. Global SUSY generator is inv. under local supersymmetry.

- B_{MN} field is related to dilatation dimension N_{MN}

$$\mathring{\triangleright}_{\underline{M}} = \mathring{\nabla}_{\underline{M}} + J^{\underline{N}}(\eta_{\underline{NM}} + B_{\underline{NM}})$$
$$B_{\underline{NM}} = (1/2) N_{[\underline{M}|}{}^{\underline{L}} \eta_{\underline{L|N}]}$$

$$\tilde{\triangleright}_{\underline{M}} = \tilde{\nabla}_{\underline{M}} - \tilde{J}^{\underline{N}}(\eta_{\underline{NM}} + \tilde{B}_{\underline{NM}})$$
$$\tilde{B}_{\underline{NM}} = L^{-1} R B R^T (L^{-1})^T$$

`15 Kamimura, Siegel & M.H.

Covariant derivative | Symmetry generator

$$\mathring{\triangleright}_M = (S_{mn}, D_\mu, P_m, \Omega^\mu, \Sigma^{mn})$$

$$\left\{\begin{array}{lcl} S_{mn} & = & \frac{1}{\sqrt{2}}\mathring{\nabla}_S \\ D_\mu & = & \frac{1}{\sqrt{2}}(\mathring{\nabla}_D - \frac{1}{2}J^\Omega) \\ P_m & = & \frac{1}{\sqrt{2}}(\mathring{\nabla}_P + J^P) \\ \Omega^\mu & = & \frac{1}{\sqrt{2}}(\mathring{\nabla}_\Omega + \frac{3}{2}J^D) \\ \Sigma^{mn} & = & \frac{1}{\sqrt{2}}(\mathring{\nabla}_\Sigma + 2J^S) \end{array}\right.$$

$$\tilde{\triangleright}_M = (\tilde{S}_{mn}, \tilde{D}_\mu, \tilde{P}_m, \tilde{\Omega}^\mu, \tilde{\Sigma}^{mn})$$

$$\left\{\begin{array}{lcl} \tilde{S}_{mn} & = & \frac{1}{\sqrt{2}}(\tilde{\nabla}_S - 2(\tilde{J}^\Sigma + \cdots)) \\ \tilde{D}_\mu & = & \frac{1}{\sqrt{2}}(\tilde{\nabla}_D + \frac{3}{2}(\tilde{J}^\Omega + \cdots)) \\ \tilde{P}_m & = & \frac{1}{\sqrt{2}}(\tilde{\nabla}_P - (\tilde{J}^P + \cdots)) \\ \tilde{\Omega}^\mu & = & \frac{1}{\sqrt{2}}(\tilde{\nabla}_\Omega - \frac{1}{2}(\tilde{J}^D + \cdots)) \\ \tilde{\Sigma}^{mn} & = & \frac{1}{\sqrt{2}}\tilde{\nabla}_\Sigma \end{array}\right.$$

★Dilatation dimensions appear in $\mathring{\triangleright}_{\underline{M}}$ & $\tilde{\triangleright}_{\underline{M}}$.

$\Rightarrow$ Covariant derivatives are used for superstring action later.

III-2. Global type II superalgebra & dimensional reduction constraints

- Type II superalgebra $\quad (q^{\rm NS}, q^{\rm RR})$ string algebra

$$\begin{cases} \{Q_{1;\mu}, Q_{1;\nu}\} &= P_{\rm m}\gamma^{\rm m}_{\mu\nu} + Q^{\rm NS}_{\mu\nu} &= \int (p_{\rm m} + q^{\rm NS}\partial_\sigma X_{\rm m})\gamma^{\rm m}_{\mu\nu} \\ \{Q_{1;\mu}, Q_{2;\nu}\} &= Q^{\rm RR}_{\mu\nu} &= \int q^{\rm RR}\partial_\sigma X_{\rm m}\gamma^{\rm m}_{\mu\nu} \\ \{Q_{2;\mu}, Q_{2;\nu}\} &= P_{\rm m}\gamma^{\rm m}_{\mu\nu} - Q^{\rm NS}_{\mu\nu} &= \int (p_{\rm m} - q^{\rm NS}\partial_\sigma X_{\rm m})\gamma^{\rm m}_{\mu\nu} \end{cases}$$

- Doubled superalgebra

$$\begin{cases} \{\tilde{D}_\mu, \tilde{D}_\nu\} &= -\tilde{P}_m\gamma^m_{\mu\nu} \\ \{\tilde{D}_\mu, \tilde{D}_{\nu'}\} &= -\tilde{\Upsilon}_{\mu\nu'} \\ \{\tilde{D}_{\mu'}, \tilde{D}_{\nu'}\} &= +\tilde{P}_{m'}\gamma^{m'}_{\mu'\nu'} \end{cases}$$

Now compare !

III-2. Global type II superalgebra & dimensional reduction constraints

- Physical symmetry generators

$$\tilde{D}_\mu \leftrightarrow Q_{1;\mu},\ \ i\tilde{D}_{\mu'} \leftrightarrow Q_{2;\mu},\ \ \tilde{P}_m + \tilde{P}_{m'} \leftrightarrow P_{\rm m}$$

All constraints are 1st class !

- Dimensional reduction constraints

$$\tilde{P}_m - \tilde{P}_{m'}(\leftrightarrow Q_{\rm NS}) = 0,\ \ \tilde{\Upsilon}_{\mu\nu'}(\leftrightarrow Q_{\rm RR}) = 0 \text{ for flat}$$

$$\tilde{\Sigma}^{mn} = \tilde{\Sigma}^{m'n'} = \tilde{\Omega}^{\mu} = \tilde{\Omega}^{\mu'} = \tilde{F}^{\mu\nu'} = 0 \text{ for flat}$$

- Coset constraints : $S_{mn} = S_{m'n'} = 0$
- Kappa symmetric Virasoro constraints:

$$(\not{P}D)^{\mu} + (\not{S}\Omega)^{\mu} = (\not{P}'D)^{\mu'} + (\not{S}'\Omega)^{\mu'} = 0$$

$$\frac{1}{2}({P_m}^2 + \Sigma^{mn}S_{mn}) + \Omega^{\mu}D_{\mu} = \frac{1}{2}({P_{m'}}^2 + \Sigma^{m'n'}S_{m'n'}) + \Omega^{\mu'}D_{\mu'} = 0$$

IIA ⇔IIB by parity transformation

- Discrete T-duality interchanging p ⇔ dX is parity transf. in doubled space (after dim. reduction)
  - Ex. $p_9 \leftrightarrow \partial_\sigma X^9$ $\qquad P_{9'} \to -P_{9'}$

$$\begin{cases} P_m &= (p_{\rm m} + \partial_\sigma X^{\rm m})/\sqrt{2} \\ P_{m'} &= (p_{\rm m} - \partial_\sigma X^{\rm m})/\sqrt{2} \end{cases}$$

$$P_{\underline{m}} \to P_{\underline{m}} - 2n_{\underline{m}} P \cdot n$$
$$n^{\underline{m}} = (0; n^{m'}) = (0; 0, \cdots, 0, 1)$$

  - Flip the chirality of $D_{\mu'}$ i.e. interchange IIA & IIB

$$D_{\underline{\mu}} = \begin{pmatrix} D_\mu \\ D_{\mu'} \end{pmatrix} = \begin{cases} \text{IIA} \begin{pmatrix} D_{1;\mu} \\ D_2{}^\mu \end{pmatrix} \\ \text{IIB} \begin{pmatrix} D_{1;\mu} \\ D_{2;\mu} \end{pmatrix} \end{cases}$$

Flip chirality

$$D_{\underline{\mu}} \to (D\gamma^{m'} n_{m'})^{\underline{\mu}}$$
$$\{D\!\!\!/ n\!\!\!/, D\!\!\!/ n\!\!\!/\} = P\!\!\!/ - 2n\!\!\!/ P \cdot n$$

SUGRA in type II superspace with manifest T-duality

1. Superspace is given by flat space covariant derivatives:

$$\overset{\circ}{\triangleright}_{\underline{M}}$$

2. Introduce SUGRA fields in curved space cov. derivatives:

$$\triangleright_{\underline{A}} = E_{\underline{A}}{}^{\underline{M}} \overset{\circ}{\triangleright}_{\underline{M}}$$

3. Torsions include curvature tensors: ex. $T_{\underline{PP}}{}^{\underline{S}} = R_{\underline{mn}}{}^{\underline{lk}}$

$$[\triangleright_{\underline{A}}, \triangleright_{\underline{B}}] = -iT_{\underline{AB}}{}^{\underline{C}} \triangleright_{\underline{C}} \delta(2{=}1) - i\eta_{\underline{AB}} \partial_\sigma \delta(2{=}1)$$

4. Kappa-Virasoro gives torsion constrains & Bianchi identity includes field eq.

$$D_{[\underline{A}} T_{\underline{BCD}]} + T_{[\underline{AB}}{}^{\underline{E}} T_{(\underline{CD}) \underline{E}} = 0$$

5. Reduce dimensions.

$$\tilde{P}_m - \tilde{P}_{m'} = \tilde{\Omega}^\mu = \tilde{\Omega}^{\mu'} = \tilde{\Sigma}^{mn} = \tilde{\Sigma}^{m'n'} = 0$$

Superflat DOB: DNA
2015
Takashi Murakami

IV. SUPERSTRING ACTION WITH MANIFEST T-DUALITY

IV. Type II superstring with manifest T-duality

- Gauge invariant action
  - Hamiltonian form action

2d. diffeo. is needed for kappa-symmetry!

$$I = \int d\tau d\sigma [\partial_\tau Z^{\underline{M}} \partial_{\underline{M}} - \mathcal{H}],\ \mathcal{H} = \underline{\lambda_\tau} \mathcal{H}_\tau + \underline{\lambda_\sigma} \mathcal{H}_\sigma + \lambda^i \phi_i$$

  - 1st class constraints

$$\mathcal{H}_\tau = \frac{1}{2} \mathring{\triangleright}_{\underline{M}} \eta^{\underline{MN}} \mathring{\triangleright}_{\underline{N}} = 0 \qquad \mathcal{H}_\sigma = \frac{1}{2} \mathring{\triangleright}_{\underline{M}} \hat{\eta}^{\underline{MN}} \mathring{\triangleright}_{\underline{N}} = 0$$

2d. diffeo.

$$\phi_i = \begin{cases} (\not{P} D)^\mu + (\not{S} \Omega)^\mu = (\not{P}' D)^{\mu'} + (\not{S}' \Omega)^{\mu'} = 0 \\ S_{mn} = S_{m'n'} = 0 \\ \tilde{P}_m - \tilde{P}_{m'} = \tilde{\Omega}^\mu = \tilde{\Omega}^{\mu'} = \tilde{\Sigma}^{mn} = \tilde{\Sigma}^{m'n'} = 0 \end{cases}$$

Kappa-sym.

local Lorentz

suppress auxiliary dim.

IV. Type II superstring with manifest T-duality

- Gauge invariant action

`15 Kamimura, Siegel & M.H.

$$I = \int d\tau d\sigma [\mathcal{L}_0 + \mathcal{L}_{\rm WZ} + \mathcal{L}_{\rm WZ;0}]$$

Written in terms of double coordinate

2-d. diffeo. inv. even for chiral field action!

$$\left\{ \begin{array}{rcl} \mathcal{L}_0 &=& \underline{\sqrt{-h}h^{ij}} J_i^+ J_j^+ - \epsilon_{ij} J_i^+ J_j^- \\ \mathcal{L}_{\rm WZ} &=& -J_0^N J_1^M B_{MN} + J_0^{N'} J_1^{M'} B_{M'N'} \\ \mathcal{L}_{\rm WZ;0} &=& J_{[0}^{\Sigma} J_{1]}^S + J_{[0}^{\Omega} J_{1]}^D - J_{[0}^{\Sigma'} J_{1]}^{S'} - J_{[0}^{\Omega'} J_{1]}^{D'} \end{array} \right.$$

Winding coordinate component

$$J_i^{\pm} = J_i^P \pm J_i^{P'} (R^{-1}L)_{P'}{}^{P'} (L^{-1}R)_P{}^P$$

`98 Sakaguchi (in simple gauge $R^{-1}L=1$, $J^s=0, Y_m=0$)

$\Rightarrow$ Green-Schwarz action in simple gauge & winding coordinate =0

V. Conclusions

- We have presented
  - covariant derivatives to define the type II superspace with manifest T-duality
  - cov. der. algebra which gives gauge transf. of type II SUGRA fields
  - symmetry generators which give the global type II SUSY algebra
  - dimensional reduction constraints, κ -Virasoro & coset constraints
  - a type II superstring gauge invariant action with manifest T-duality
- Future problems
  - Curved space example ex. type II chiral affine super-AdS algebra
  - D-branes & exotic branes in the doubled superspace
  - S & U-duality extension
  - Cosmological application with manifest T-duality
  - more ･･･