Part I: Introduction

A brief Introduction of Fe-based SC

Yunkyu Bang (*Chonnam National Univ., Kwangju, Korea)*

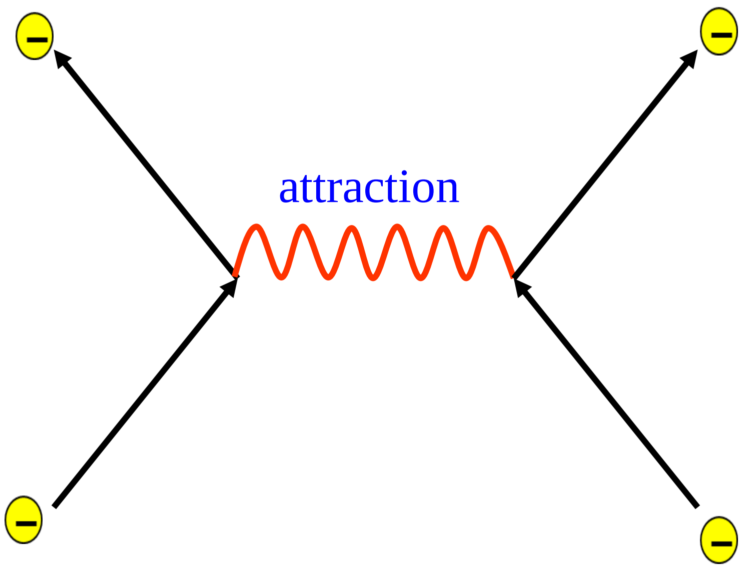
Lecture 1: Introduction
1. Overview
2. What is sign-changing s-wave gap : +/-s-wave gap

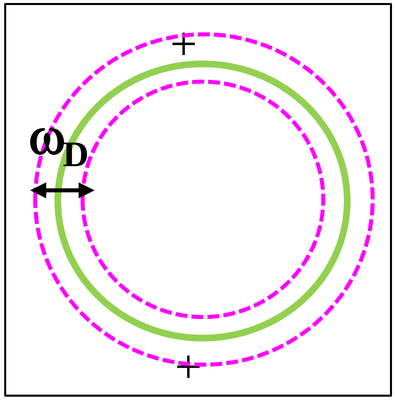
Lecture 2: Superconducting properties of +/-s-wave gap
1. Impurity effects on the +/-s-wave gap
2. NMR : Knight shift and T1 relaxation rate
3. Penetration depth versus T
4. Volovik effect – general principle
5. Volovik effect on +/-s-wave gap – thermal conductivity

Lecture 3: Specific heat jump and Condensation E vs. Tc
1. Specific heat jump vs. Tc : BNC scaling
2. Condensation E in BCS superconductor
3. Condensation E in multi-band superconductor
4. Pairing mechanism

Lecture 4: FeSe system (Tc~100K)
1. Possible phonon contribution
2. Renormalization of pairing cutoff in incipient band superconductors
3. Outlook.

BCS theory:
Only one proven Theory of Superconductivity





S-wave gap

$$T_c = W_c exp[-1/\lambda]$$

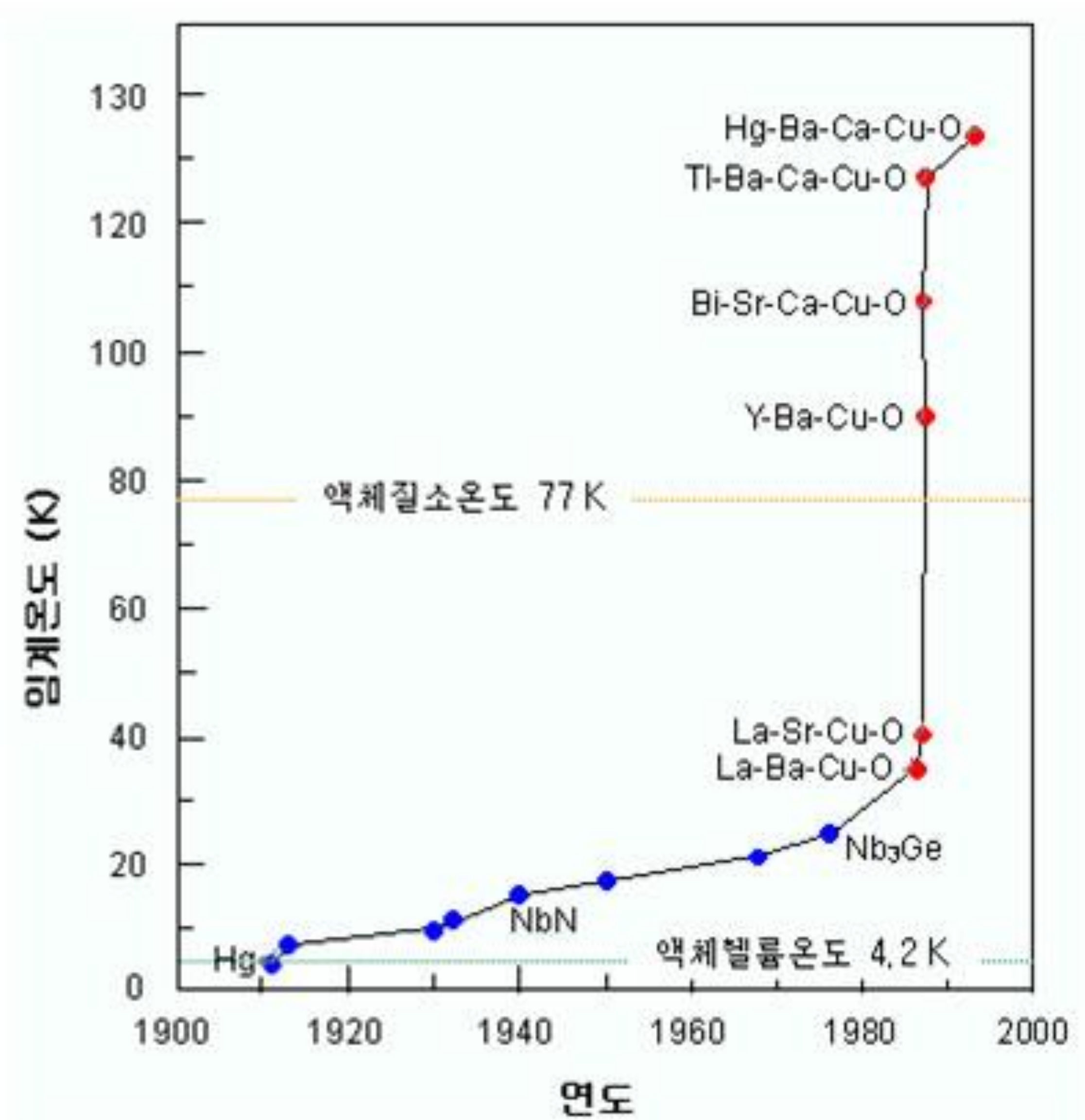
W_c : energy scale of coupling boson
λ : coupling constant

A stronger glue can increase **T_c:**
But it causes the material unstable (***general stability problem***) :
too much strong phonon attraction will cause a lattice collapse.

→ Maybe $\mathbf{T^{max}_c}$ **< 30K** (Anderson & Cohen)

In 1986, Discovery of Cuprate SC: Tc ~100K

100K higher !!



130K = -143C

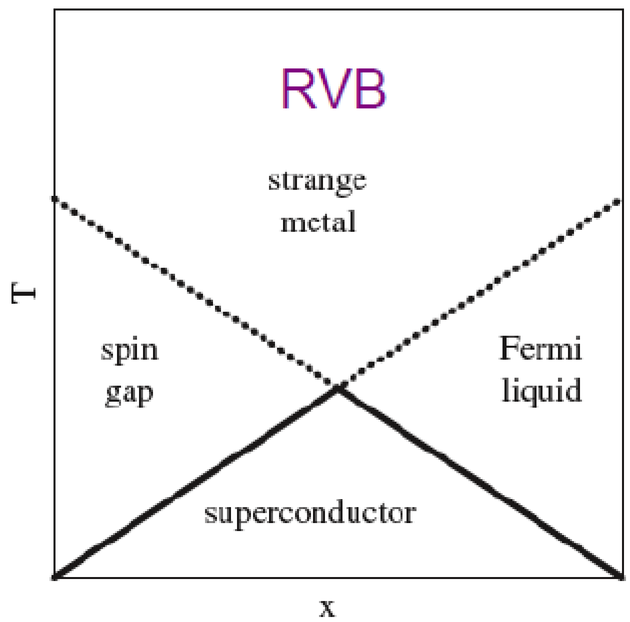
90K = -183C

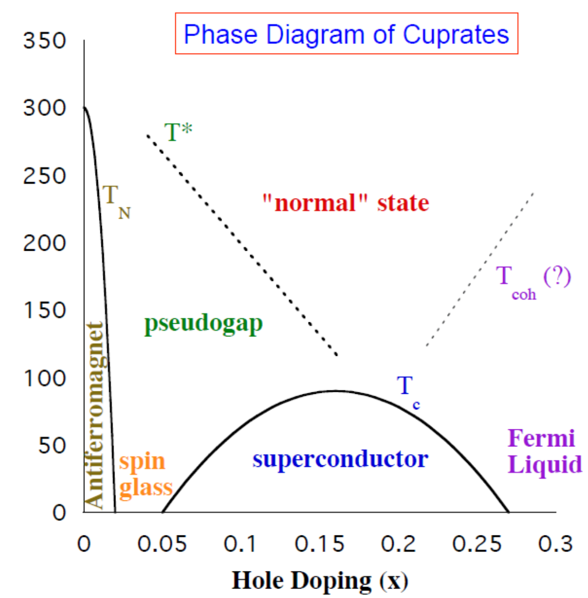
20K = -253C

Anderson declared: (In **1987** Science vol. 235 and subsequent papers)

" This is Un-conventional SC" : non-BCS, non-Fermi liquid

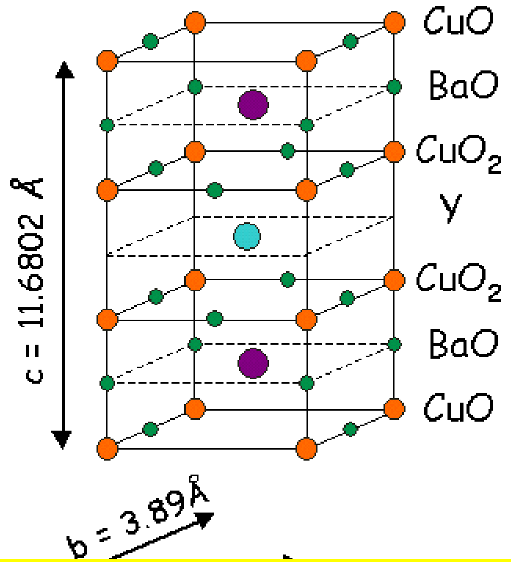
1. Cuprate-SC → (RVB + doped holes) condensation
2. Cuprate-normal state → new quantum liquid (no q.p.)
3. **high-Tc** is a natural manifestation of this new quantum liquid

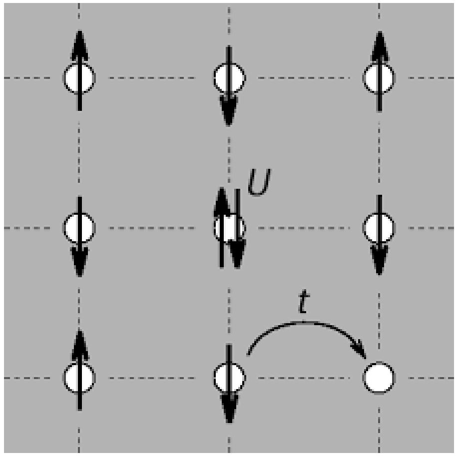




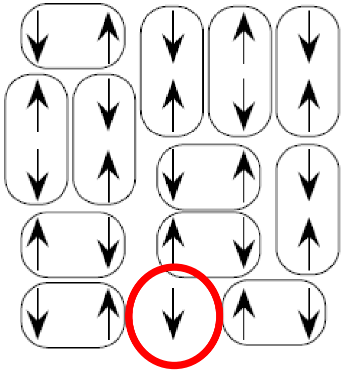
Mott Physics

Cu-O plane → 2D Hubbard model : Mott Ins.



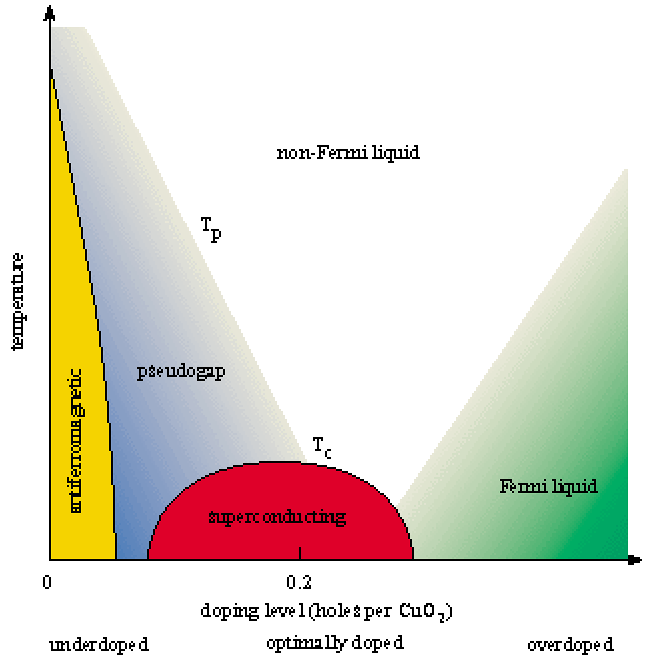


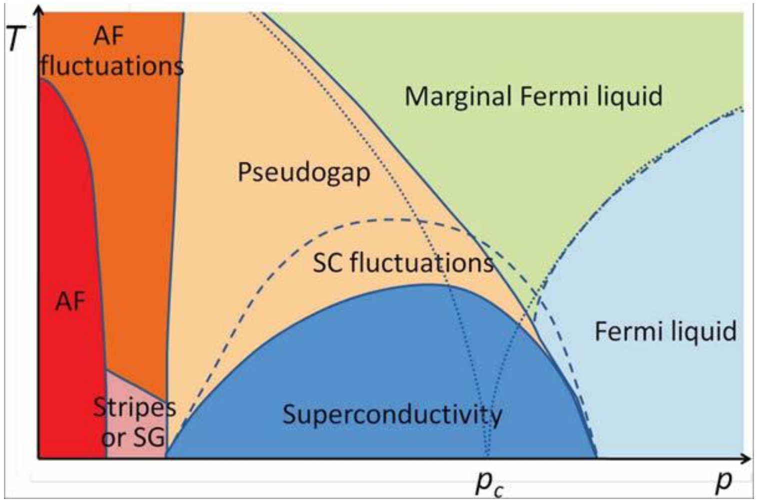
Big Question: what is the GS of the **doped** Hubbard model ?

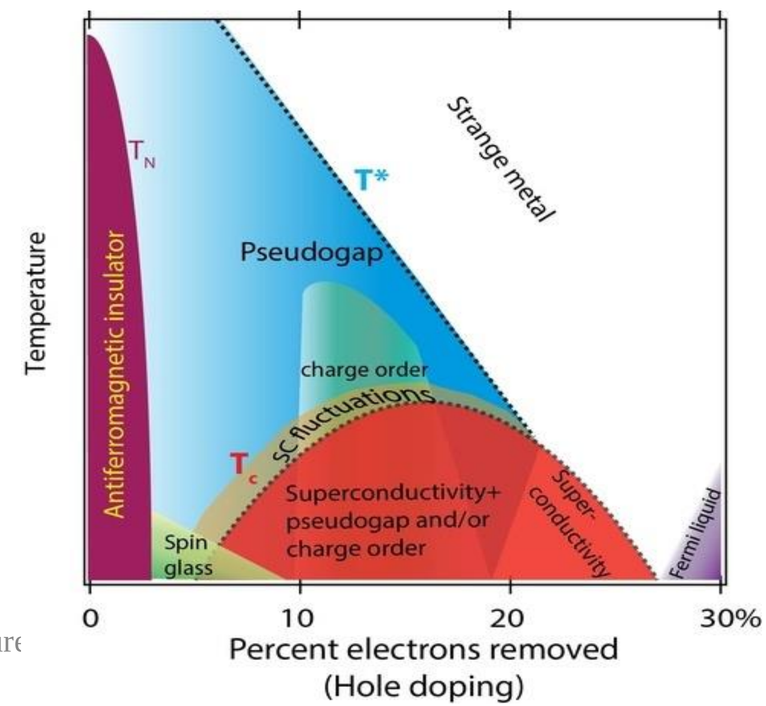


Electron q.p. = spinon + holon
(spin-charge separation, Doped Spin liquid, etc., etc)

Last 30 years = history of frustration – still on going







Theories of Cuprate SCs:

RVB (Mott Physics)
Pre-formed Cooper pairs
Spin-Fermion (BCS)
QC fluctuations
Anyon SC
Local Hartree-Fock theory
…

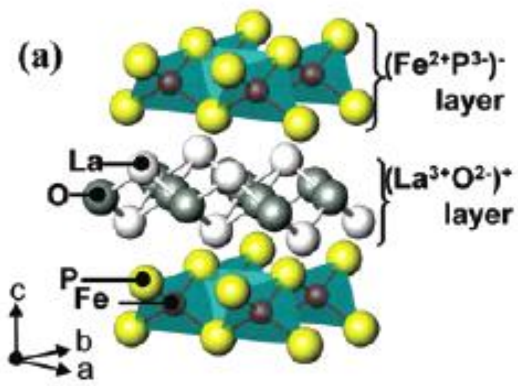
Discovery of Fe-based Superconductor (2006,2008)

2006

Iron-Based Layered Superconductor: LaOFeP

Yoichi Kamihara,† Hidenori Hiramatsu,† Masahiro Hirano,†‡ Ryuto Kawamura,§ Hiroshi Ya
Toshio Kamiya,†§ and Hideo Hosono*†‡

(Tc ~ 4K –7K)

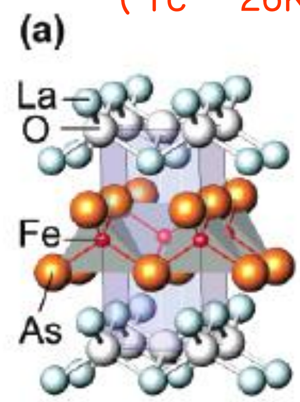


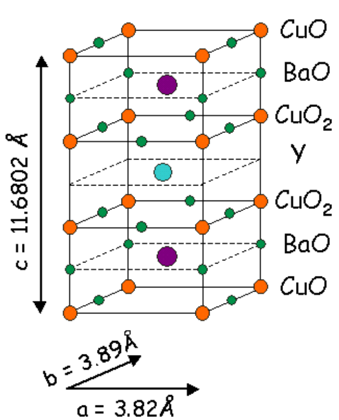
2008

Iron-Based Layered Superconductor $La[O_{1-x}F_x]FeAs$ (x = 0.05–0.12) with T_c = 26 K

Yoichi Kamihara,*† Takumi Watanabe,‡ Masahiro Hirano,†§ and Hideo Hosono†‡§

(Tc ~ 26K)

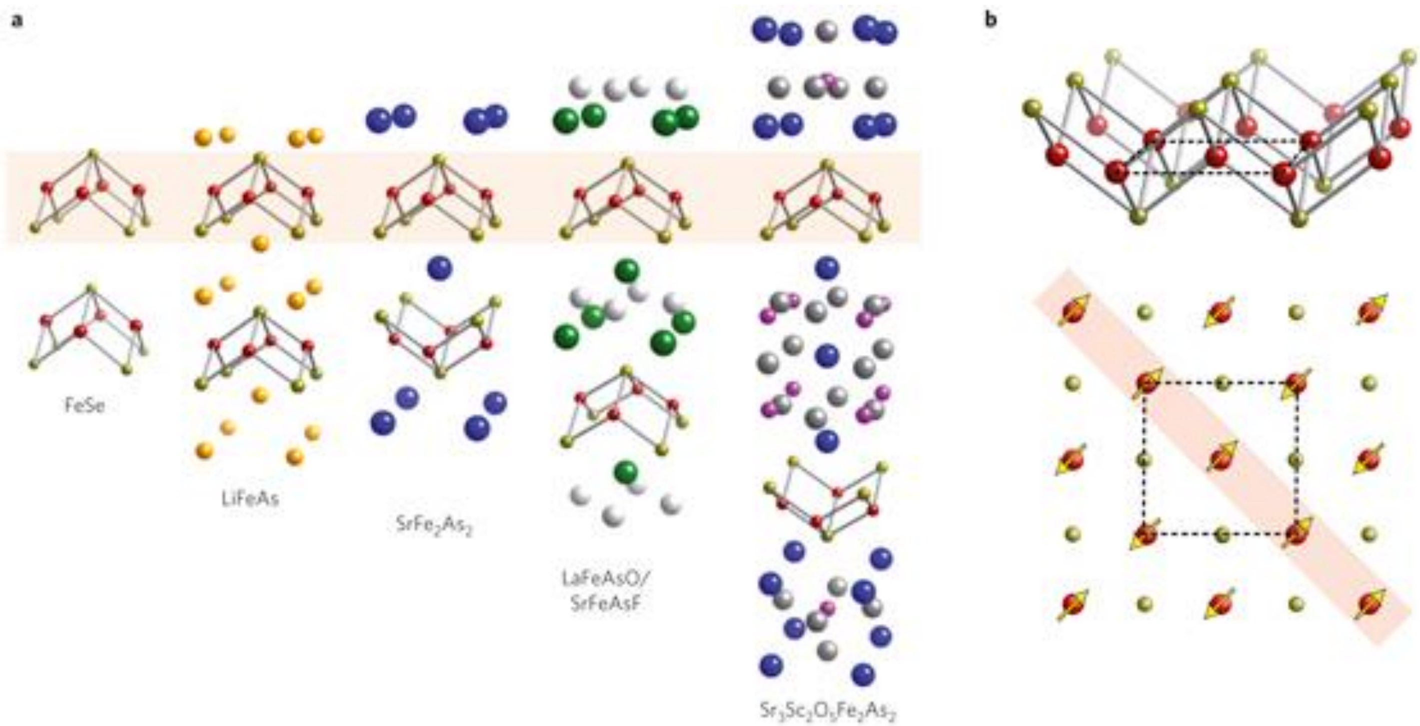




Cuprate

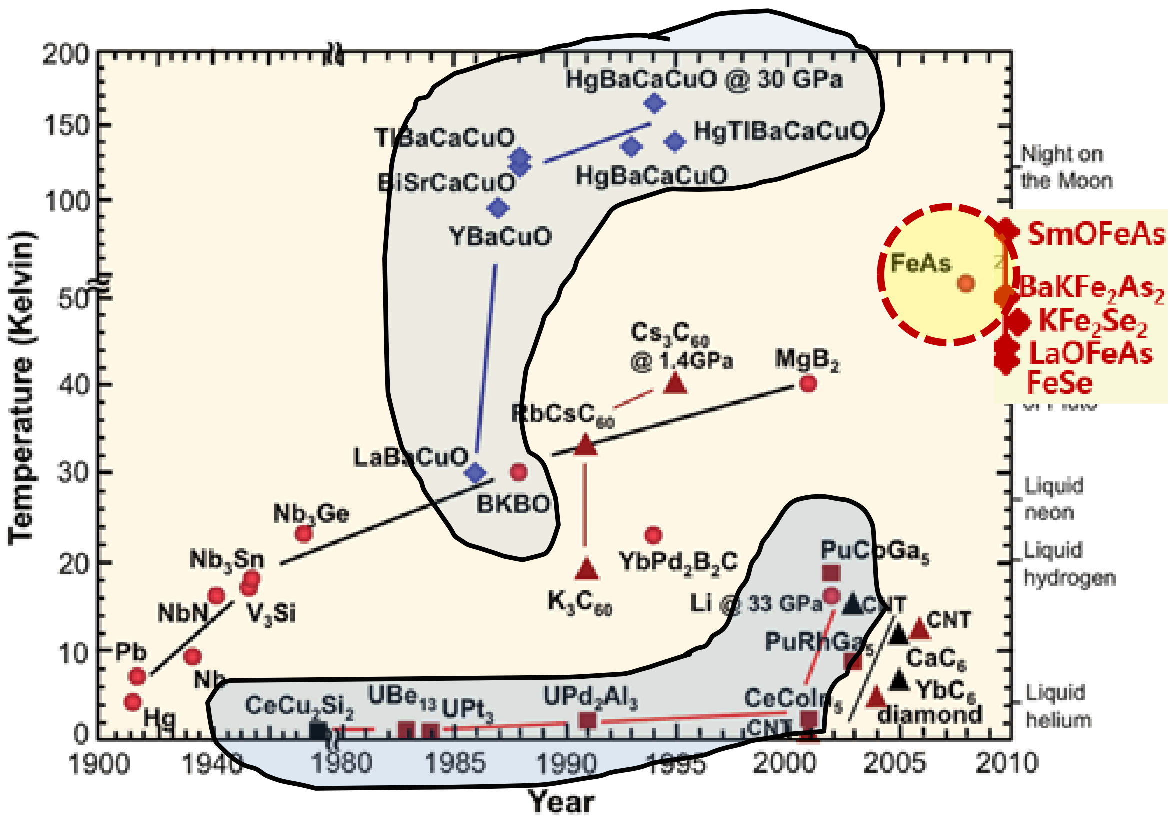
oxypnictide	T_c (K)	non-oxypnictide	T_c (K)
$LaO_{0.89}F_{0.11}FeAs$	26[8]	$Ba_{0.6}K_{0.4}Fe_2As_2$	38[9]
$LaO_{0.9}F_{0.2}FeAs$	28.5[10]	$Ca_{0.6}Na_{0.4}Fe_2As_2$	26[11]
$CeFeAsO_{0.84}F_{0.16}$	41[8]	$CaFe_{0.9}Co_{0.1}AsF$	22[12]
$SmFeAsO_{0.9}F_{0.1}$	43[8]	$Sr_{0.5}Sm_{0.5}FeAsF$	56[13]
$La_{0.5}Y_{0.5}FeAsO_{0.6}$	43.1[14]	LiFeAs	<18[15][16]
$NdFeAsO_{0.89}F_{0.11}$	52[8]	NaFeAs	9-25[17][18]
$PrFeAsO_{0.89}F_{0.11}$	52[19]	FeSe*	<27[20][21]
$GdFeAsO_{0.85}$	53.5[22]	$BaFe_{1.8}Co_{0.2}As_2$ +	25.3[23]
$SmFeAsO_{\sim 0.85}$	55[24]		

Nature Physics (2010)
J Paglione R L. Greene



Crystallographic and magnetic structures of the iron-based superconductors.

Three main groups of Un-conventional Superconductors



Key Unconventional Superconductors:

1. High-Tc cuprate SC
2. Heavy Fermion SC
3. Iron-based SC

What do we mean by “Unconventional-SC” ?
→ Probably, non-BCS SC

When did it start ? → because of high-T_c cuprates (1986 Bednorz & Meuller)
Why did it start ? → because of PW Anderson, partly

Ideas of **truly non-BCS** superconductors: e.g.

1. RVB SC : based on spin-charge separation
2. Anyon SC : based on new q.p. (neither fermion nor boson)

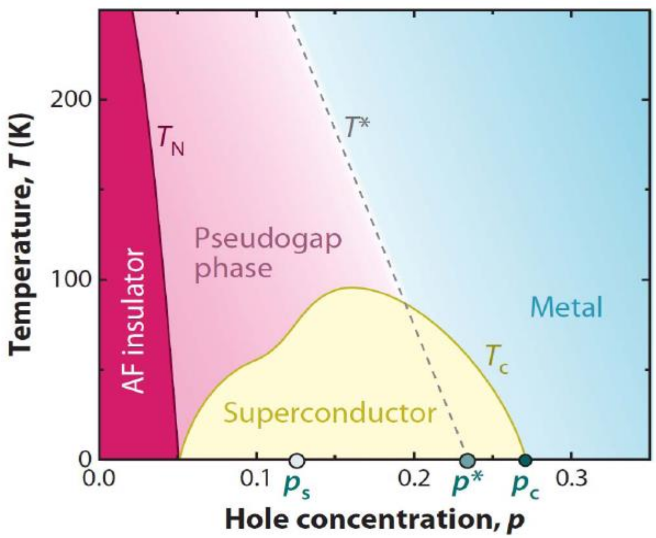
Interesting ideas but didn't pass exp. tests. →
at least, the cuprate and all other SC seems to consist of **Cooper pairs.**
(*Josephson tunneling* + *many other conventional SC properties*)

But still many people wish to find a novel (non-BCS) theory.

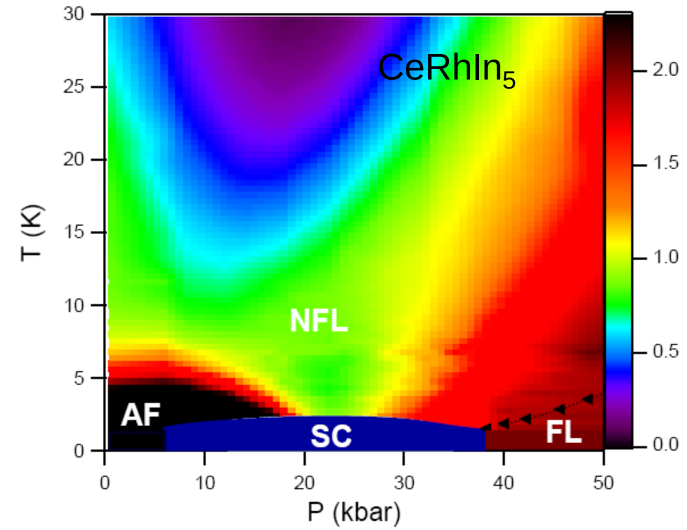
→ But, besides RVB, Anyon theories, what non-BCS theory ?

Common Phase diagram of unconventional SCs

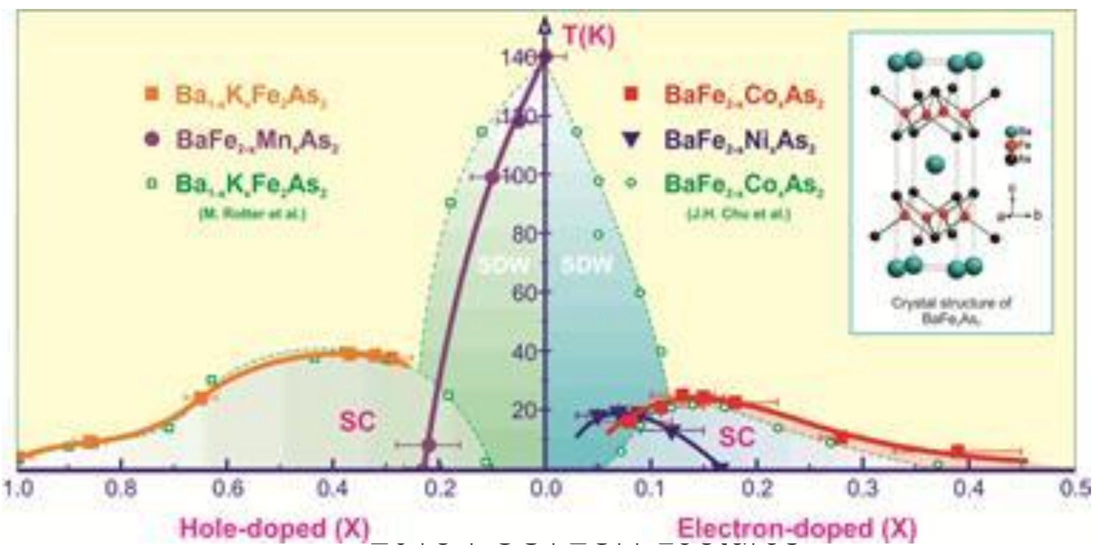
cuprate



heavy fermion



Fe-122(BaFe2 As2)



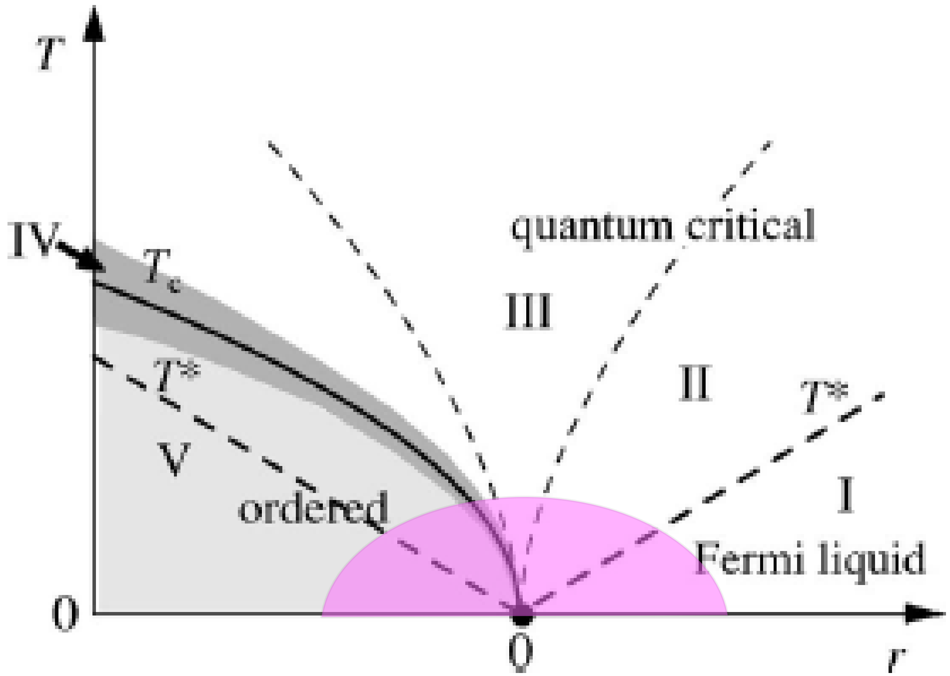
Common Belief of paradigm:

-- *Strong correlation* → ***QCP*** *(?)*

→ *no q.p* → *Non-Fermi Liquid normal state*

→ *Un-conventional SC (perhaps high-T_c)*

→ *Un-conventional pairing mechanism* ***(non-BCS type ?)***

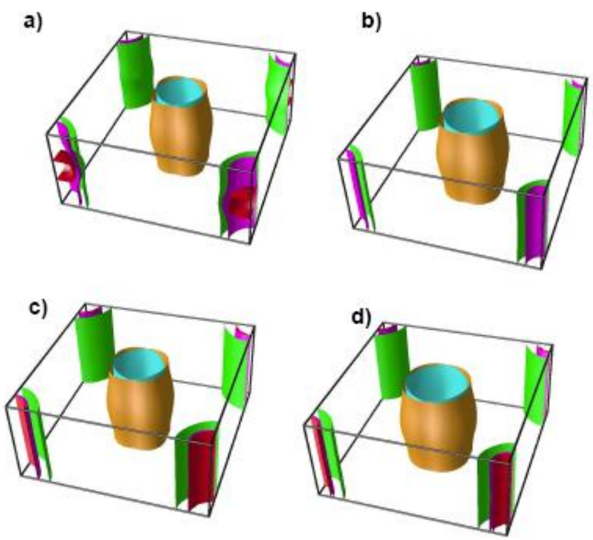


BCS theory of Fe-based Superconductor:
Sign-changing S-wave state or **±S-wave**

How much can we understand Fe-based SC with BCS theory ?

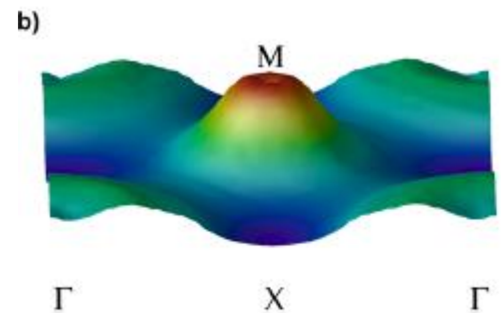
Early theories (2008) just after the discovery of La(OF)FeAs (26K)

Mazin et al, PRL 101, 057003; Kuroki et al., 101, 087004 (later many more)



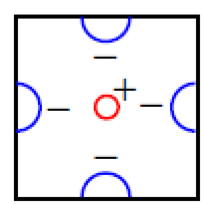
2D FS

+

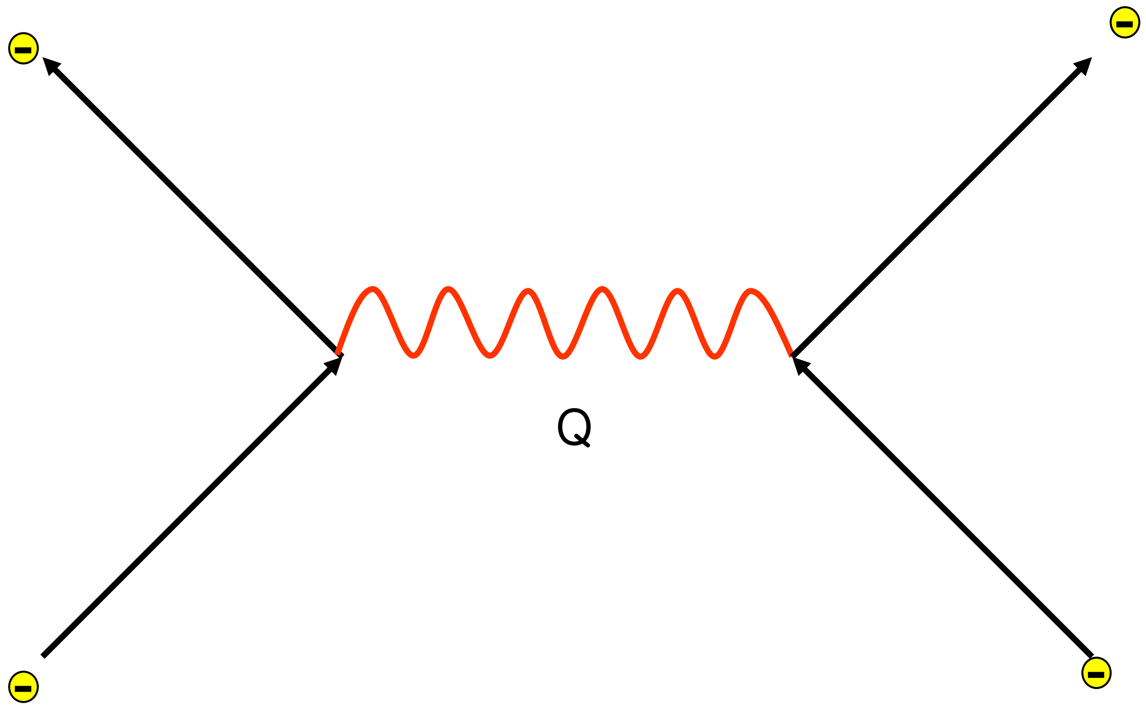


Strongly peaked SF
χ**(q)** at **q=(π,π) : AFM fluc.**
→ All repulsive !!

→ ±S gap solution
(BCS theory)



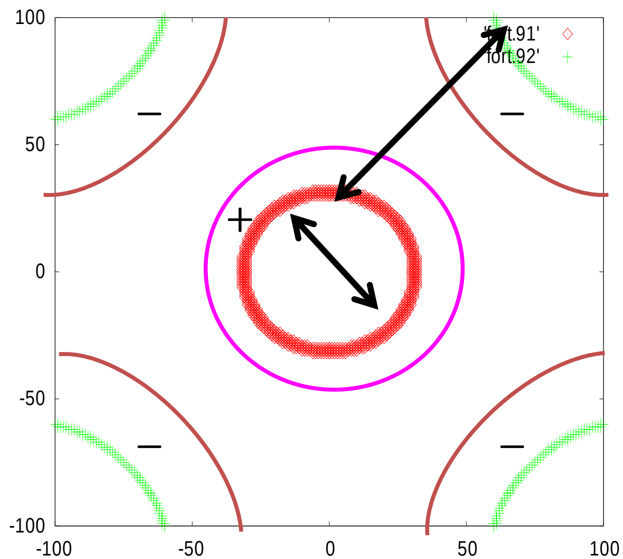
Cartoon picture of the BCS theory of Fe-SC



Replace V_{ph} by $V(Q)_{mag}$

Sign-changing S-wave solutions = multi-bands + AFM fluctuations

±S-wave



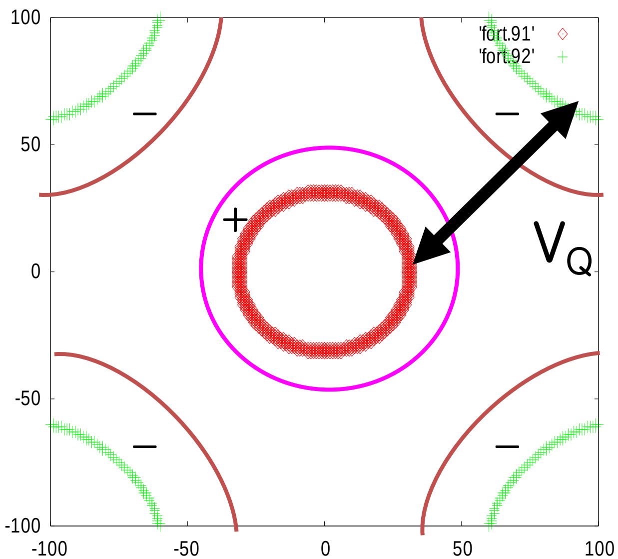
$V_{int}(q)$: All repulsive interaction

Two band BCS gap Equation

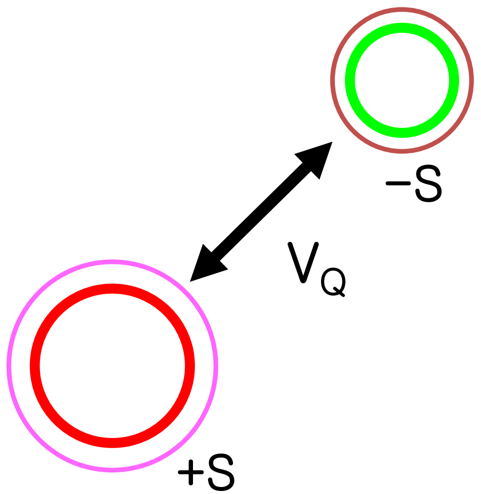
$$
\begin{aligned}
\Delta_h(k) &= \qquad (5)\\
&- \sum_{k'} [V_{hh}(k,k')\Delta_h(k')\chi_h(k') + V_{he}(k,k')\Delta_e(k')\chi_e(k')],\\
\Delta_e(k) &= \\
&- \sum_{k'} [V_{eh}(k,k')\Delta_h(k')\chi_h(k') + V_{ee}(k,k')\Delta_e(k')\chi_e(k')].\\
& \qquad (6)
\end{aligned}
$$

Sign-changing S-wave solutions for multi-bands + AFM interaction

±S-wave pairing

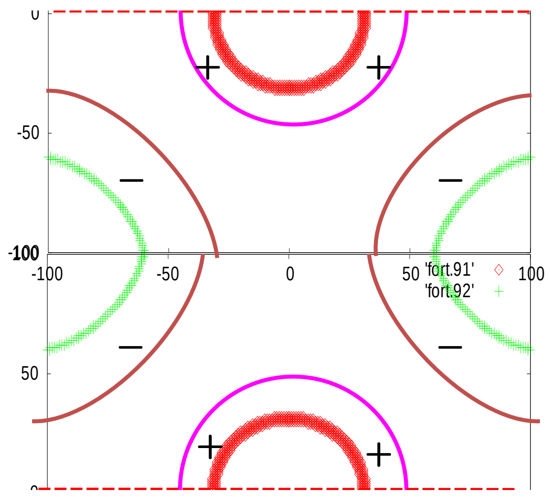


→



Two band Model

±s-wave = d-wave



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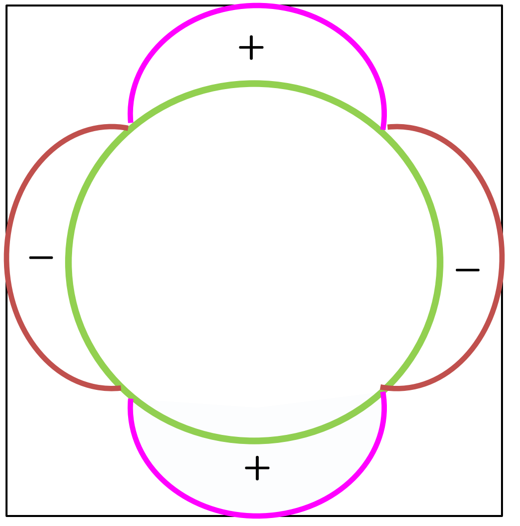
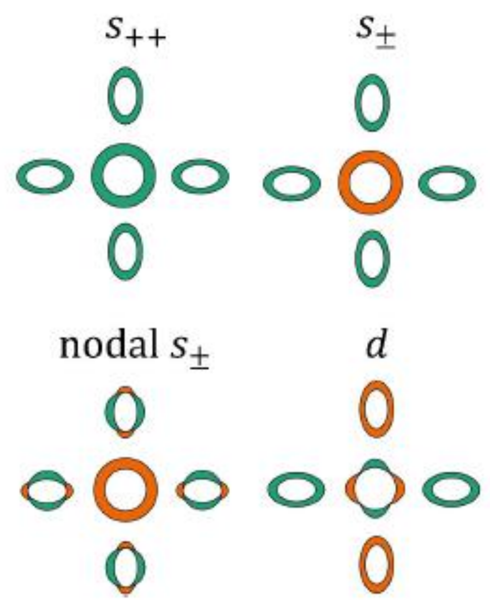


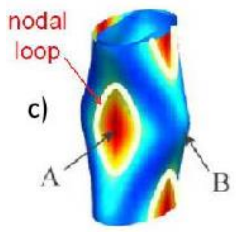
Universal Pairing Mechanism of AFM spin fluctuation V(Q)

All BCS theory

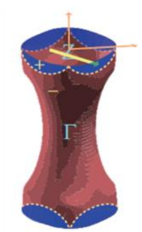
More realistic calculations with **orbital** d.o.f., 3D $\mathbf{V_{spin}(q)}$, FS shapes, etc found that **±s-gap** is a dominant solution.
But with some parameters, other solutions are possible:
accidental nodes, horizontal node, d-wave, s++, etc.

P. Hirschfeld
I. Mazin
I. Eremin
A. Chubukov
H. Kontani
K. Kuroki
Z. Tesanovic
J. Hu
D. H. Lee
R. Thomale
T. Maier
T. Das
A.V. Balatsky
…





Matsuda et al 2010

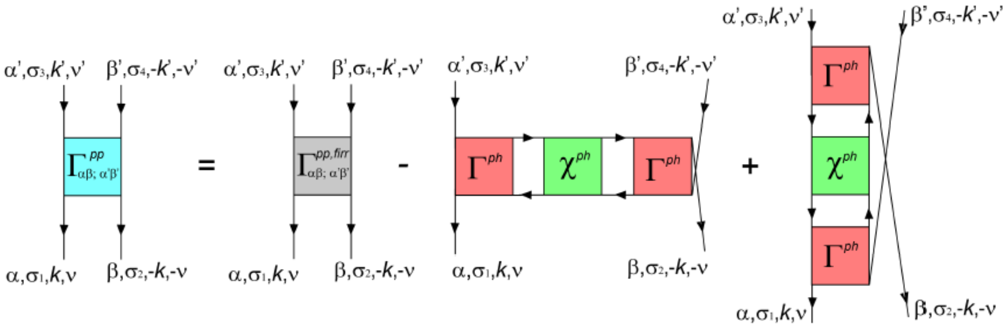


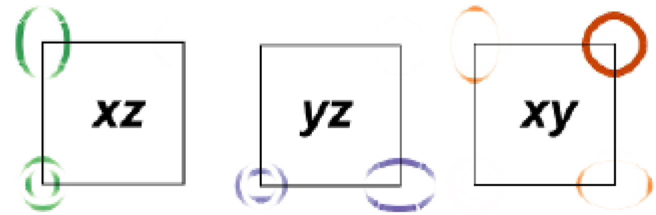
Kuroki et al JPSJ 2010

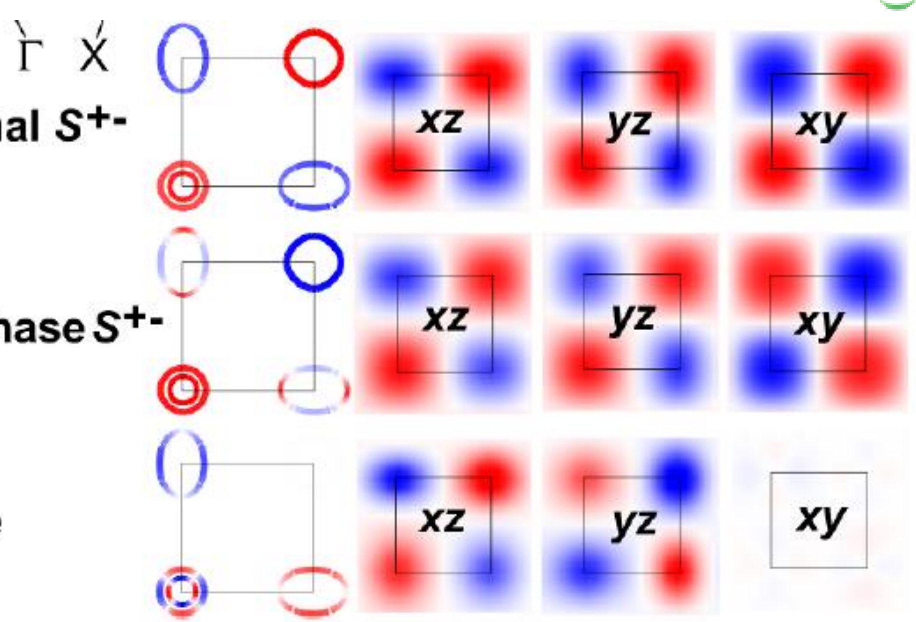
*P. J. Hirschfeld, M. M. Korshunov, I. I. Mazin, **Rep. Prog. Phys.** 74, 124508 (2011)* an orbital-antiphase pairing symmetry in iron-based superconductors

Z. P. Yin,* K. Haule, and G. Kotliar

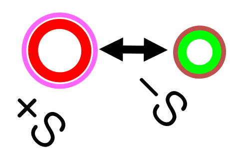
More variation



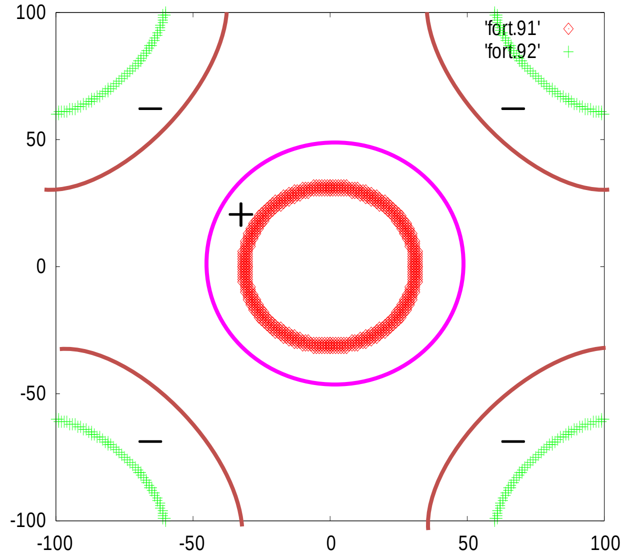




Closely competing



Standard Paradigm of Pnictide SC



Q: Is it consistent with experiments ?

A: Almost Yes.

Experimental situation of Fe-based SC

1. Most of Experiments (90 % or more)
(ARPES, Raman, Penetration depth, Specific heat, Thermal Conductivity, NMR, etc)
are all consistent with the **±s-wave gap**.

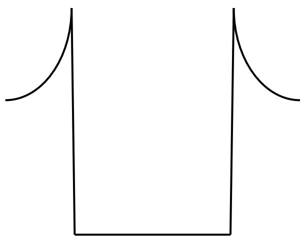
2. Problem is : it is too boring BCS (also s-wave) SC.

Part II:

Superconducting properties of ±s-wave gap

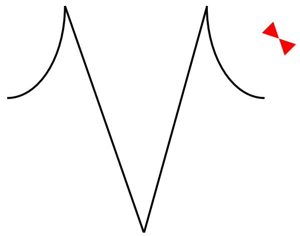
Probing **Pairing Mechanism** is **Religion**, but

Probing **Pairing symmetry** is **Science !!** → low energy DOS **N(ω, T, H)**



$$\sim e^{-T/\Delta}$$

Activated laws



$$\sim T^{\alpha}$$

Power laws

Many probes for **N(ω, T, H):**

- **C(T,H)/T**
- **Thermal conductance κ(T, H)**
- **Penetration depth λ(T)**
- **NMR 1/T_1 (T,H)**
- **Tunneling**
- **Etc.**

Hallmarks of D-wave SC (Cuprates)

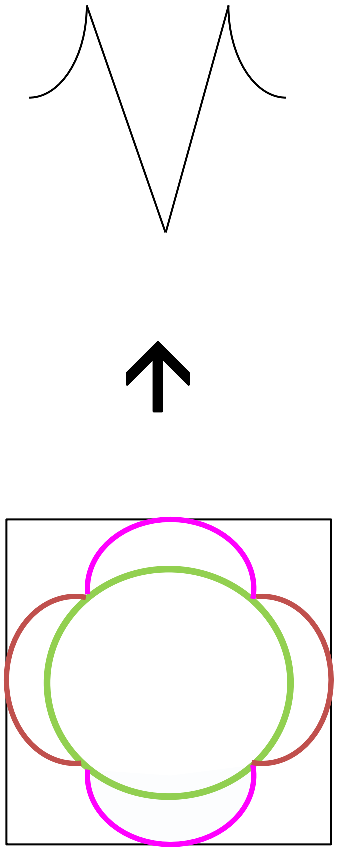
NMR $1/T_1 \sim T^3$

Pene. Depth $\lambda(T) \sim T$

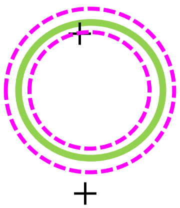
Superfluidity density $\Delta\rho(T) \sim T$

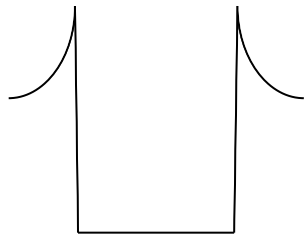
Thermal conduc. $\kappa(H) \sim H^{1/2}$

\+ *various power laws* T^{α}



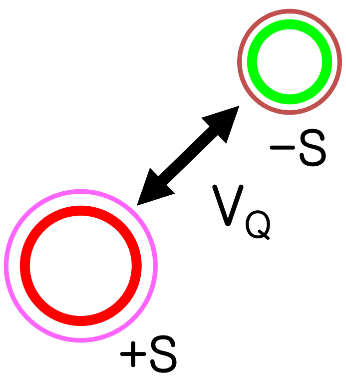
SC properties of **±s gap** (Fe-SC) : exponential behaviors

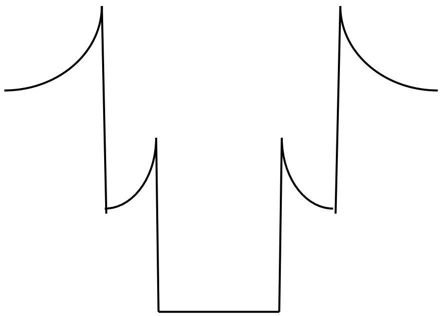




Simple **s-wave SC**

$$\sim e^{-T/\Delta}$$

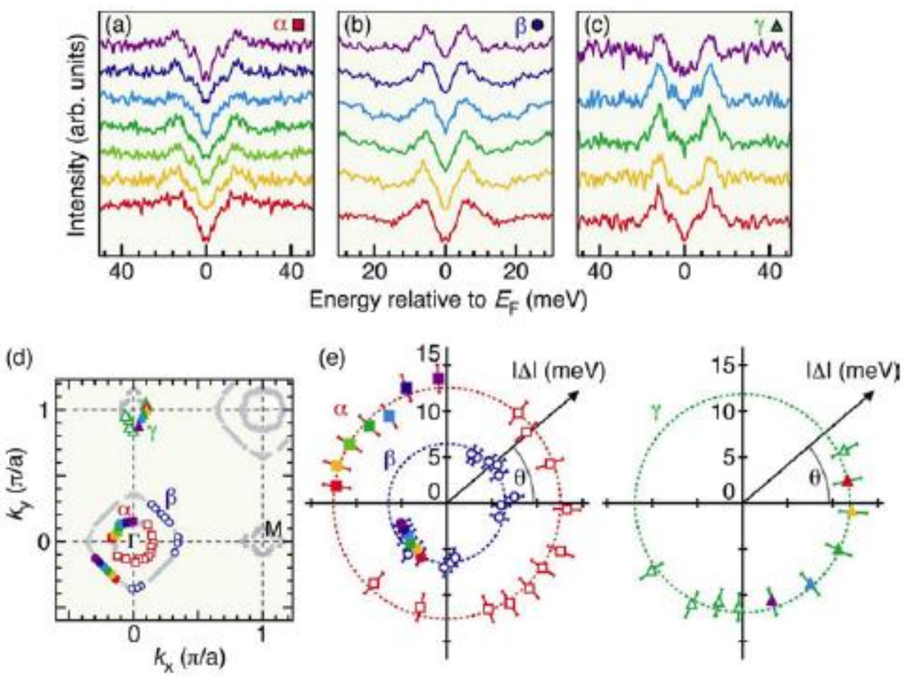




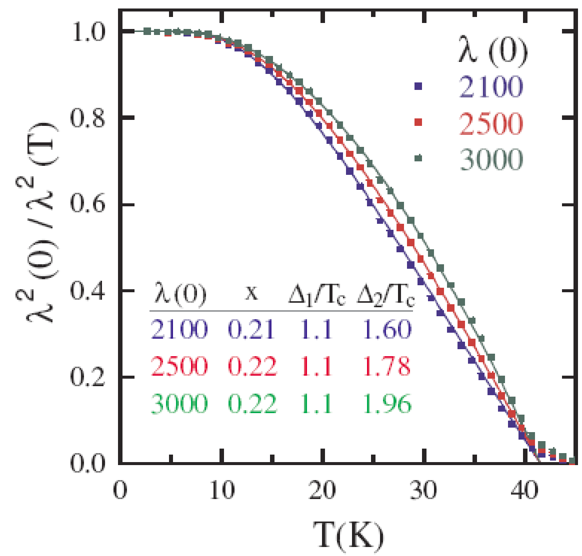
$$\sim e^{-T/\Delta_{small}}$$

Direct Evidences of isotropic S-wave gaps

$Ba_{0.6}K_{0.4}Fe_2As_2$ H.Ding et al



$SmFeAsO_{1-x}F_y$



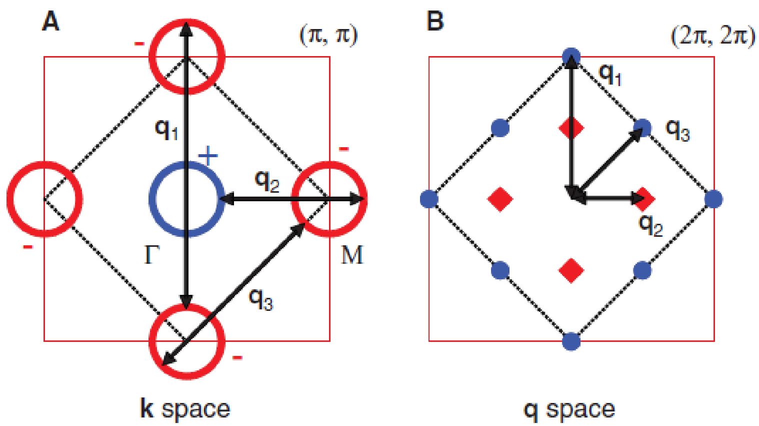
L. Malone,

Evidence for isotropic full gaps

Not so Direct Evidence of sign-changing S-wave gaps

Unconventional *s*-Wave Superconductivity in Fe(Se,Te)

T. Hanaguri,[1,2]* S. Niitaka,[1,2] K. Kuroki,[2,3] H. Takagi[1,2,4]



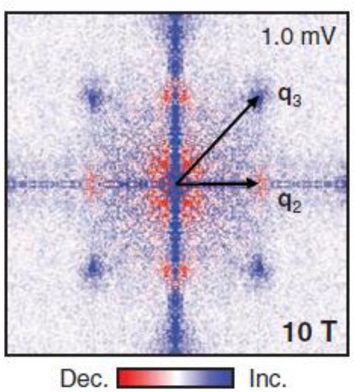


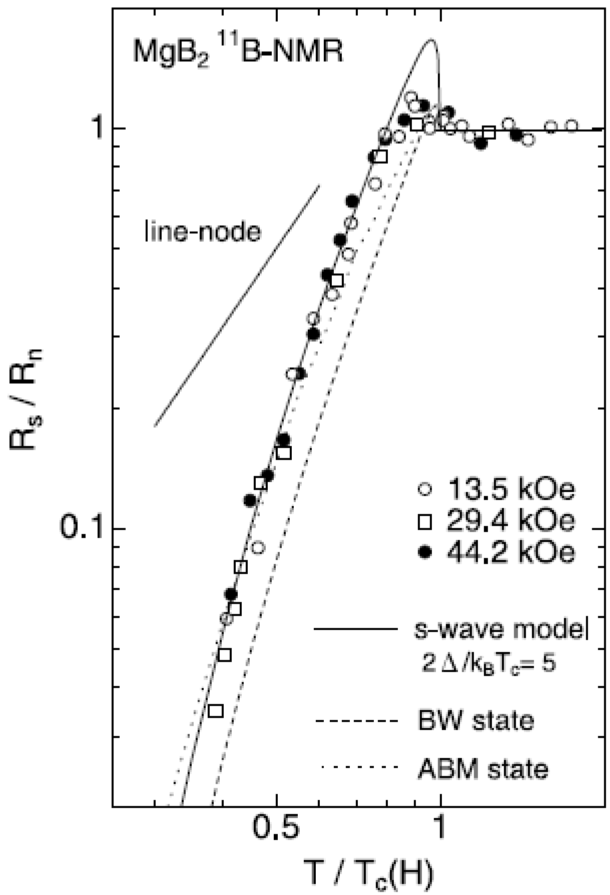
Fig. 4. Magnetic field–induced change in QPI intensities indicates the $s_{\pm}$-wave symmetry. Differ-

q_2 point signal enhances or decreaces with B fields

(1) First challenge for full s-wave gap

NMR spin relaxation: T_1 time

S-wave SC



d-wave like

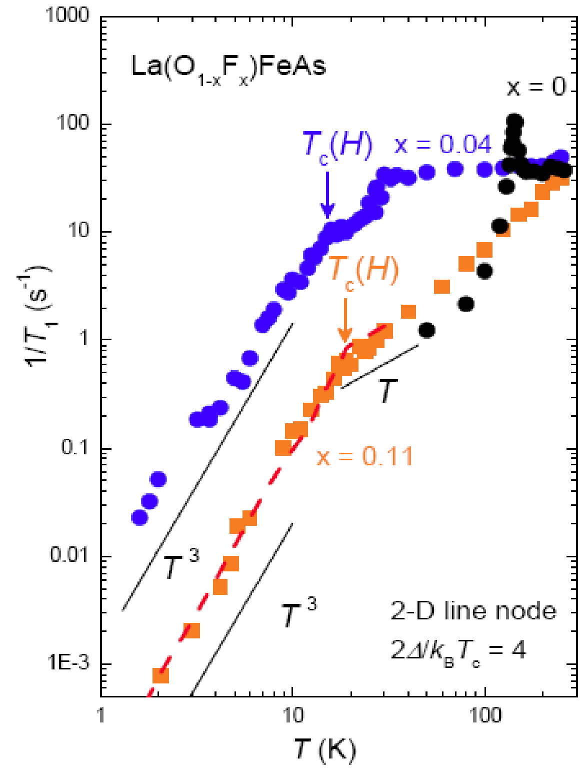
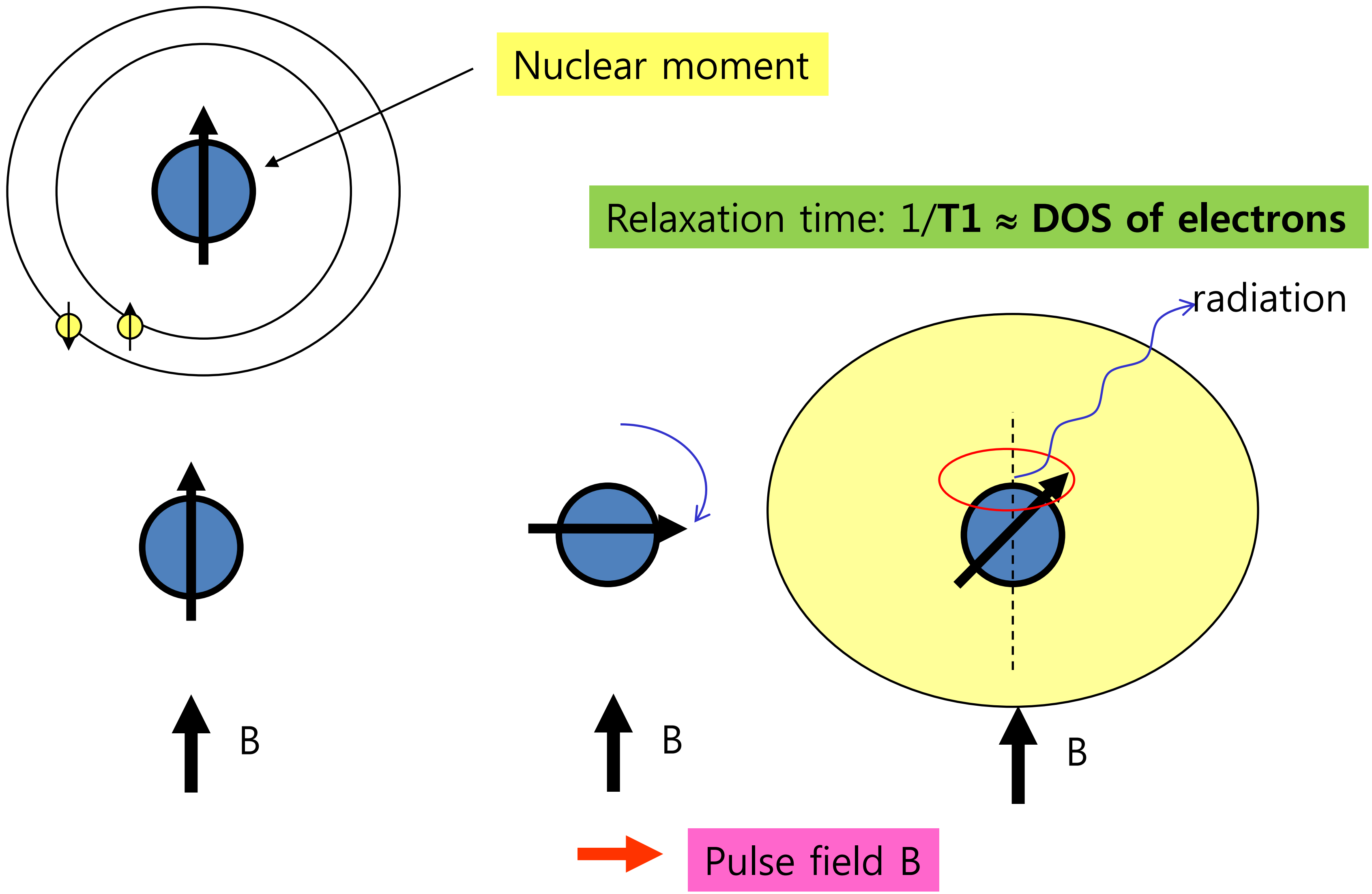


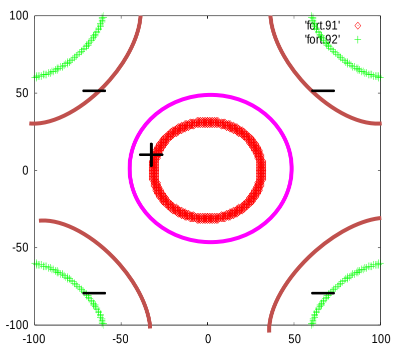
Fig. 7. T-dependence of T^{-1} under $H \sim 9.9$ T for x =0.0, 0.04 ($T_c(H) = 16K$) and 0.11 ($T_c(H) = 20K$). The dashed curve is a calculation with assuming a line node gap $\Delta(\phi) = \Delta_0 \sin(2\phi)$ with $\Delta_0 = 2k_B T_c$ (see text).

Evidence for nodal gap

NMR cartoon picture

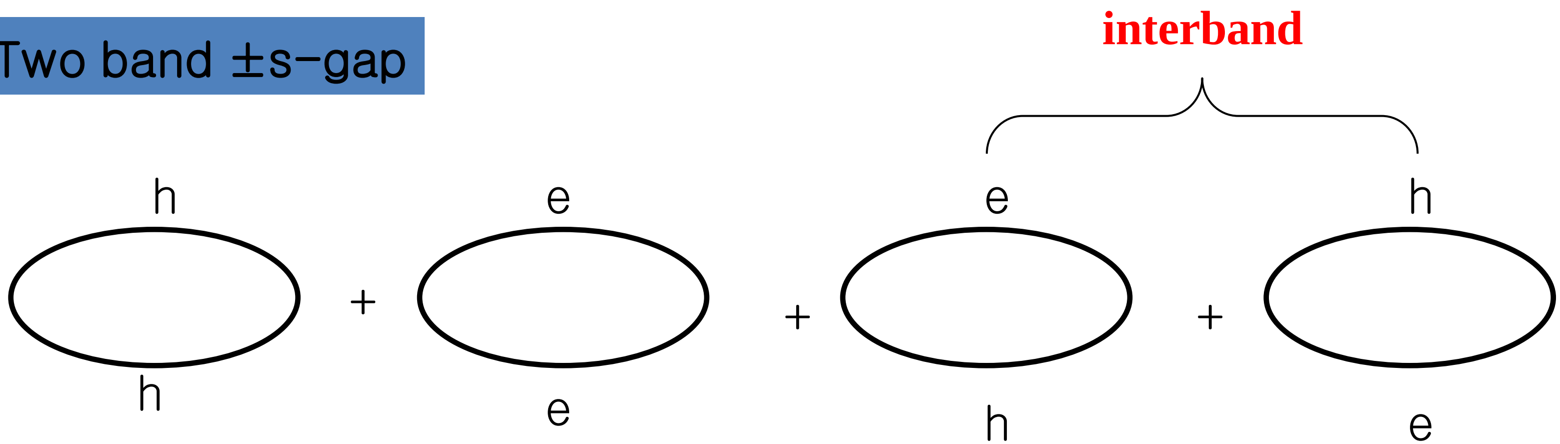


Spin-lattice relaxation rate $1/T_1$



$$\frac{1}{T_1 T} \propto \lim_{\omega \to 0} \sum_{\mathbf{q}} \frac{1}{\omega} \mathrm{Im}\, \chi(\mathbf{q}, \omega).$$

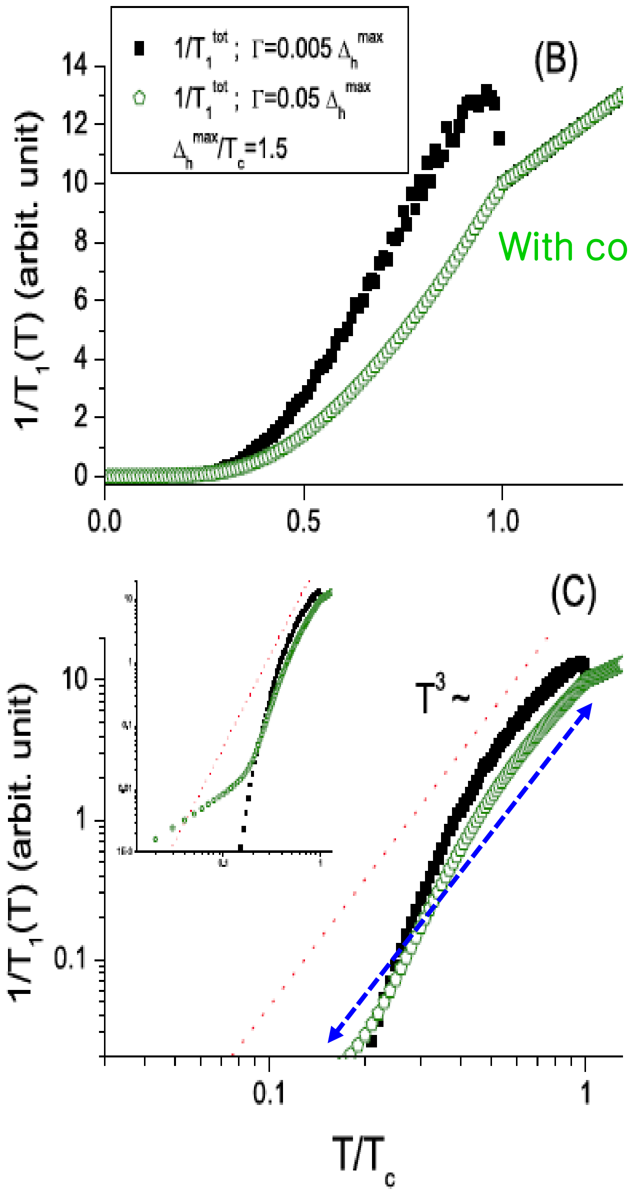
Two band ±s-gap



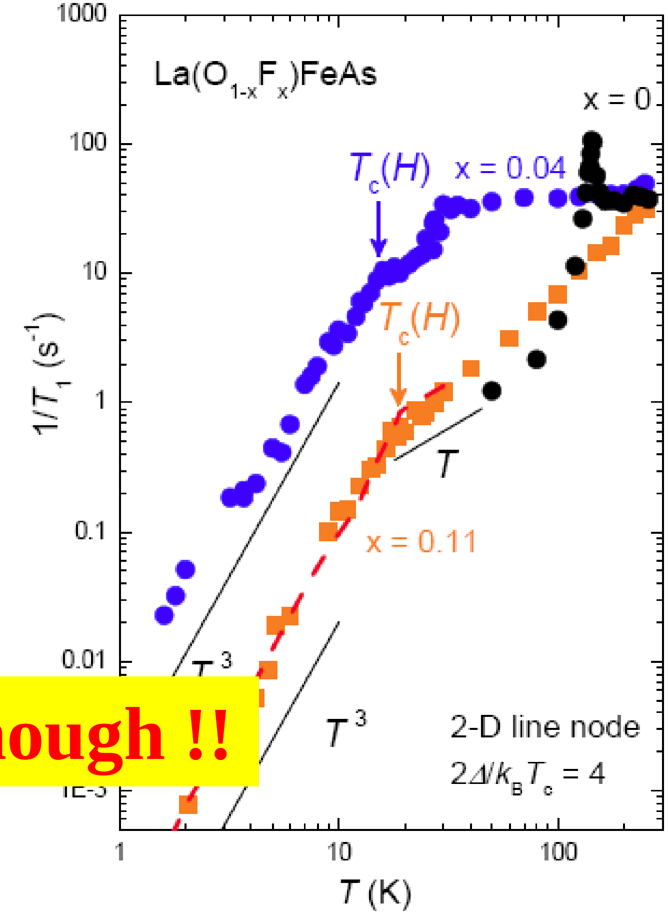
Coherence factor

$$\frac{1}{2}\left(1 + \frac{\Delta_a \Delta_b}{E_a(k) E_b(k)}\right)$$

±Δ interband term gives (−)suppression → **No Hebel-Slichter peak**



With constant damping



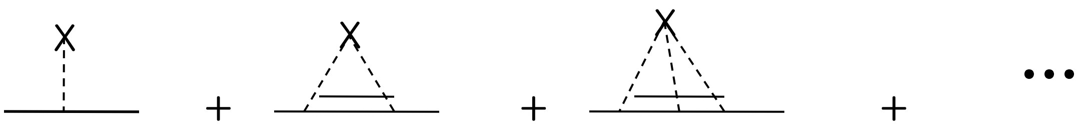
Not good enough !!

Fig. 7. T-dependence of T^{-1} under $H \sim 9.9$ T for x =0.0, 0.04 ($T_c(H) = 16K$) and 0.11 ($T_c(H) = 20K$). The dashed curve is a calculation with assuming a line node gap $\Delta(\phi) = \Delta_0 \sin(2\phi)$ with $\Delta_0 = 2k_BT_c$ (see text).

Impurity effect

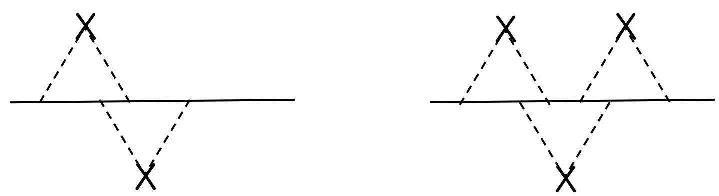
T-matrix approximation for imp. scattering

(summation of a infinite series of single impurity scattering) x (imp. concentration n_{imp})



$\sim O(n_{imp})$

Not include

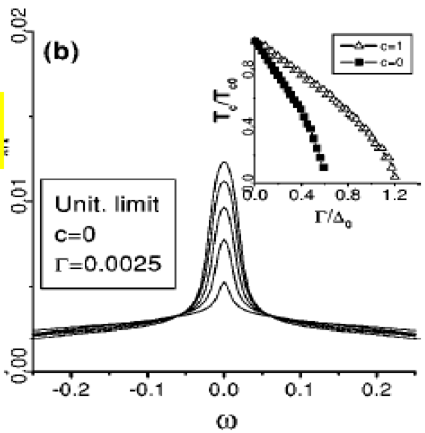


$O(n^2_{imp})$ $O(n^3_{imp})$

1. Low concentration expansion.
2. Interaction strength can be strong or weak.

Zero energy resonance for d-wave

$\mathrm{Im}\Sigma(\omega)$



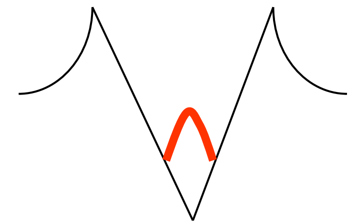
$$T_a^i(\omega_n) = \frac{G_a^i(\omega_n)}{D} \qquad (i=0,1;\ a=h,e),$$
$$D = c^2 + [G_h^0 + G_e^0]^2 + [G_h^1 + G_e^1]^2,$$

Unitary scattering (c=0)

D−wave case

For D-wave, $G_D^1 = \left\langle \frac{\tilde{\Delta}(\theta)}{\sqrt{\tilde{\omega}_n^2 + \tilde{\Delta}^2(\theta)}} \right\rangle_\theta = 0.$

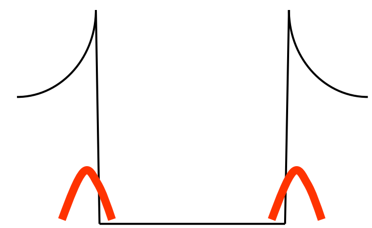
D=0



Ordinary S−wave case

If $[G_s^1] \sim 1$

D~1

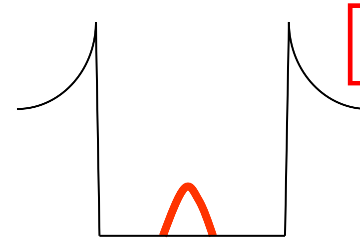


Artificial ±S-wave case

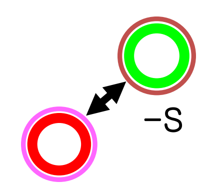
Magnetic impurity (Shiba state)

If $[G^1] = 0$

D=0



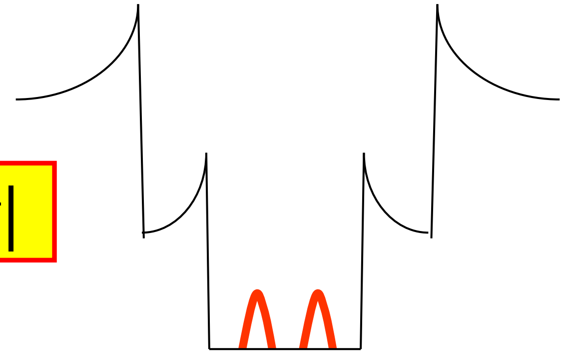
$|\Delta^+| = |\Delta^-|$

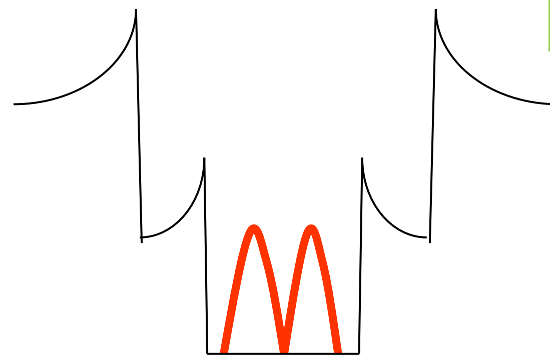


Impurity effect (unitary limit)

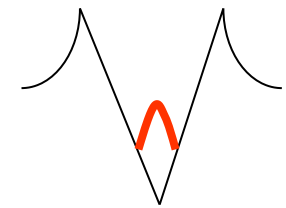
In ±s-wave gap,
Off-centered resonance

$|\Delta^{+}| \neq |\Delta^{-}|$

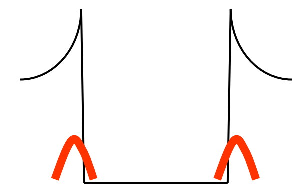


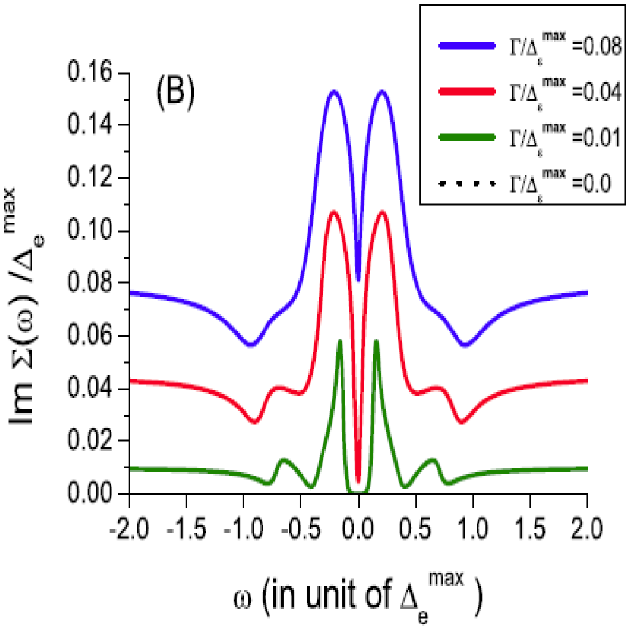


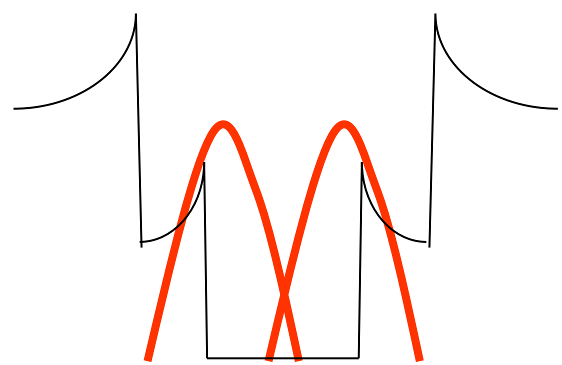
D-wave case



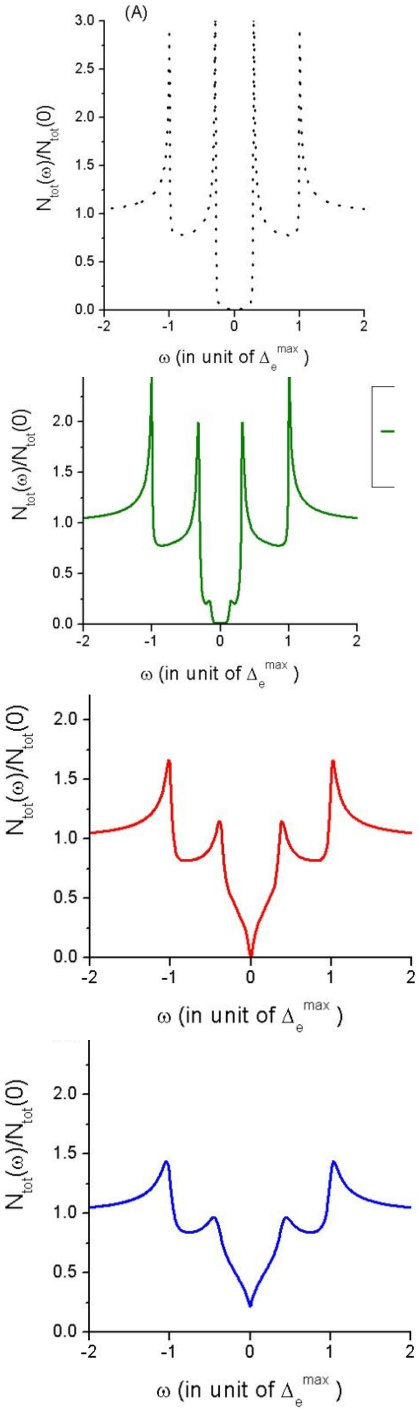
Ordinary S-wave case







Increasing impurities

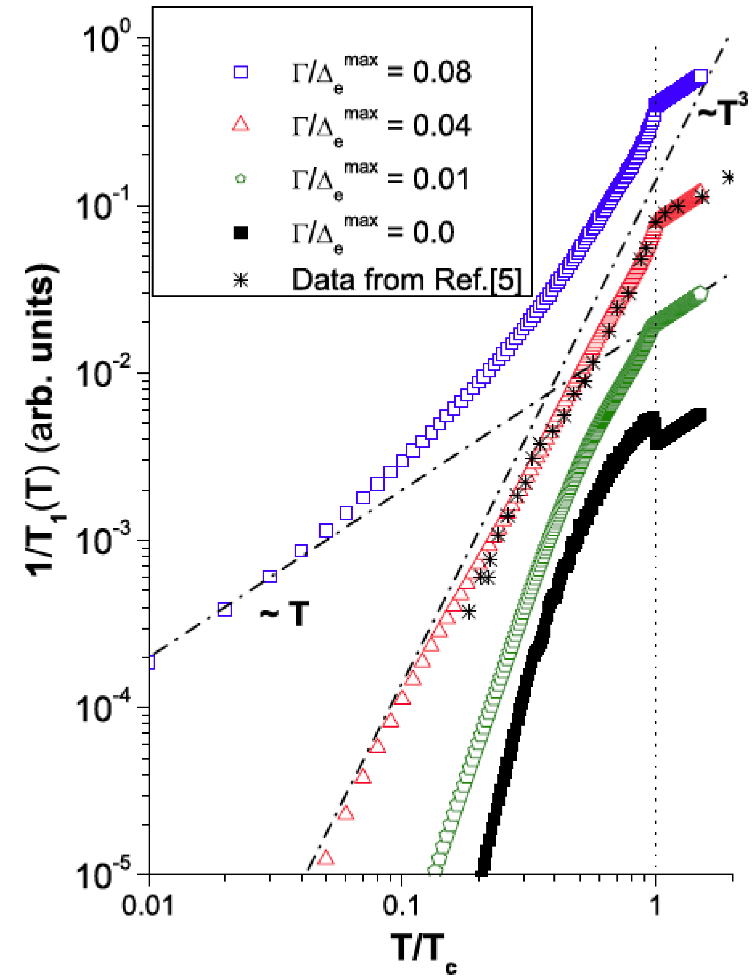


Full gaps

Reduced gap

Pure d-wave like

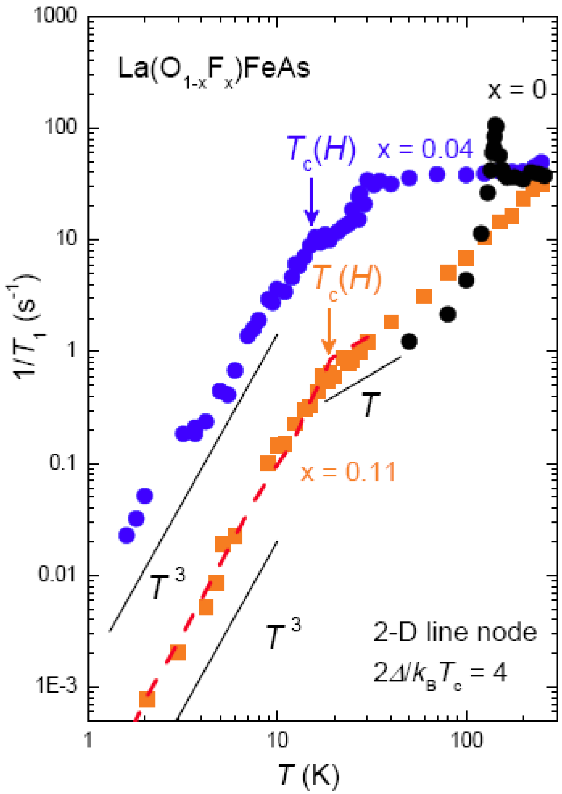
Disordered d-wave like



$LaFeAsO_{0.92}F_{0.08}$

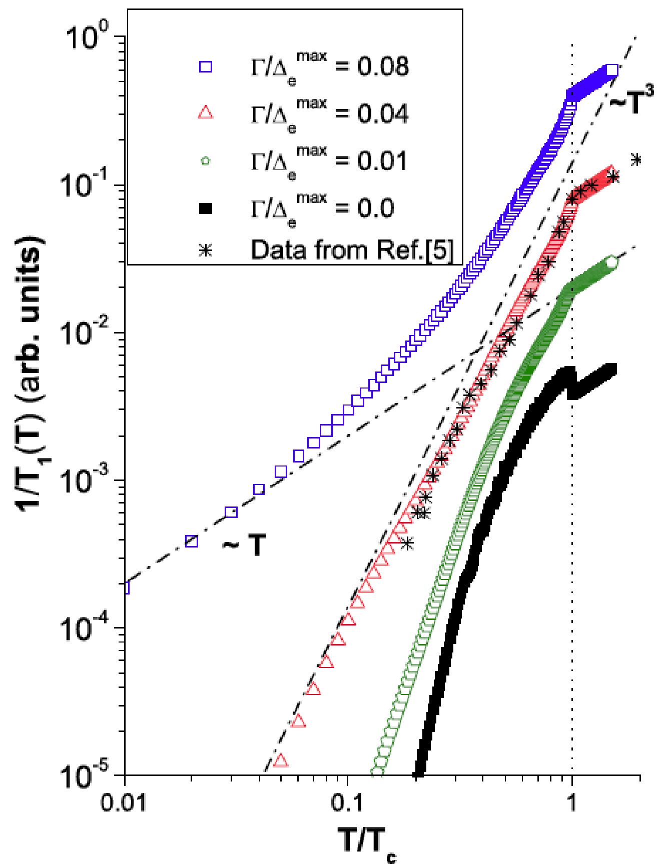
(2) Spin-lattice relaxation rate $1/T_1$ → T^3 → lines of node (D-wave)

Y.Bang, PRB 79, 054529 (2009)



D-wave evidence

Y. Nakai et al, PRB 79, 212506 (2009)



More consistent with ±S-wave

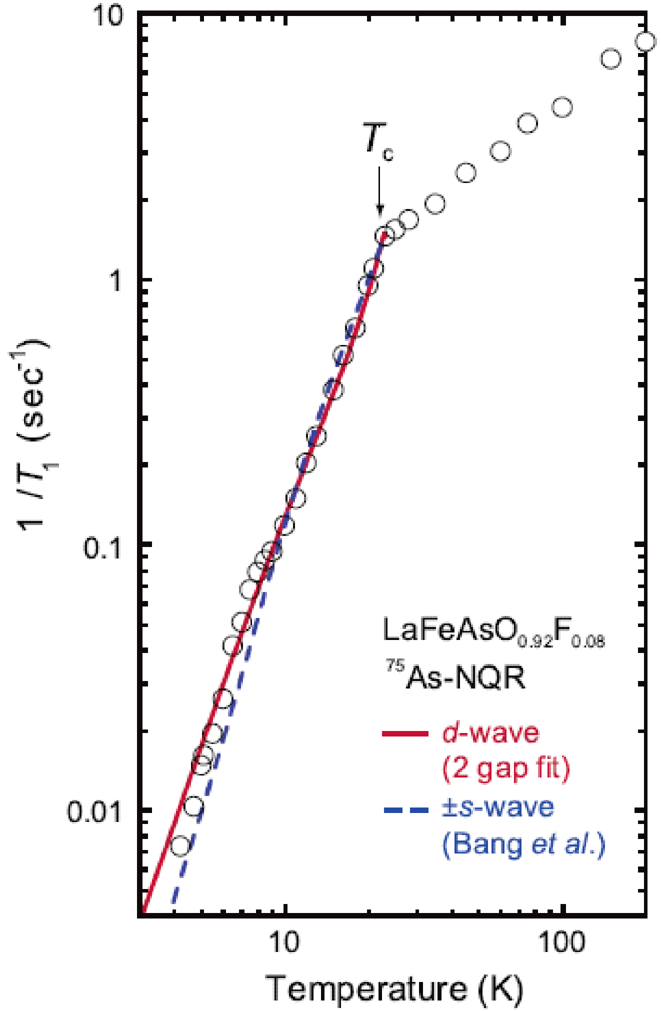


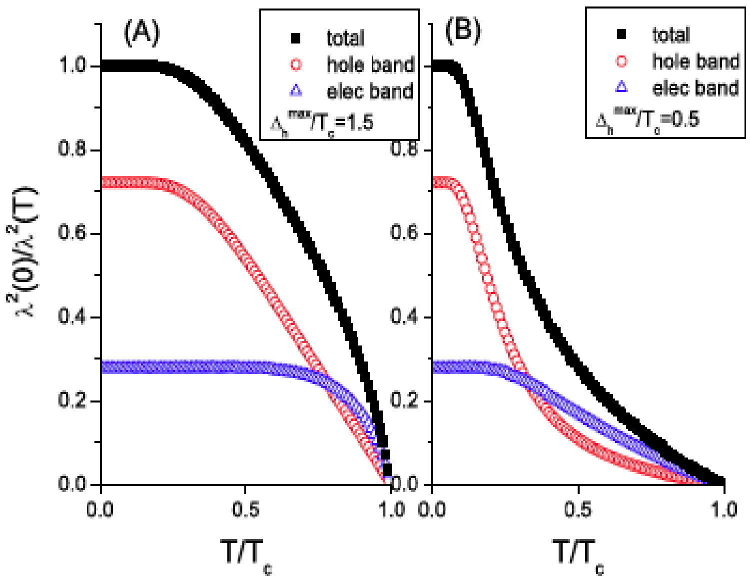
FIG. 3: (color online) The temperature dependence of $^{75}(1/T_1)$ in $LaFeAsO_{0.92}F_{0.08}$ measured at zero magnetic field. The solid curve is a two gap fit assuming a d-wave symmetry with parameters, $\Delta_1(0) = 4.2k_BT_c$, $\Delta_2(0) = 1.6k_BT_c$, and $\alpha = 0.6$ (see text). The dotted curve is a simulation assuming two s-wave gaps that change signs with parameters, $\Delta_1(0) = 3.75k_BT_c$, $\Delta_2(0) = 1.5k_BT_c$, and $\alpha = 0.38$ referred from literature.[30,31] The solid arrow indicates T_c.

Penetration depth $\lambda(T)$

2^{nd} challenge

Penetration depth (pure case)

L. Malone,



C. Martin, R. T. C
P. C. Canfield,

$SmFeAsO_{1-x}F_y$

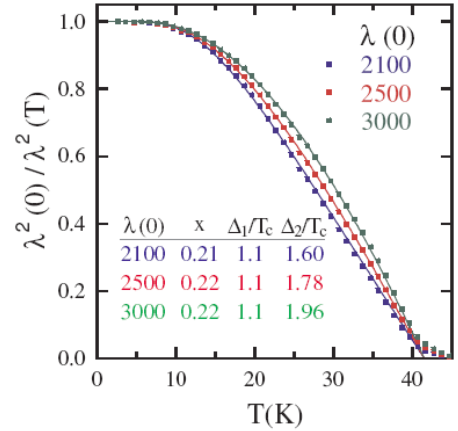
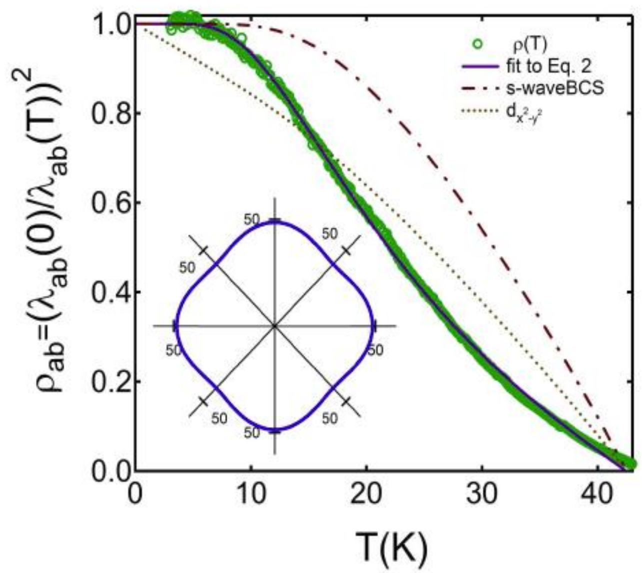


FIG. 2: (color online). Calculated normalized superfluid density $[\lambda^2(0)/\lambda^2(T)]$ versus temperature for sample #1, using different assumed values for the zero temperature penetration depth $\lambda(0)$. The solid lines are fits to the two gap model described in the text, and the inset table shows the fit parameters.

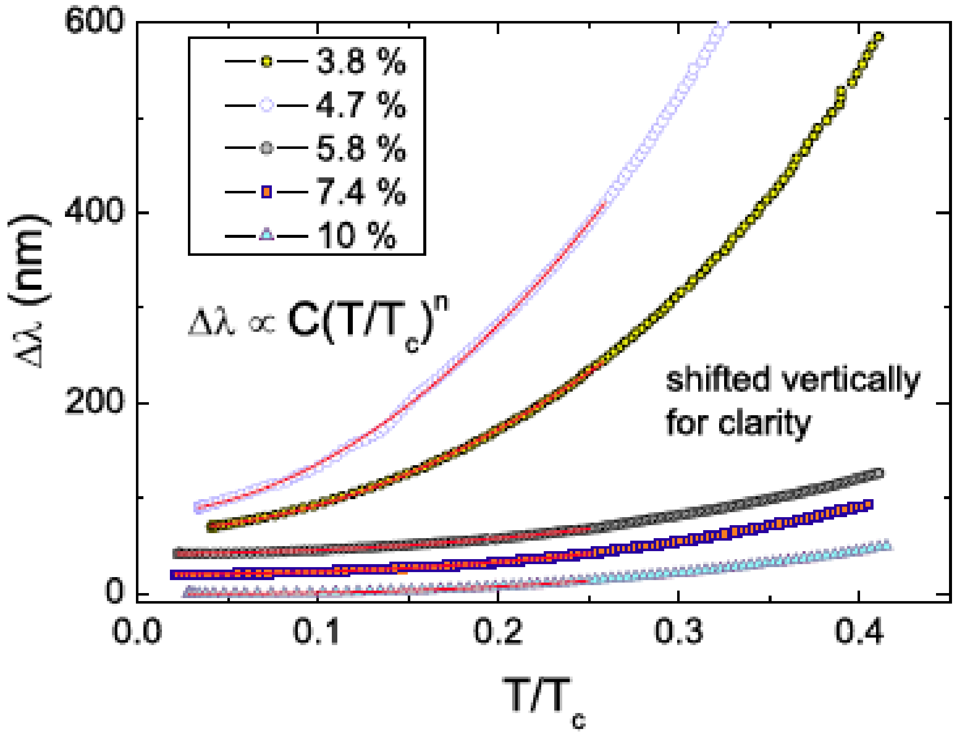
$NdFeAsO_{0.9}F_{0.1}$

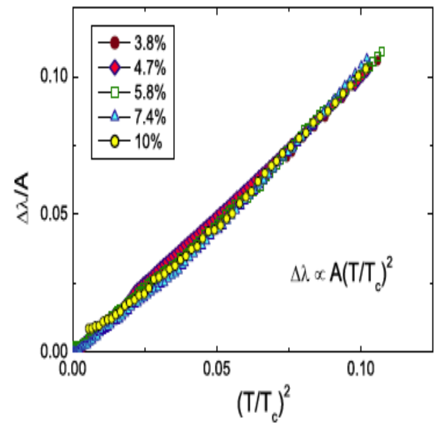


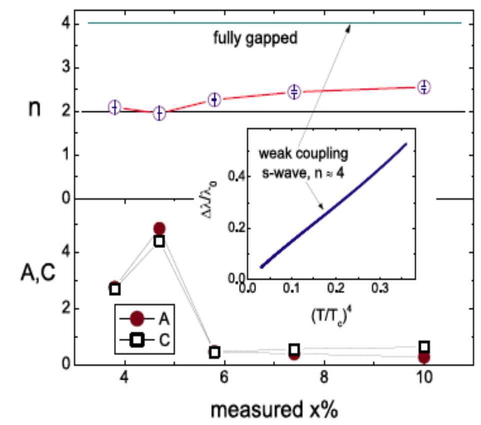
Prozorov et al

$Ba(Fe_{1-x}Co_x)_2As_2$

n=2 −2.5

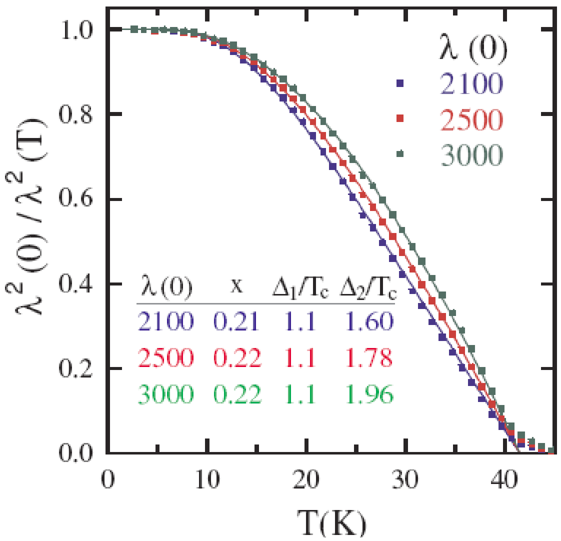


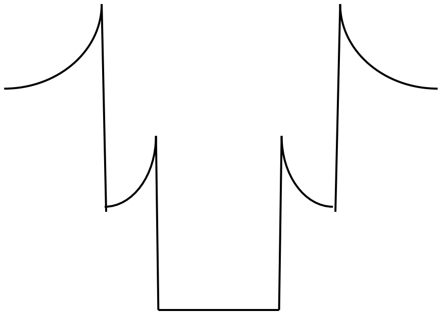




$1/\lambda^2(T) = n_{tot} - n_{qp}(T)$

Classically, $n_{qp}(T) \sim$ N(T)

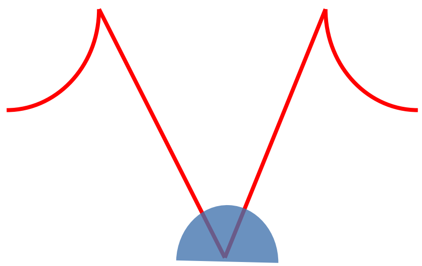




$$N(T) \sim e^{-T/\Delta_{small}}$$

This is not exactly true with impurities.

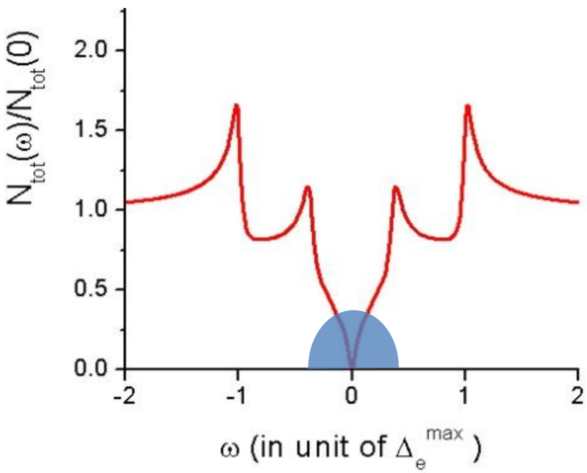
D-wave gap : kinematically driven V-shape DOS



$\delta\lambda_v(T) \sim T$

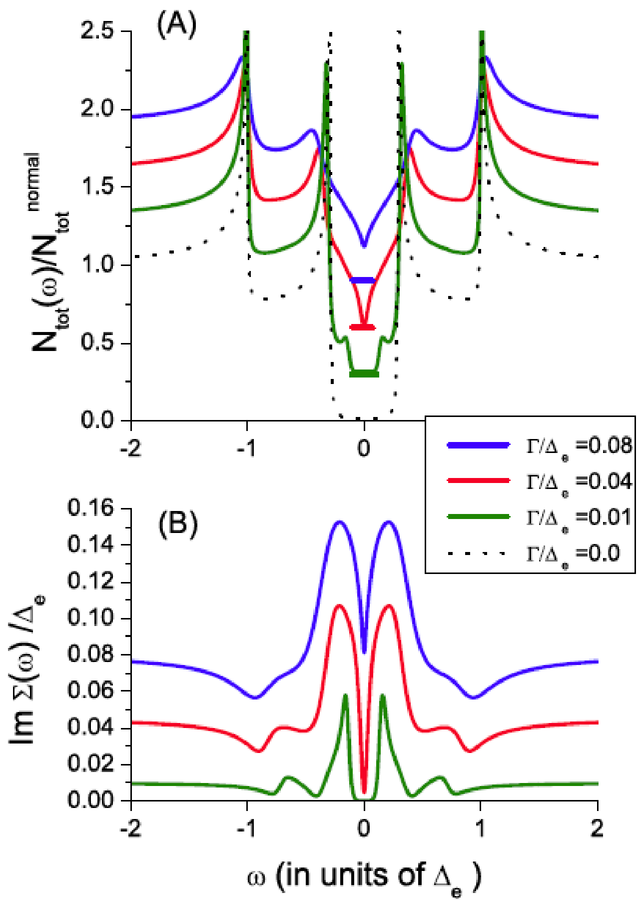
$\delta\lambda_{const}(T) \sim T^2$

±S-wave gap : dynamically driven V-shape DOS



$\delta\lambda_v(T) \sim T^3$

$\delta\lambda_{const}(T) \sim T^2$



$$K(T) = \frac{e^2}{c} \sum_{a=h,e} N_a 2\pi T \sum_n \left\langle v_{a\|}^2 \mathrm{Re} \frac{\tilde{\Delta}_a^2}{(\tilde{\omega}_n^2 + \tilde{\Delta}_a^2)^{3/2}} \right\rangle,$$

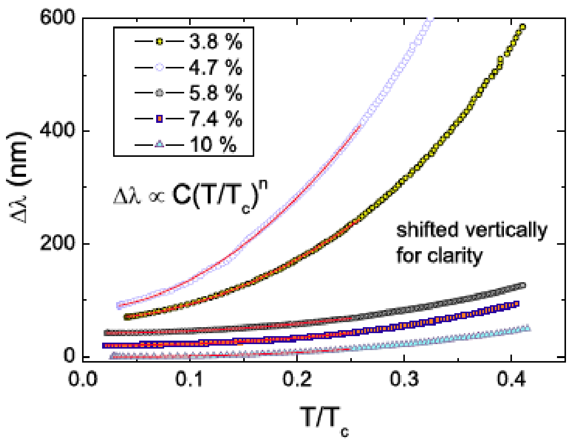
$$\tilde{\omega}_n = \omega_n + \Sigma_h^0(\omega_n) + \Sigma_e^0(\omega_n),$$
$$\tilde{\Delta}_{h,e} = \Delta_{h,e} + \Sigma_h^1(\omega_n) + \Sigma_e^1(\omega_n),$$
$$\Sigma_{h,e}^{0,1}(\omega_n) = \Gamma \cdot \mathcal{T}_{h,e}^{0,1}(\omega_n), \quad \Gamma = \frac{n_{imp}}{\pi N_{tot}}.$$

$$\delta K(T) = -\frac{2e^2}{c} \sum_{a=h,e} N_a \left\langle v_{a\|}^2 \int_0^\infty d\omega f(\omega) \mathrm{Re} \frac{\tilde{\Delta}_a^2}{(\tilde{\omega}^2 - \tilde{\Delta}_a^2)^{3/2}} \right\rangle$$

$$\tilde{\omega} \approx a\omega + i\beta\omega + i\gamma \ldots$$

$$\delta K_a(T) \approx -\frac{2e^2 N_a v_{a\|}^2}{c} \times \left[\frac{\pi^2 \Delta_a^2 a\gamma}{4(\gamma^2 + \Delta_a^2)^{5/2}} T^2 + \frac{4\Delta_a^2 a\beta}{(\gamma^2 + \Delta_a^2)^{5/2}} \left(1 - \frac{5\gamma^2}{(\gamma^2 + \Delta_a^2)} \right) (1.35231)\, T^3 \right] \ldots$$

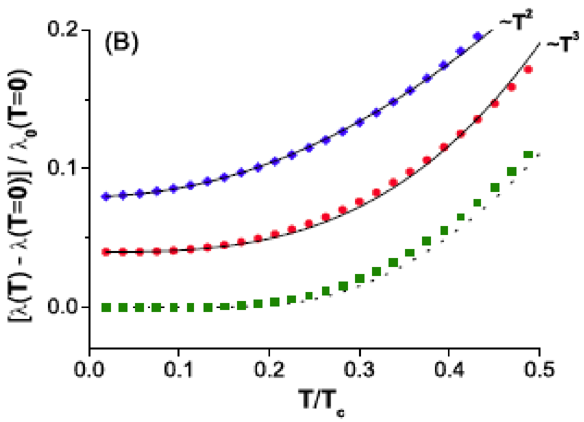
Penetration depth $\delta\lambda(T)$



$Ba(Fe_{1-x}Co_x)_2As_2$

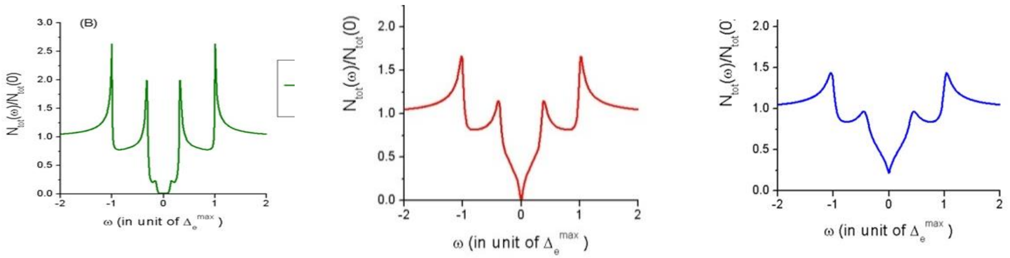
n=2 −2.5

Prozorov et al, PRB 79, 100506 (2009)



Bang, EPL 86, 47001 (2009)

Exponentially flat → T^3 → T^2



Some challenge for nodal gap ?

LaFePO, $BaFe_2(As_{0.67}P_{0.33})_2$, KFe_2As_2

Some evidence for nodal gap: λ(T) ~ T

PRL **102**, 147001 (2009) PHYSICAL REVIEW LETTERS week ending 10 APRIL 2009

Evidence for a Nodal-Line Superconducting State in LaFePO

J. D. Fletcher,[1] A. Serafin,[1] L. Malone,[1] J. G. Analytis,[2] J.-H. Chu,[2] A. S. Erickson,[2] I. R. Fisher,[2] and A. Carrington[1]
[1]*H. H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, BS8 1TL, United Kingdom*
[2]*Geballe Laboratory for Advanced Materials and Department of Applied Physics, Stanford University, Stanford, California 94305-4045, USA*

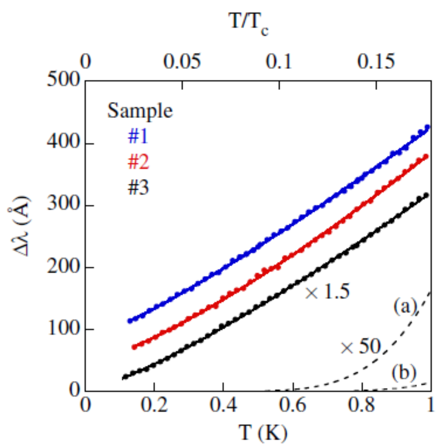


FIG. 2 (color online). Temperature dependence of the in-plane penetration depth $\Delta\lambda_a$ in three single crystals of LaFePO. Data for sample 3 has been multiplied by 1.5. The curves are offset for clarity. Solid lines are power-law fits giving an exponent of 1.2 ± 0.1. The dashed lines are the behavior expected for a fully gapped Fermi surface (a) with parameter relevant to Sm-1111 [5] and (b) the conventional isotropic BCS case. Both curves have been multiplied by 50 for clarity.

PRL **103**, 127003 (2009) PHYSICAL REVIEW LETTERS week ending 18 SEPTEMBER 2009

Evidence for a Nodal Energy Gap in the Iron-Pnictide Superconductor LaFePO from Penetration Depth Measurements by Scanning SQUID Susceptometry

Clifford W. Hicks,[1] Thomas M. Lippman,[1] Martin E. Huber,[2] James G. Analytis,[1] Jiun-Haw Chu,[1] Ann S. Erickson,[1] Ian R. Fisher,[1] and Kathryn A. Moler[1]
[1]*Geballe Laboratory for Advanced Materials, Stanford University, Stanford, California, 94305, USA and Stanford Institute for Materials and Energy Sciences, SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, California 94025, USA*
[2]*Departments of Physics and Electrical Engineering, University of Colorado Denver, Denver, Colorado, 80217, USA*

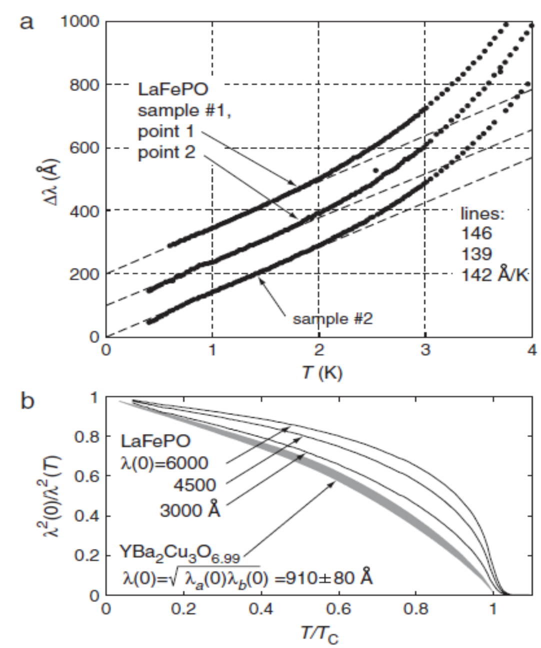
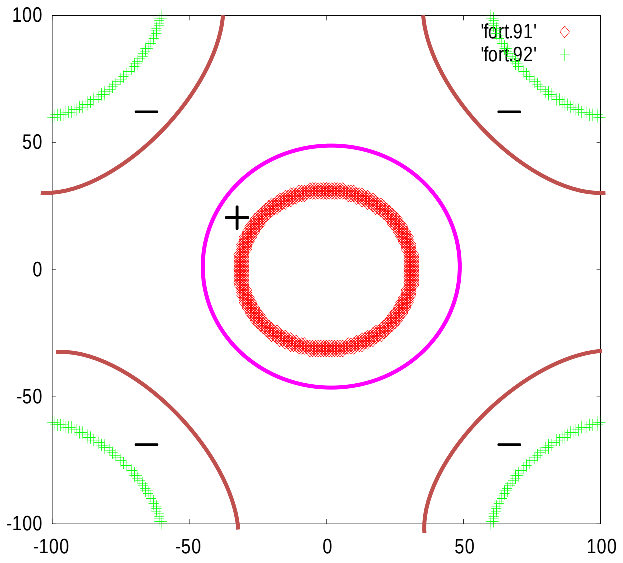
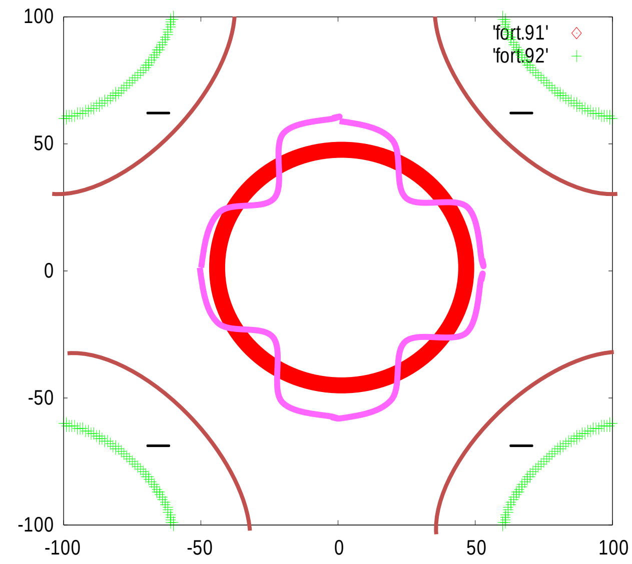


FIG. 3. Top: $\Delta\lambda$ of two LaFePO specimens, at the points indicated in Fig. 4. The lines were fit over $0.7 < T < 1.6$ K. Bottom: black lines are possible superfluid densities for LaFePO sample #1, point 2, with different $\lambda(0)$. Shaded area: superfluid density of $YBa_2Cu_3O_{6.99}$ $[1/(\lambda_a\lambda_b)]$, from [30,31]; the width of the shaded area reflects uncertainty in $\lambda(0)$.

Continuous evolution from ±S gap → Nodal gap which is the same Mechanism.





±S

g+S : nodal gap (A1g)

Another challenge for nodal gap

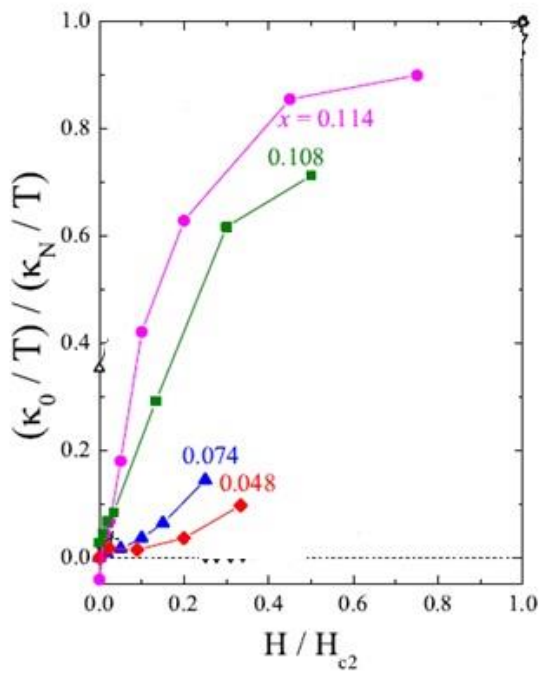
Volovik Effect : thermal conductivity $\kappa(H)$

Volovik Effect

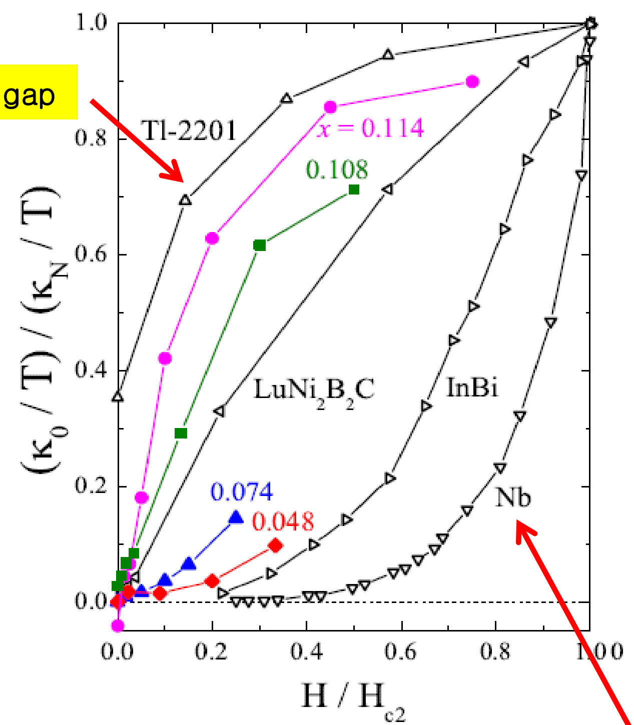
C(T,H)/T and κ(T,H) : probe N(0,H) in mixed state with mag. field H

M. Tanatar et al, PRL, 104, 067002 (2010)

$Ba(Fe_{1-x}Co_x)_2As_2$



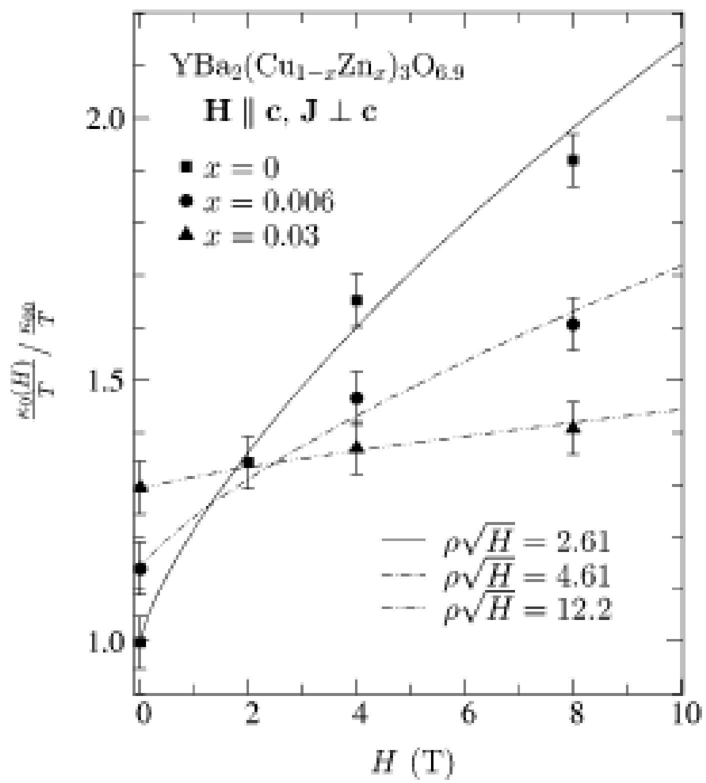
D-wave gap



single s-wave gap

$$C(H)/T, \kappa(T)/T \sim \sqrt{H}, \text{ hall mark of D-wave gap.}$$

D-wave : nodal gap → $\kappa(H) \sim \sqrt{H}$ **Volovik effect**



L. Taillefer et al

But, strong field dependence is not unique with d-wave gap.
Some S-wave also show strong field dependence.

THERMAL CONDUCTIVITY OF SINGLE-CRYSTALLINE MgB_2

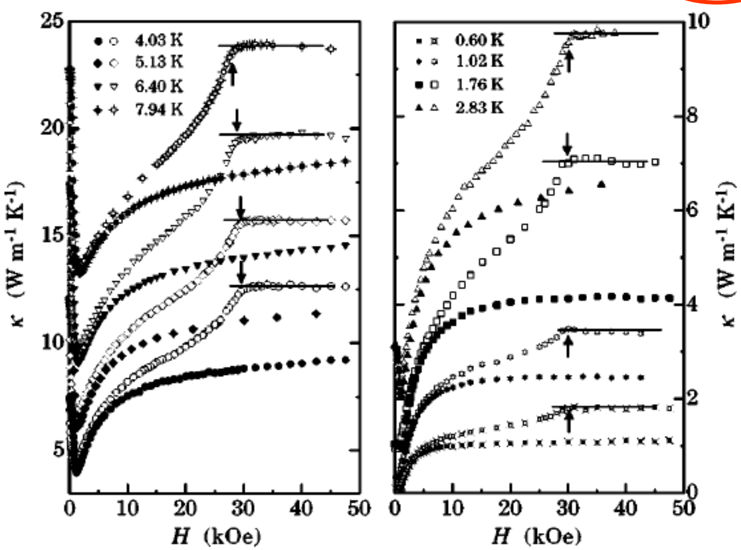
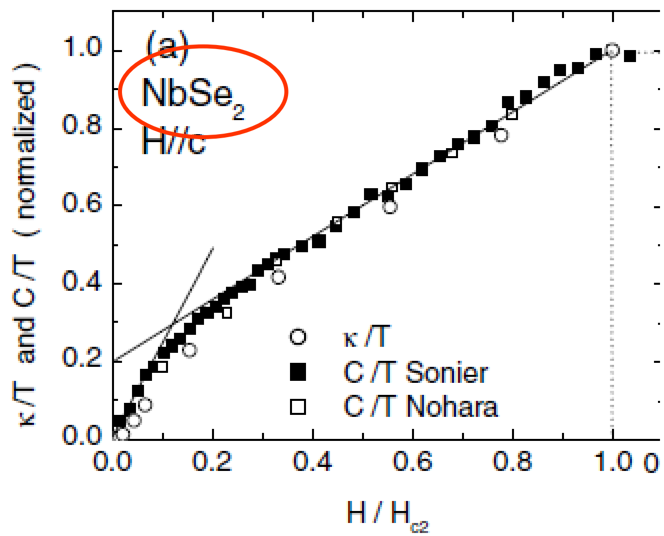


FIG. 4. Thermal conductivity in the basal plane of MgB_2 vs H at several fixed temperatures. The arrows denote the upper critical field H_{c2} for $H \| c$. The solid and open symbols correspond to the field direction perpendicular and parallel to the c axis, respectively.



$$C(T,H)/T \propto \int d\omega \left(-\frac{\partial f}{\partial \omega}\right) N(\omega, H)$$

$$\kappa(T,H) \propto \int d\omega \left(-\frac{\partial f}{\partial \omega}\right) S(\omega, H)$$

$$N(\omega, H) = \sum_{n} \delta(\omega - E_n(H))$$

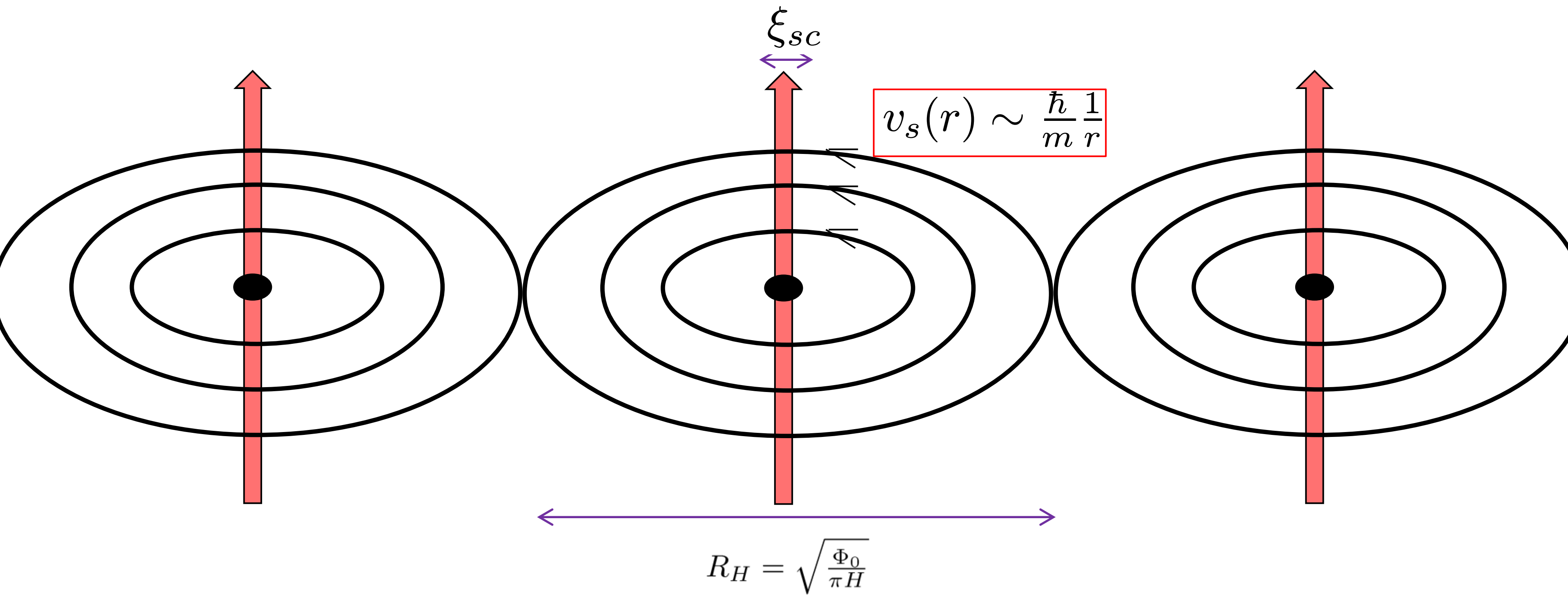
$$S(\omega, H) = \sum_{n,m} v_x^2 \delta(\omega - E_n(H)) \delta(E_n - E_m)$$

Volovik effect :

Fully Q.M treatment *(Tesanovic et al, Mishra et al.)*

semiclassical approximation valid for $H_{c1} < H << H_{c2}$.

Replace $\mathbf{A}(\mathbf{r})$ by $v_s(\mathbf{r})$ and have Doppler shift of q.p. energy ξ_k.

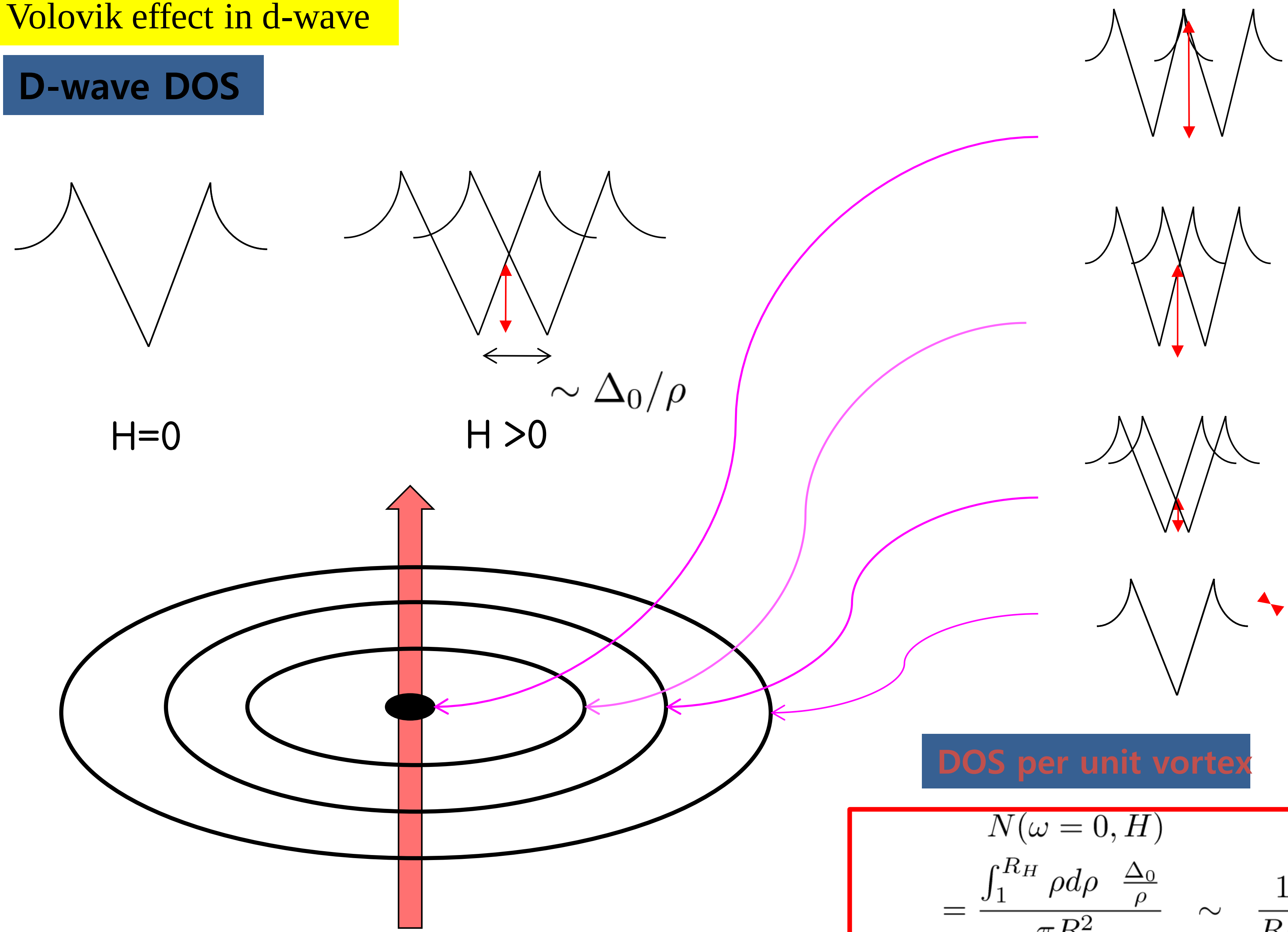


$$\Big(\epsilon(k), \epsilon(-k)\Big) \text{ pairs} \quad \rightarrow \Big(\epsilon(k) + \underline{k \cdot v_s(r)}, \;\; \epsilon(-k) - \underline{k \cdot v_s(r)}\Big) \text{ pairs}$$

Pair breaking due to Doppler shift of q.p. energy

Volovik effect in d-wave

D-wave DOS



DOS per unit vortex

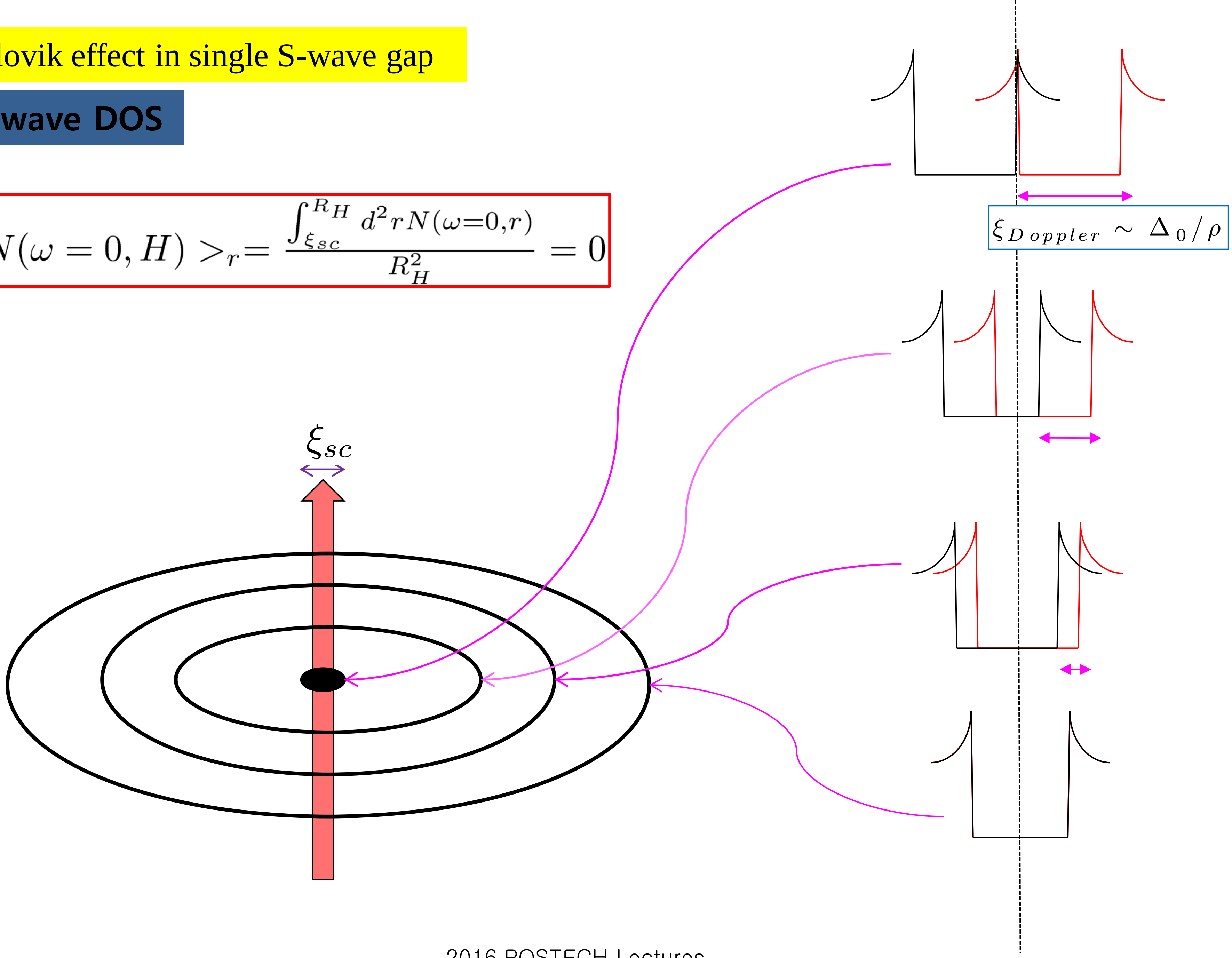
$$N(\omega = 0, H) = \frac{\int_1^{R_H} \rho d\rho \;\; \frac{\Delta_0}{\rho}}{\pi R_H^2} \sim \frac{1}{R_H} \sim \sqrt{H}$$

$$where \quad R_H = \sqrt{\frac{\Phi_0}{\pi H}}$$

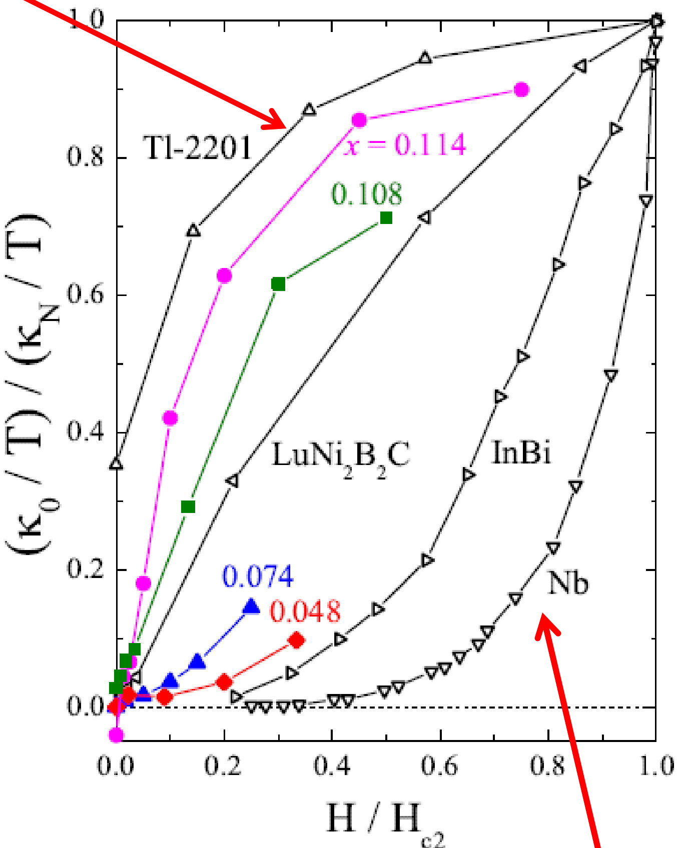
Volovik effect in single S-wave gap

S-wave DOS

$$< N(\omega = 0, H) >_r = \frac{\int_{\xi_{sc}}^{R_H} d^2 r N(\omega{=}0, r)}{R_H^2} = 0$$



D-wave gap

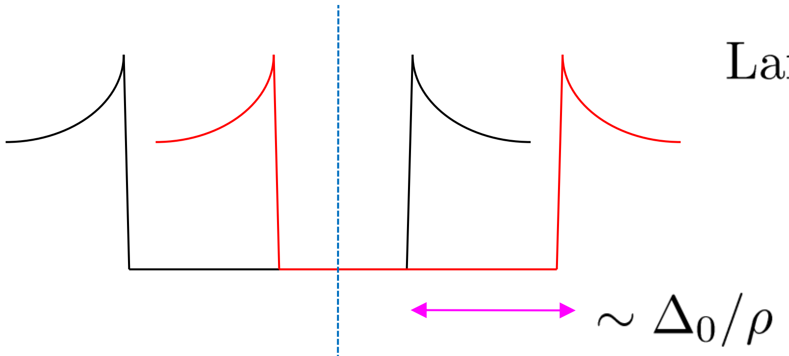


single s-wave gap

Two gaps $\Delta_{large} \neq \Delta_{small}$ in ±S-wave

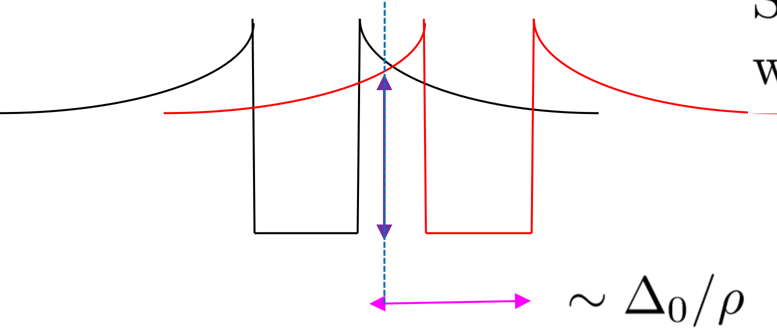
for $0 < H < H_{c2}$

Large gap



Large gap: $N(0, H) = 0$ for all ρ

Small gap



Small gap: $N(0, H, r) = N_s^{normal}$ for $1 < \rho < \rho^*$

where $\frac{\Delta_l}{\Delta_s} = \rho^*$

Small Gap OP doesn't collapse because Δ_{large} and Δ_{small} are coupled each other.

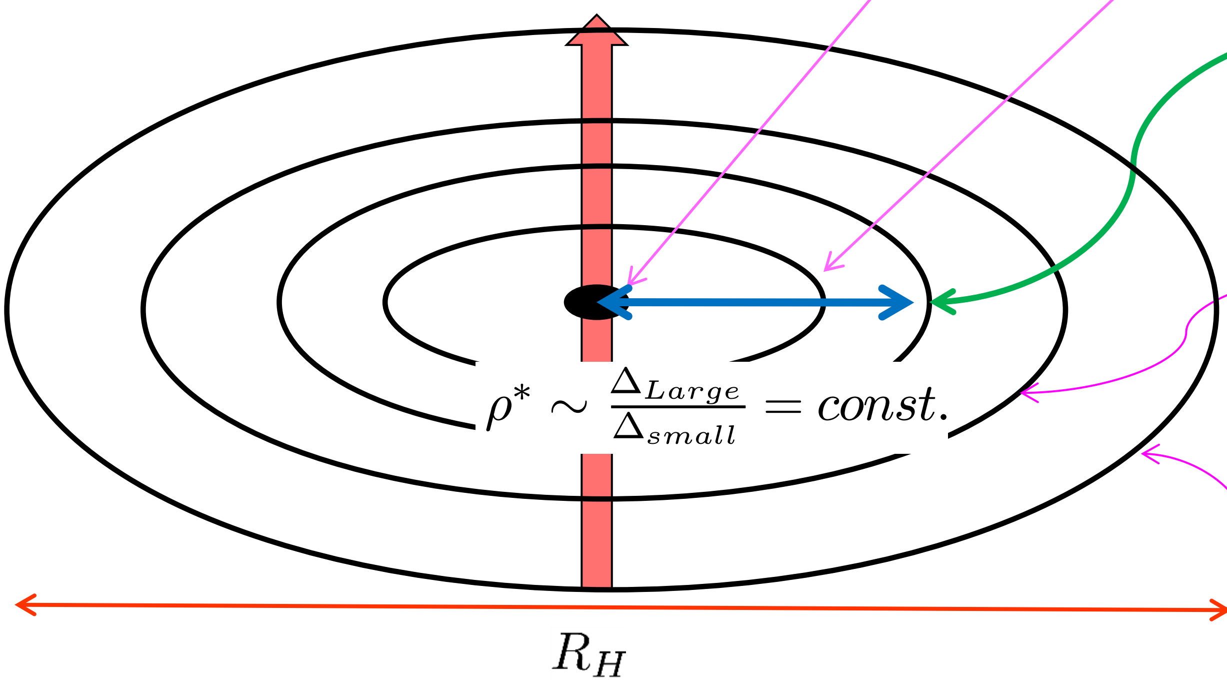
Volovik effect in two s-wave gap

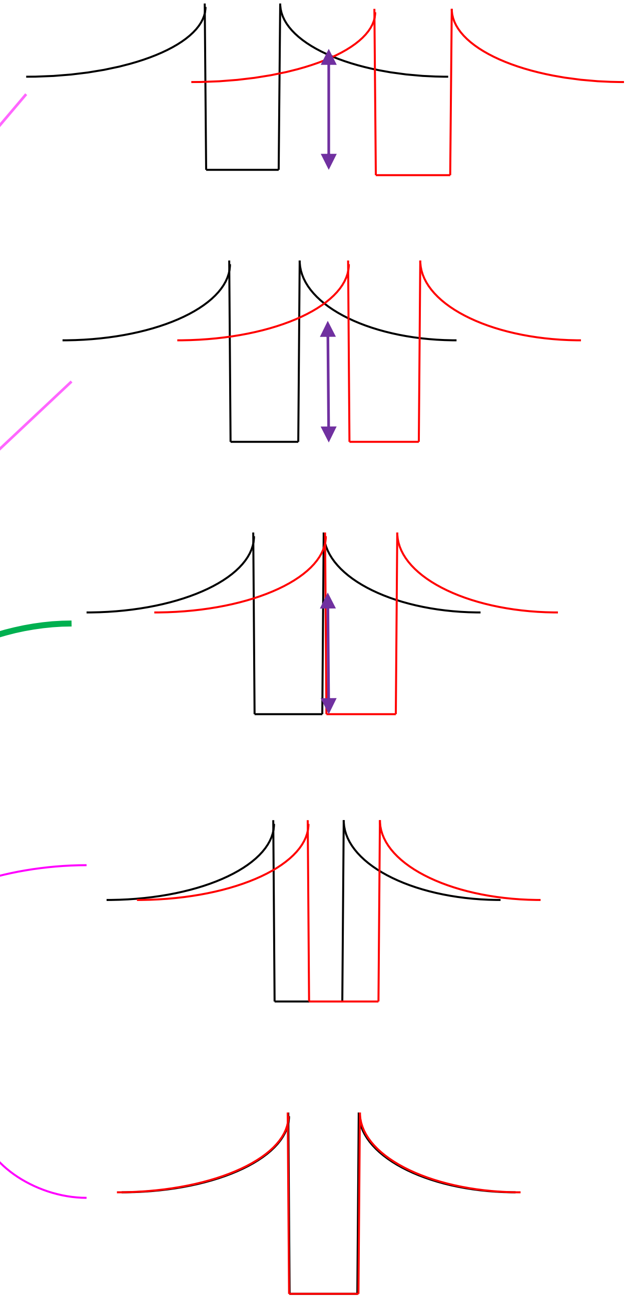
Only small gap contributes.

$$\langle N_{small}(\omega = 0, H) \rangle$$

$$= \frac{\int_1^{\rho^*} \rho d\rho \quad N_s^{normal}}{\pi R_H^2} = \frac{const}{R_H^2} \sim H$$

$$where \qquad R_H = \sqrt{\frac{\Phi_0}{\pi H}}$$





$$C(T,H) = \int d\omega < N(\omega,H) > \frac{\omega^2}{T^2} sech^2(\frac{\omega}{2T})$$

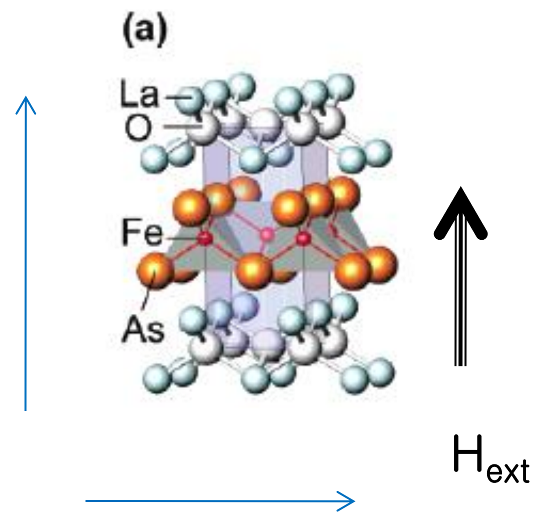
$$\kappa(T,H,r) \sim \frac{1}{T^2} \int_0^\infty d\omega \omega^2 sech^2(\frac{\omega}{2T}) K_i(\omega,T,H,r)$$

where, $K_i(\omega,T,H,r) = \frac{1}{Im\sqrt{\tilde{z}^2 - \tilde{\Delta}_i^2}} \times \left(1 + \frac{\tilde{z}^2 - \tilde{\Delta}_i^2}{|\tilde{z}^2 - \tilde{\Delta}_i^2|}\right)$

and $\tilde{z} = \tilde{\omega} + \mathbf{v}_s(\mathbf{r}) \cdot \mathbf{k_F}$

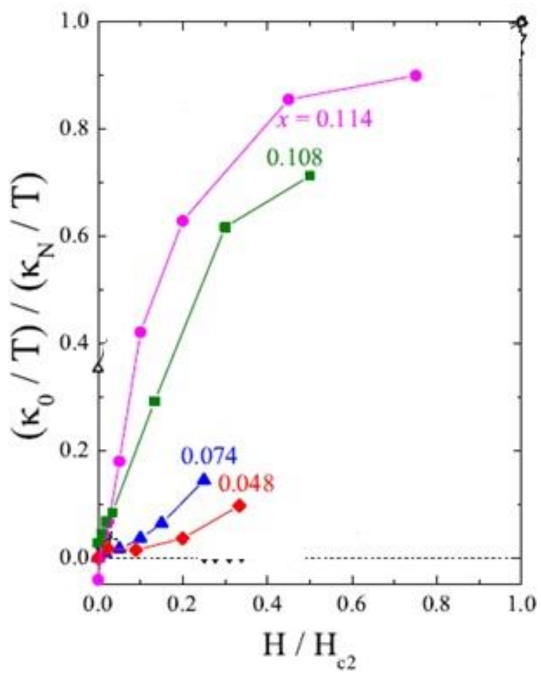
$$\kappa_{\|}(T,H) = \int dr^2 \kappa(T,H,r)/\pi R_H^2$$

$$\kappa_{\perp}^{-1}(T,H) = \int dr^2 \kappa^{-1}(T,H,r)/\pi R_H^2$$



Thermal conductivity κ(H) : probe **N(0,H)** in mixed state with mag. field H

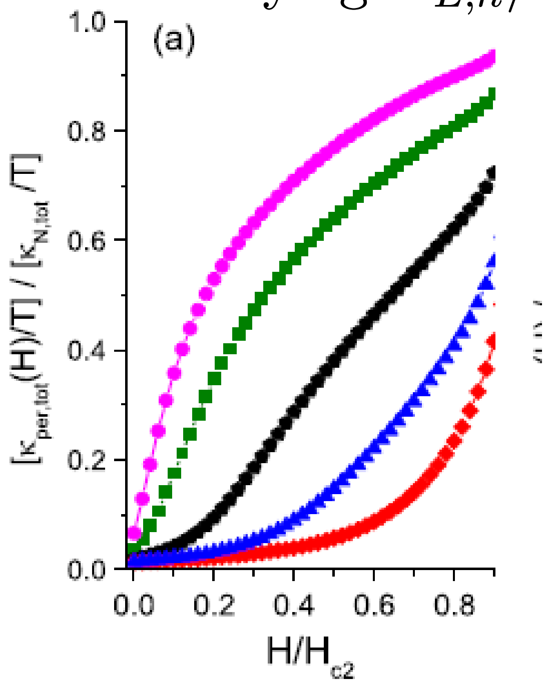
$Ba(Fe_{1-x}Co_x)_2As_2$



D-wave evidence

M. Tanatar et al, PRL, 104, 067002 (2010)

Varying $\Delta_{L,h}/\Delta_{S,e}$



Absolute evidence for ±S-wave

Y Bang, PRL,, 104, 217001 (2010)

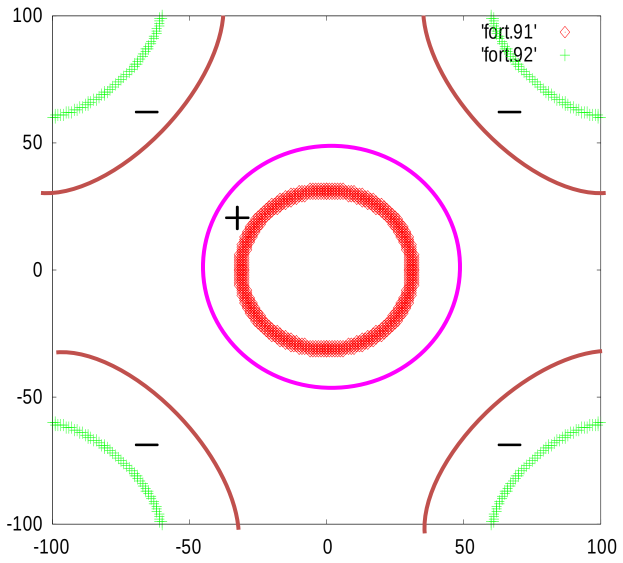
1. We have no consensus yet for a microscopic theory of IBS.

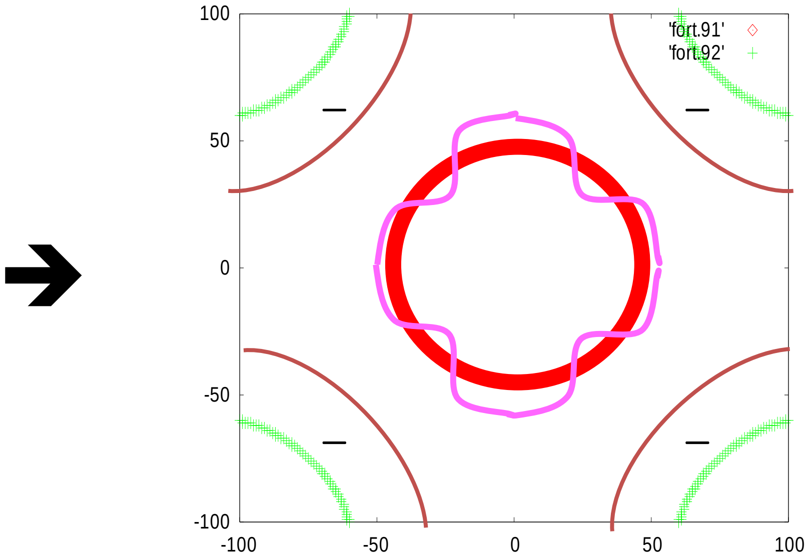
2. But, the **±s-wave gap** symmetry conforms with most of Experiments (90 % or mo
(ARPES, Raman, Penetration depth, Specific heat, Thermal Conductivity, NMR, etc)

3. Problem is : it is too boring BCS (also s-wave) SC.

4. Some people likes to find, at least, some nodes.

Continuous evolution from ±S gap → Nodal gap which is the same Mechanism.



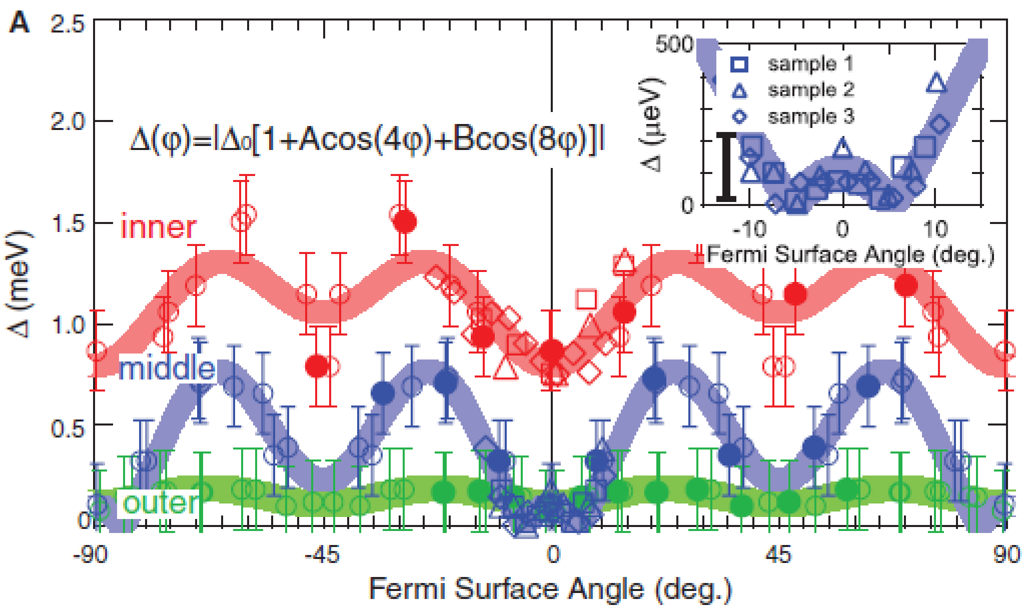


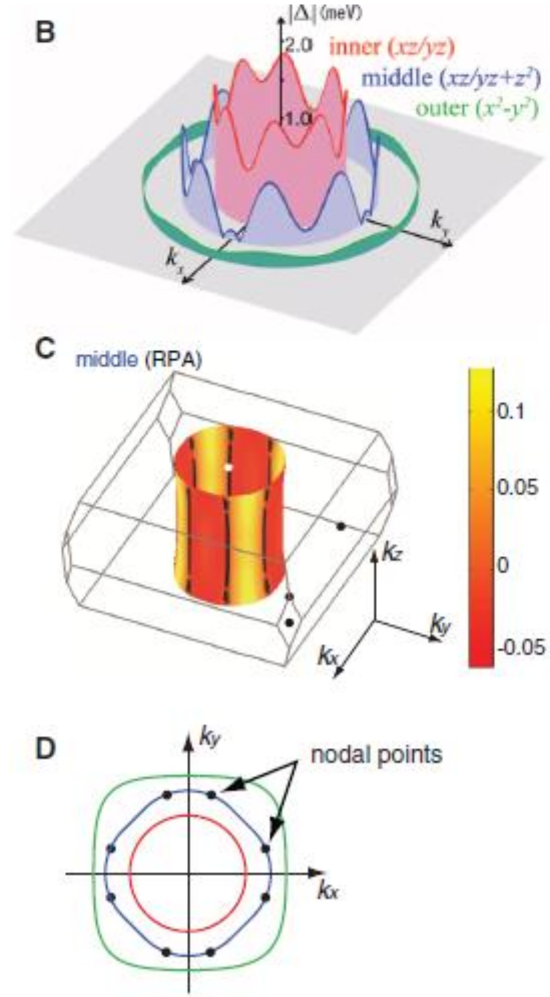
±S

g+S : nodal gap (A1g)

KFe_2As_2

Octet-Line Node Structure of Superconducting Order Parameter in KFe2As2
K. Okazaki *et al.*
Science **337**, 1314 (2012);





We are obsessed with Un-conventional Superconductivity in the past 30 yrs.
Many people have different meaning with it.
We need to clarify our questions and wishes more clearly.

1. Normal state properties show abundant NFL, QC behaviors, and Mott physics.
2. But, all SC properties are very conventional with Cooper pairs.
3. Do we think a Non-BCS theory is only option for explaining high Tc ?
4. Do we have any idea of non-BCS SC theory ?