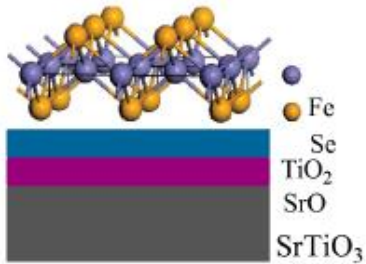


## Part IV: one last challenge of IBS

# Pairing Mechanism for FeSe systems: **HEDIS** (highly electron doped FeSe )

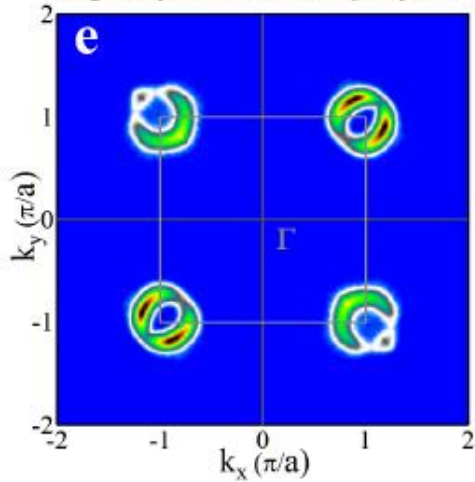
# FeSe Problem ?

**b**



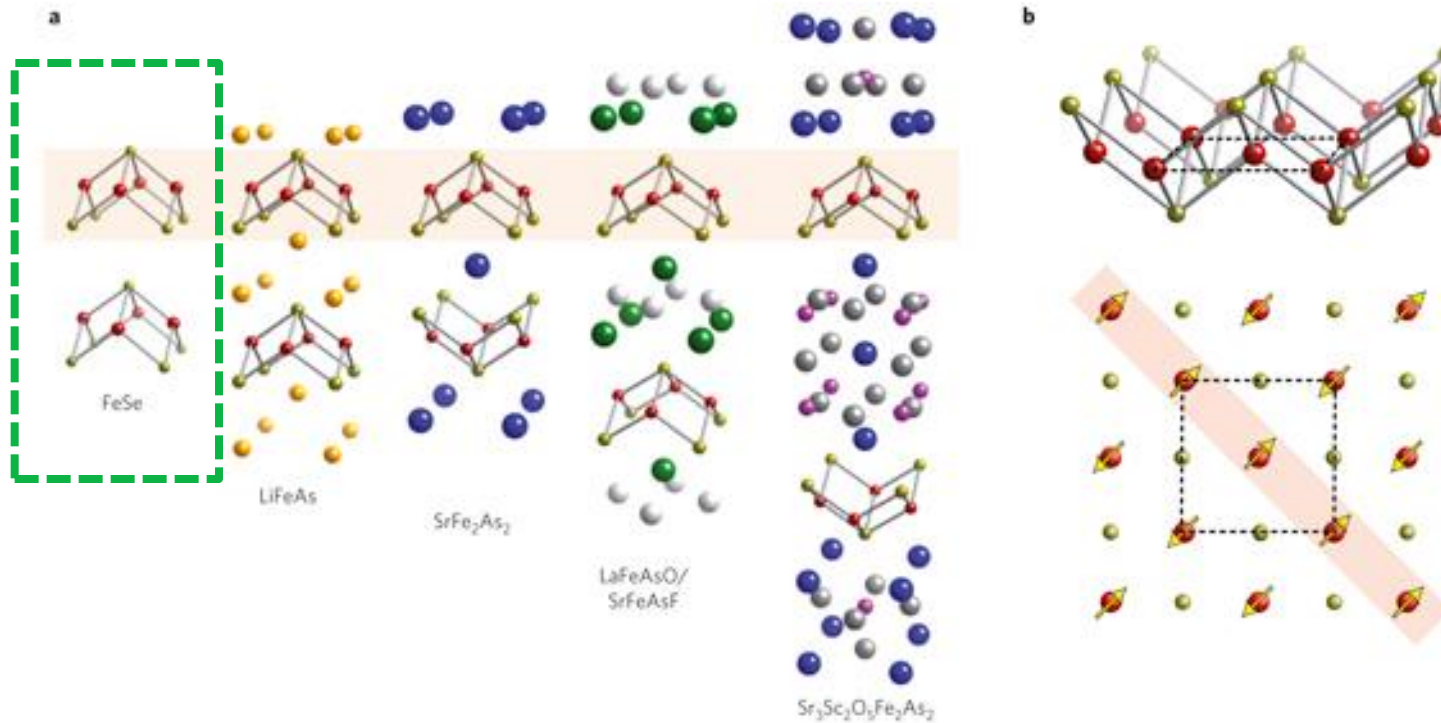
One layer system.  
Only Electron pockets.  
 $T_c \sim 100\text{K}$

Single-layer FeSe/SrTiO<sub>3</sub>,  $T_c > 65\text{K}$



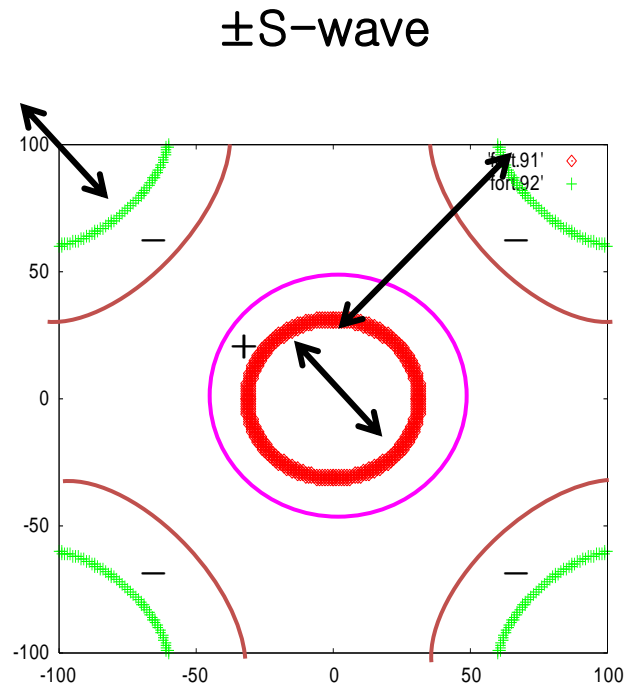
# Standard Paradigm of IBS: $\pm$ S-wave pairing

## Different family of Fe/Pnictides n Fe/Chacogenides



Crystallographic and magnetic structures of the iron-based superconductors.

# Sign-changing S-wave solutions for multi-bands + AFM interaction



$\mathbf{V}_{\text{int}}(\mathbf{q})$ : All repulsive interaction

Two band BCS gap Equation

$$\Delta_h(k) = \quad (5)$$

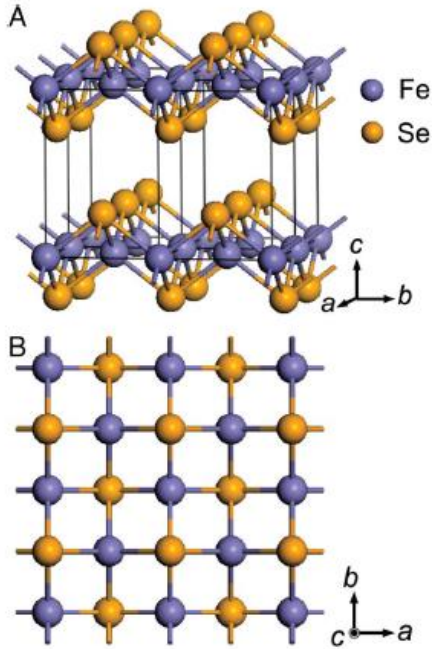
$$- \sum_{k'} [V_{hh}(k, k') \Delta_h(k') \chi_h(k') + V_{he}(k, k') \Delta_e(k') \chi_e(k')],$$

$$\Delta_e(k) =$$

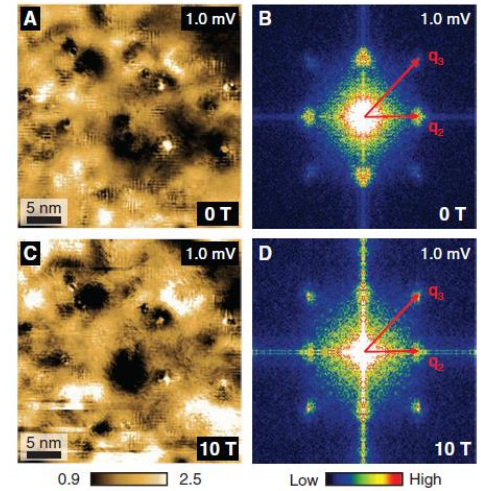
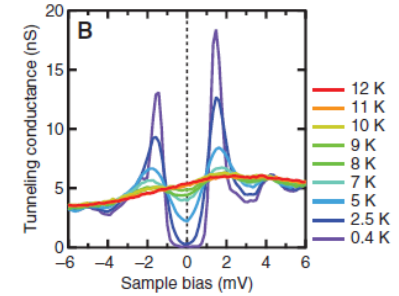
$$- \sum_{k'} [V_{eh}(k, k') \Delta_h(k') \chi_h(k') + V_{ee}(k, k') \Delta_e(k') \chi_e(k')].$$

(6)

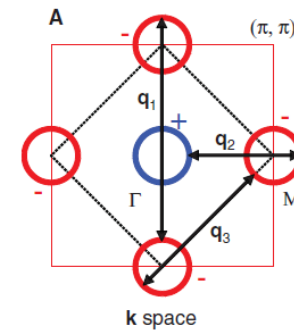
**Bulk FeSe** crystal:  $T_c \sim 8\text{K}$  (PNAS 2008 M.K. Wu et al.)



Hanaguri, Science 2010

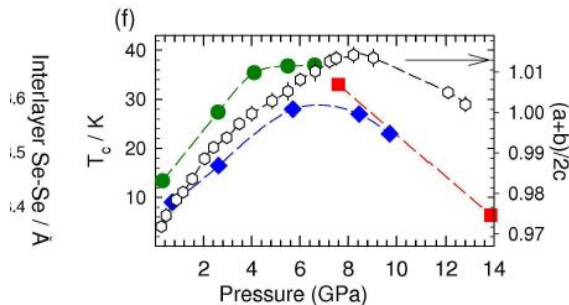
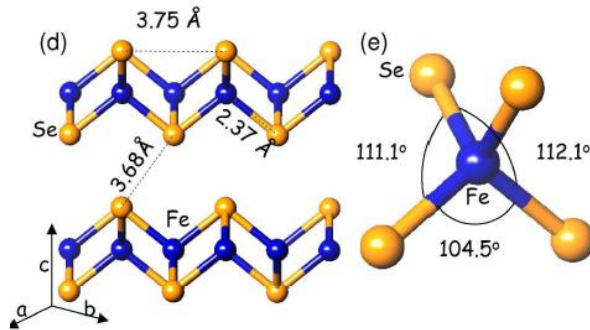


Evidence: the same  $\pm S$ -wave gap in **Bulk FeSe**



Tc increases by

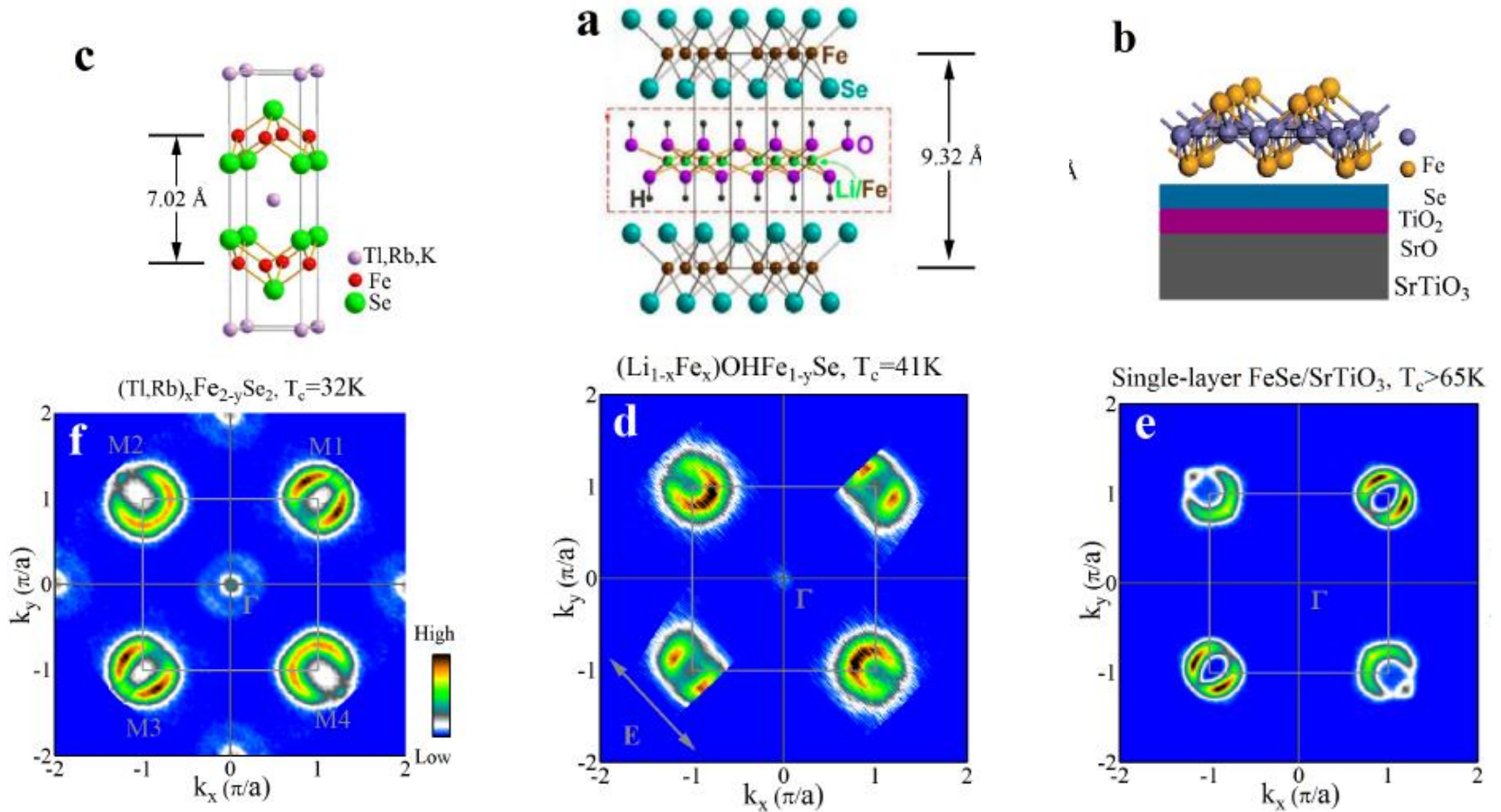
1. FeSe pressure  $\rightarrow$  37K,
2. Alkali element (K,Rb,Cs,Tl)doping  $\rightarrow$  30-40K,
3. (LiFeOH)FeSe  $\rightarrow$  40K,
4. some vacancies Fe(SeTe)  $\rightarrow$  20K,
5. FeSe monolayer  $\rightarrow$  **100K**



What happens ?

1. Fermi surface change ?  $\rightarrow$  common
2. Magnetic fluc changes (AFM1, AFM2, FM) ?
3. Phonon ?

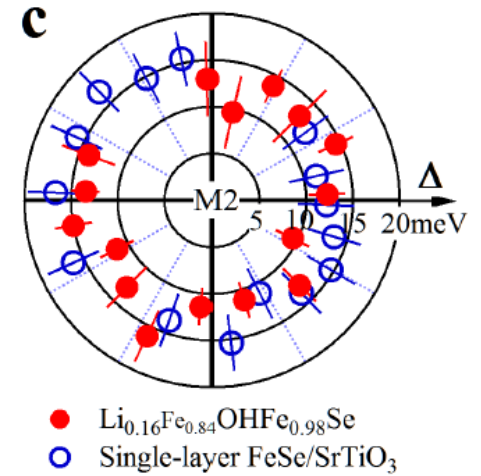
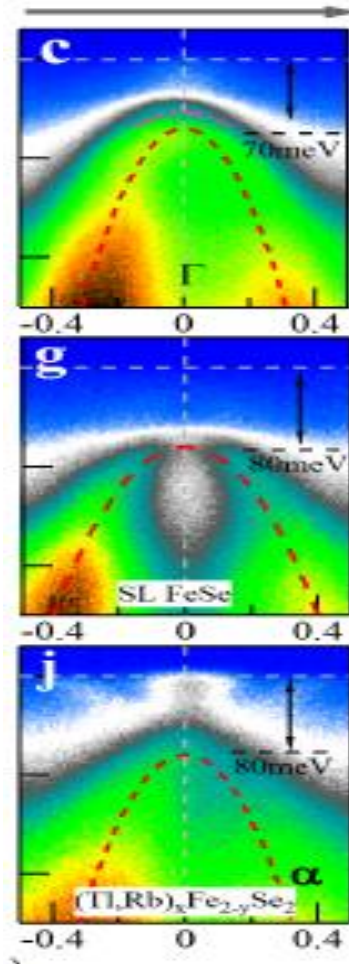
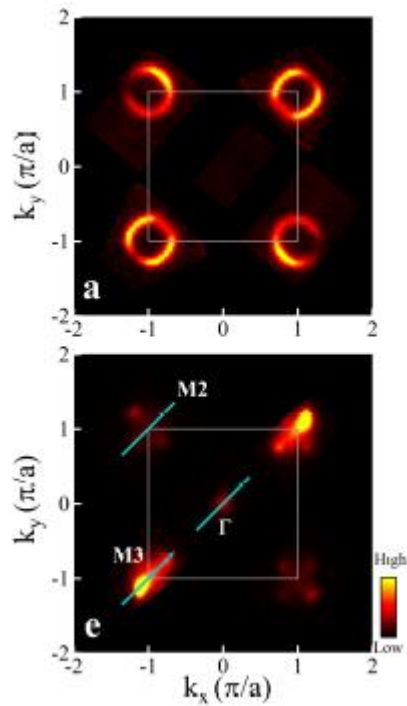
**Heavily Electron Doped FeSe systems** (X J Zhou et al)  
 → Main change: only electron pockets





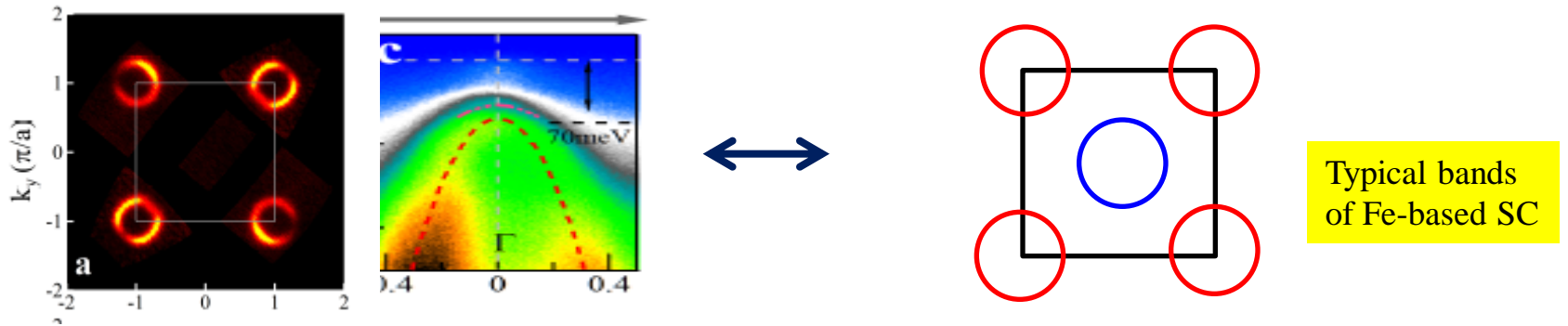
## Common features:

1. No  $\Gamma$  band + **S-wave gap** on **M** band.



## Key Questions:

1. Only Electron Band  $\rightarrow$  how to pair ?
2.  $T_c$  ( $\sim 100\text{K}$ ) is too high.



## Answers :

1. Only Electron Pockets  $\rightarrow$  (Bang, NJP 2014; [arXiv:1605.01509](https://arxiv.org/abs/1605.01509))
2.  $T_c$  ( $\sim 100\text{K}$ ) is too high  $\rightarrow$  (Bang, prb 2008, 2009)

**Phonon Boost Effect**  
**for Un-conventional Superconductivity**  
(Bang, prb 2008, 2009)

Let us separate the questions:

1. HEDIS with only electron pockets typically have  $T_{c,\max} \sim 30\text{-}40\text{K}$
2. FeSe monolayer with  $T_c \sim 100\text{K}$  needs **extra boosting mechanism**.

**No. 2 is simple:  
extra mechanism  $\rightarrow$  small angle phonon + large angle AFM repulsion**

$$T_c \simeq 1.14 \omega_{sf}^{\tilde{\lambda}_{sf}} \cdot \omega_{ph}^{\tilde{\lambda}_{ph}} e^{-1/\lambda_{tot}}$$

$$\lambda_{tot} = \lambda_{sf} + \lambda_{ph}$$

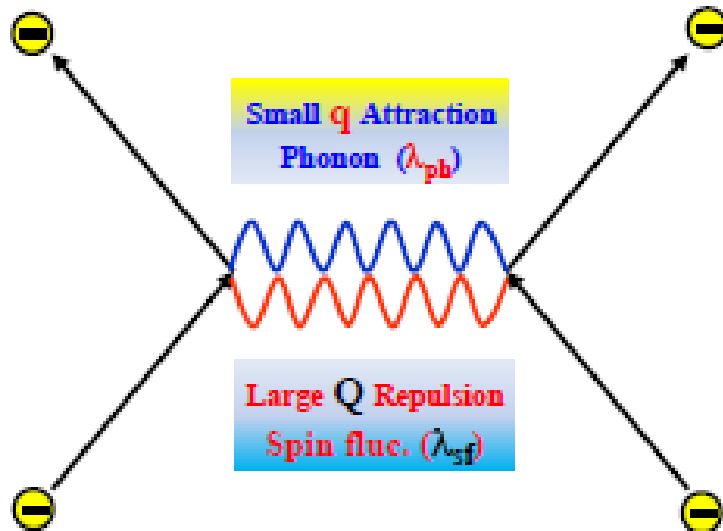
$$\tilde{\lambda}_{sf} = \lambda_{sf}/\lambda_{tot}, \tilde{\lambda}_{ph} = \lambda_{ph}/\lambda_{tot}$$

Effects of phonon interaction on pairing in high- $T_c$  superconductors

Isotope effect and the role of phonons in the iron-based superconductors

$$T_c \simeq 1.14 \omega_{sf}^{\tilde{\lambda}_{sf}} \cdot \omega_{ph}^{\tilde{\lambda}_{ph}} e^{-1/\lambda_{tot}}$$

$$\lambda_{tot} = \lambda_{sf} + \lambda_{ph}$$



**Both interactions utilize different mom. Spaces !!**

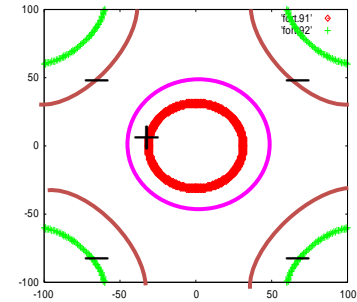
# Simple toy model

Bang, PRB 78 (2008), 79(2009)

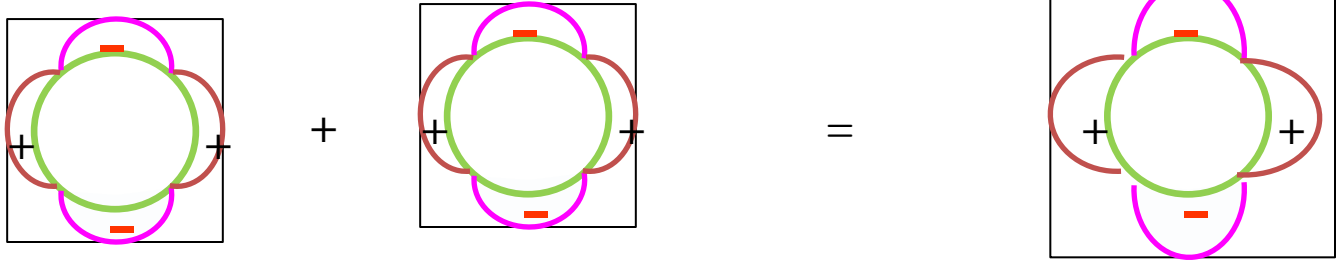
$$H = \sum_{k\sigma} \epsilon(k) c_{k\sigma}^\dagger c_{k\sigma} + \sum_{kk'\uparrow\downarrow} V_{\text{AFM}}(k, k') c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger c_{k'\downarrow} c_{-k'\uparrow},$$

$$\sum_{kk'\uparrow\downarrow} V_{\text{ph}}(k, k') c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger c_{k'\downarrow} c_{-k'\uparrow},$$

$\pm S_{\text{AFM}} + S_{\text{ph}}$  wave



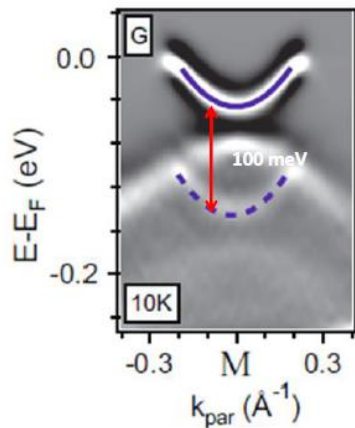
$D_{\text{AFM}} + D_{\text{ph}}$  wave



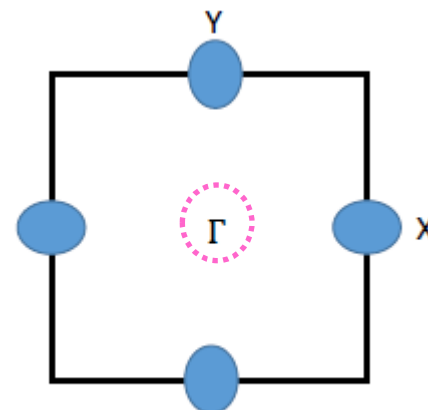
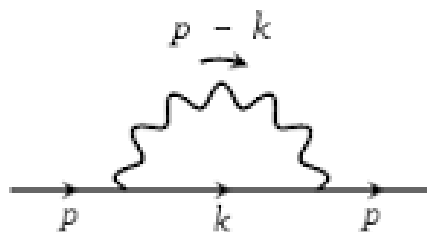
Required condition: **small angle phonon interaction**

$T_c$  ( $\sim 100\text{K}$ ) is too high.

Evidence for **small angle phonon + strong coupling**

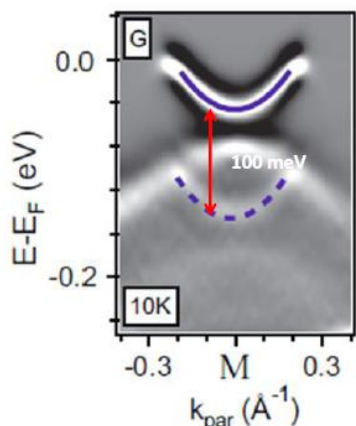
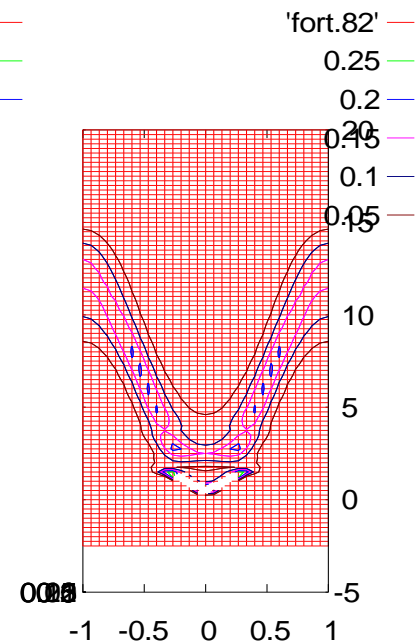
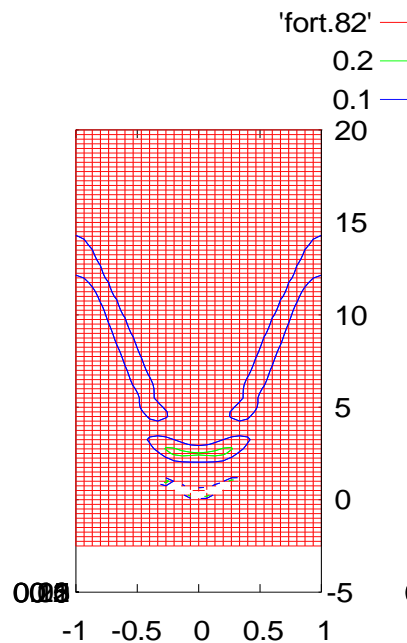
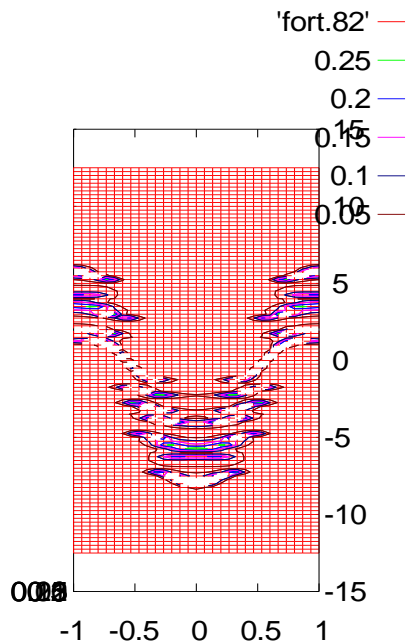
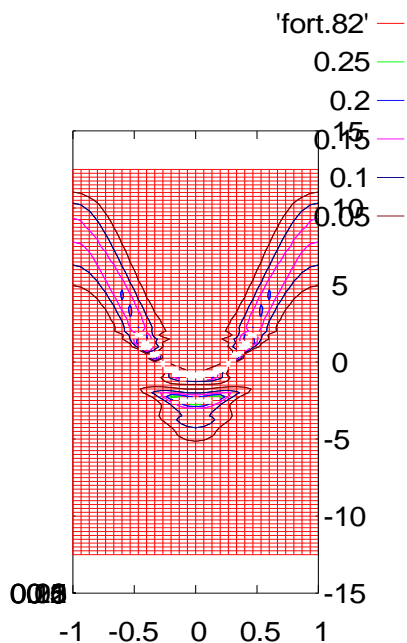
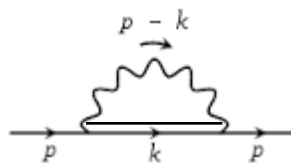
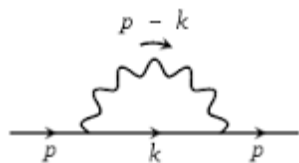


ZX Shen et al 2014



DH Lee 2015, *small angle phonon scattering*

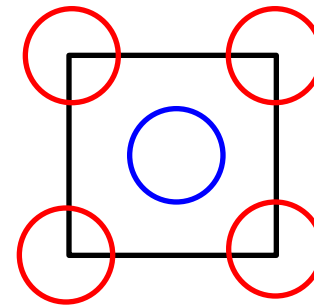
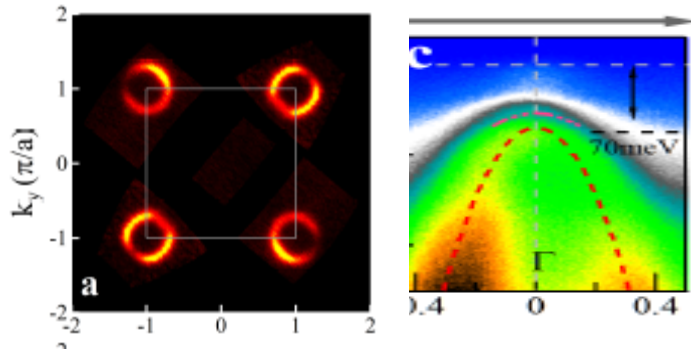
In reality,





More difficult question:

1. Pairing with only electron pockets :  $T_{c,max} \sim 30-40K$



Typical bands  
of Fe-based SC

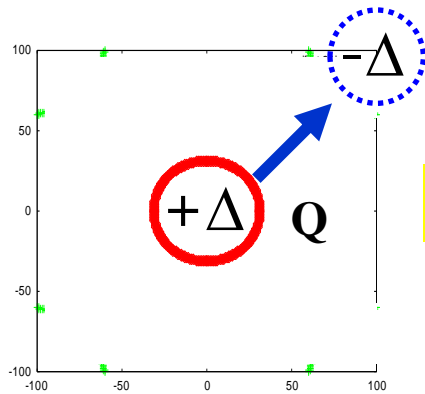
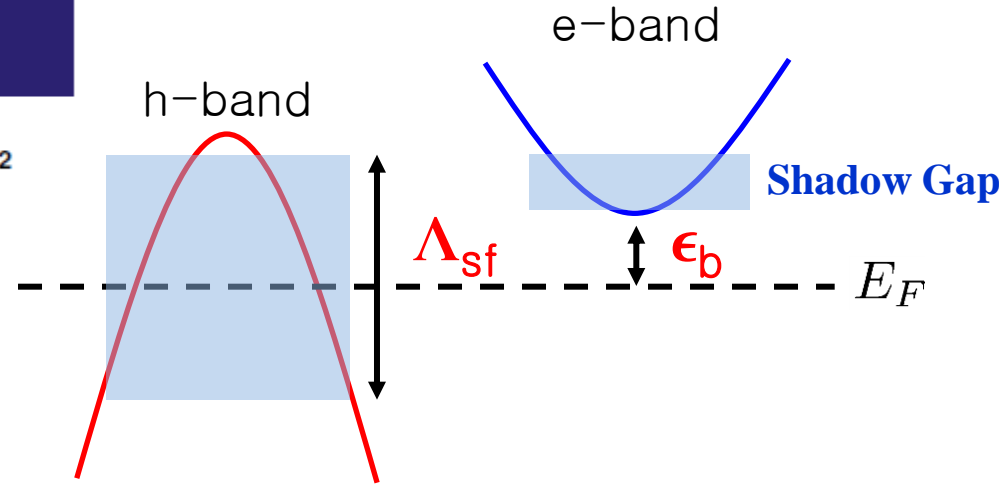
$T_{c,max} \sim 30-40K$

$T_{c,max} \sim 30-50K$

Pairing is possible with the same  $V_{sf}$  interaction, but **Tc should decrease.**  
- Bang NJP 2014

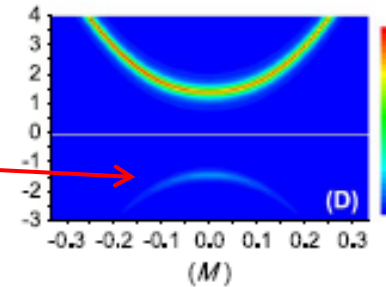
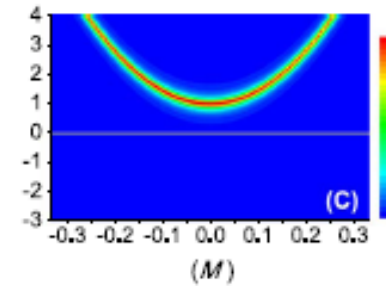
## A shadow gap in the over-doped $(\text{Ba}_{1-x}\text{K}_x)\text{Fe}_2\text{As}_2$ compound

Yunkyu Bang



Hidden  $\pm S$ -gap

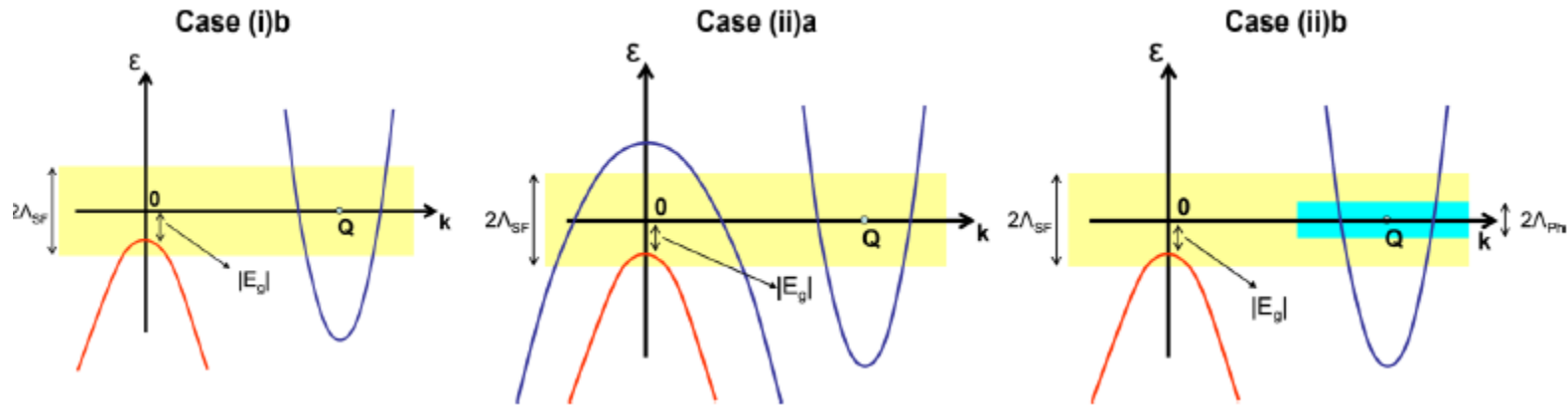
Electron pockets sink down  
Now !!.



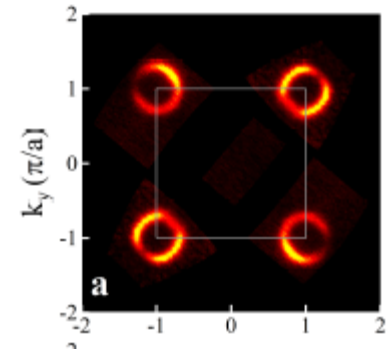
Shadow gap

# Electron pairing in the presence of incipient bands in iron-based superconductors

Xiao Chen,<sup>1</sup> S. Maiti,<sup>1,2</sup> A. Linscheid,<sup>1</sup> and P. J. Hirschfeld<sup>1</sup>



**So, only Electron Band → No problem**



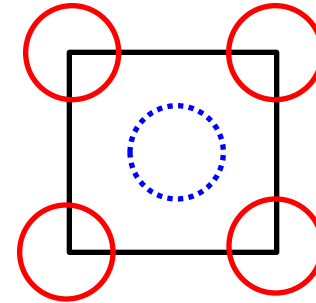
$$\Delta_h(k) = \quad (5)$$

$$- \sum_{k'} [V_{hh}(k, k') \Delta_h(k') \chi_h(k') + V_{he}(k, k') \Delta_e(k') \chi_e(k')],$$

$$\Delta_e(k) =$$

$$- \sum_{k'} [V_{eh}(k, k') \Delta_h(k') \chi_h(k') + V_{ee}(k, k') \Delta_e(k') \chi_e(k')].$$

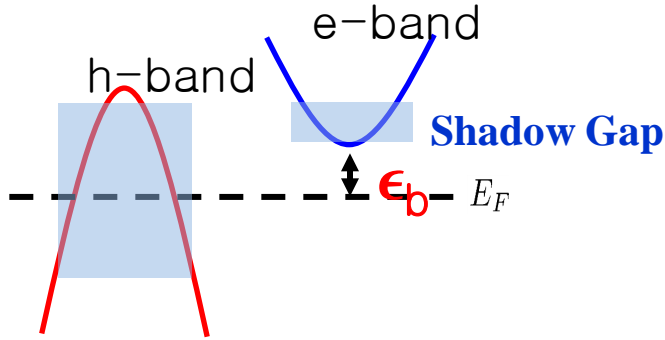
$$(6)$$



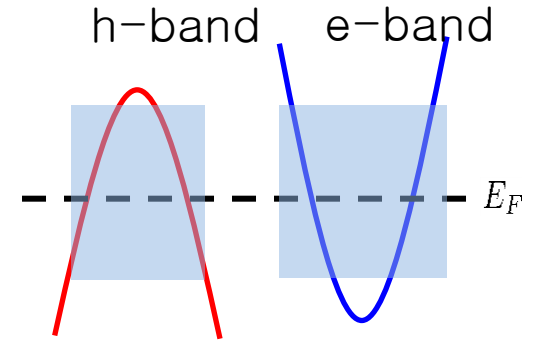
$$\chi_{h,e}(T) = \int_0^{\Lambda_{hi}} \frac{d\xi}{\xi} \tanh\left(\frac{\xi}{2T}\right) \approx \ln\left[\frac{1.14\Lambda_{hi}}{T}\right]$$

$$\chi_e = \int_{\epsilon_b}^{\Lambda_{hi}} \frac{d\xi}{\xi} \tanh\left(\frac{\xi}{2T}\right) \approx \ln\left[\frac{1.14\Lambda_{hi}}{\epsilon_b}\right]$$

**But  $T_c$  decreases** with sunken band for a given  $V_{AFM}(Q)$



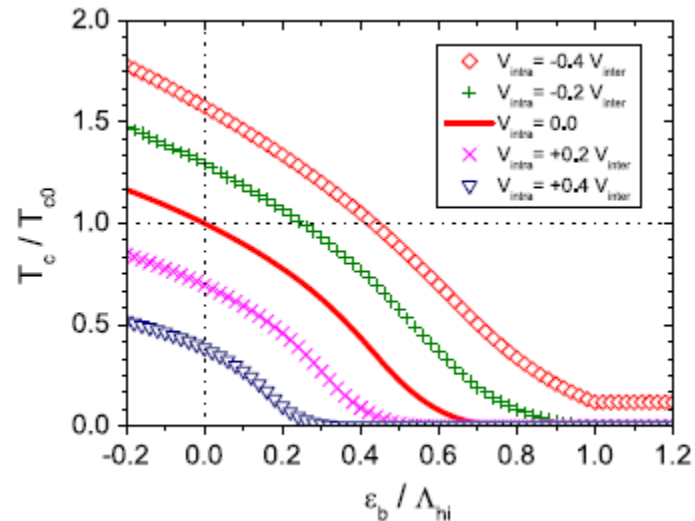
$$\tilde{\lambda}_{eff} = [\bar{V}_{he} N_e \bar{V}_{eh} N_h] \cdot \ln \left[ \frac{1.14 \Lambda_{hi}}{\epsilon_b} \right]$$



$$\tilde{\lambda}_{eff} = \sqrt{\bar{V}_{he} N_e \bar{V}_{eh} N_h}$$

Assuming only interband  $V_{he}$

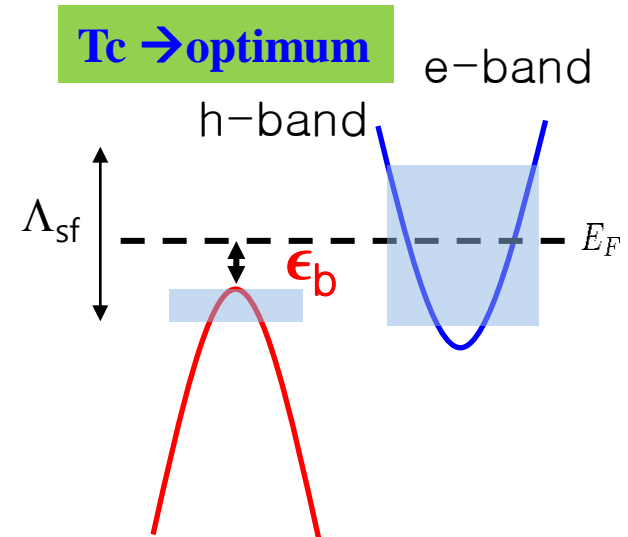
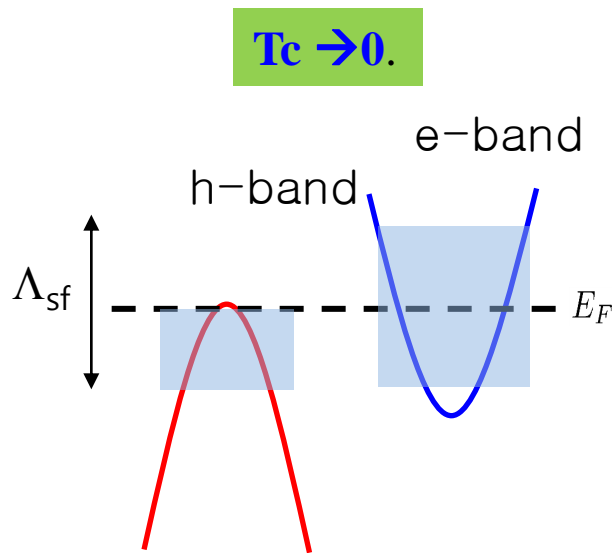
$$T_c(\epsilon_b) \approx 1.14 \Lambda_{hi} \exp \left[ -1/\tilde{\lambda}_{eff}(\epsilon_b) \right]$$



# One problem of this incipient band model: optimal $\epsilon_b \sim 60-80 \text{ meV}$ ?

In real materials,  $T_c \rightarrow 0$  as  $\epsilon_b \rightarrow 0$  ?

Opposite trend to the incipient band pairing scenario.



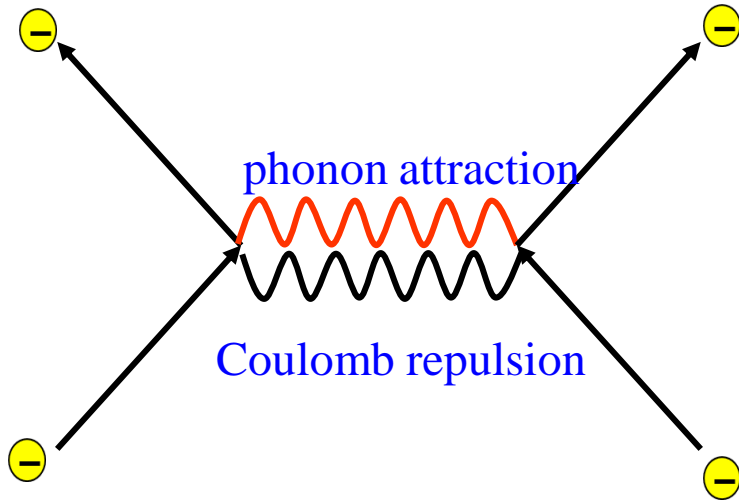
1.  $\pm S$ -gap solution with higher  $T_c$  without impurity
2. but impurity pair-breaking effect severely suppresses  $T_c \rightarrow 0$ .

1. Without impurity,  $T_c$  is lower.
2.  $\pm S$ -gap and  $++S_{ee}$ -gap solutions are degenerate, with the same  $T_c$
3.  $++S_{ee}$ -gap solution is robust against impurities.

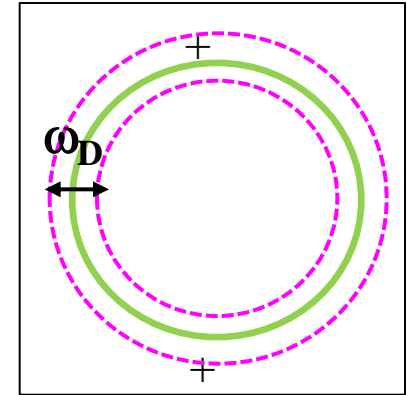
New Concept:

Dynamical Tuning of pairing Cutoff

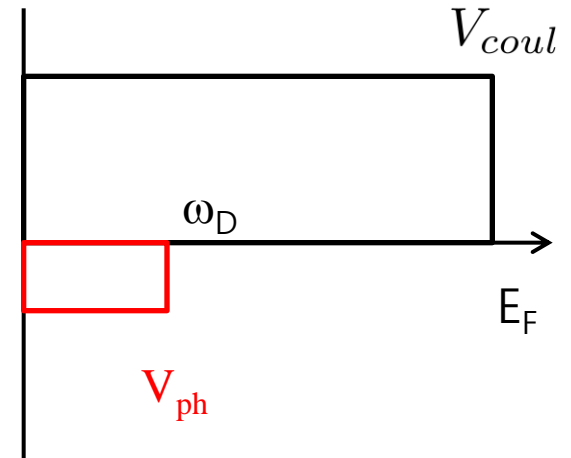
# BCS theory:



Bare elec-elec int. is strongly repulsive.



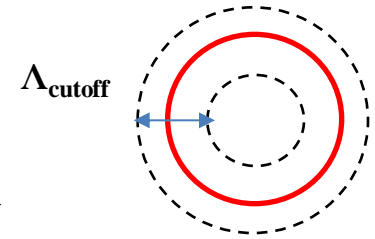
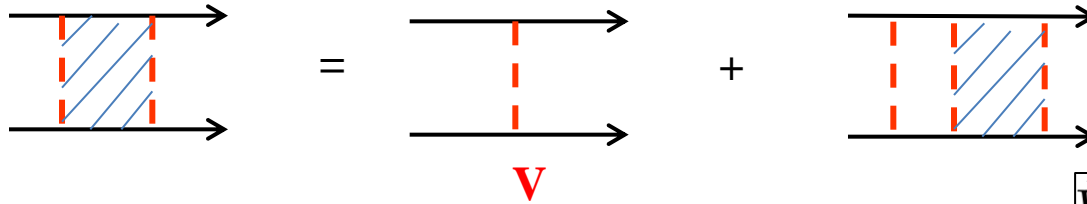
S-wave gap





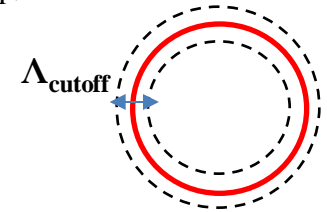
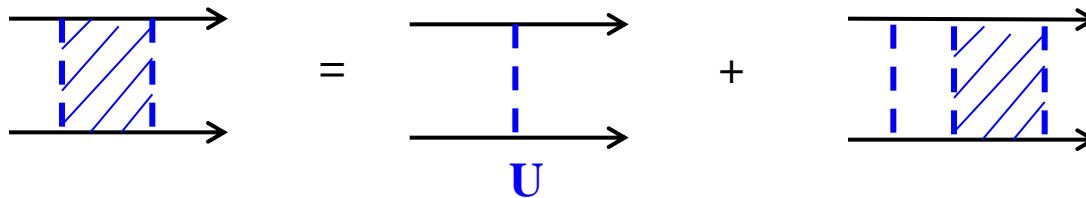
## RG scaling for $S_{ee}$ -wave gap solution

pairing interaction : Cooperon propagator



$$\tilde{V} = \frac{V}{1 + V N(0) \log \frac{E_F}{T}}$$

RG process of pairing interaction (reduce  $\Lambda_{\text{cutoff}}$  from  $E_f$  to  $\omega_m$ )

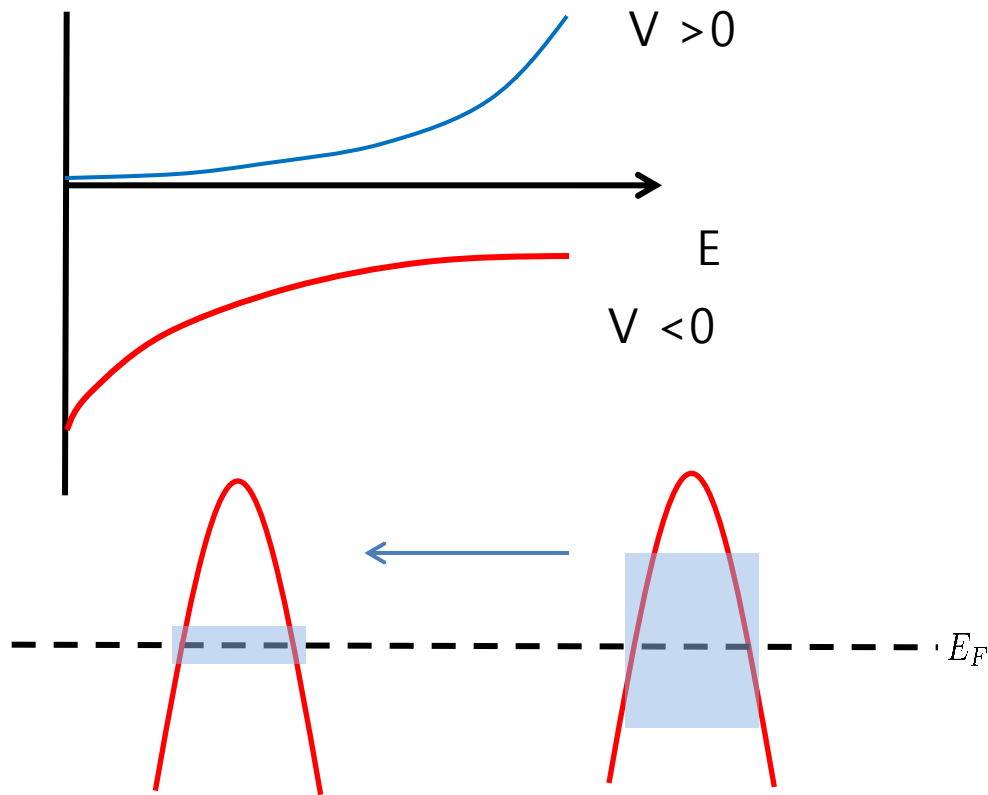


$$V_{\text{tot}} = \frac{V}{1 + V N(0) \log \frac{E_F}{T}} = \frac{U}{1 + U N(0) \log \frac{\omega_m}{T}} \quad \text{with } \omega_m < E_F$$

$$U = \frac{V}{1 + V N(0) \log \frac{E_F}{\omega_m}}$$

Renormalized int.  $U$  from original  $V$ .

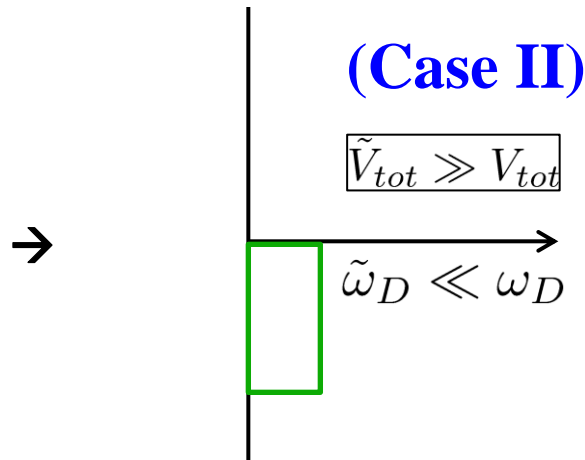
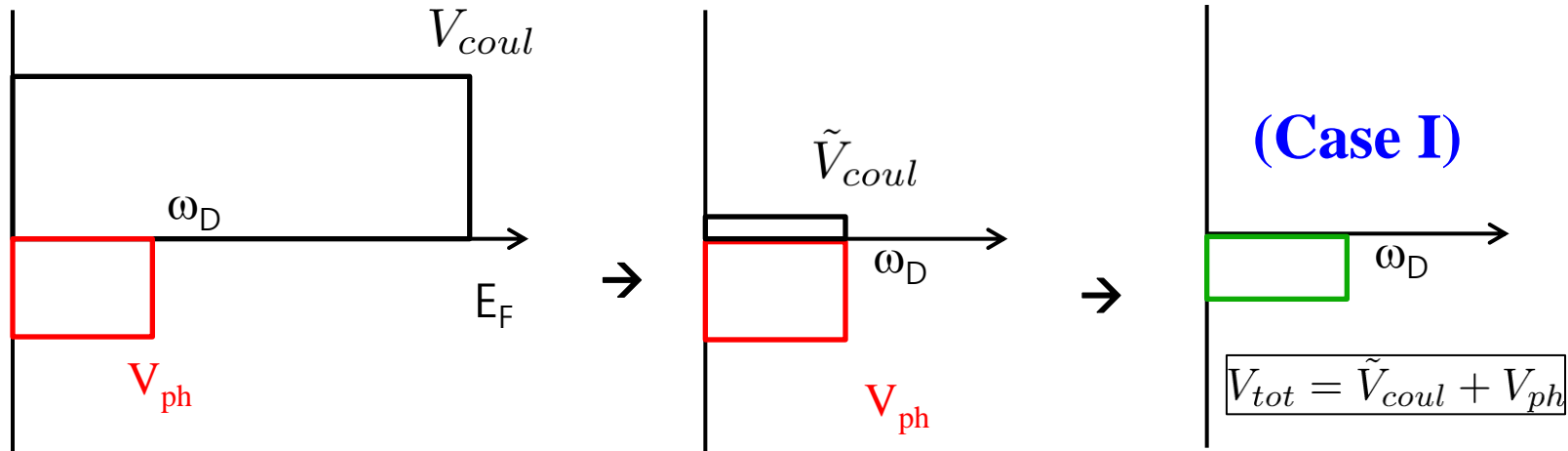
$$U = \frac{V}{1 + V N(0) \log \frac{E_F}{\omega_m}}$$



Lowering cutoff energy  $\Lambda_{\text{cut}}$

$$U = \frac{V}{1 + VN(0) \log \frac{E_F}{\omega_m}}$$

$V_{tot} < 0$  for low energy of  $E < \omega_D$

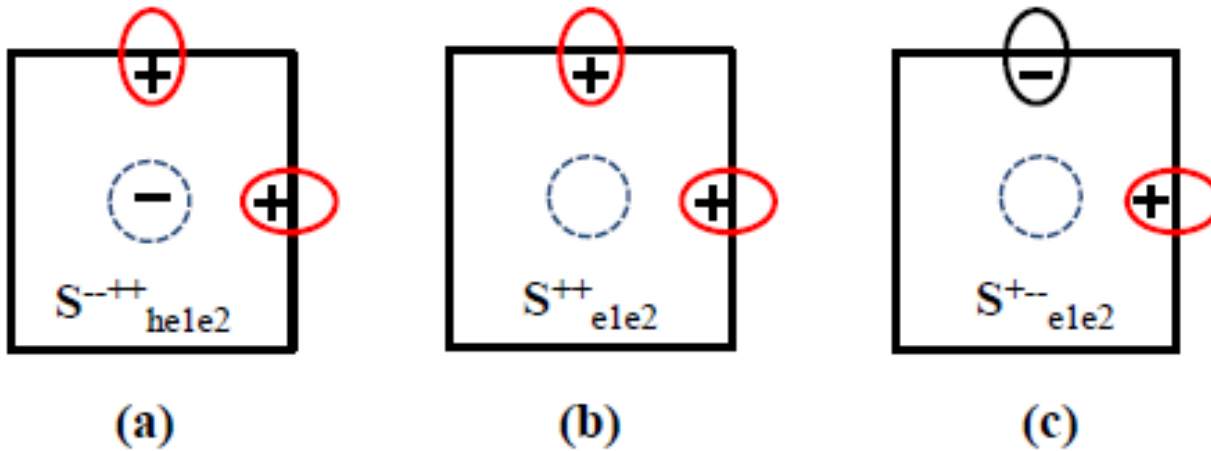


$$T_c = 1.14\omega_D e^{-1/N_0 V_{tot}}$$

$$T_c = 1.14\tilde{\omega}_D e^{-1/N_0 \tilde{V}_{tot}}$$

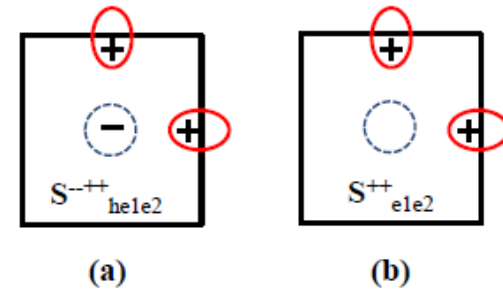
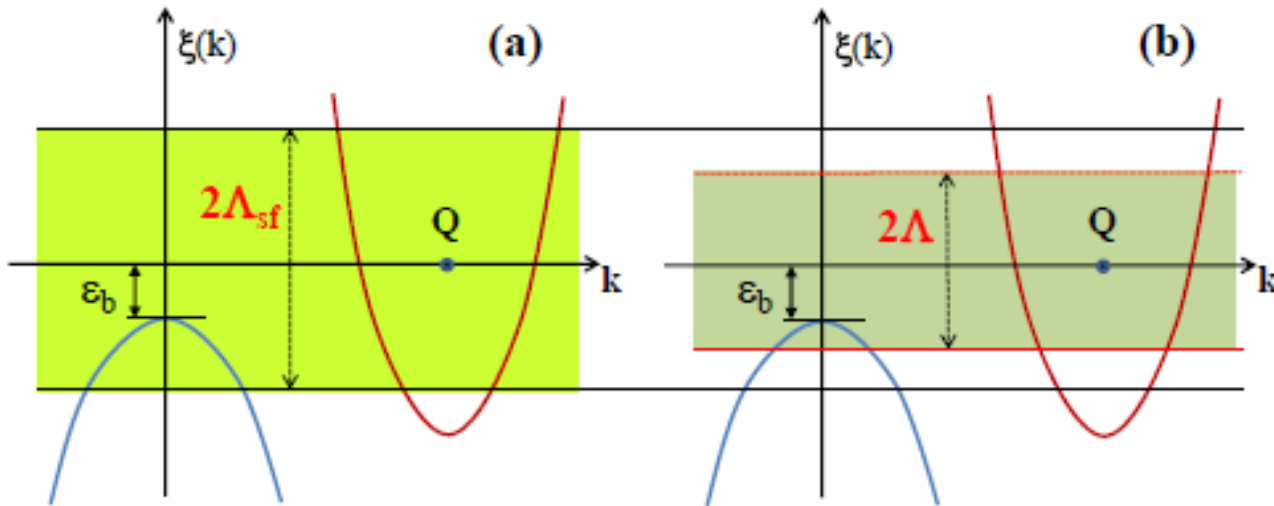
**Case I and Case II** have the same  $T_c$ .

Possible pairing solutions in Incipient band model

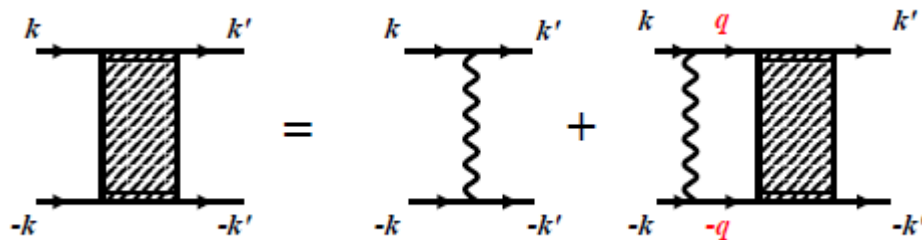


First, compared the solutions **(a)** and **(b)**.

# Scaling the cutoff $\Lambda$



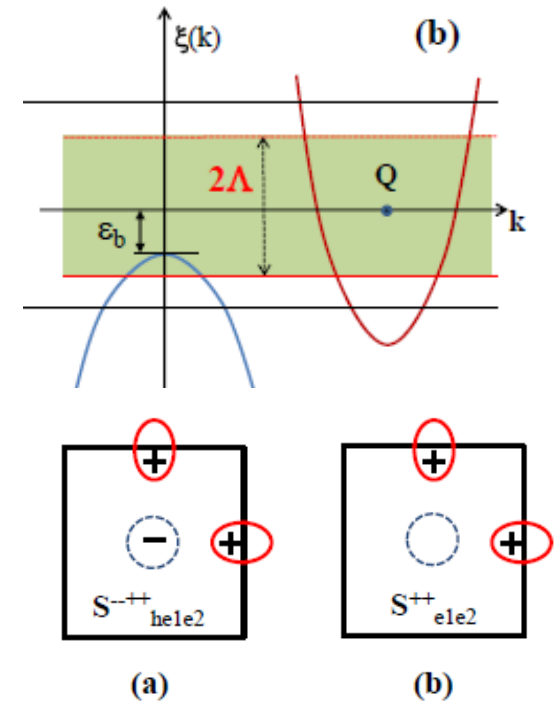
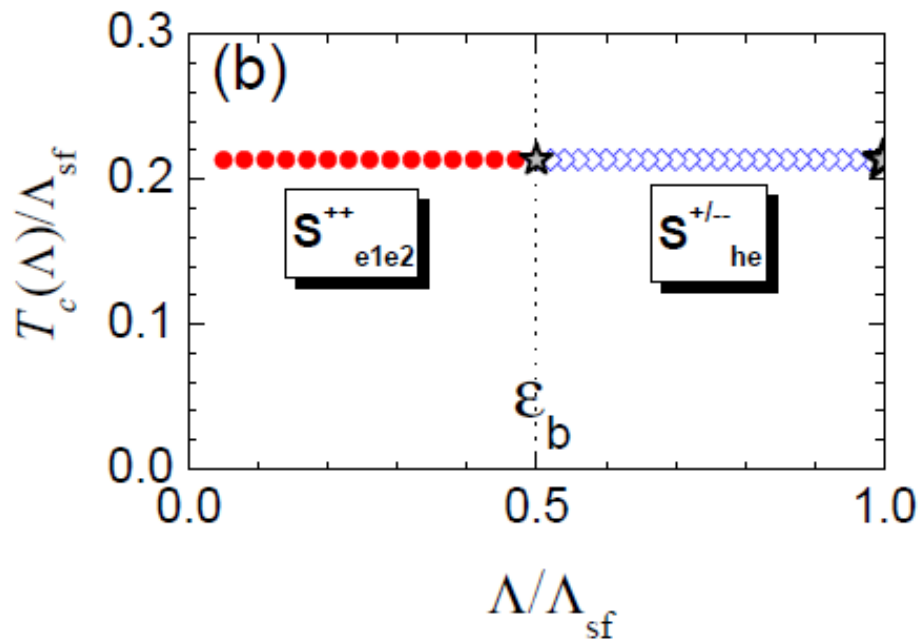
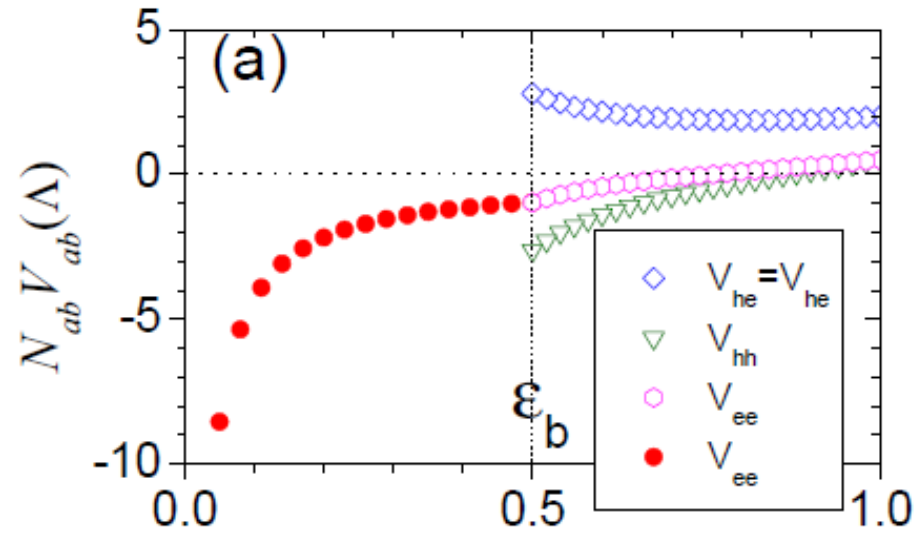
# RG of $\mathbf{V}_{ab}(\Lambda)$



$$\hat{V}(\Lambda) = \hat{V}^0 + \hat{V}^0 \cdot \hat{\chi}(\Lambda_{sf}; \Lambda) \cdot \hat{V}(\Lambda)$$

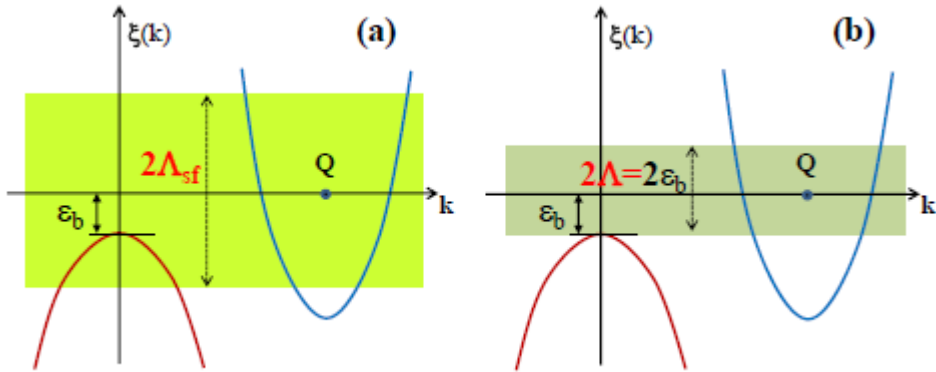
$$\hat{V}^0 = \begin{pmatrix} V_{hh}^0 & V_{he}^0 \\ V_{eh}^0 & V_{ee}^0 \end{pmatrix}$$

$$\hat{\chi}(\Lambda_{sf}; \Lambda) = \begin{pmatrix} \chi_h & 0 \\ 0 & \chi_e \end{pmatrix}$$

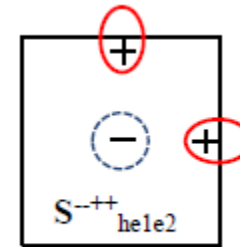


Tc is the same !!

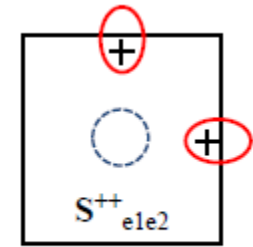
Pairing solution changes across  $\epsilon_b$ .



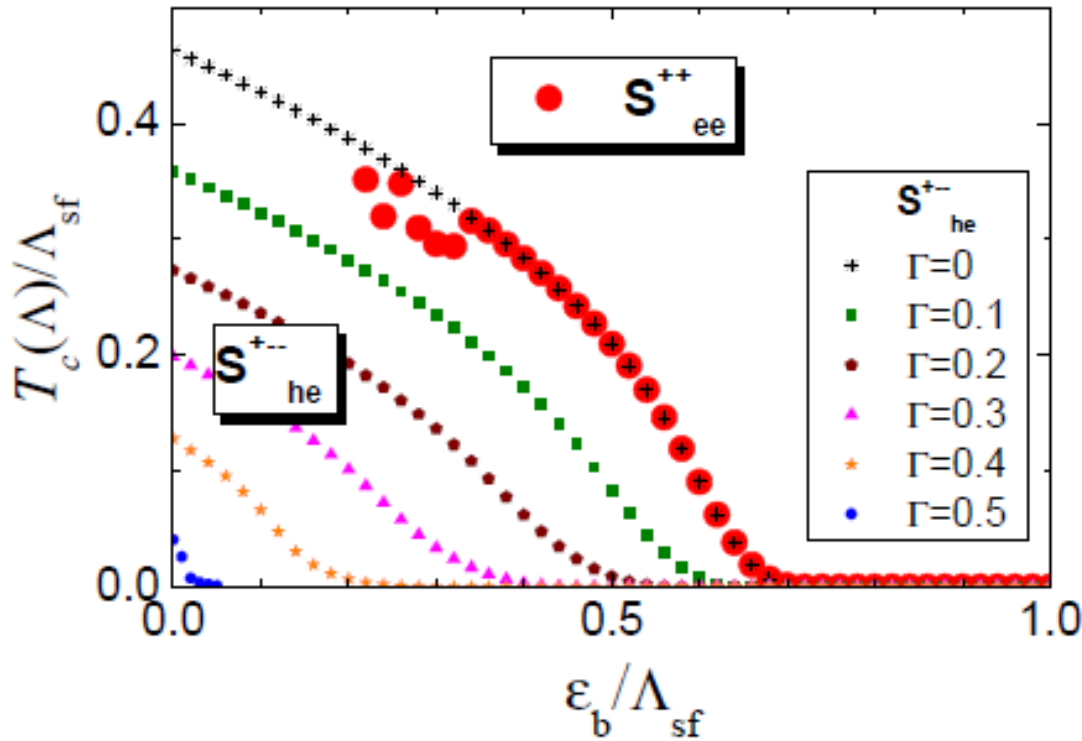
Now consider only two solutions:



(a)



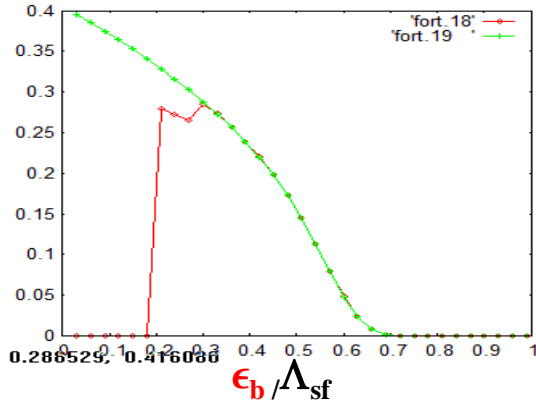
(b)



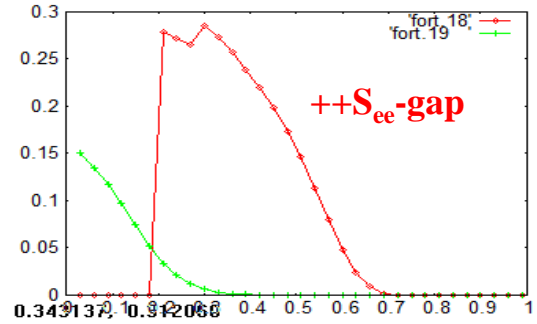
Non-mag impurities kill only  $S^{+-}_{he}$  state.

# Imp. Effect on $\pm S$ -gap n $++S_{ee}$ -gap

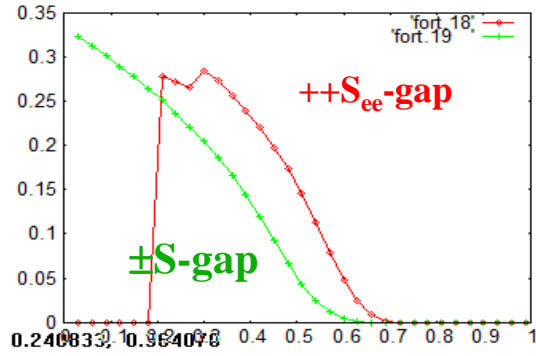
$T_c$



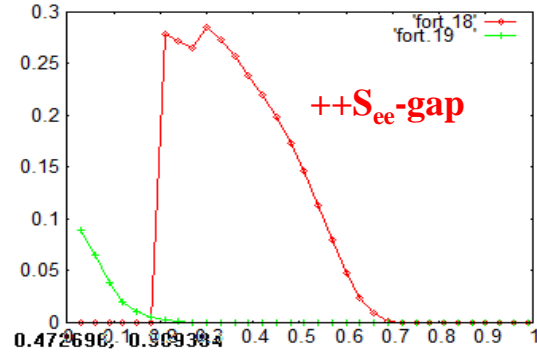
Imp.  
 $\Gamma=0.0$



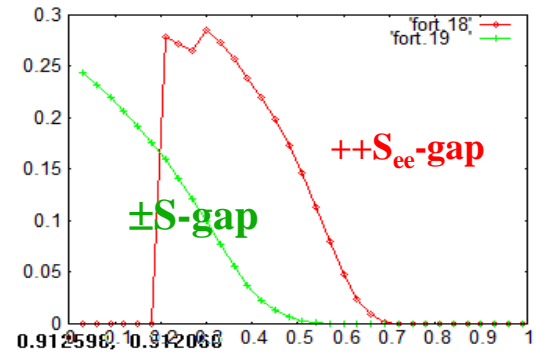
Imp.  
 $\Gamma=0.3$



Imp.  
 $\Gamma=0.1$



Imp.  
 $\Gamma=0.35$

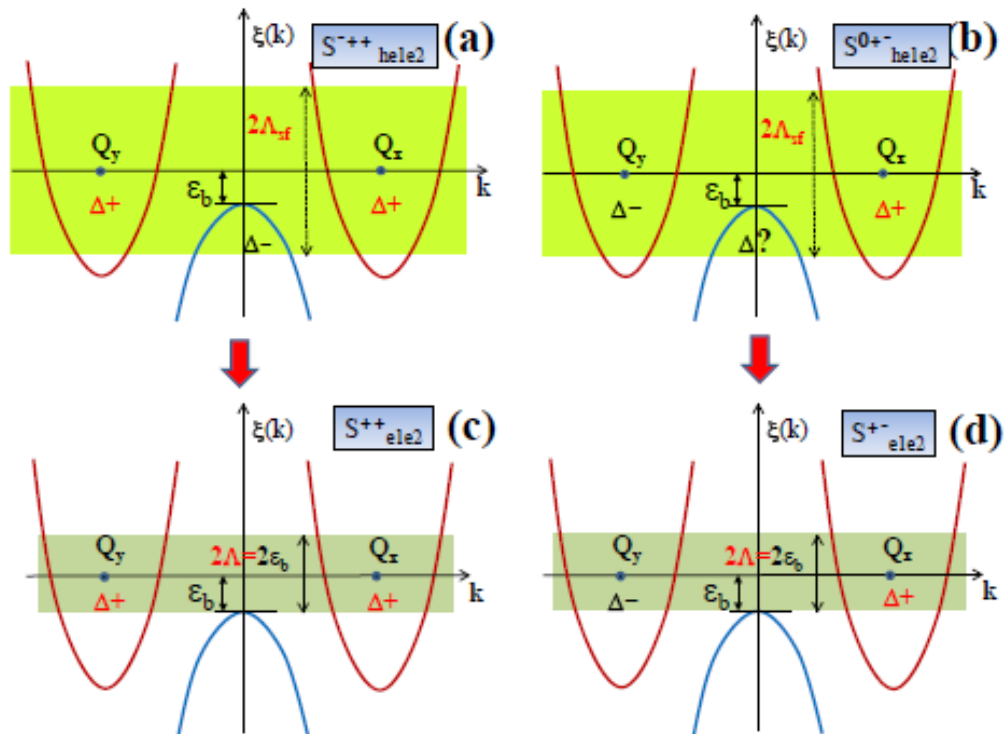
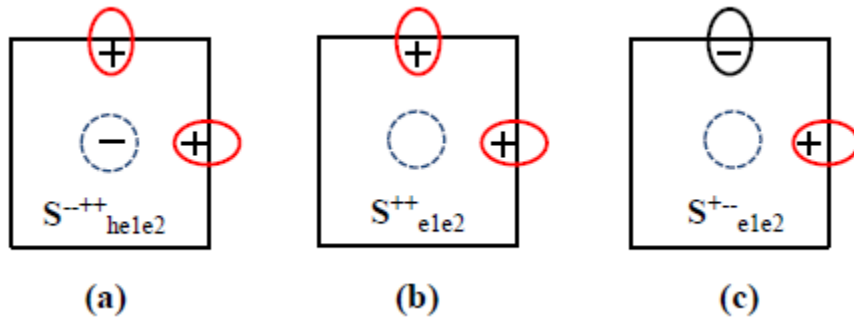


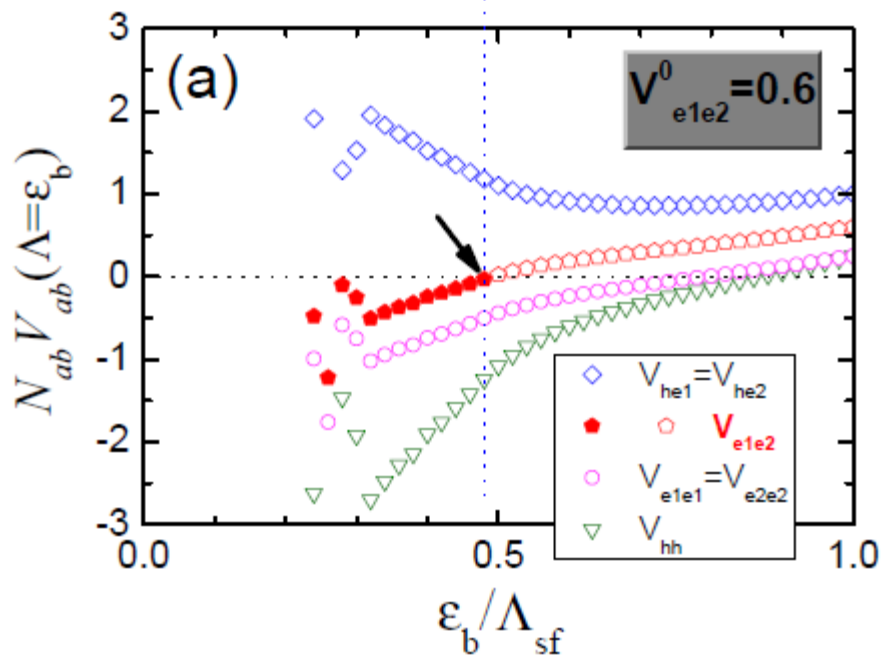
Imp.  
 $\Gamma=0.2$

Impurity pair-breaking only for  $\pm S$ -gap,  
 $++S_{ee}$ -gap is robust against imp. scattering.

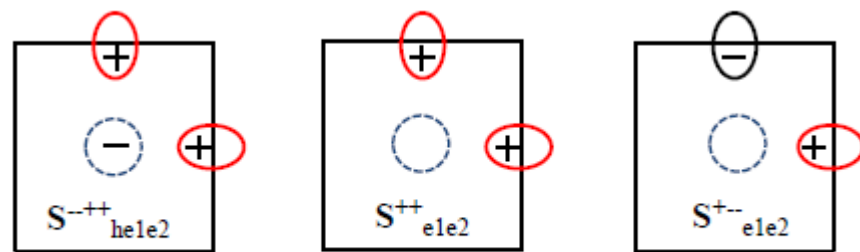


# Three band model





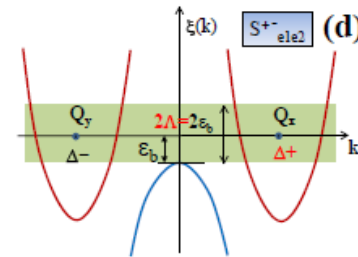
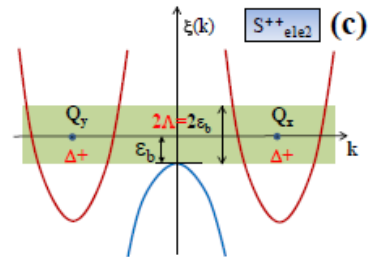
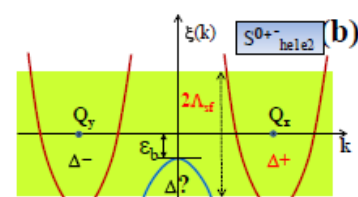
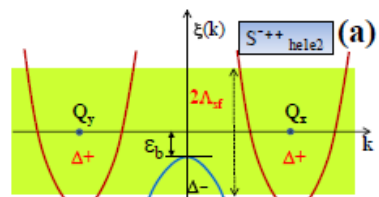
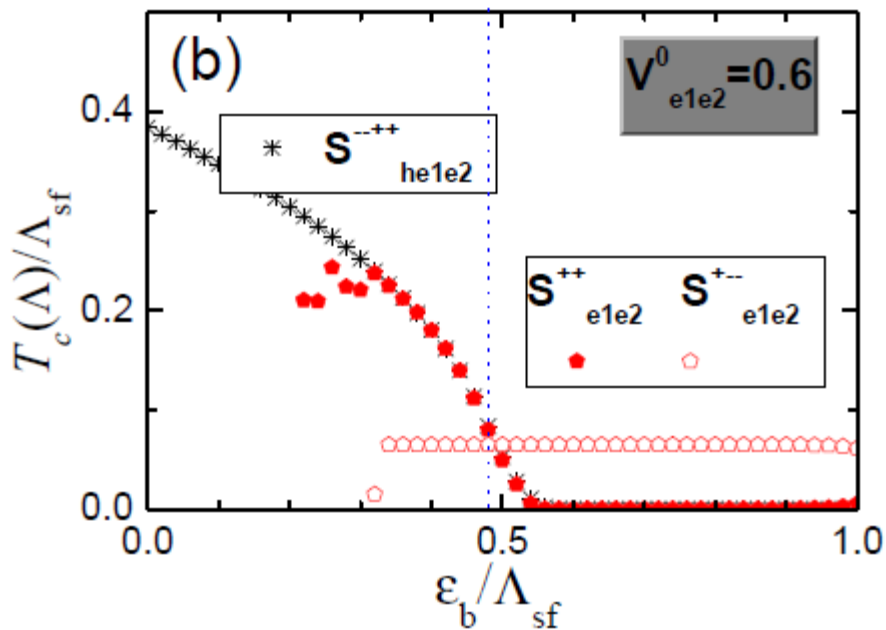
$$V = \begin{bmatrix} V_{hh} & V_{he1} & V_{he2} \\ V_{e1h} & V_{e1e1} & V_{e1e2} \\ V_{e2h} & V_{e2e1} & V_{e2e2} \end{bmatrix}$$



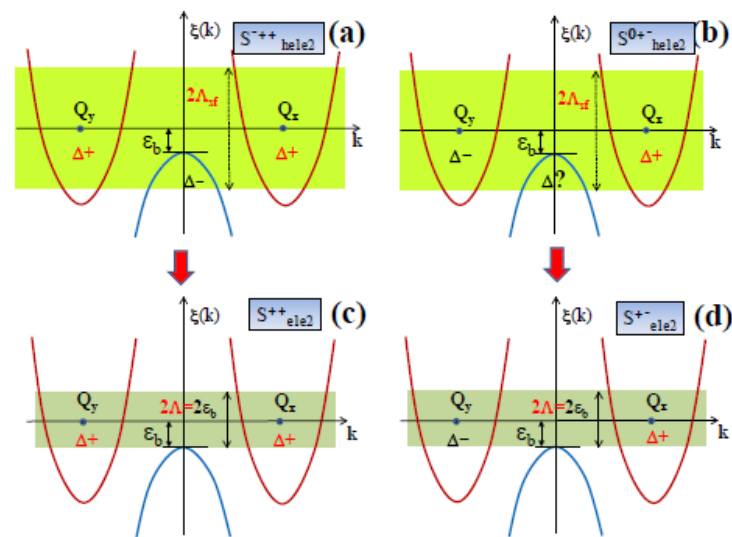
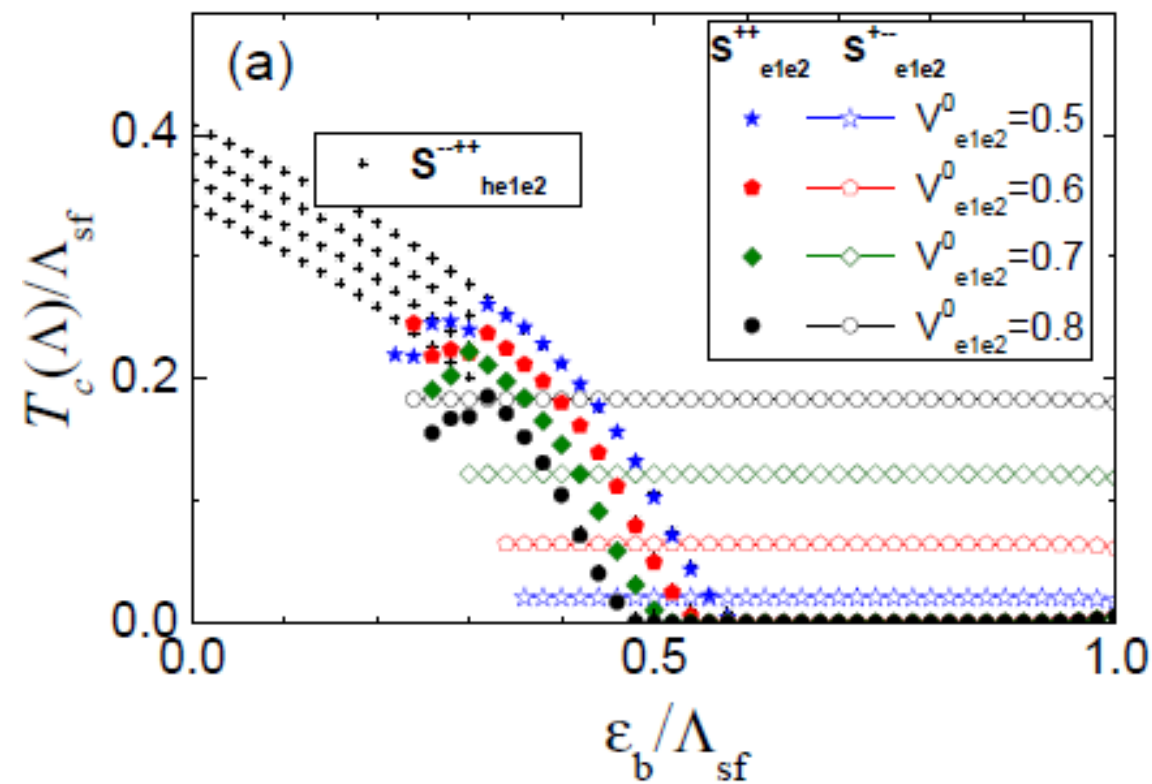
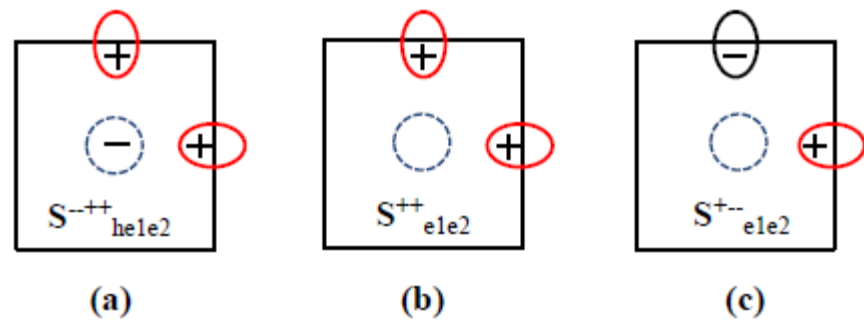
(a)

(b)

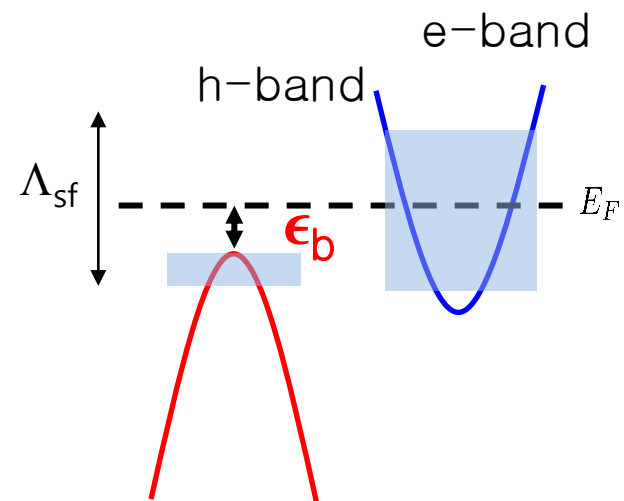
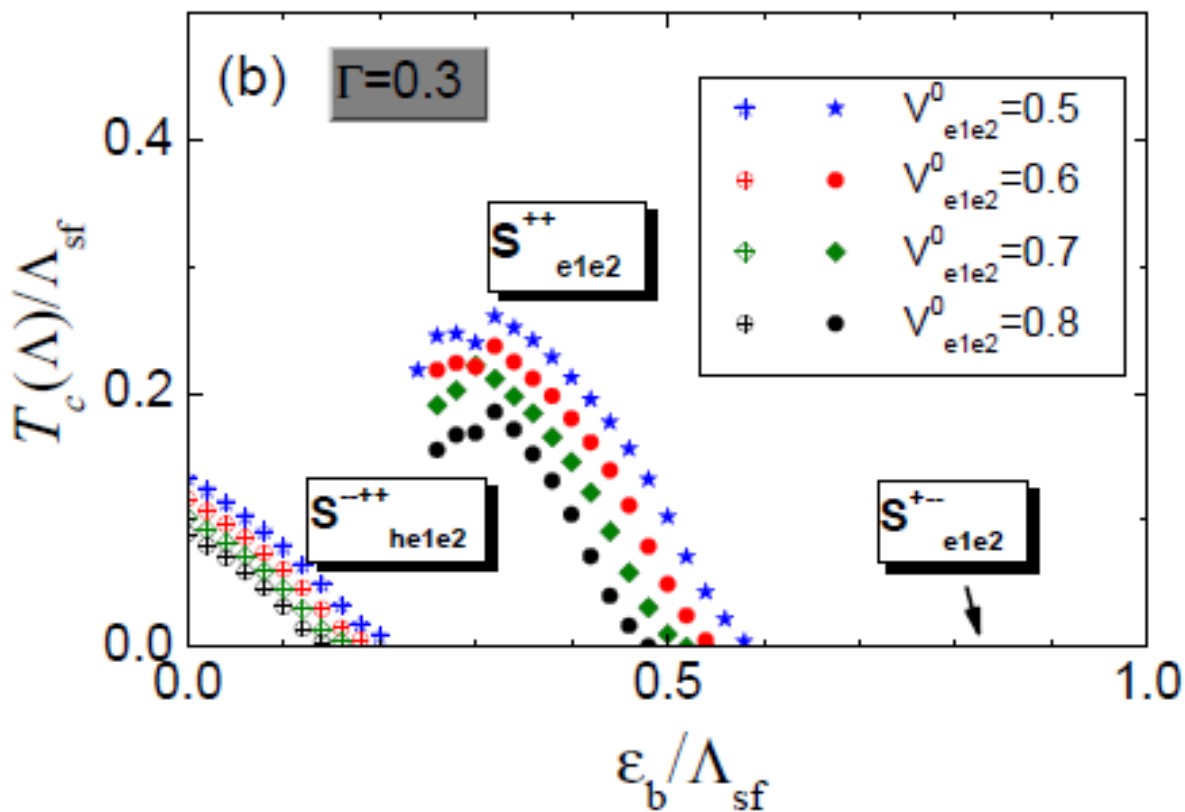
(c)



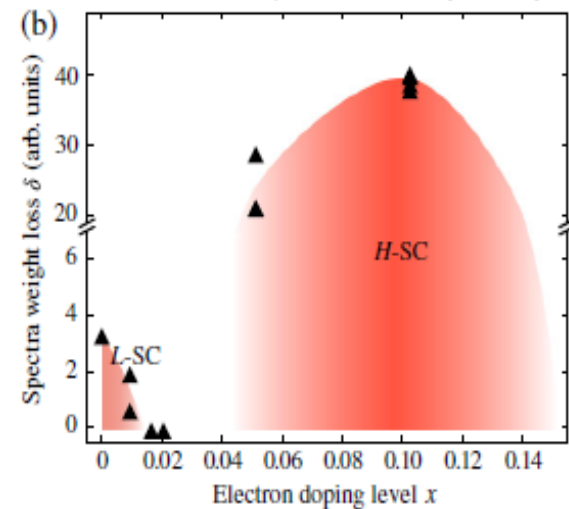
# Varying $V_{e1e2}$



With non-magnetic impurities.



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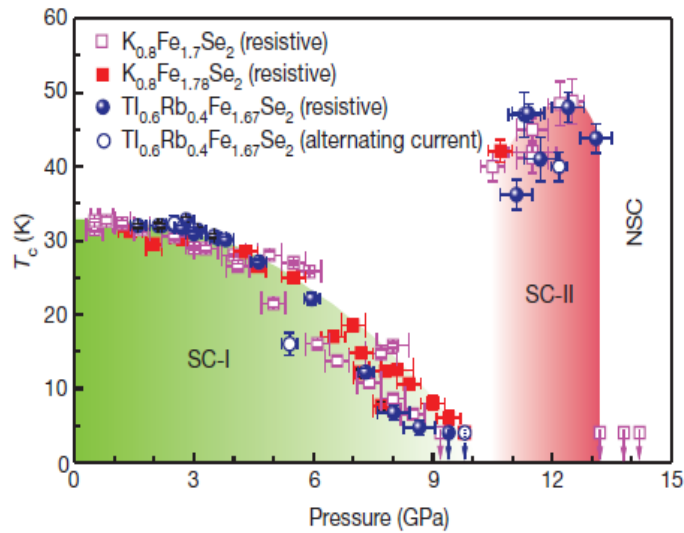
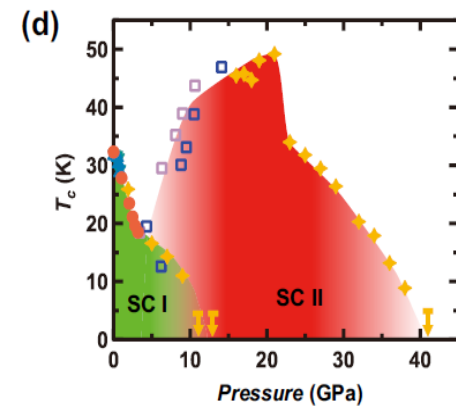
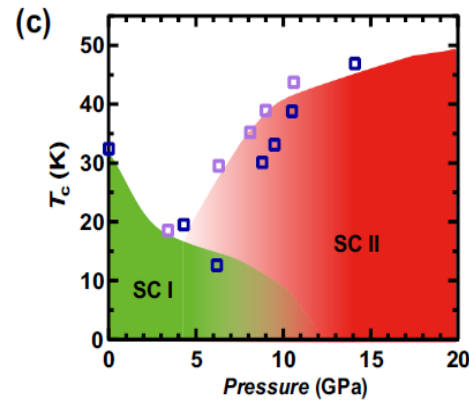
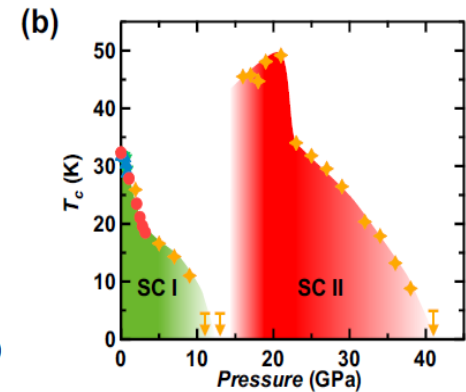
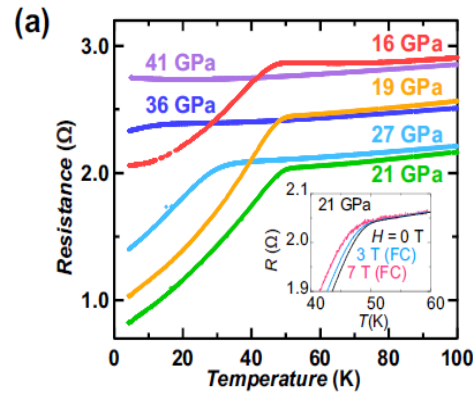
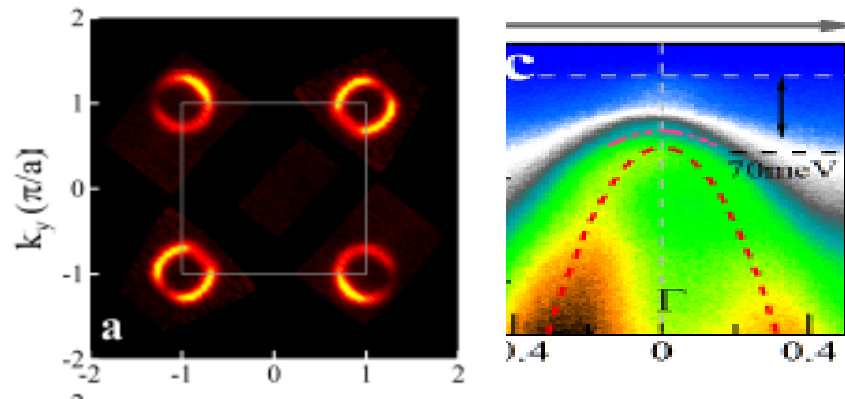


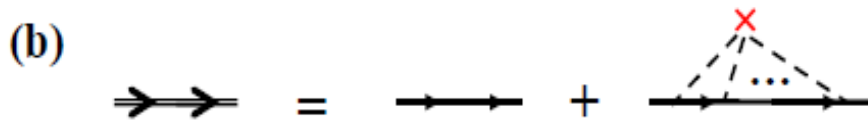
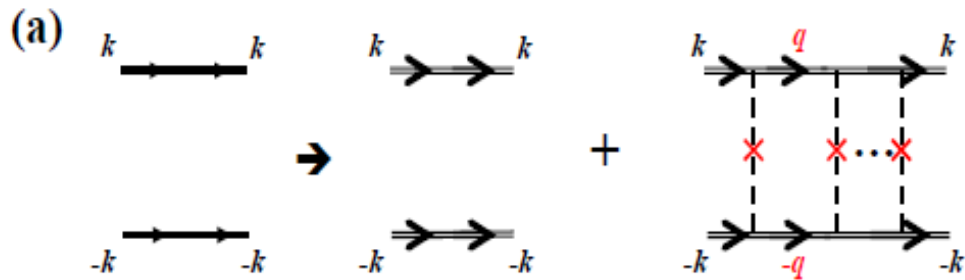
Figure 4 | Pressure dependence of the  $T_c$  for  $Ti_{0.6}Rb_{0.4}Fe_{1.67}Se_2$ ,  $K_{0.8}Fe_{1.7}Se_2$  and  $K_{0.8}Fe_{1.78}Se_2$ . The symbols represent the pressure-temperature



## Conclusions:

1. All HEDIS system with only electron pockets has  **$++S_{ee}$ -gap**
2. **FeSe monolayer is special.** Need small angle phonon boost
3. Pairing cutoff energy is dynamically tuned by RG to  $\epsilon_b$ .
4.  **$\pm S_{he}$ -pairing** paradigm still governs FeSe system.





$$\tilde{\chi}_e(\Lambda_{sf}; \Lambda) = -T_c \sum_n 2N_e \int_{\Lambda}^{\Lambda_{sf}} d\xi \frac{\eta_v}{\tilde{\omega}_n^2 + \xi^2},$$

$$\tilde{\chi}_h(\Lambda_{sf}; \Lambda) = -T_c \sum_n N_h \int_{-\Lambda_{sf}}^{-\Lambda} d\xi \frac{\eta_v}{\tilde{\omega}_n^2 + \xi^2}.$$

When  $\eta_v = \eta_l$ , Cooperon is invariant w.r.t non-mag. Impurity scatt.

