

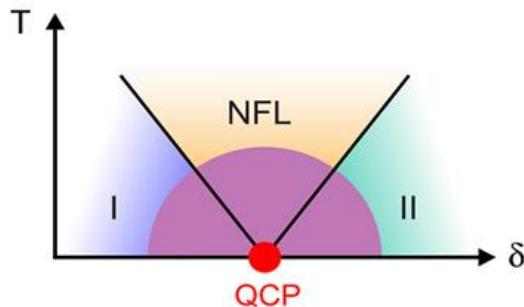
Beyond Landau Fermi liquid and BCS superconductivity near quantum criticality

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Karlsruhe Institute of Technology*

Lecture Series at APCTP, Pohang

May 23-27, 2016



Lecture 1.

Overview of the lecture series – General introduction to phase transitions

Contents of Lecture 1

Overview over the lecture series

Landau theory of phase transitions

Static critical behavior, universality and scaling –
concept of the renormalization group

New universality classes?

Dynamic critical behavior

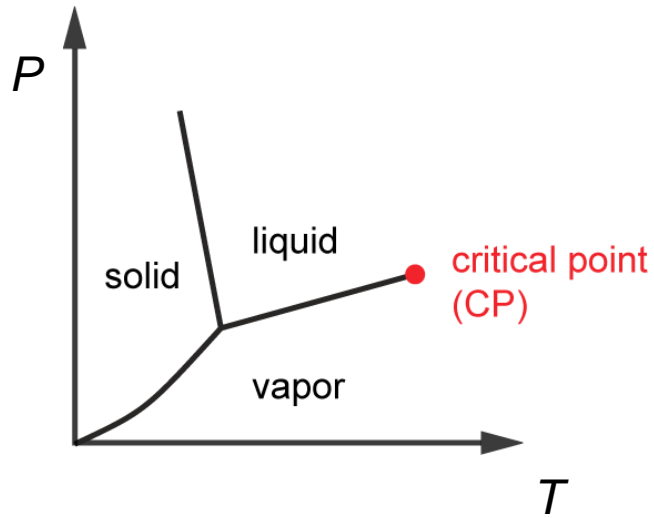
Classical vs. quantum phase transitions

Different types of quantum phase transitions

Overview over the lecture series

How does matter change its state?

Phase transitions in H₂O



Types of phase transitions

- discontinuous (1st order)
e.g. melting
- continuous (2nd order, ...)
e.g. vapor-liquid transition
at critical point

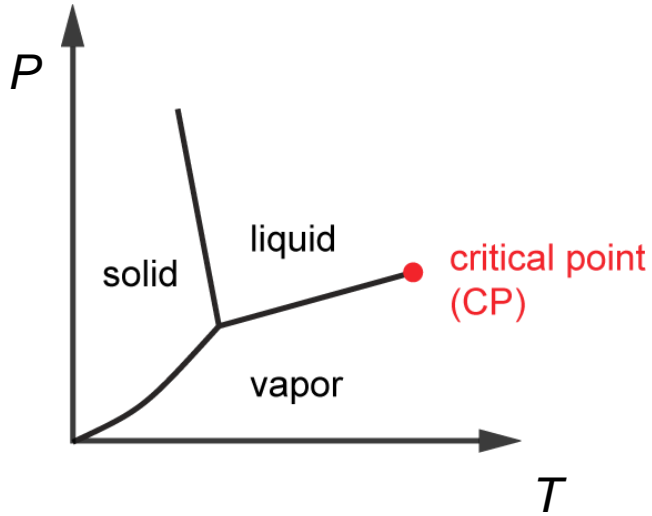
Typical of 2nd order transitions:
continuous growth of ordered
regions

⇒ critical opalescence

Example: demixing transition

How does matter change its state?

Phase transitions in H₂O



Types of phase transitions

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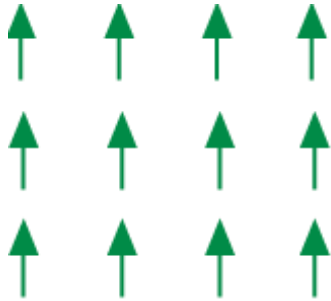
⇒ critical opalescence

Example: demixing transition

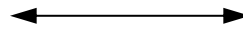


Magnetic instabilities in metals

Magnetic order

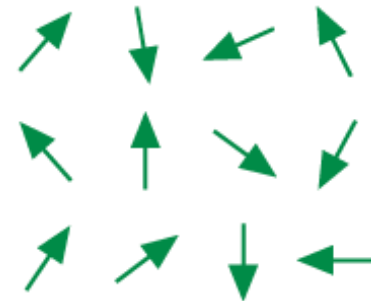


control parameter

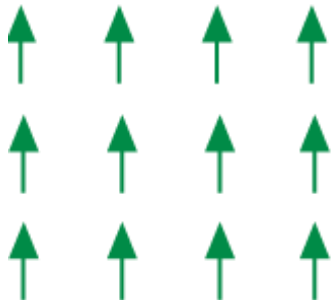


temperature,
pressure,
....

disordered magnetic moments



Magnetic order

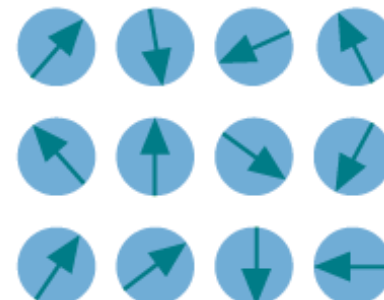


control parameter



fluctuation energies,
hybridization,
....

magnetic moments „lost“



scenarios:

charge fluctuations, Kondo effect, itinerant magnetism

The Standard Model of phase transitions: Ginzburg-Landau-Wilson theory



V. Ginzburg



L. D. Landau



G. K. Wilson

Universality of finite- T second-order transitions

Static critical behavior (exponents $\alpha, \beta, \gamma, \nu, \dots$) depends only on spatial dimension of the system and symmetry of the order parameter because correlation length diverges at T_c , $\xi \sim |T - T_c|^{-\nu}$

correlation time: $\tau \sim \xi^z$ (“critical slowing down”).

Dynamic properties are “less” universal, e. g., difference between FM and AF dynamics

The Standard Model of phase transitions: Ginzburg-Landau-Wilson theory



V. Ginzburg



L. D. Landau

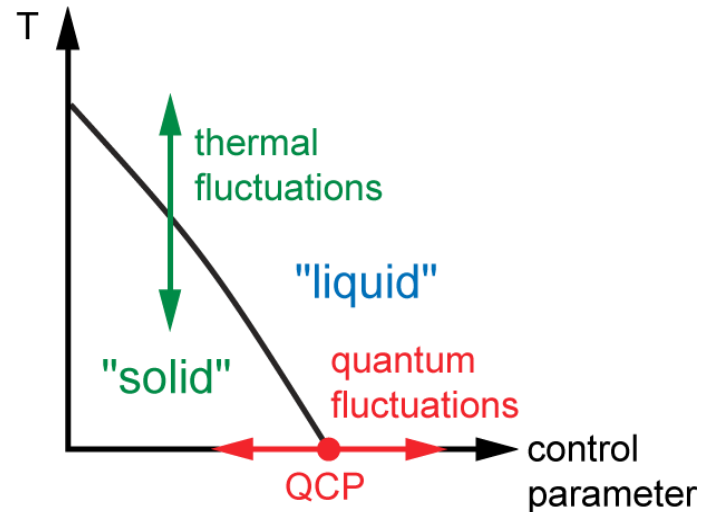


G. K. Wilson



J. A. Hertz

What happens if one can drive a second-order transition to zero by a non-thermal control parameter?

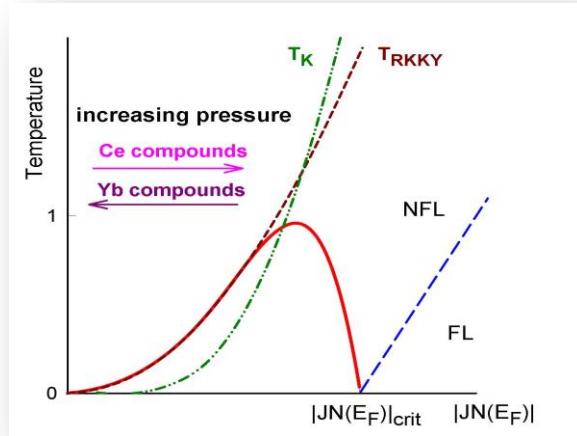


$T_c \rightarrow 0$: energy of fluctuations \hbar/τ important:
temperature sets the system size
in the time direction: $d \rightarrow d + z$

Problem: low-energy fermions

Competing interactions with the possibility of quantum phase transitions

Heavy-fermion metals: Kondo vs. RKKY interaction



onsite – intersite competition

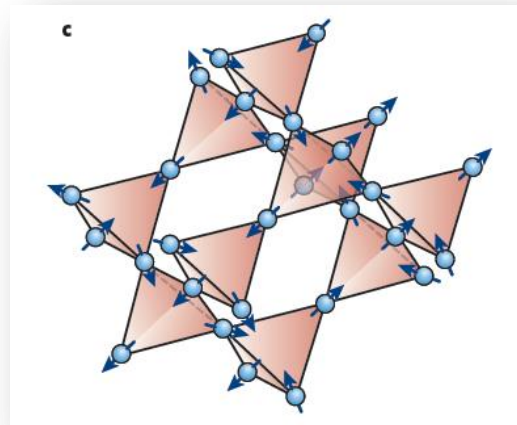
Doniach “phase diagram”

S. Doniach, Physica 91B, 231 (1977)

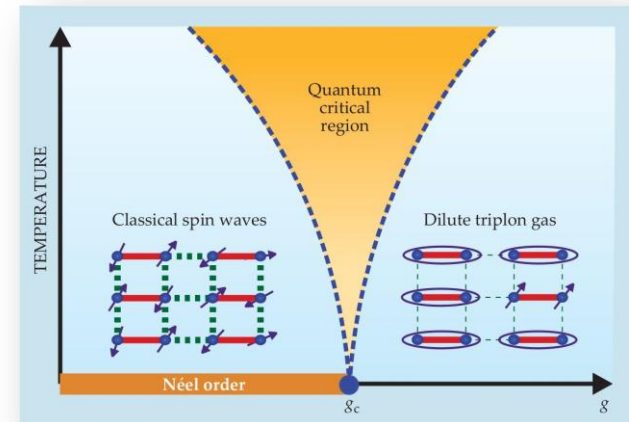


Competing nn or nn/nnn interactions in insulating magnets

Geometric frustration of nn interactions



Competing nn and nn interactions



S. Sachdev, B. Keimer, Physics Today: 64 (2), 29 (2011)

Conduction electrons in a metal: noninteracting Fermi gas



A. Sommerfeld



W. Pauli

Specific heat

$$C = (\pi^2/3) n k_B^2 N(E_F) T = \gamma_0 T$$

Wilson ratio $\gamma_0 / \chi_0 = \text{const}$ (universal for free-electron metals)

Electron-electron scattering: resistivity $\Delta\rho \sim AT^2$ with $A/\gamma_0 \approx \text{const}$

Pauli spin susceptibility χ_0

$$\chi_0 = \mu_0 \mu_B^2 N(E_F) = \text{const}$$

The Standard Model of metals: Landau Fermi-liquid theory



L. D. Landau

Electron-electron interactions parametrized by few parameters m^* , F_0^a , F_0^s , ...

$$C = \gamma T = \frac{m^*}{m_0} \gamma_0 T; \quad \chi = \frac{m^*}{m_0} \frac{1}{1 + F_0^a} \chi_0$$

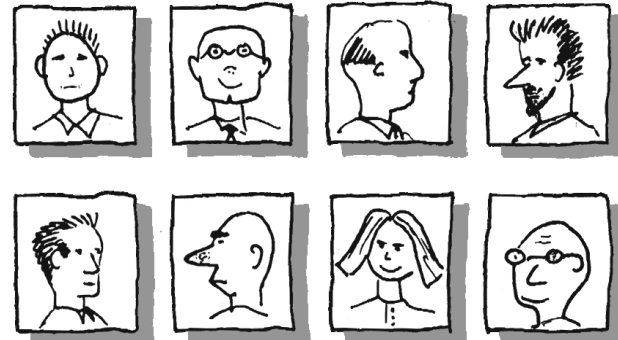
$$\Delta\rho \sim T^2$$

Since ~ 1990: many systems show deviations:

“non-Fermi liquids“

1:1 correspondence between
excitations of interacting and
noninteracting systems:

“Fermi liquid“



Non-Fermi liquid behavior

Specific heat: often $\gamma = C/T \sim \ln(T_0/T)$

Resistivity: $\rho = \rho_0 + A'T^\alpha$ with $\alpha < 2$

NFL behavior can arise from
distinctly different physical origins:

- Single-ion (impurity) effects

 - Multichannel Kondo effect

 - Distribution of Kondo temperatures

- Collective effects

 - Quantum phase transitions

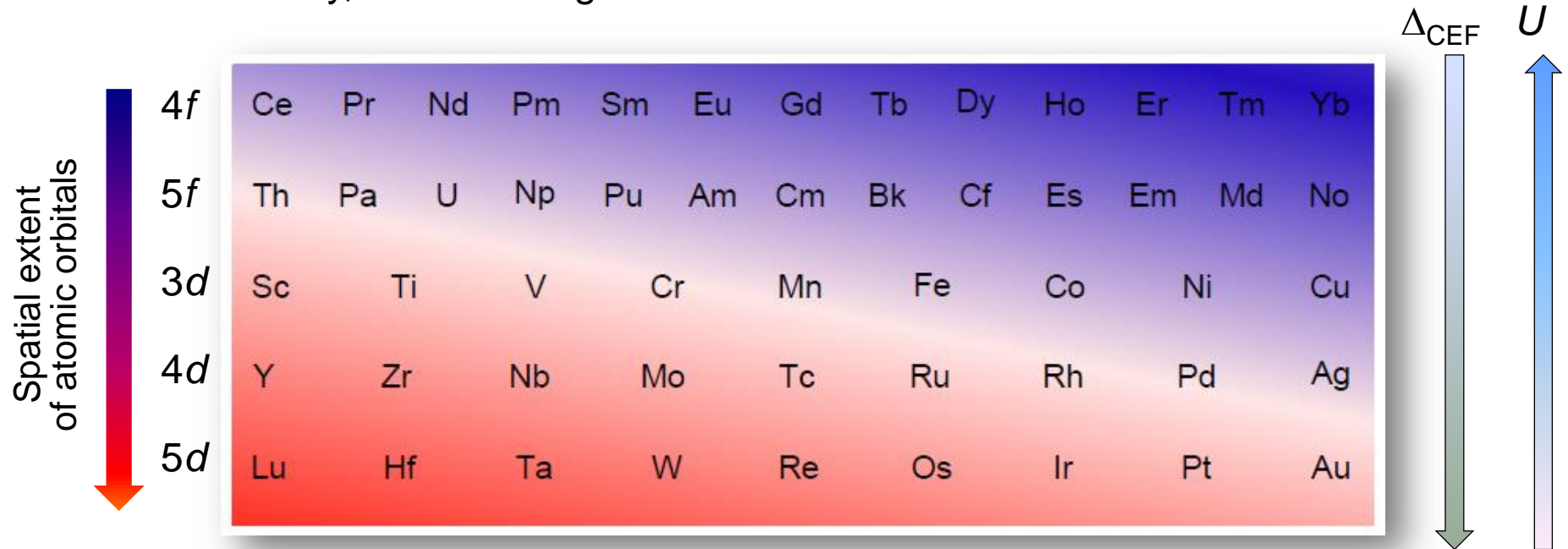
Note: “single-ion“ effects, such as the Kondo effect,
are in reality many-body phenomena.

There is no such thing as “the“ non-Fermi liquid!

Partially filled d and f shells of the elements

*J. L. Smith and E. A. Kmetko,
J. Less-Comm. Met. 90, 83 (1983)*

Generally, rather strong correlations between these electrons



different energy scales:

crystalline electric field Δ_{CEF} and intra-atomic repulsion U

Magnetic quantum phase transitions in metals: important issues

Non-Fermi-liquid behavior:

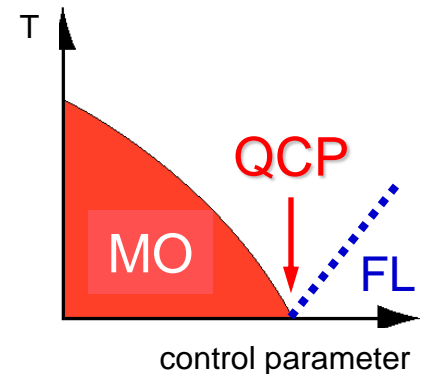
What happens to the Fermi surface?

Low energy scales, otherwise masked by magnetic interaction,
may become important as $T_c \rightarrow 0$:

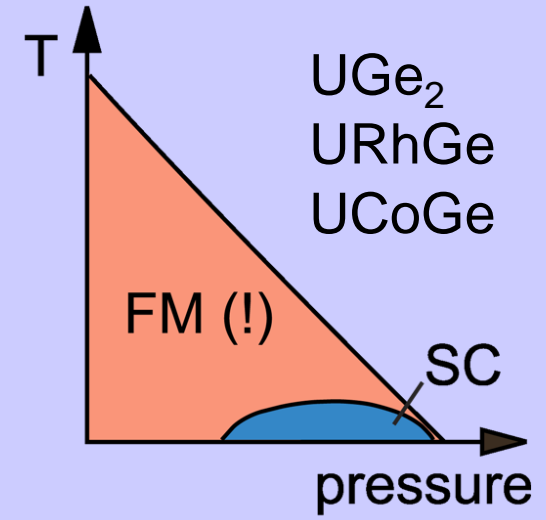
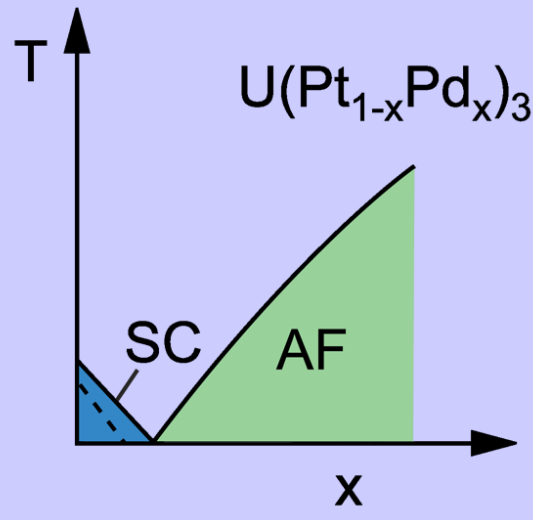
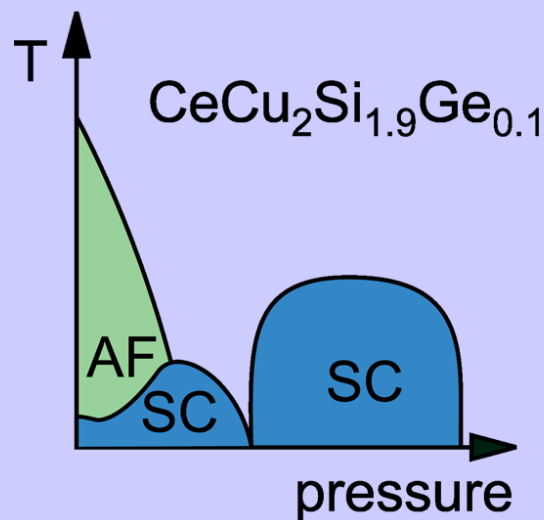
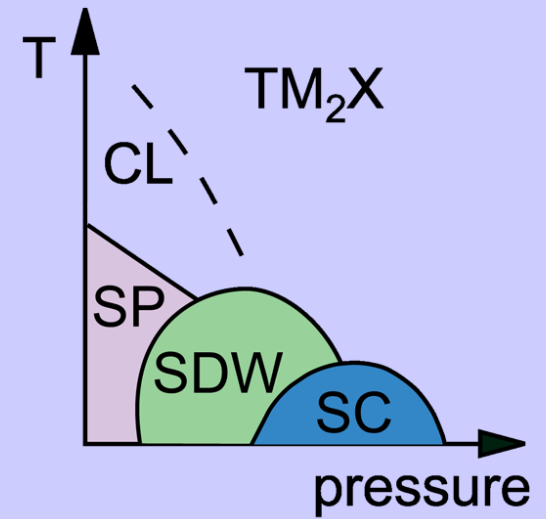
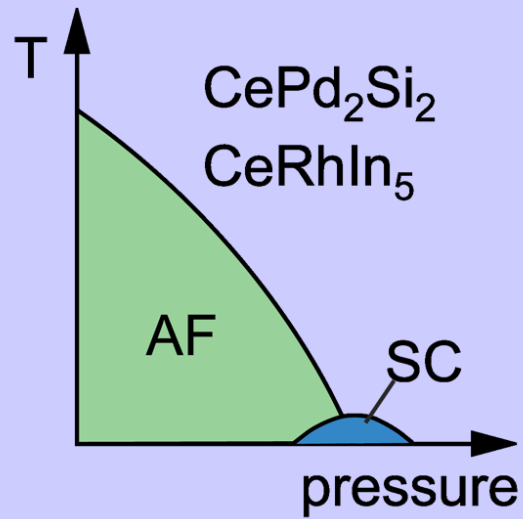
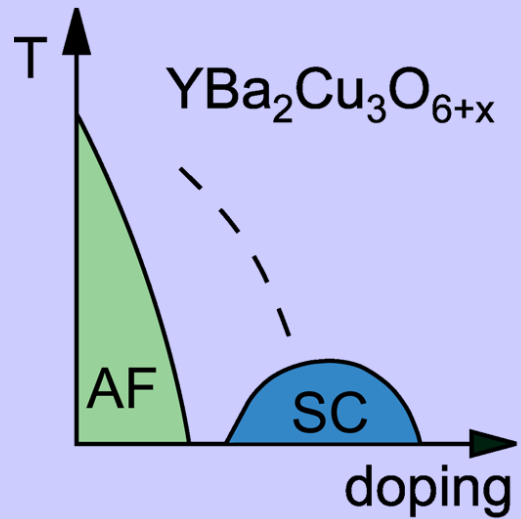
Possibility of novel phases

Examples

- Superconductivity
in the vicinity of quantum critical points
- Low-dimensional fluctuations
over an extended x -range in $\text{CeCu}_{6-x}\text{Au}_x$
- Partial order in MnSi



Interplay of superconductivity and magnetism near quantum critical points



Outline of the lectures

1. Overview of the lecture series and General introduction to phase transitions (4h)

Overview over the lectures – Landau theory of phase transitions – Static critical behavior, universality and scaling; concept of the renormalization group – New universality classes? – Dynamic critical behavior – Classical vs. quantum phase transitions – Different types of quantum phase transitions

2. Fermi liquids and heavy-fermion systems (2h)

Landau Fermi liquids – Kondo effect: Concept of a local Fermi liquid – Heavy-fermion systems – Quantum phase transitions in heavy-fermion systems

3. Ce-Cu-Au: a case study for heavy fermion quantum criticality (2h)

Introduction to the $\text{CeCu}_{6-x}\text{Au}_x$ system – Non-Fermi liquid thermodynamic and transport properties near the quantum critical point at $x = 0.1$ – Measurement of quantum-critical fluctuations by inelastic neutron scattering – Fate of the Kondo energy scale at the concentration-tuned QCP in $\text{CeCu}_{6-x}\text{Au}_x$? – Role of the tuning parameter: composition, hydrostatic pressure, magnetic field – Thermal expansion: a sensitive thermodynamic probe of quantum criticality – Determining the entropy landscape near quantum criticality

4. Further examples of quantum critical *d*- and *f*-electron systems (2h)

CePdAl, a partially frustrated heavy-fermion system – Approaching quantum criticality of CePdAl with Ni substitution – Frustrated Ce moments in CePdAl: a two-dimensional spin liquid? – Classical and quantum phase transitions in the itinerant ferromagnet $\text{Sr}_{1-x}\text{Ca}_x\text{RuO}_3$ – Evolution of criticality with x – Unusual slow quantum critical dynamics

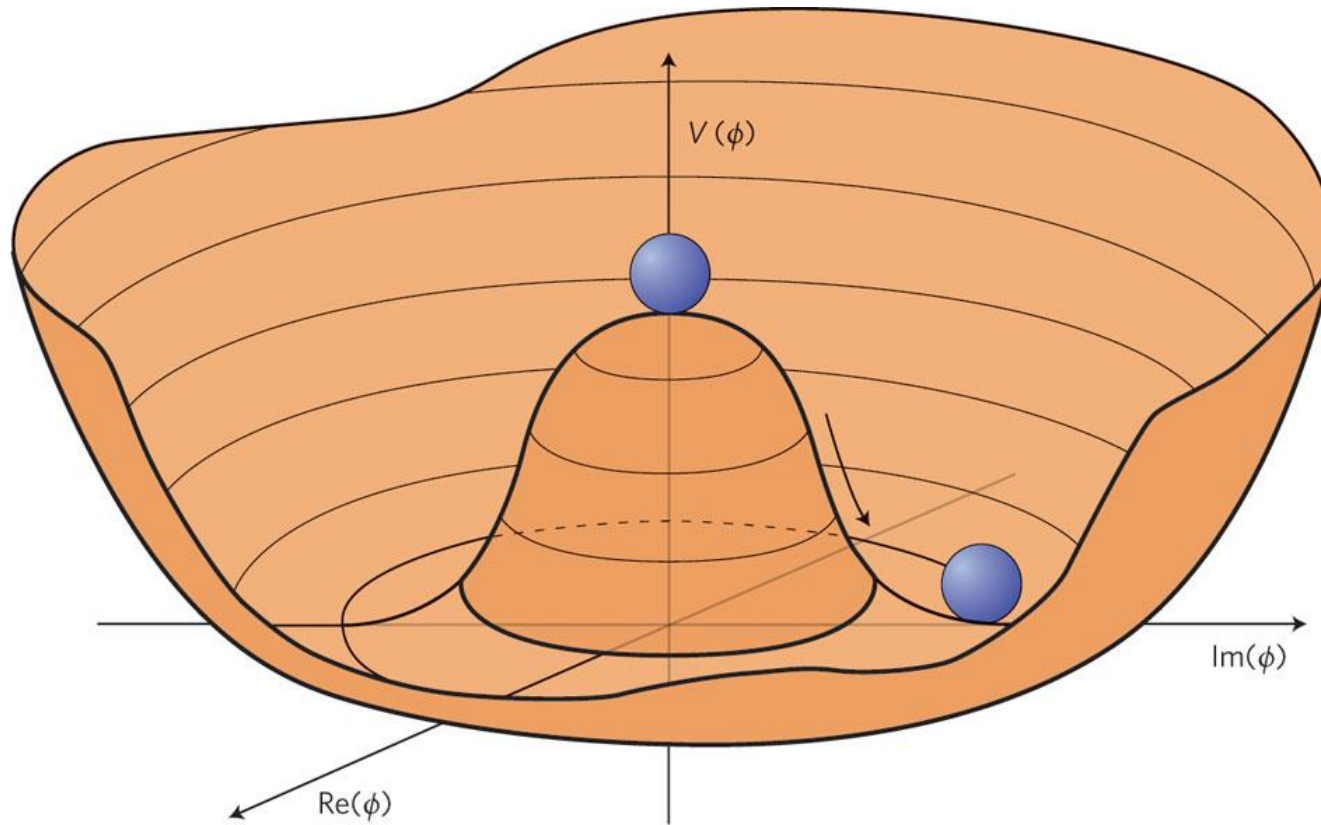
Landau theory of phase transitions

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See standard textbooks on Statistical Mechanics,

Also: P. M. Chaikin, T. C. Lubensky, *Principles of Condensed Matter Physics*,
Cambridge University Press, 1995, ch. 4

“Mexican hat” potential



Static critical behavior,
universality and scaling –
Concept of the
Renormalization Group

Blackboard script not reproduced here

See standard textbooks on Statistical Mechanics

Also: P. M. Chaikin, T. C. Lubensky, *Principles of Condensed Matter Physics*,
Cambridge University Press, 1995, ch. 5

Static critical exponents

from P. M. Chaikin and T. C. Lubensky-

Exponent	α	β	γ	ν	η
Property	specific heat	order parameter	susceptibility	coherence length	correlation function
Definition	$C \sim t^{-\alpha}$	$\langle \phi \rangle \sim t^\beta$	$\chi \sim t^{-\gamma}$	$\xi \sim t^{-\nu}$	$G(q) \sim q^{-2+\eta}$
Mean-field	0	0.5	1	0.5	0
3D theory					
$n = 0$ (SAW)	0.24	0.30	1.16	0.59	
$n = 1$ (Ising)	0.11	0.32	1.24	0.63	0.04
$n = 2$ (xy)	-0.01	0.35	1.32	0.67	0.04
$n = 3$ (Heisenberg)	-0.12	0.36	1.39	0.71	0.04
Experiment					
3D $n = 1$	$0.11^{+0.01}_{-0.03}$	$0.32^{+0.16}_{-0.04}$	$1.24^{+0.16}_{-0.04}$	$0.63^{+0.04}_{-0.04}$	0.03 - 0.06
3D $n = 3$	$0.1^{+0.05}_{-0.04}$	$0.34^{+0.04}_{-0.04}$	$1.4^{+0.07}_{-0.07}$	$0.7^{+0.03}_{-0.03}$	
2D $n = 1$	$0.0^{+0.01}_{-0.003}$	$0.3^{+0.04}_{-0.04}$	$1.82^{+0.7}_{-0.07}$	$1.02^{+0.07}_{-0.07}$	

Experiments on 3D $n = 1$ compiled from liquid-gas, binary fluid, ferromagnetic, and antiferromagnetic transitions.

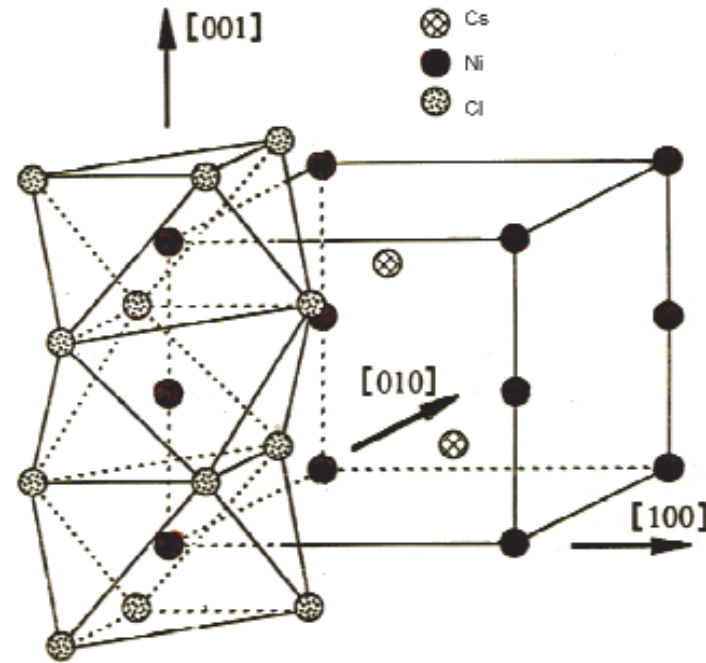
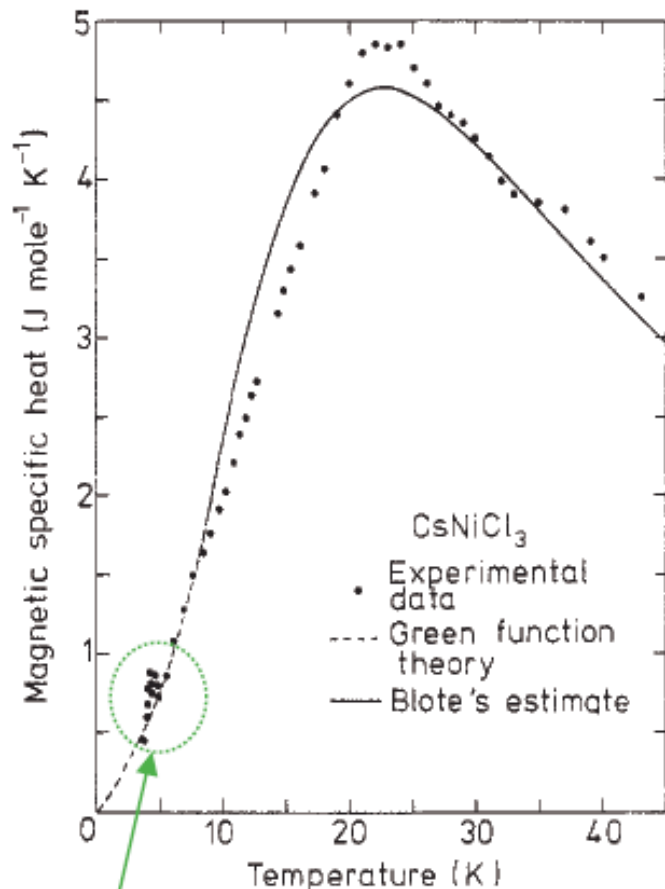
Experiments on 3D $n = 3$ transitions compiled from some ferromagnetic and antiferromagnetic transitions.

Experiments on 2D $n = 1$ compiled from some antiferromagnetic transitions.

New universality classes?

Quasi-one-dimensional Ising antiferromagnet: CsNiCl_3

Magnetic specific heat



triangular ab planes of Ni^{2+} ions
magnetic easy axis $[001]$

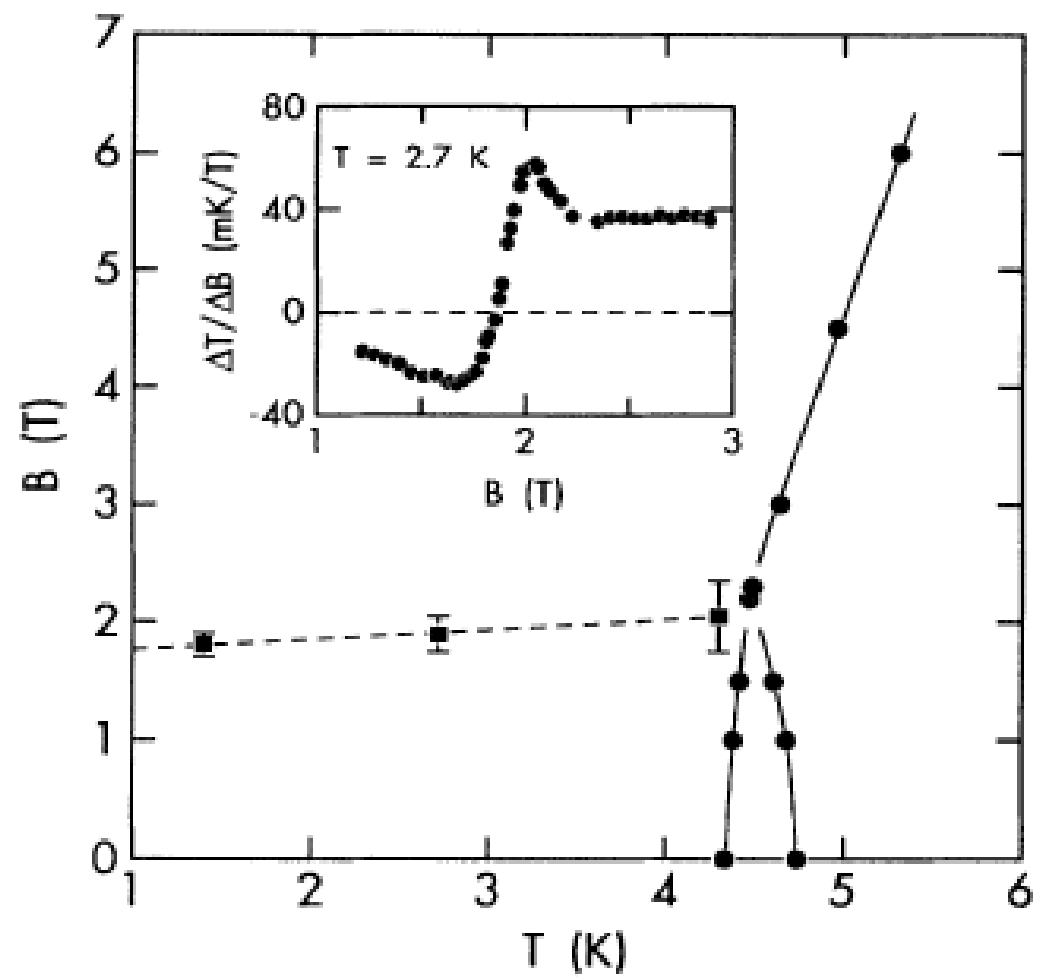
no phase transition,
only short range correlations

3D magnetic order due to weak interaction between chains

Quasi-one-dimensional Ising antiferromagnet: CsNiCl_3

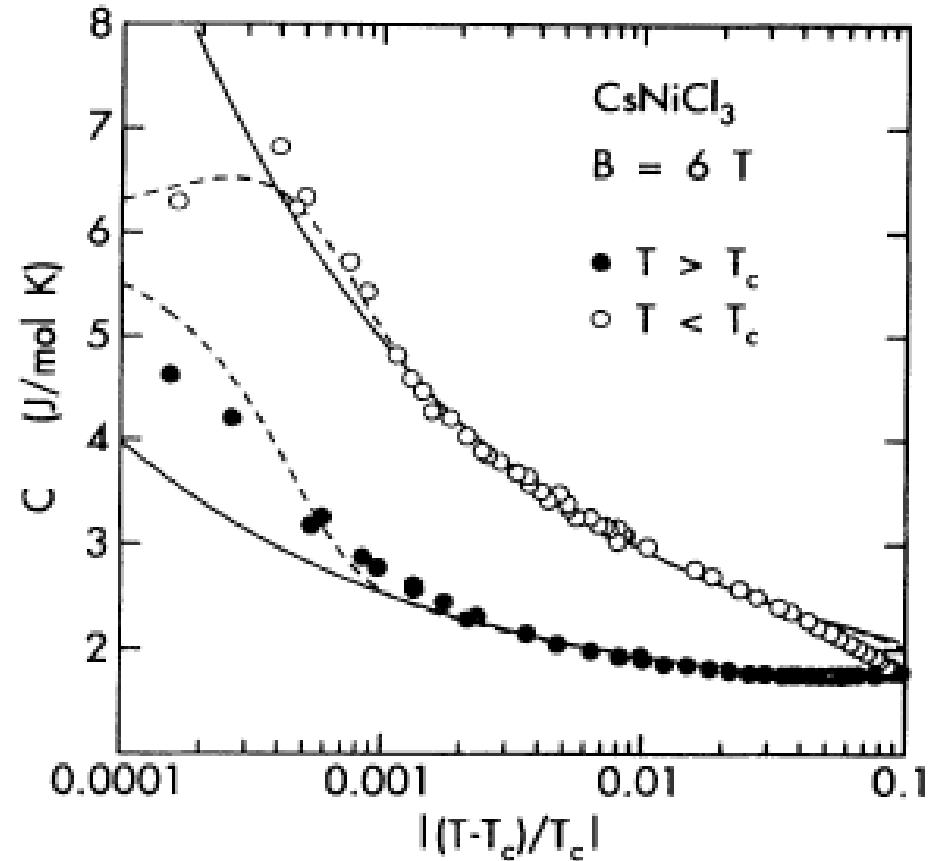
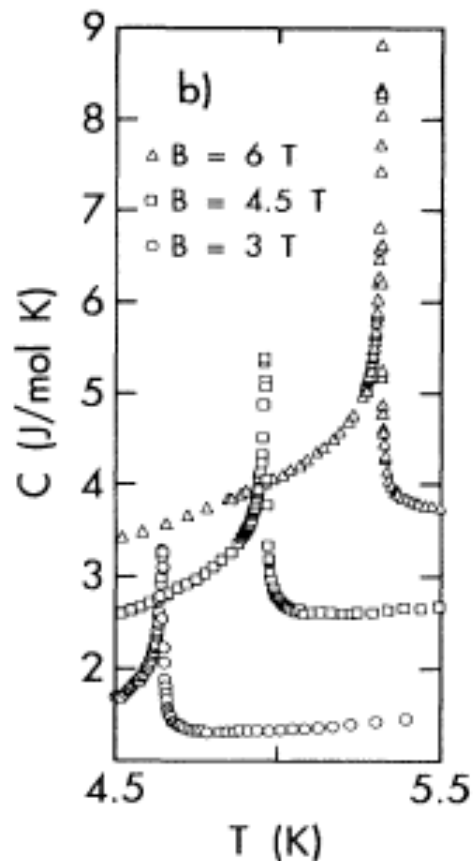
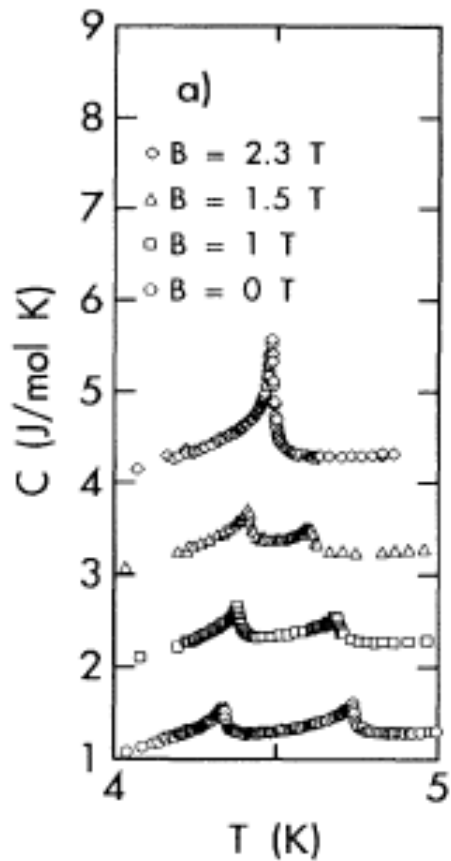
D. Beckmann et al., PRL 78, 2129 (1993)

B - T Phase diagram



Magnetic ordering of CsNiCl₃ at low temperature

D. Beckmann et al., PRL 78, 2129 (1993)



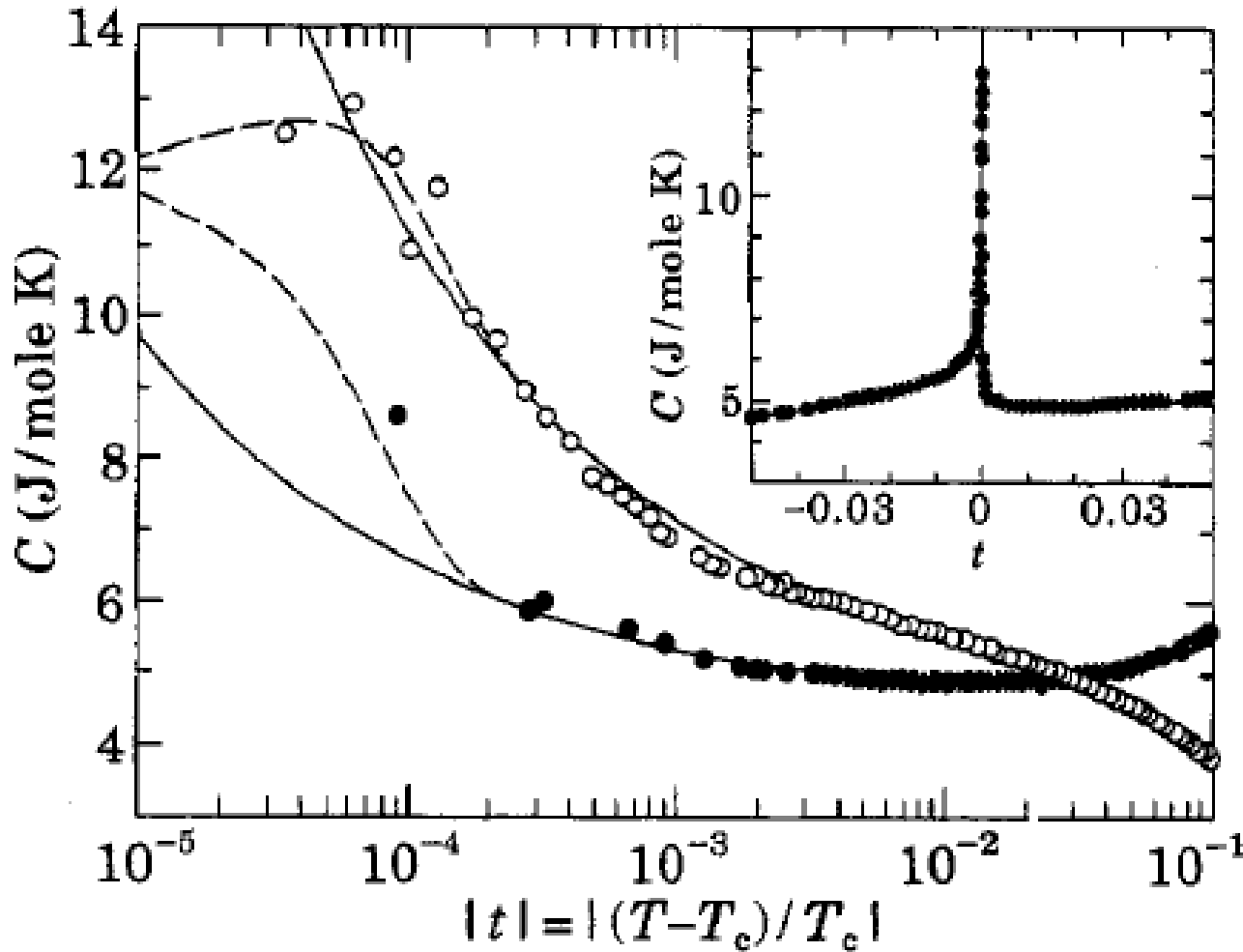
$B = 2.3 \text{ T}: \alpha = 0.25$

$B = 6 \text{ T}: \alpha = 0.37$

Quasi-one-dimensional Ising antiferromagnet: CsMnBr₃

Easy-plane system

Specific heat



Large critical exponents α suggest the possibility of new universality classes due to the discrete Z_2 chiral symmetry being broken at the magnetic transition

*R. Deutschmann et al.,
EPL 17, 637 (1992)*

Best fit for the specific-heat exponent yields $\alpha = 0.4$, Gauss broadening $\Delta T_c/T_c = 5 \cdot 10^{-5}$

Dynamic critical behavior

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See standard textbooks on Statistical Mechanics,

Specifically: P. C. Hohenberg and B. I. Halperin,
Theory of dynamic critical phenomena,
Rev. Mod. Phys. **49**, 435 (1977)

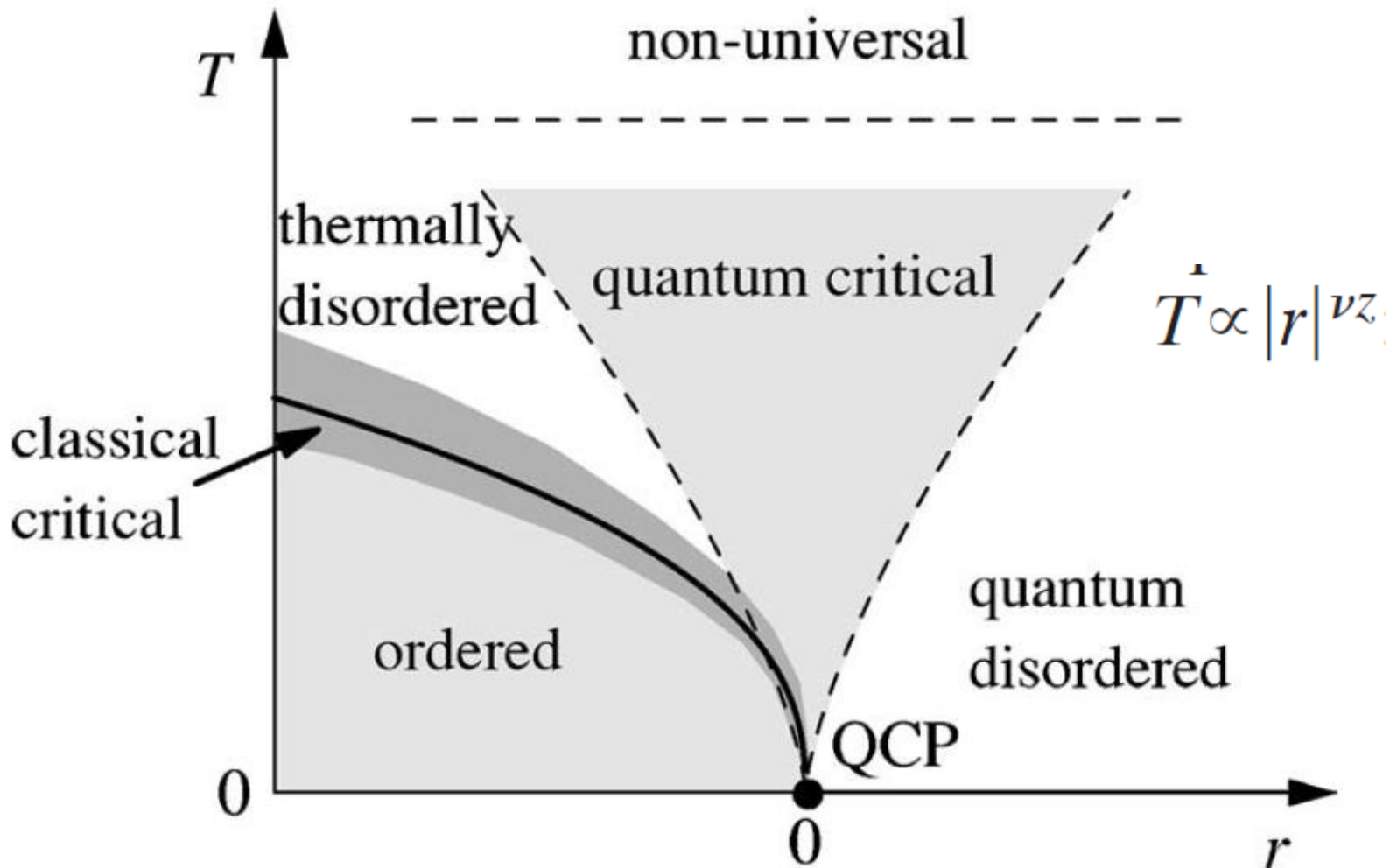
Dynamical models studied by RG models

Halperin and Hohenberg, RMP 435 (1977)

Model	Designation	System	Dimension order of parameter	Non-conserved fields	Conserved fields	Non-vanishing Poisson bracket
Relaxational	A	Kinetic Ising anisotropic magnets	n	ψ	None	None
	B	Kinetic Ising uniaxial ferromagnet	n	None	ψ	None
	C	Anisotropic magnets structural transition	n	ψ	m	None
Fluid	H	Gas-liquid binary fluid	1	None	ψ, j	$\{\psi, j\}$
Symmetric planar magnet	E	Easy-plane magnet, $h_z = 0$	2	ψ	m	$\{\psi, m\}$
Asymmetric planar magnet	F	Easy-plane magnet, $h_z \neq 0$ superfluid helium	2	ψ	m	$\{\psi, m\}$
Isotropic antiferromagnet	G	Heisenberg antiferromagnet	3	ψ	m	$\{\psi, m\}$
Isotropic ferromagnet	J	Heisenberg ferromagnet	3	None	ψ	$\{\psi, \psi\}$

Classical vs. quantum phase transitions

Generic phase diagram near a continuous quantum phase transition



Spin-fluctuation scenario of a quantum critical point

Hertz, Millis, Moriya, Lonzarich, Rosch

		specific-heat coefficient	electrical resistivity
		C/T	$\Delta\rho$
$d = 3$	FM ($z = 3$)	$\sim \ln (T_0/T)$	$\sim T^{5/3}$
	AF ($z = 2$)	$\sim 1 - a\sqrt{T}$	$\sim T^{3/2}$ (dirty limit)
$d = 2$	FM ($z = 3$)	$\sim T^{-1/3}$	$\sim T^{3/2}$
	AF ($z = 2$)	$\sim \ln (T_0/T)$	$\sim T$

CeCu_{6-x}Au_x: quasi-twodimensional fluctuations?
 \Rightarrow inelastic neutron scattering

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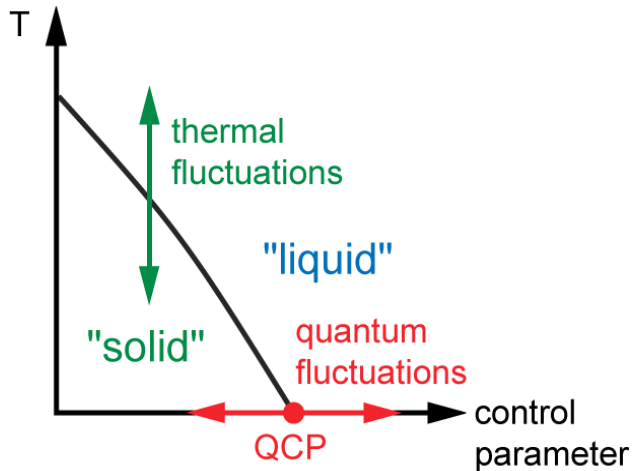
See: P. C. Hohenberg and B. I. Halperin,
Theory of dynamic critical phenomena,
Rev. Mod. Phys. **49**, 435 (1977), in particular p. 472-573

H. v. Löhneysen, A. Rosch, M. Vojta, and P. Wölfle,
Fermi liquid instabilities at magnetic quantum phase transitions,
Rev. Mod. Phys. **79**, 1015 (2007) , In particular p.1027-1035

Different types of quantum critical points

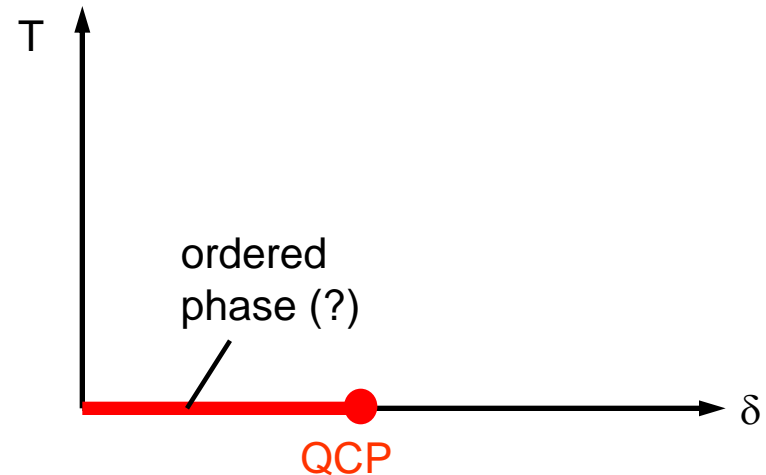
Scenarios for quantum phase transitions

Transition at finite temperature driven to absolute zero by control parameter δ



Ginzburg-Landau theory for (magnetic) quantum phase transition in metals: but experiments differ strongly!

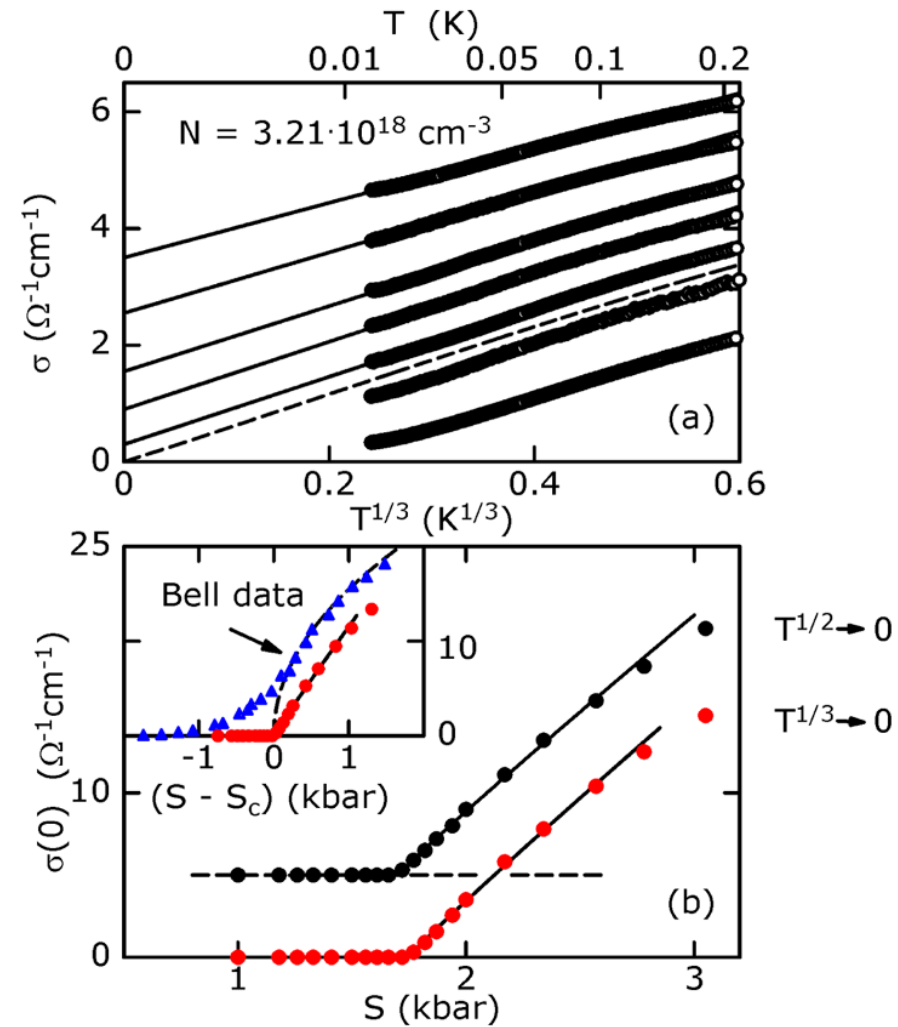
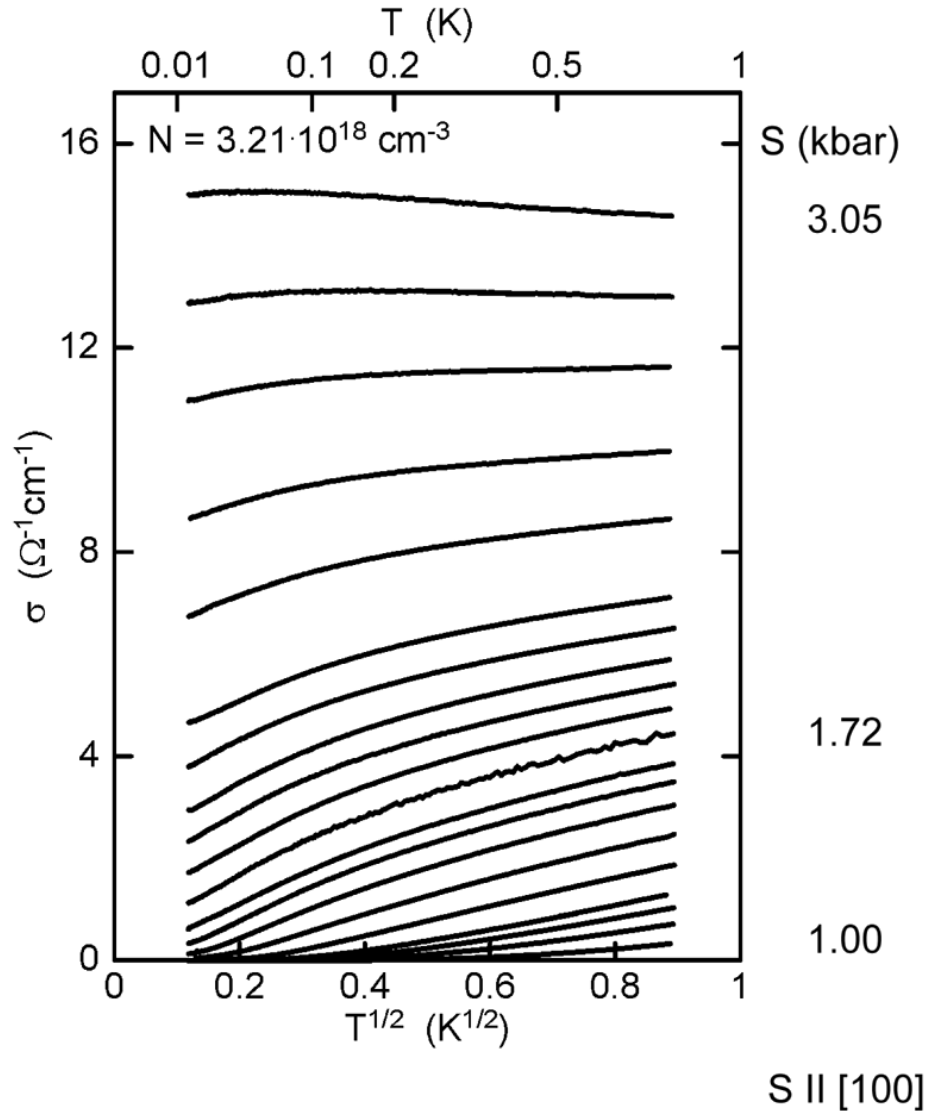
No finite-temperature transition, critical line of zero-temperature transitions terminating in QCP



Metal-insulator transition in the presence of disorder and interactions: what is the order parameter?

Stress-tuning of the metal-insulator transition in Si:P

S. Waffenschmidt et al., PRL 83, 3005 (1999)



Dynamic scaling of the conductivity at the metal-insulator transition of P-doped silicon

S. Waffenschmidt et al., PRL 83, 3005 (1999)

$$\sigma(S, T) = \sigma_c(T) \cdot F \left(\frac{|S - S_c|}{T^y} \right)$$

$$\sigma_c(T) = \sigma(S_c, T) = aT^x(1+dT^w)$$

$$x = \frac{\mu}{\nu z}, \quad y = \frac{1}{\nu z}$$

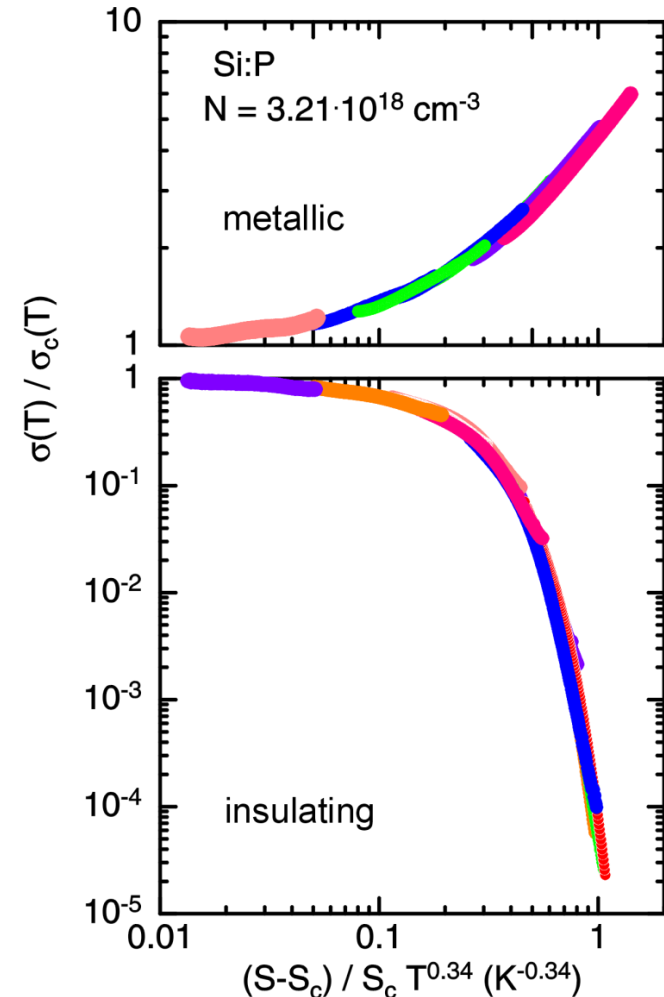
from $\sigma(S, T \rightarrow 0) \sim |S - S_c|^\mu$: $\mu = 1$

Wegner scaling: $\mu = (d-2)\nu$

dynamical exponent z : $\xi_t \sim \xi^z$

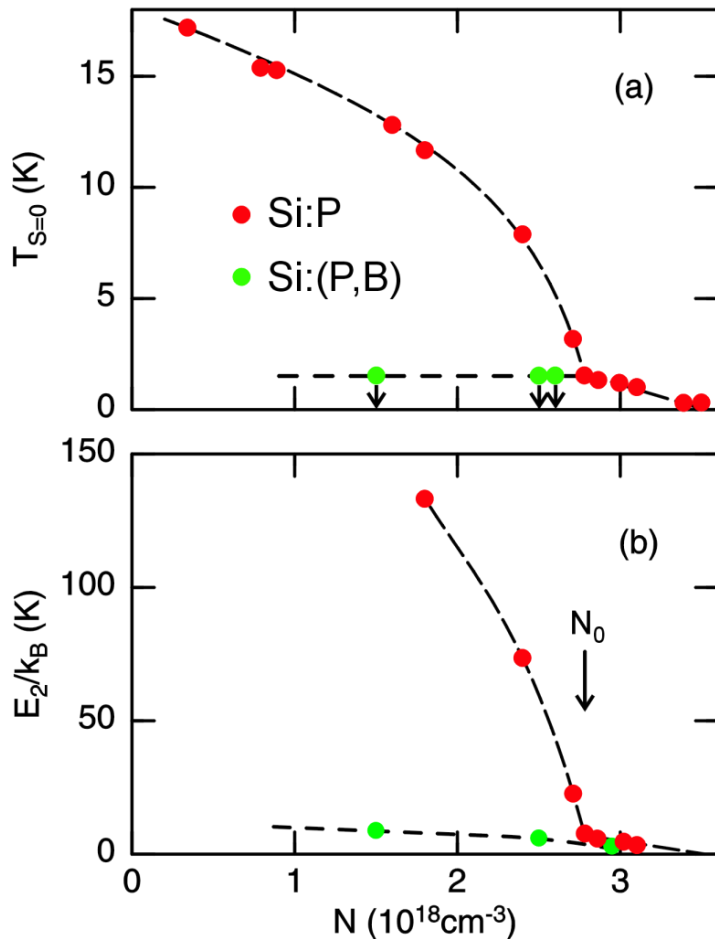
expt. scaling with $x = 1/3$ and $y = 0.34$

$$\mu = 1, \quad z = 3$$

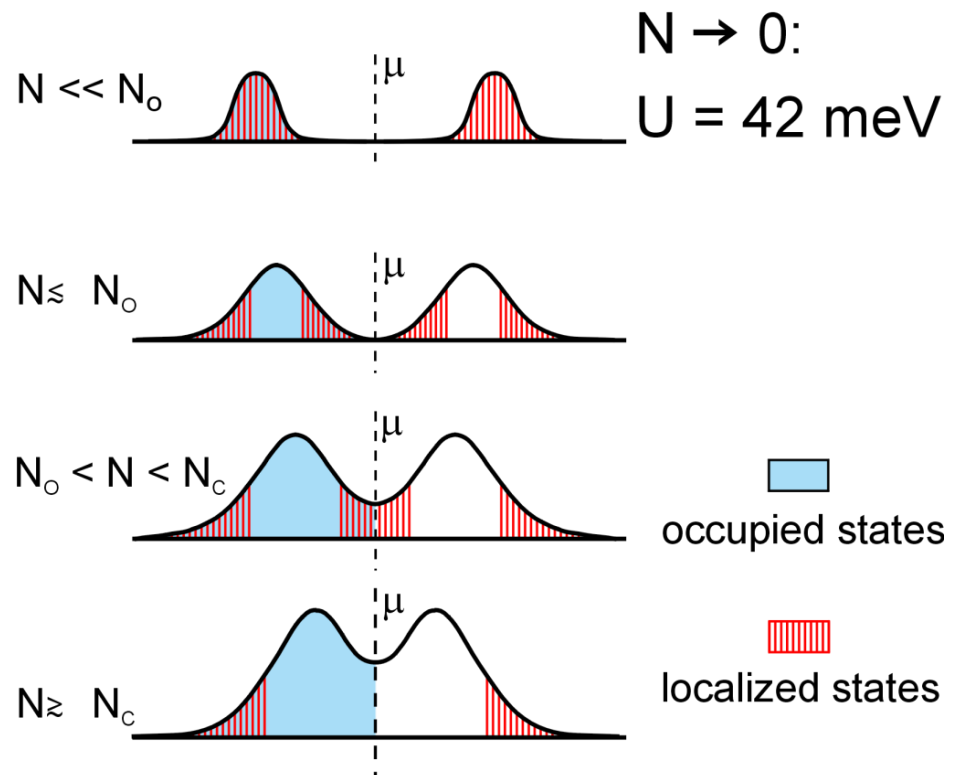


Hubbard splitting of the impurity band in Si:P

Simple activated conductance below N_0 in uncompensated Si:P only



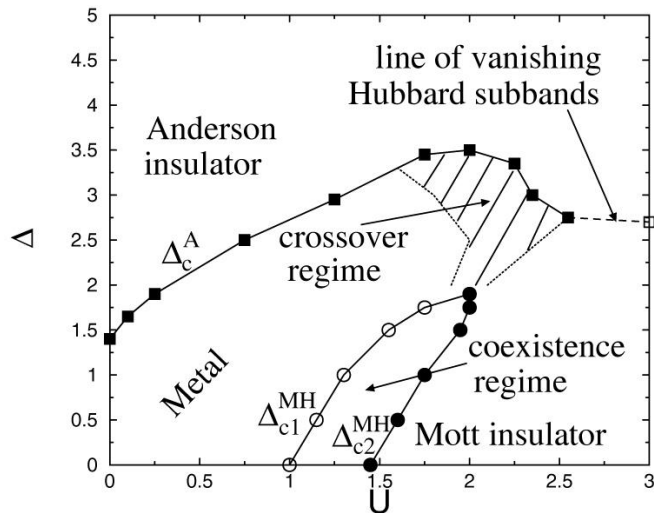
1s(A_1) band at half-filling



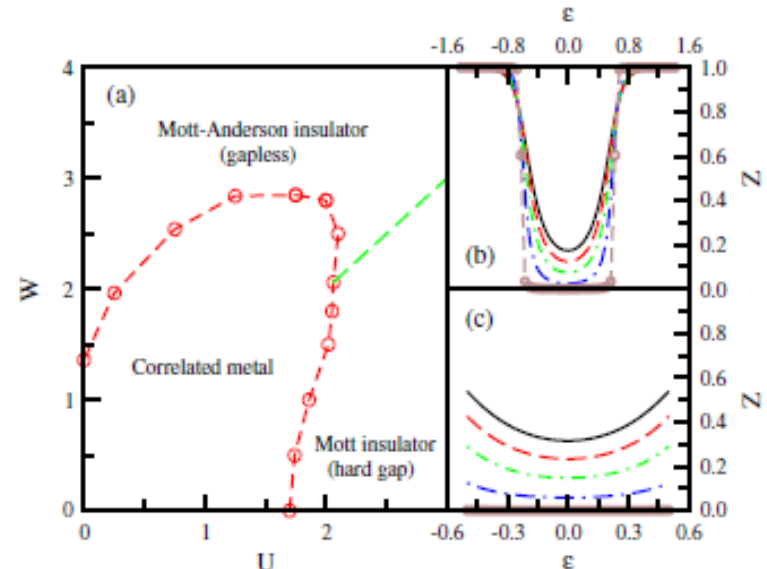
X. Liu et al., Phys.Rev. Lett. **77**, 3395 (1996)
HvL, Philos. Trans. Roy. Soc. (London) **A356**, 139 (1998);
Ann. Phys. (Leipzig) **523**, 599 (2011)

Interplay of disorder and on-site interactions

Solution of Anderson-Hubbard model (TMF-DMFT),
on-site interaction favors delocalization



Byczuk et al., PRL 94, 056404 (2005)



Aguiar et al., PRL 102, 156402 (2009)

Role of long-range Coulomb interaction?

Coulomb interaction between carriers

$$\nu \approx 1.3$$

Harashima and Slevin, Phys. Rev. B 89, 205108 (2014)

Critical endpoint driven to $T = 0$ by a nonthermal parameter

