

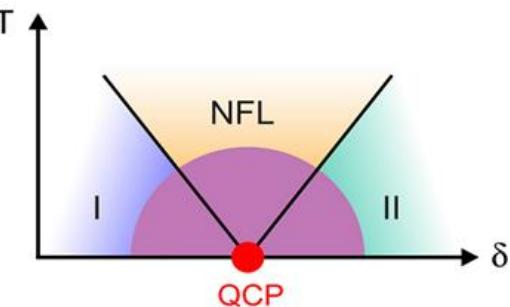
# Beyond Landau Fermi liquid and BCS superconductivity near quantum criticality

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Karlsruhe Institute of Technology*

Lecture Series at APCTP, Pohang

May 23-27, 2016



# Lecture 2

## Fermi liquids, Kondo effect and heavy fermions

### **Contents of Lecture 2**

Landau Fermi liquids

Kondo effect: Concept of a local Fermi liquid

Heavy-fermion systems

Quantum phase transitions in heavy fermion systems

# Landau Fermi liquids

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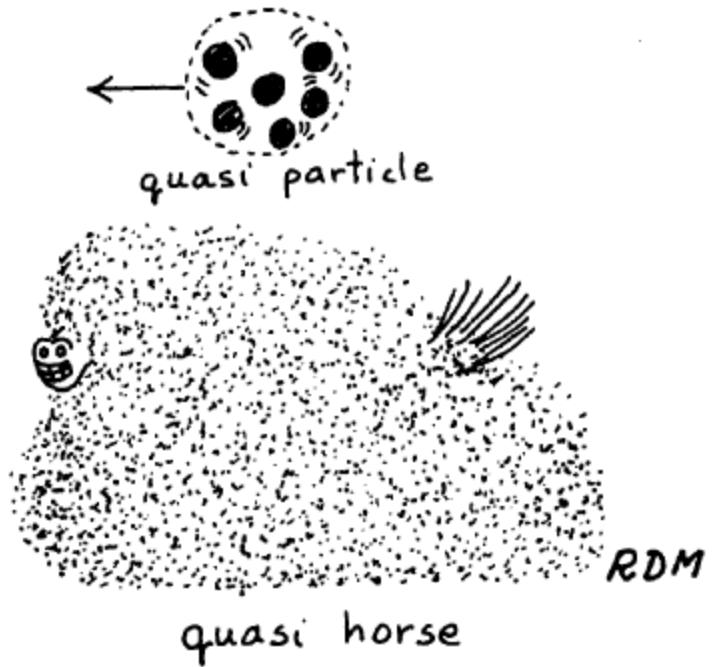
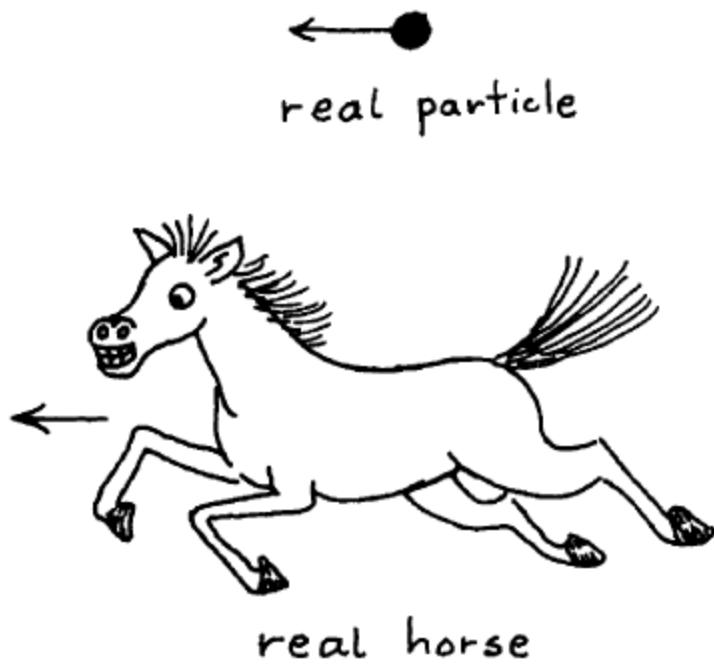
See standard textbooks on interacting electron systems, e. g.,

G.Baym, C, Pethick, *Landau Fermi-liquid Theory: Concepts and applications*, Wiley 1991 (also available online)

See also: H. v. Löhneysen, A. Rosch, M. Vojta, and P. Wölfle, *Fermi liquid instabilities at magnetic quantum phase transitions*, Rev. Mod. Phys. **79**, 1015 (2007) , In particular p.1018-1027

See also N. W. Ashcroft, N. D. Mermin, *Solid State Physics*, Saunders College Publishing1976, in particular ch. 17

# Noninteracting independent free particles and quasiparticles



R. D. Mattuck, in: *A guide to Feynman diagrams in the many-body problem*,  
McGraw-Hill 1967;2nd edition, Dover 1992

# Kondo effect: concept of a local Fermi liquid

# THE ELECTRICAL RESISTANCE OF GOLD BELOW 1°K

by W. J. DE HAAS, H. B. G. CASIMIR and G. J. VAN DEN BERG

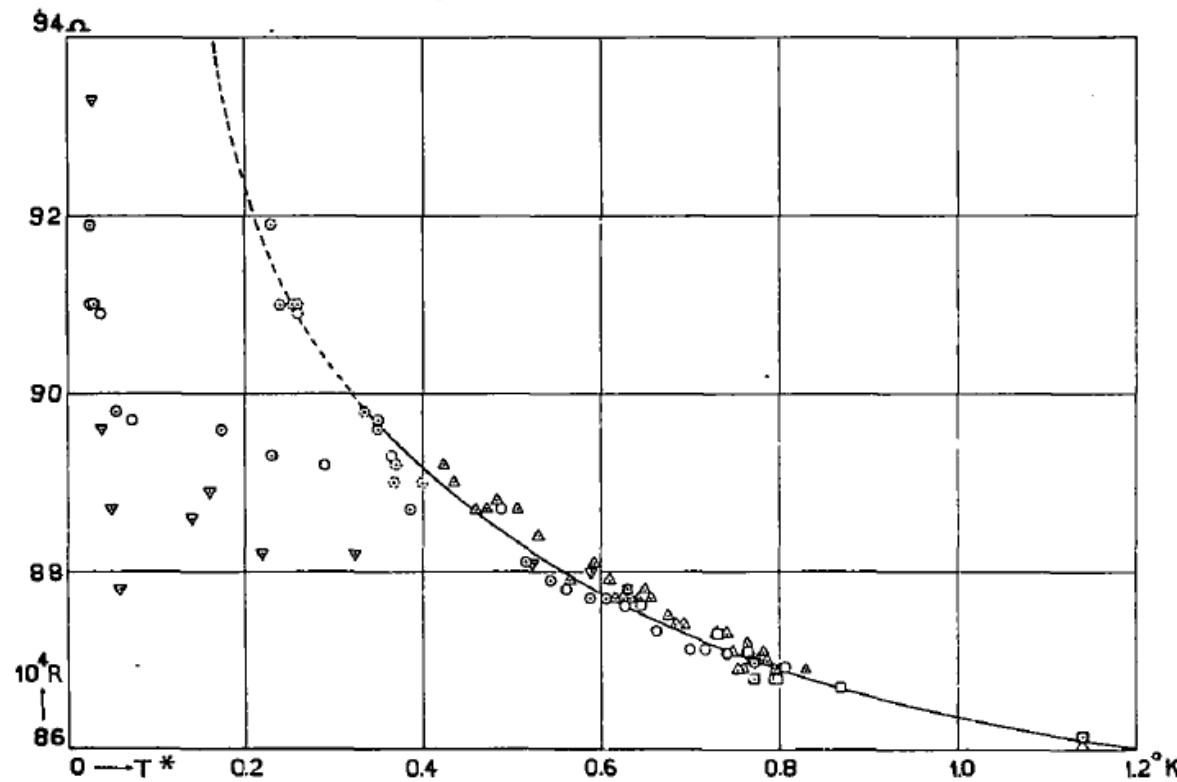
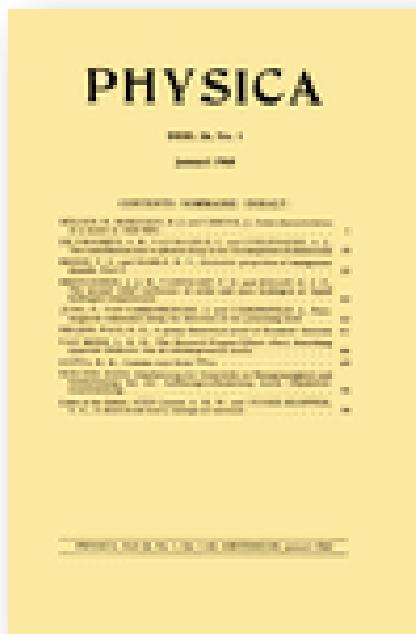
Communication No. 251c from the Kamerlingh Onnes Laboratory at Leiden

Dedicated to Professor Max Planck on the occasion  
of his eightieth birthday

## Summary

The resistance of gold was determined at temperatures below 1°K, obtained by adiabatic demagnetization of iron-ammonium-alum. The increase observed at ordinary liquid helium temperatures is much more pronounced below 1°K and our results suggest, that the resistance may become infinite at the absolute zero-point.

Physica 5, 225 (1938)



# J. Kondo 1964: explanation of the resistance minimum in metals

Progress of Theoretical Physics, Vol. 32, No. 1, July 1964

## Resistance Minimum in Dilute Magnetic Alloys

Jun KONDO

*Electro-technical Laboratory  
Nagatacho, Chiyodaku, Tokyo*

(Received March 19, 1964)

Based on the  $s$ - $d$  interaction model for dilute magnetic alloys we have calculated the scattering probability of the conduction electrons to the second Born approximation. Because of the dynamical character of the localized spin system, the Pauli principle should be taken into account in the intermediate states of the second order terms. Thus the effect of the Fermi sphere is involved in the scattering probability and gives rise to a singular term in the resistivity which involves  $c \log T$  as a factor, where  $c$  is the concentration of impurity atoms. When combined

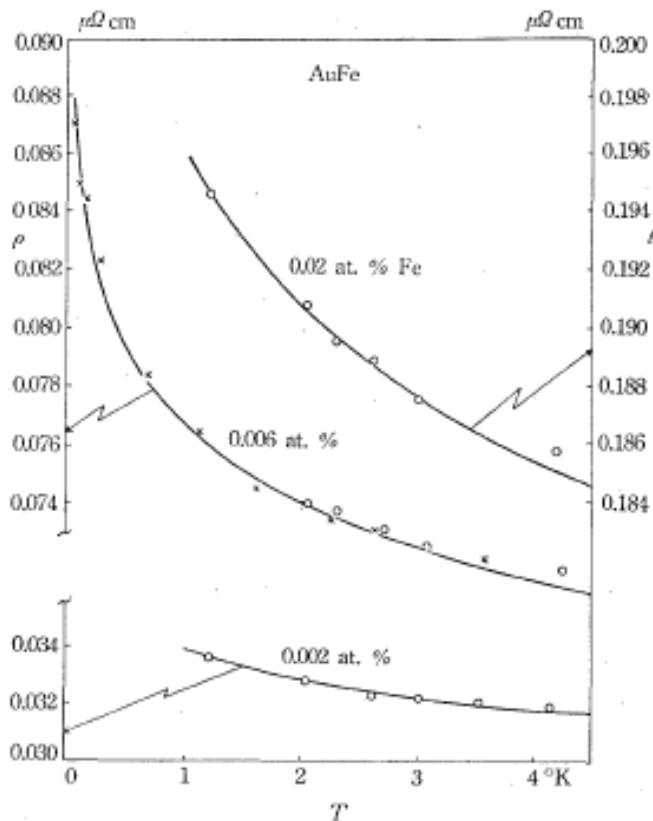


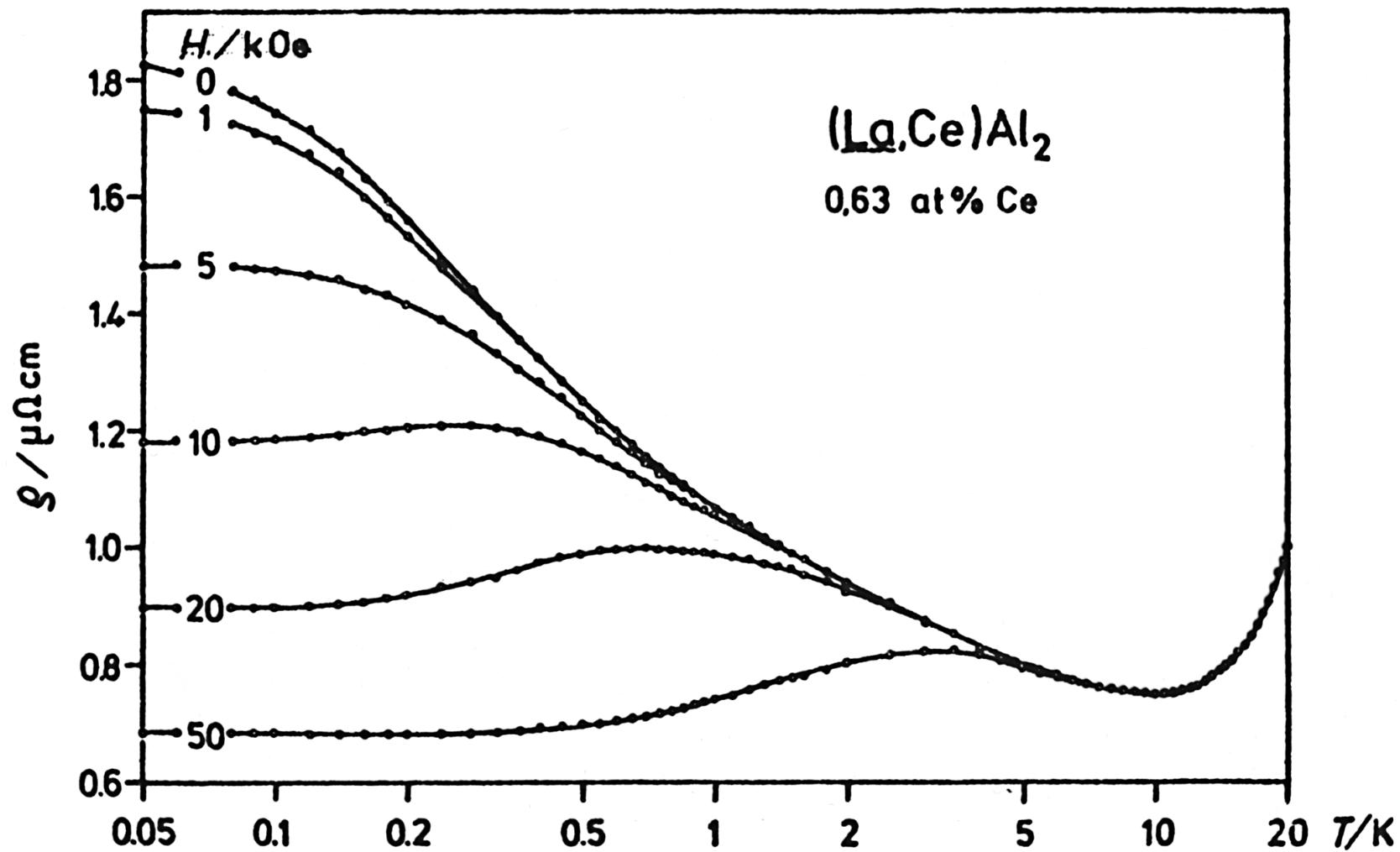
Fig. 1. Comparison of experimental and theoretical  $\rho$ - $T$  curves for dilute AuFe alloys.

## Kondo Hamiltonian

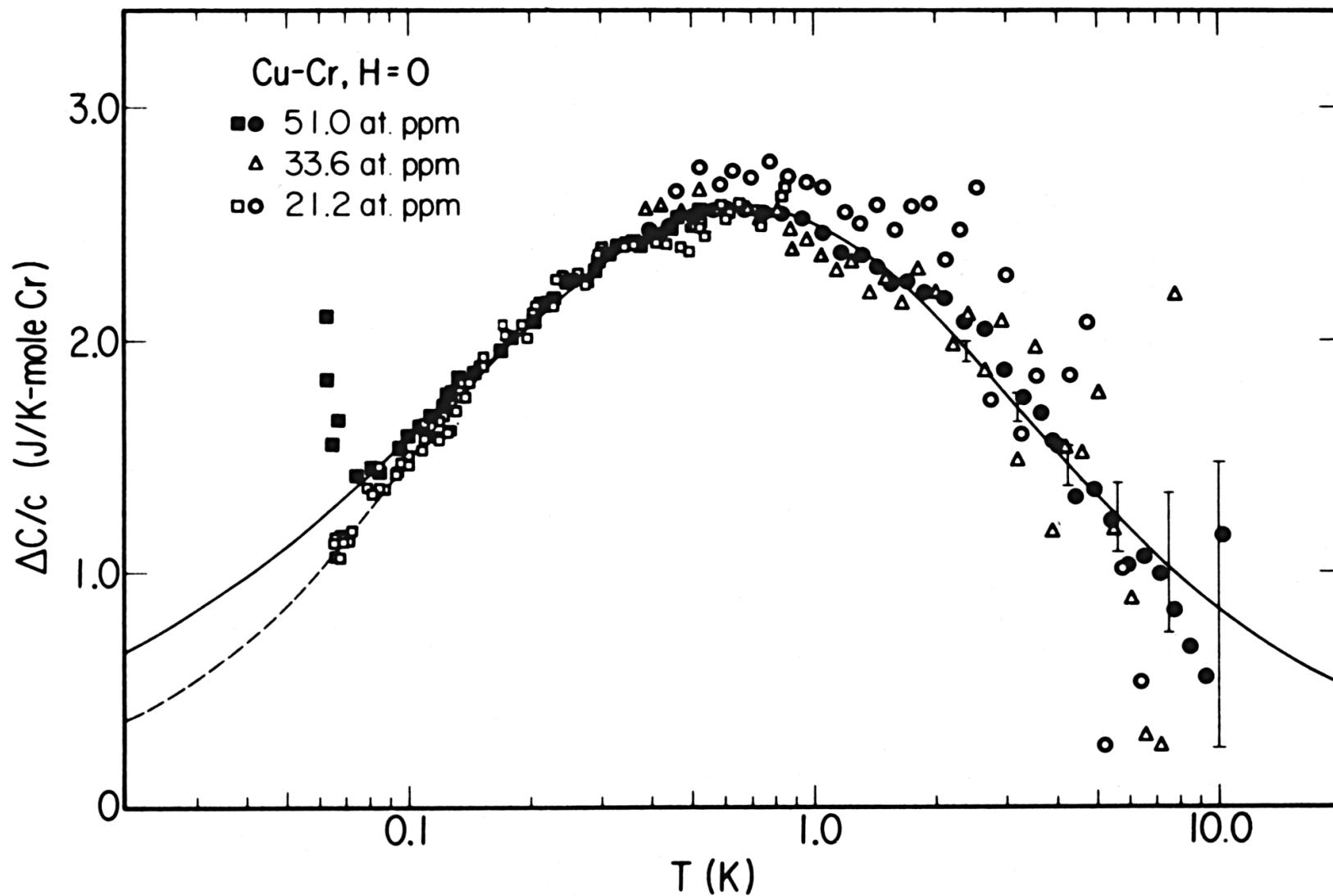
$$\mathcal{H} = -2J \mathbf{S} \cdot \mathbf{s}$$



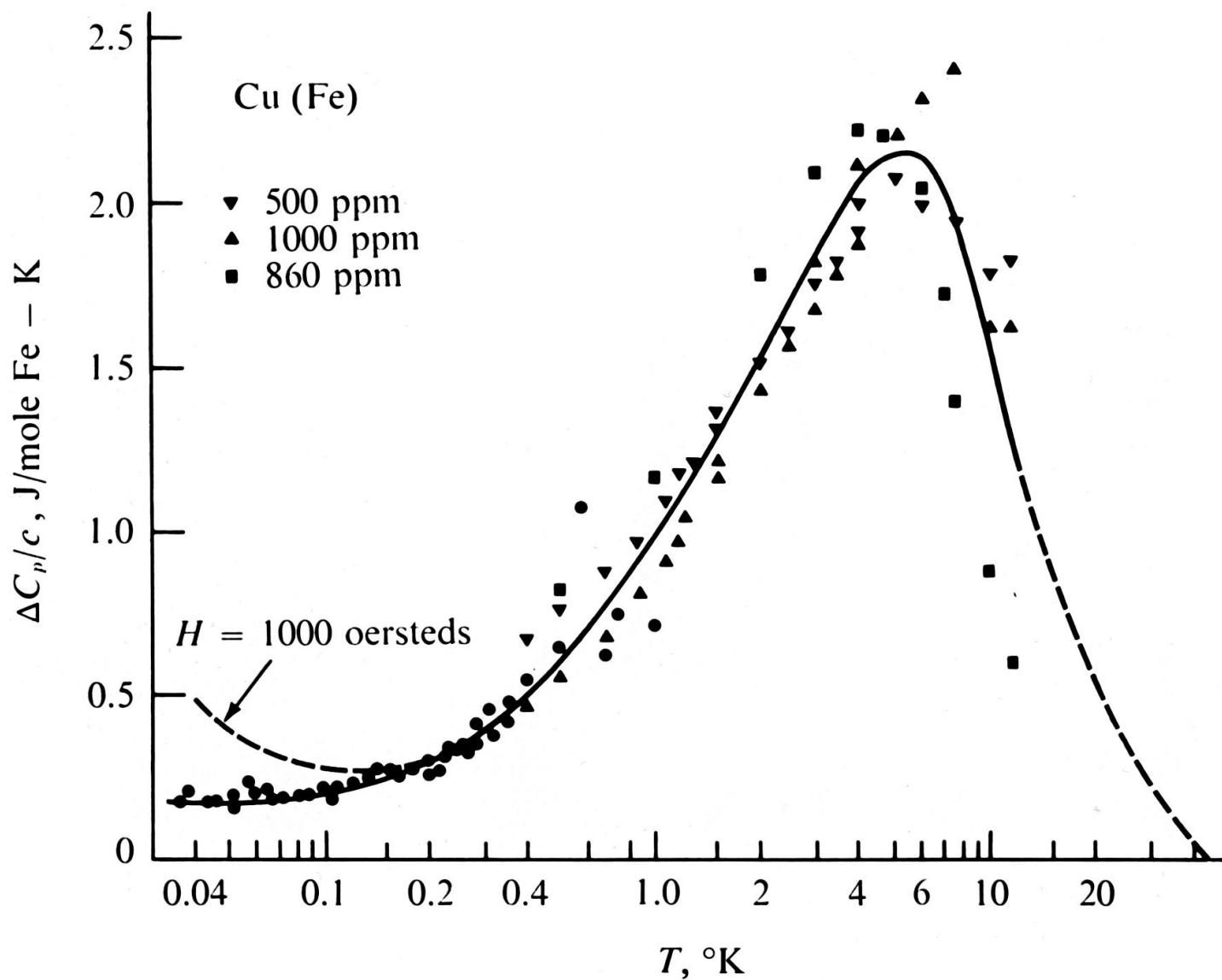
# Electrical resistivity of $(\text{La,Ce})\text{Al}_2$



# Kondo anomaly of the specific heat of Cu-Cr

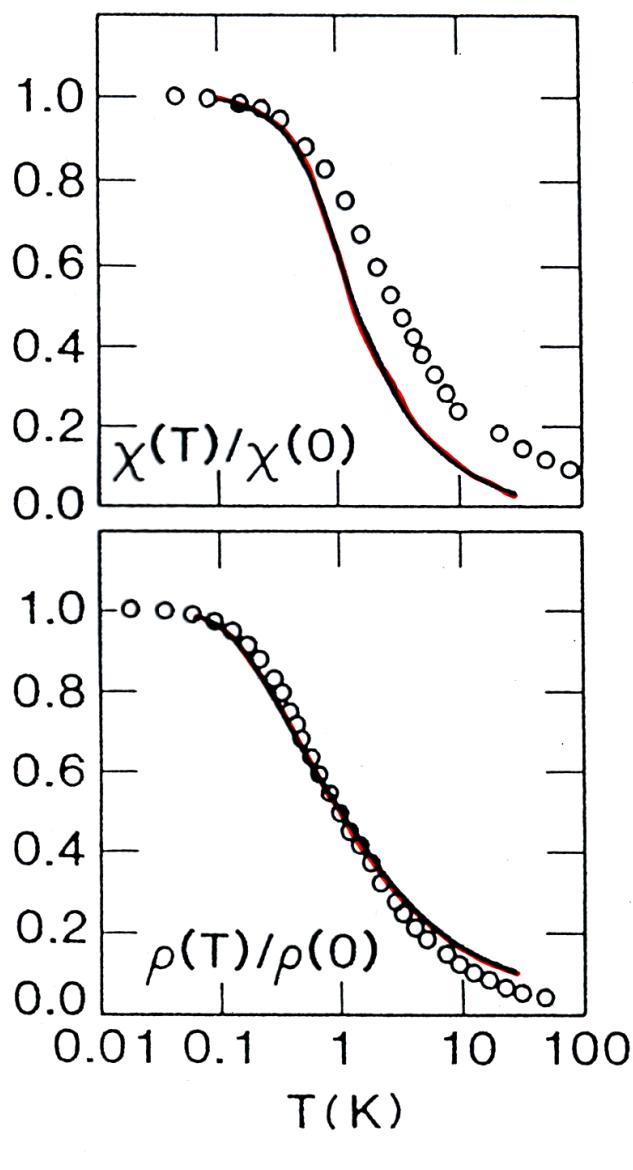


# Kondo anomaly of the specific heat of Cu-Fe

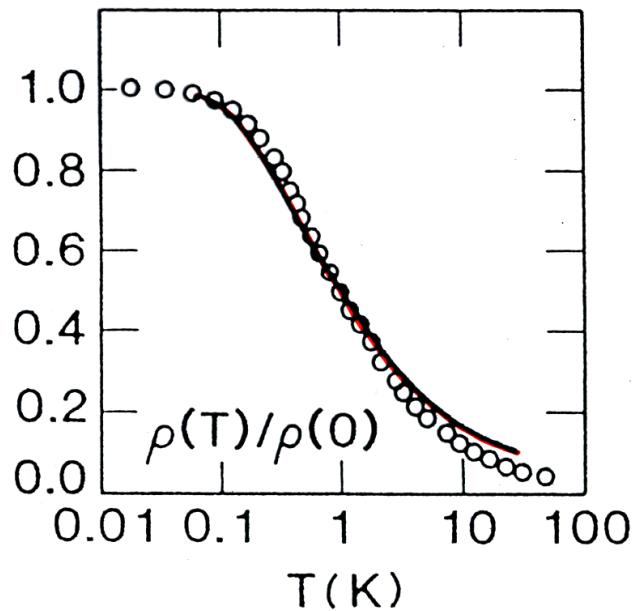


# Kondo anomalies of $(\text{La}, \text{Ce})\text{B}_6$

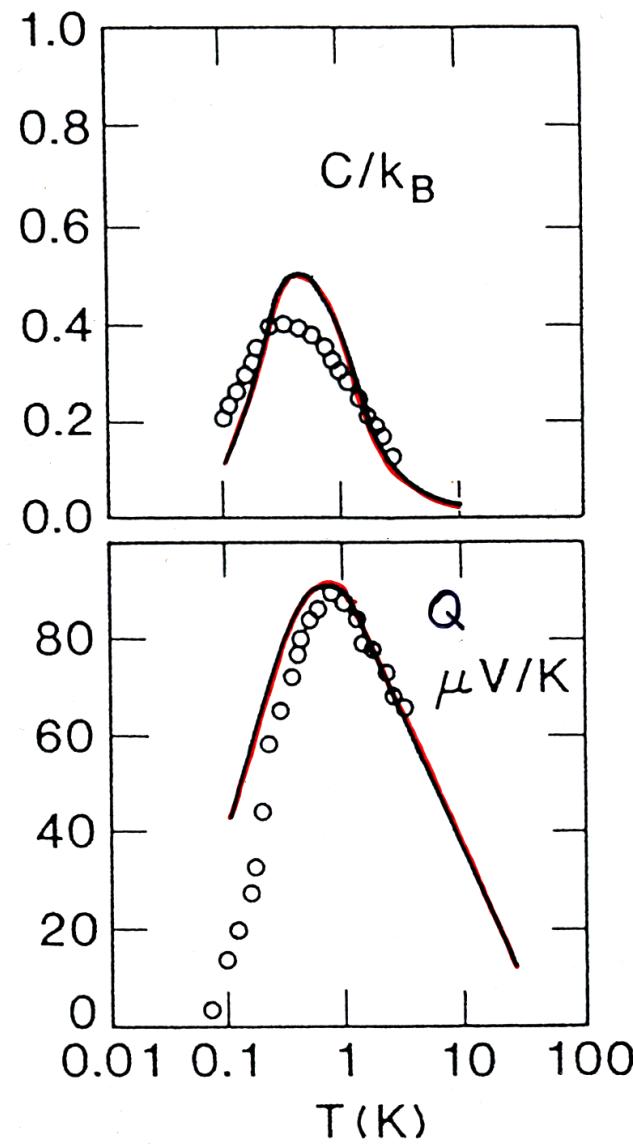
Magnetic susceptibility



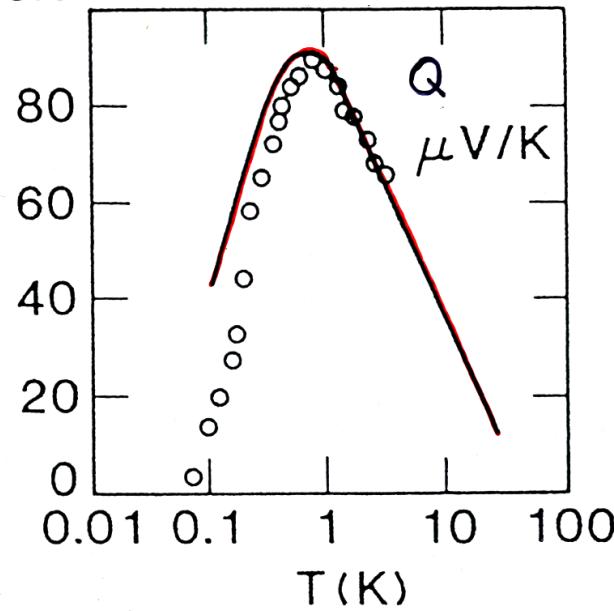
Electrical resistivity



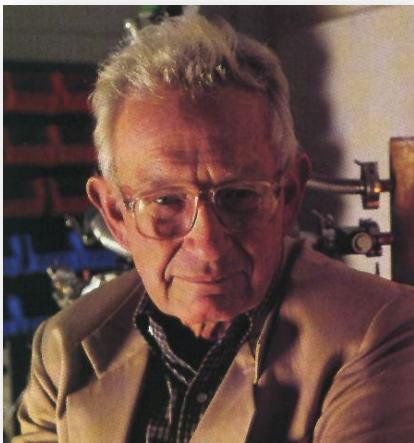
Specific heat



Thermo-electric power

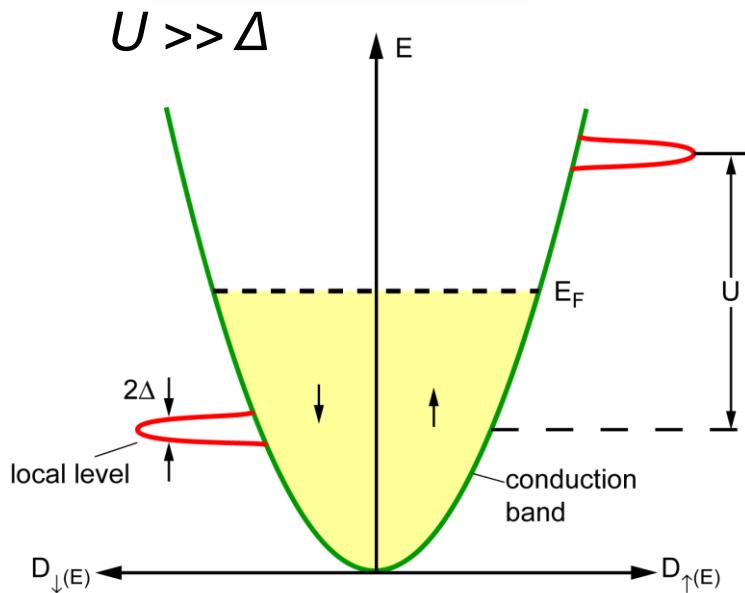


# Anderson model for a magnetic impurity in a metal

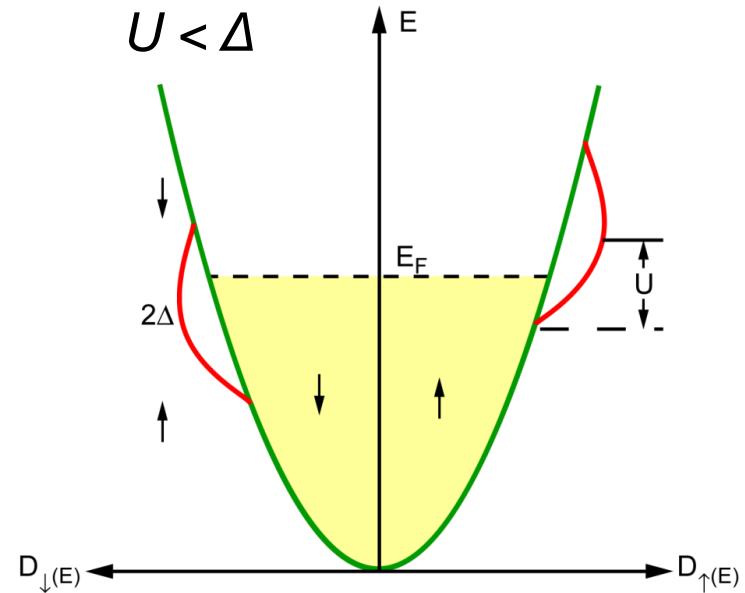


P.W. Anderson, Phys. Rev. 124 (1961)

Two important parameters:  
hybridization with conduction electrons:  
⇒ broadening  $\Delta$  of local level  
on-site Coulomb repulsion  $U$  on local level  
⇒ tendency to single occupancy, i. e., local moment

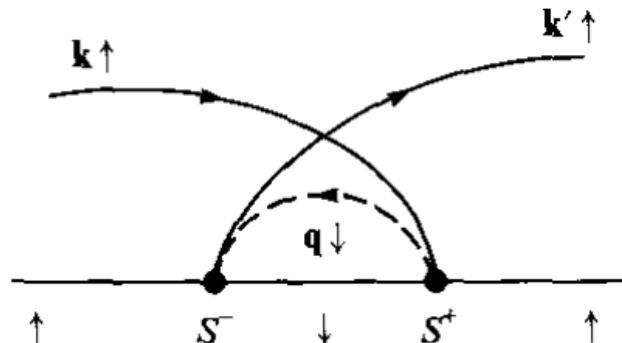
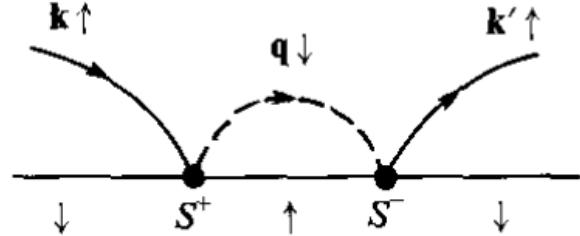


stable magnetic moment

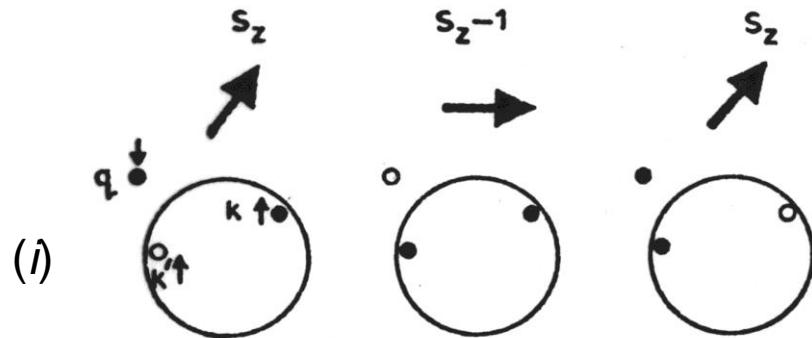


local magnetic moment „lost“

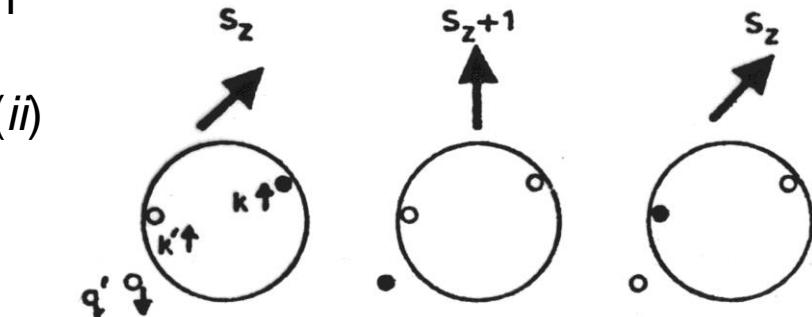
# Second-order scattering processes of conduction electrons by magnetic impurities (Kondo scattering)



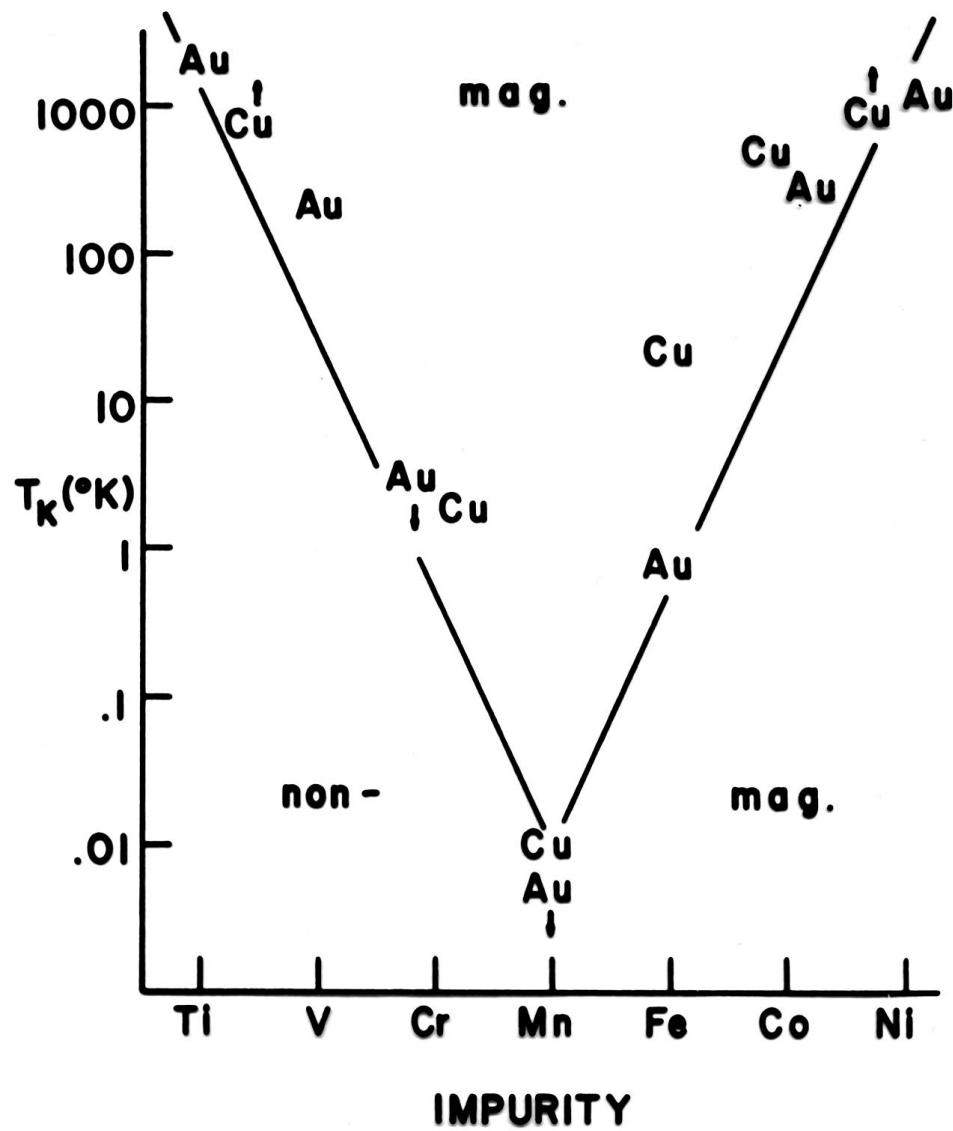
Intermediate state involving a conduction-band particle (*i*) or hole (*ii*)



Illustration



# Kondo temperature $T_K$ of 3d transition-metal impurities in noble-metal hosts



# Relation between Anderson and Kondo models

Schrieffer-Wolff transformation (1966) maps Anderson impurity Hamiltonian onto an effective spin Hamiltonian

$$\mathcal{H} = -2J \mathbf{S} \cdot \mathbf{s} \quad \text{with } J = J_0 + J_1$$

$J_0$  : Heisenberg exchange integral between  
localized and conduction electrons  $> 0$

$$J_1 = \langle V_{lc} \rangle^2 U / (E_l - E_F) (E_l - E_F + U) < 0$$

if hybridization sufficiently strong, then  $|J_0| < |J_1|$   
 $\Rightarrow$  antiferromagnetic exchange  $J < 0$  (spin-singlet formation)

Note: in rare-earth metals angular momentum not quenched,  
e.g.,  $\text{Ce}^{3+}$   $4f^1$  configuration:  ${}^2F_{5/2}$  Hund's rule ground state,  
six-fold degeneracy lifted by crystal field

Modified Kondo Hamiltonian

*B. Coqblin, J. R. Schrieffer, Phys. Rev. **185**, 847 (1969)*

# Origin of heavy masses $m \approx 100 m_0$ in rare-earth alloys: Kondo effect

Two “ingredients“ for Kondo effect of isolated 4f impurities in metals

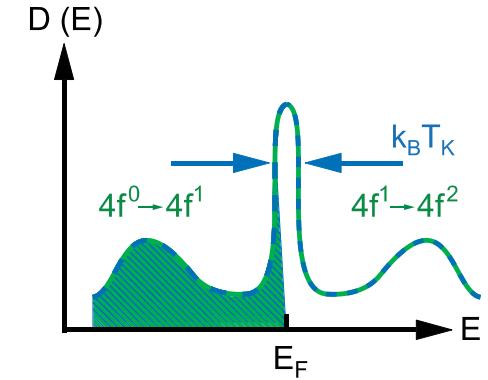
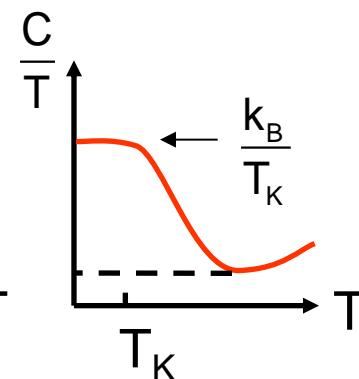
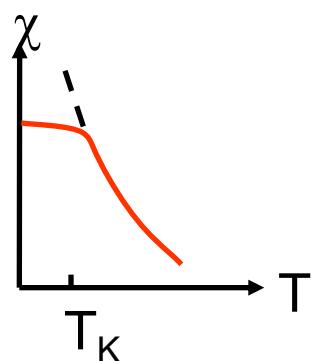
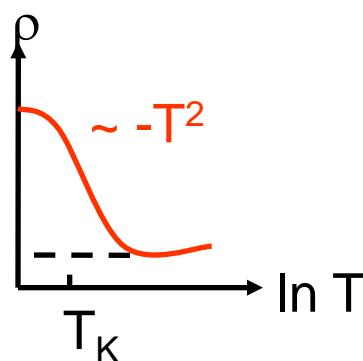
- hybridization of 4f and conduction electrons
- strong on-site electron repulsion in 4f state

singly occupied lowest 4f state will be screened by conduction electrons:  
singlet formation

Resonance at  $E_F$  due to virtual excitations from 4f state to  $E_F$

Kondo resonance

“Kondo anomalies“ at low  $T$



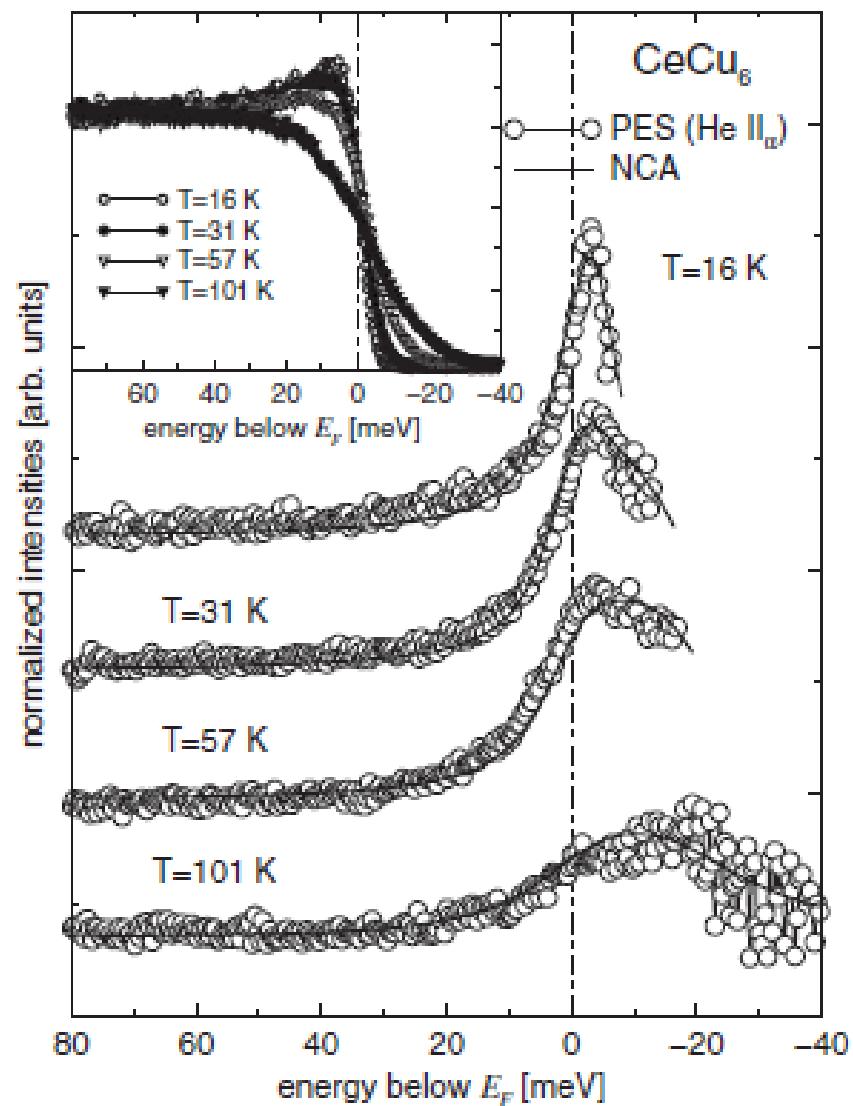
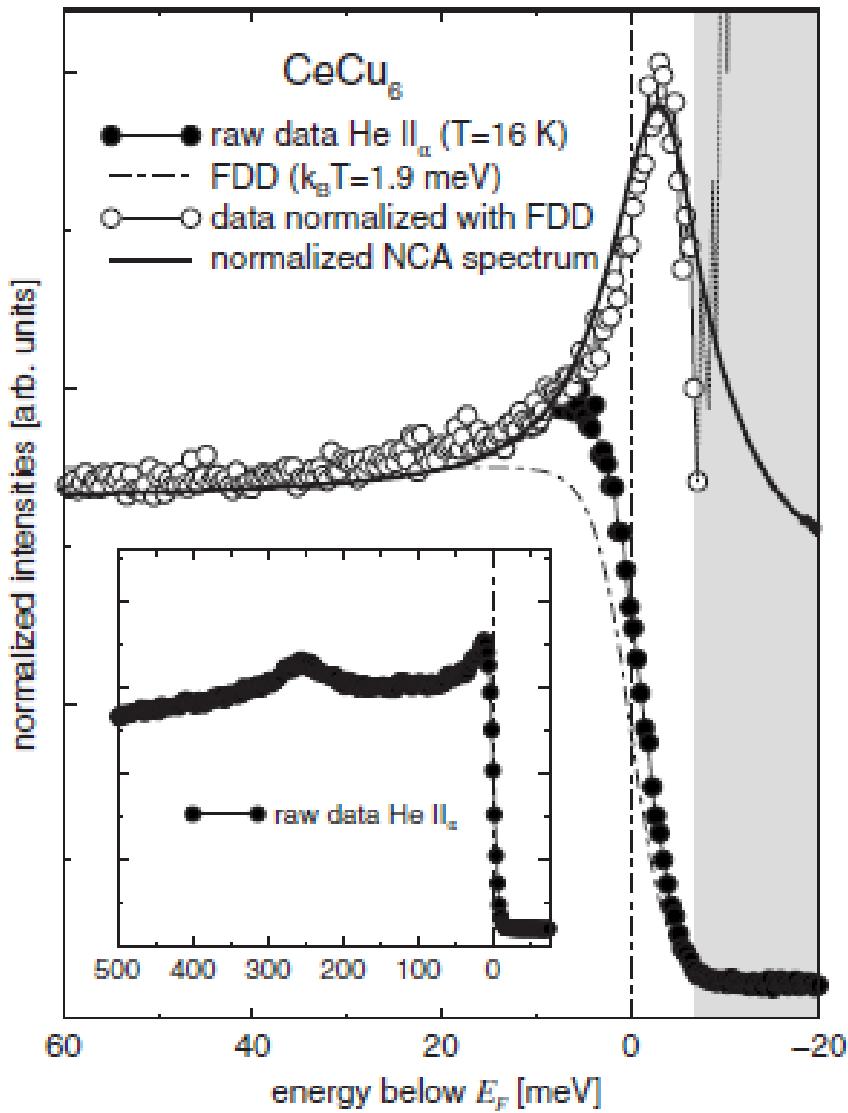
“local Fermi liquid“

Heavy-fermion system: lattice-coherent superposition of Kondo anomalies

# Heavy-fermion systems

# Kondo resonance in the heavy-fermion system $\text{CeCu}_6$

D. Ehm et al., Phys. Rev. B **76**, 04511 (2007)

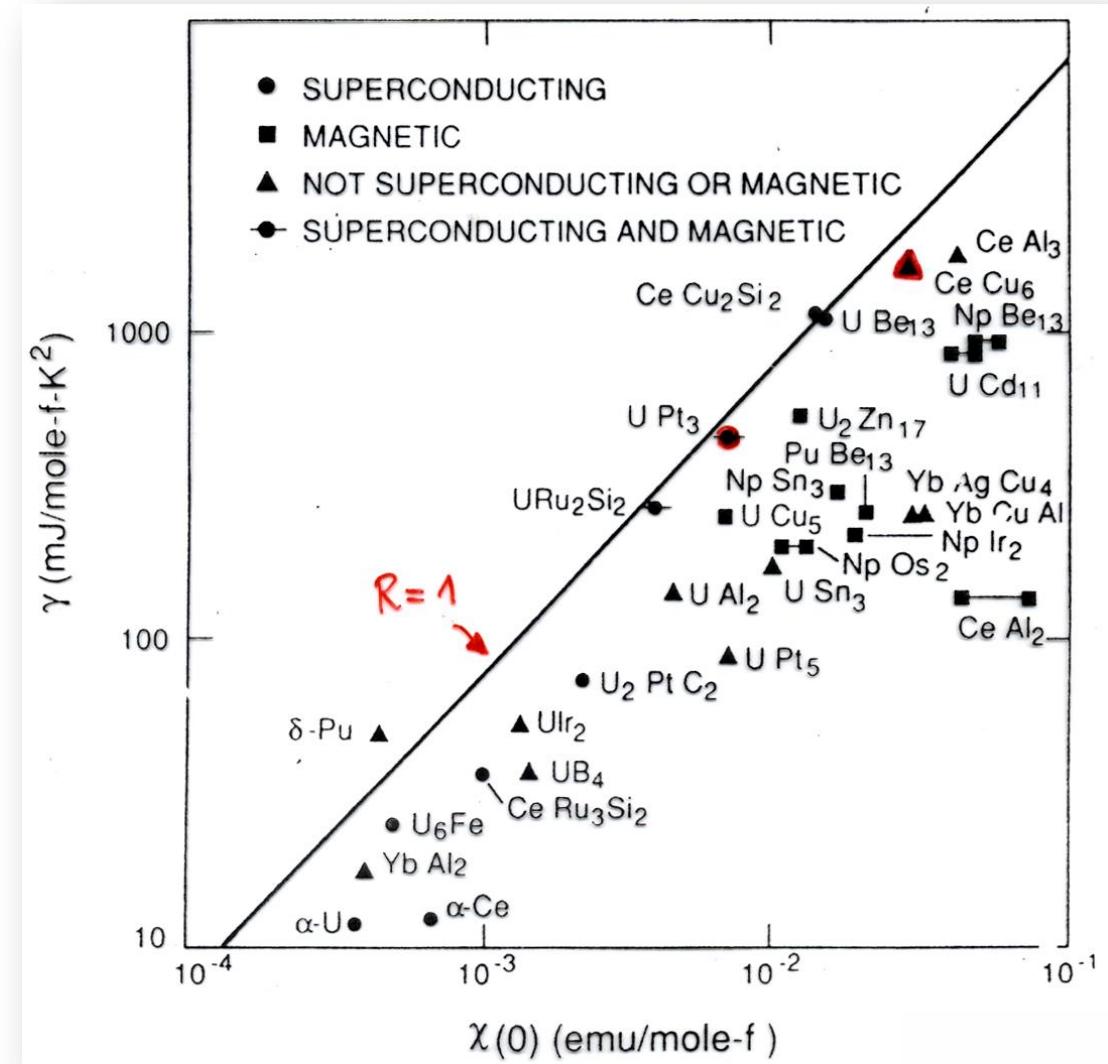


# Wilson relation between the specific-heat coefficient $\gamma$ and the Pauli susceptibility

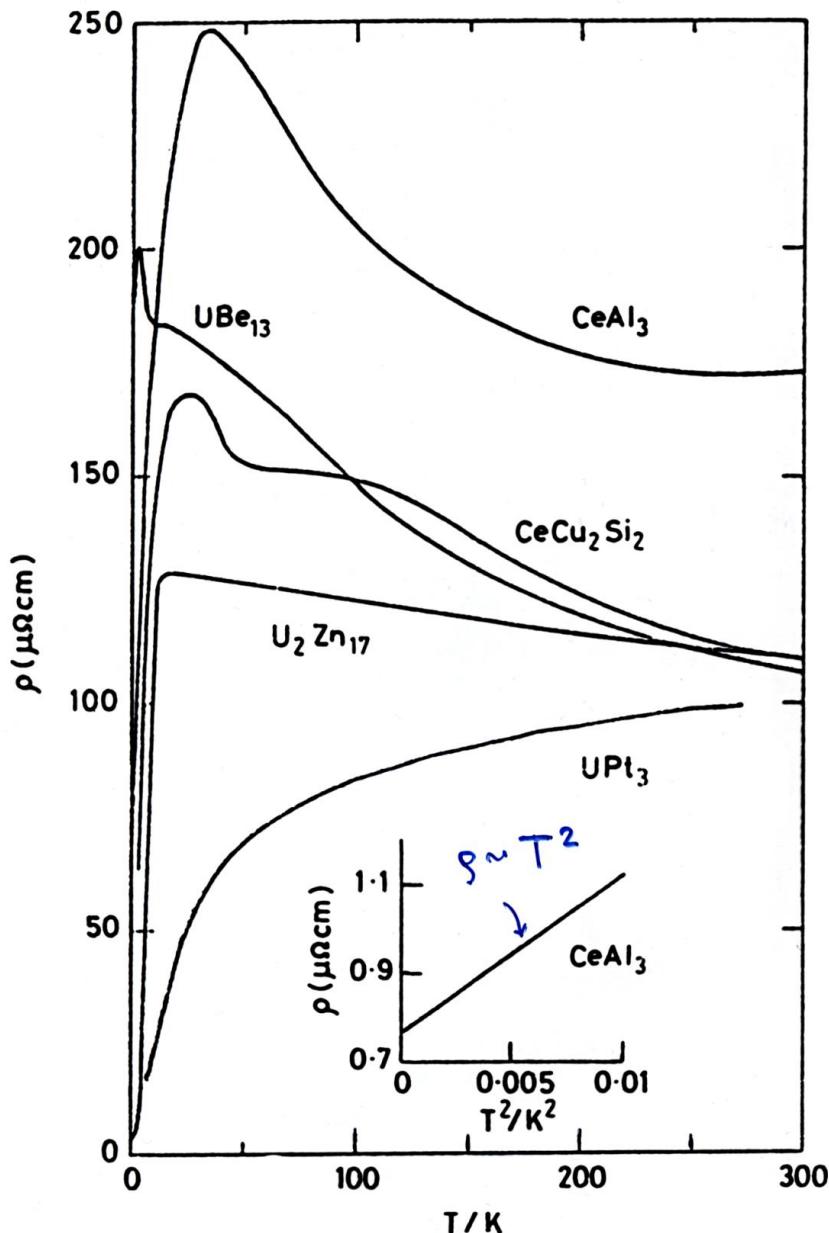
Z. Fisk et al.

Heavy-fermion systems:

$$F_0^a \sim -0.5$$



# Electrical resistivity of heavy-fermion systems



Kondo-like increase of  $\rho(T)$  toward low  $T$

Coherence maximum below  $T_K$ , cf. Bloch theorem

At lowest  $T$ ,  $\rho(T) = \rho + AT^2$ ,  
cf. Fermi liquid

# Kadowaki-Woods ratio of the specific-heat coefficient $\gamma$ and the coefficient $A$ of the $T^2$ resistivity

## Specific heat

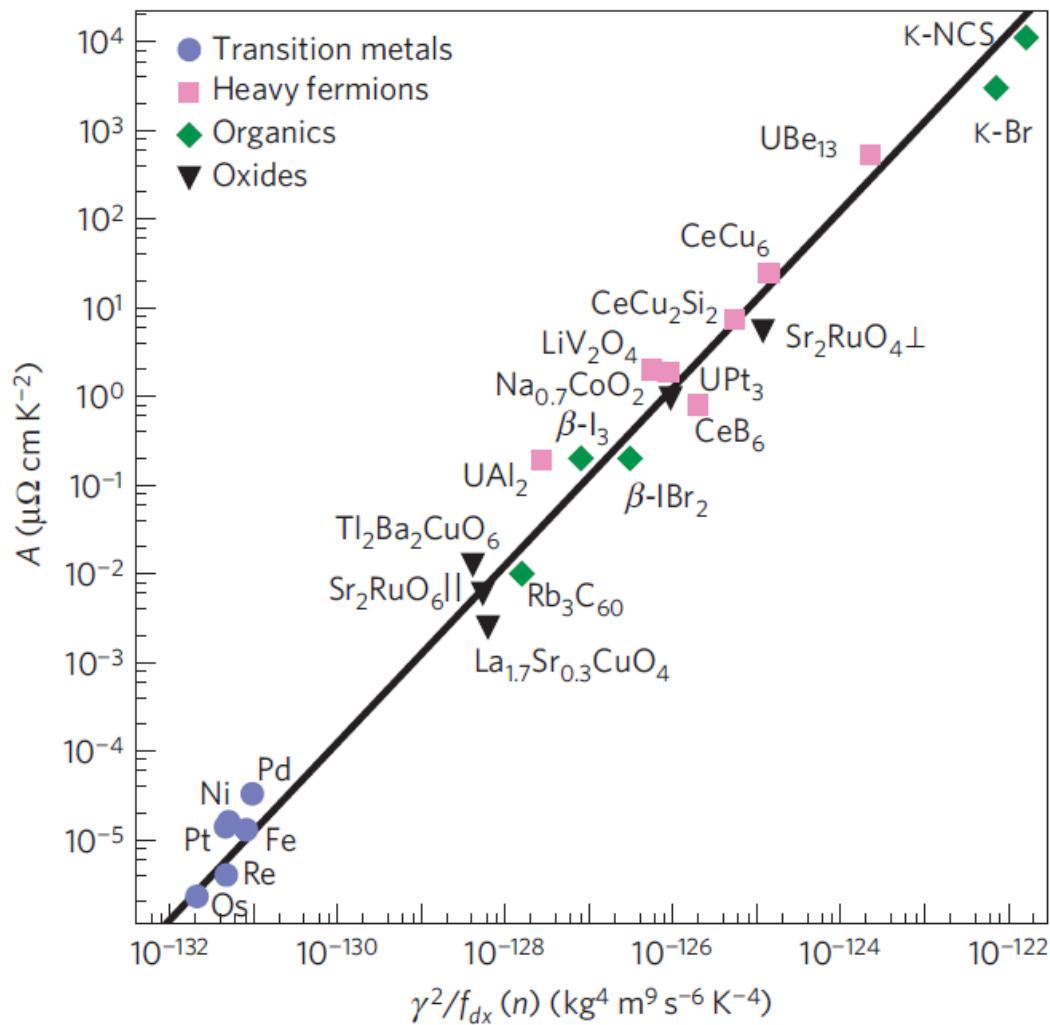
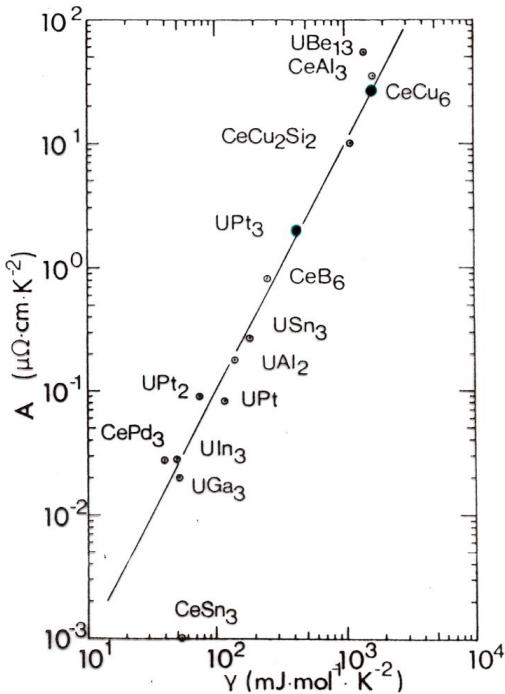
$$C = \gamma T, \quad \gamma \sim m^*/m_0 = E_F^0/E_F$$

# Electrical resistivity

$$\rho = \rho_0 + A(k_B T/E_F)^2$$

$$\Rightarrow A \sim \gamma^2$$

*K. Kadowaki, S.B. Woods,  
Solid State Commun. 58, 507  
(1986)*



A. C. Jacko et al., *Nature Phys.* **5**, 422 (2009)

# Quantum phase transitions in heavy-fermion systems

# Competition between Kondo effect and RKKY interaction Leading to quantum criticality in heavy-fermion systems

Kondo effect

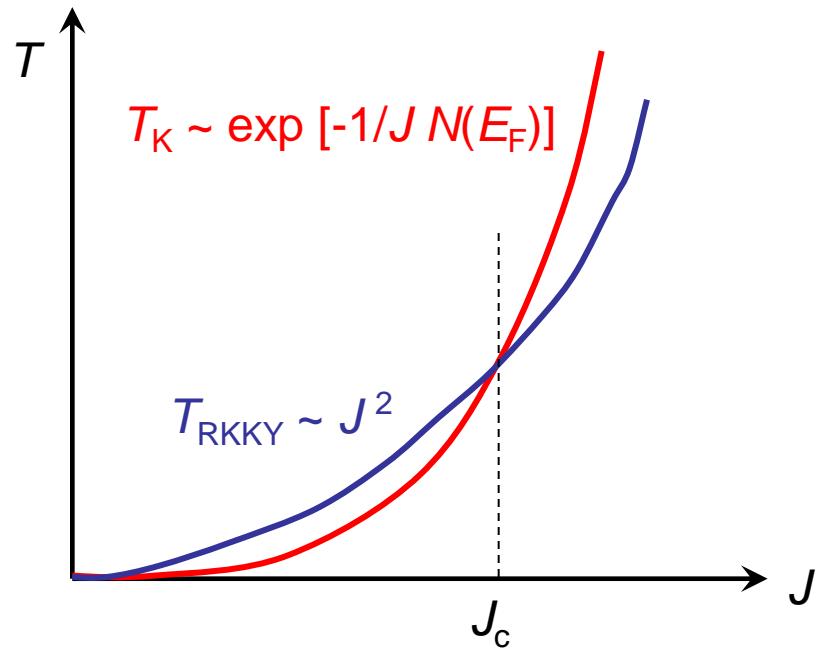
Formation of local singlets  
with conduction electrons

$\Rightarrow$  nonmagnetic groundstate

RKKY interaction

Polarization of conduction  
electrons via  $J$  is sensed by  
another magnetic ion

$\Rightarrow$  tendency towards  
magnetic order



S. Doniach, Physica **91B**, 231 (1977)

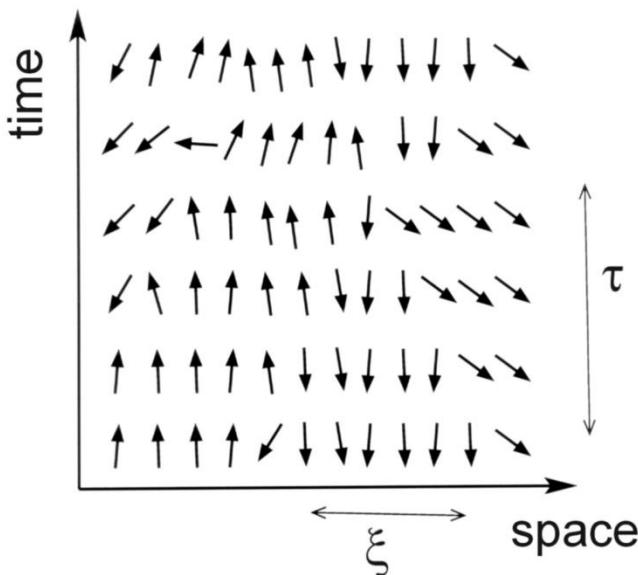
# Quantum phase transitions (2<sup>nd</sup> order)

Energy scale of quantum effects:  $\frac{\hbar}{\tau}$

classical case:  $\frac{\hbar}{\tau} \ll k_B T \approx k_B T_c$ , quantum fluctuations negligible

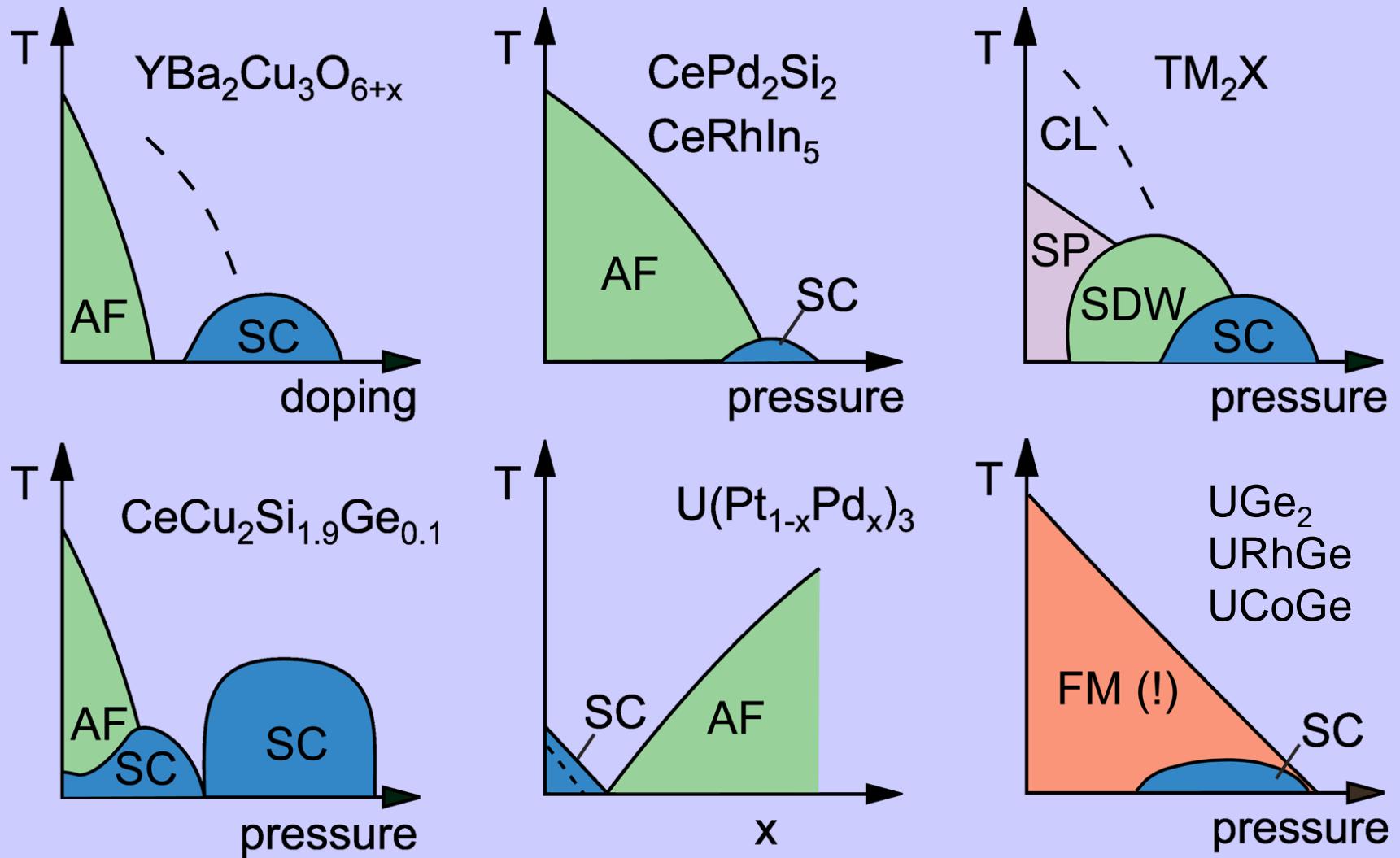
quantum case:  $\frac{\hbar}{\tau} \geq k_B T$ , possible if  $T_c \rightarrow 0$

spatial *and* temporal fluctuations determine dynamics,  $\tau \sim \xi^z$

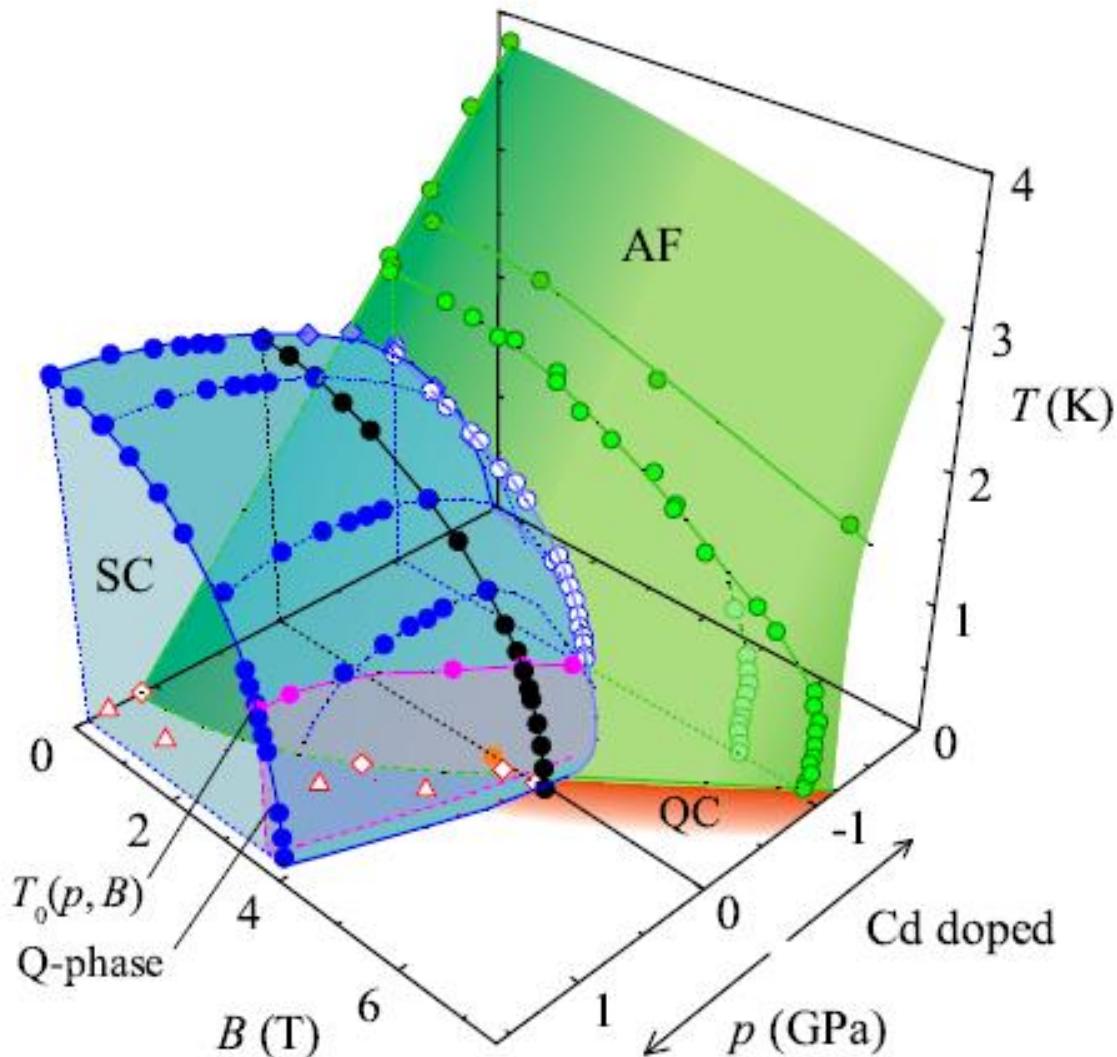


effective dimension  $d_{\text{eff}} = d + z$   
new universality classes  
role of temperature  
control parameter  
thermal excitations via  $k_B T$   
finite system size  $\tau \leq \hbar/k_B T$

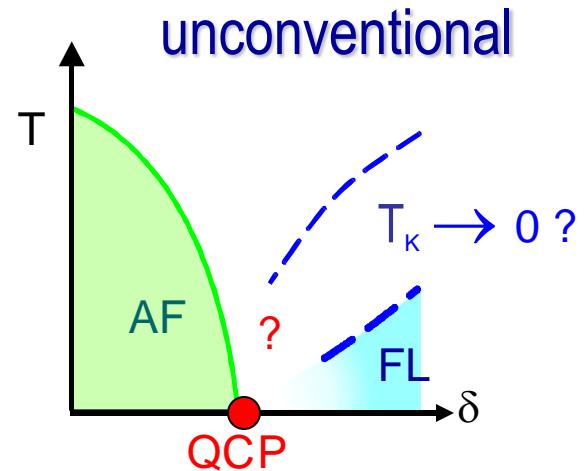
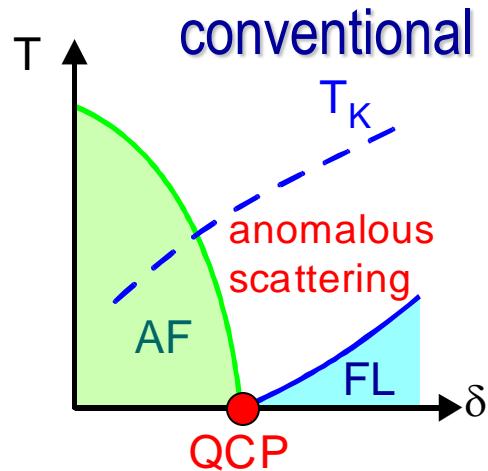
# Interplay of superconductivity and magnetism near quantum critical points



# Interplay of superconductivity and antiferromagnetism at quantum criticality in CeCoIn<sub>5</sub>

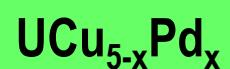


# Scenarios for quantum criticality in heavy-fermion systems



Scattering of heavy quasiparticles  
by spin fluctuations: diverging  $m^*$   
for 3D FM and 2D AF

Unbinding of composite  
heavy quasiparticles:  
change of Fermi volume



Multiple energy scales?  
Dimensionality ?  
Disorder effects ?