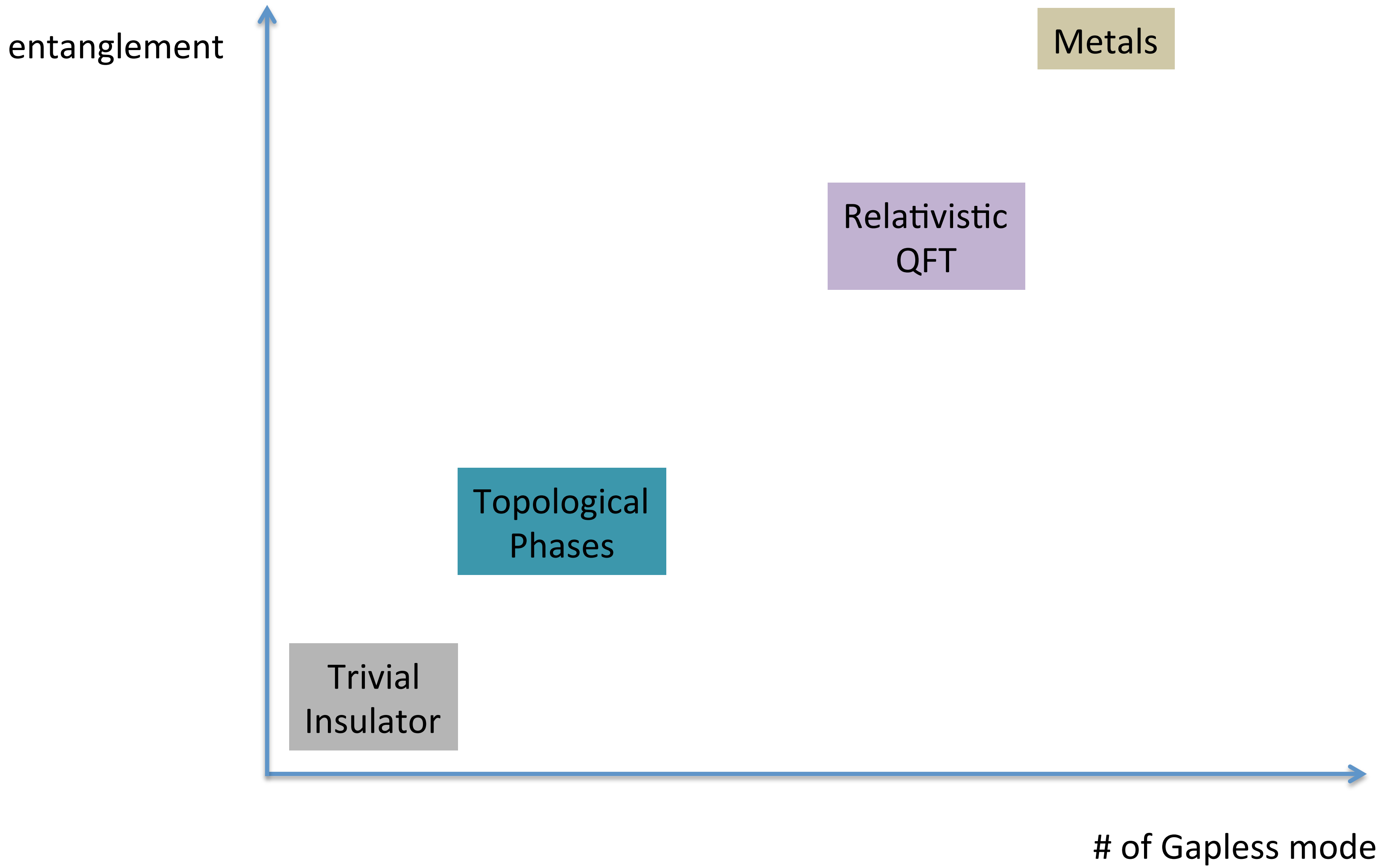
Low Energy Effective Theories for non-Fermi liquids

Sung-Sik Lee
McMaster University
Perimeter Institute

Different phases of matter

- Gapped states (no IR d.o.f.)
  - (trivial) insulator
- Topological states (sub-extensive IR d.o.f.)
  - Quantum Hall liquids, topological insulator
- Gapless states (extensive gapless modes)
  - Relativistic CFT (z=1; graphene, Ising critical point)
  - Fermi surface (metal)

Different phases of matter



Goal

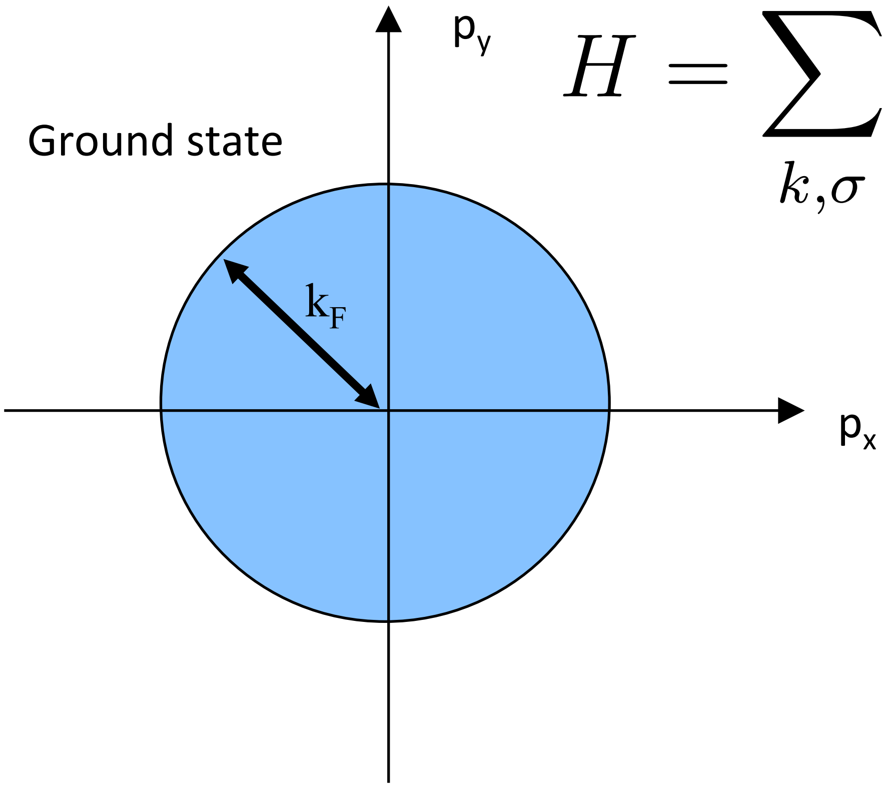
Understanding universal properties of metals based on low energy effective field theories

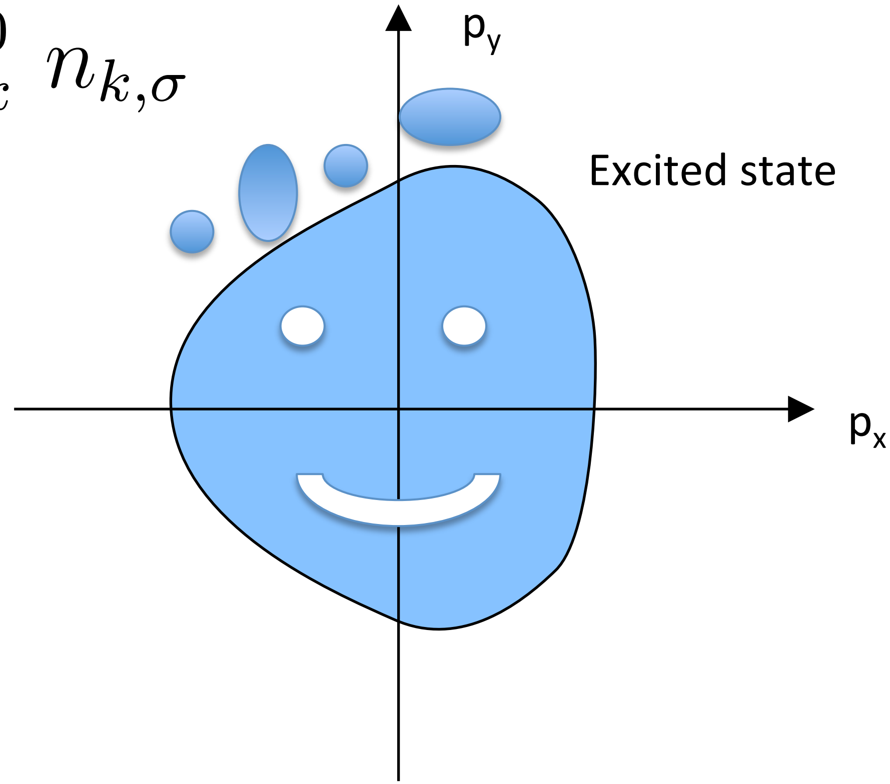
Plan

- Fermi liquid theory
- Routes to non-Fermi liquids
- Attempts toward controlling quantum fluctuations in non-Fermi liquids
  - Large N
  - Dynamical modification
  - Dimensional regularization

Fermi Gas

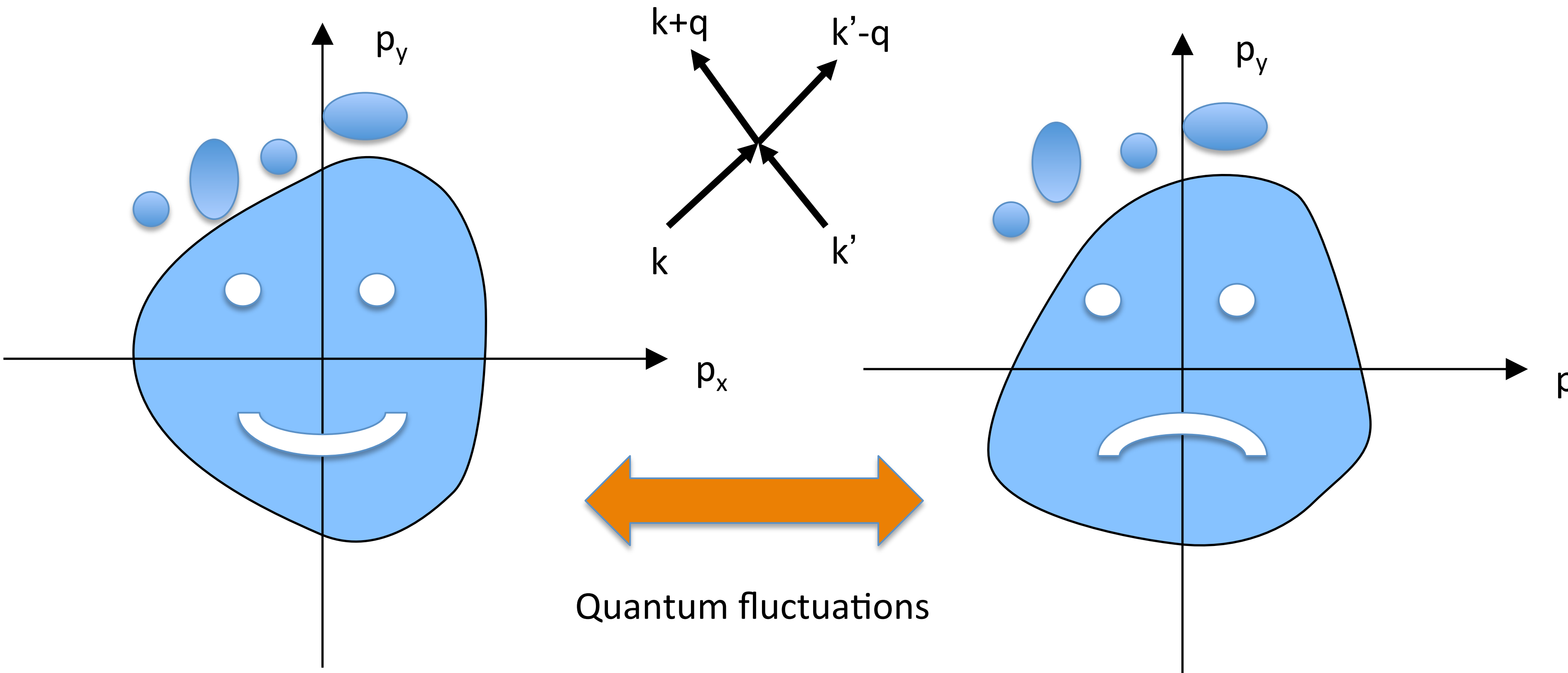
$$H = \sum_{k,\sigma} \epsilon_k^0 \, n_{k,\sigma}$$





Many-body eigenstates are labeled by a set of occupation numbers of single-particle states $|n_{k_1,\sigma_1}, n_{k_2,\sigma_2}, \ldots >$

Interacting Fermions



$|n_{k_1,\sigma_1}, n_{k_2,\sigma_2}, \ldots >$ is no longer an eigenstate

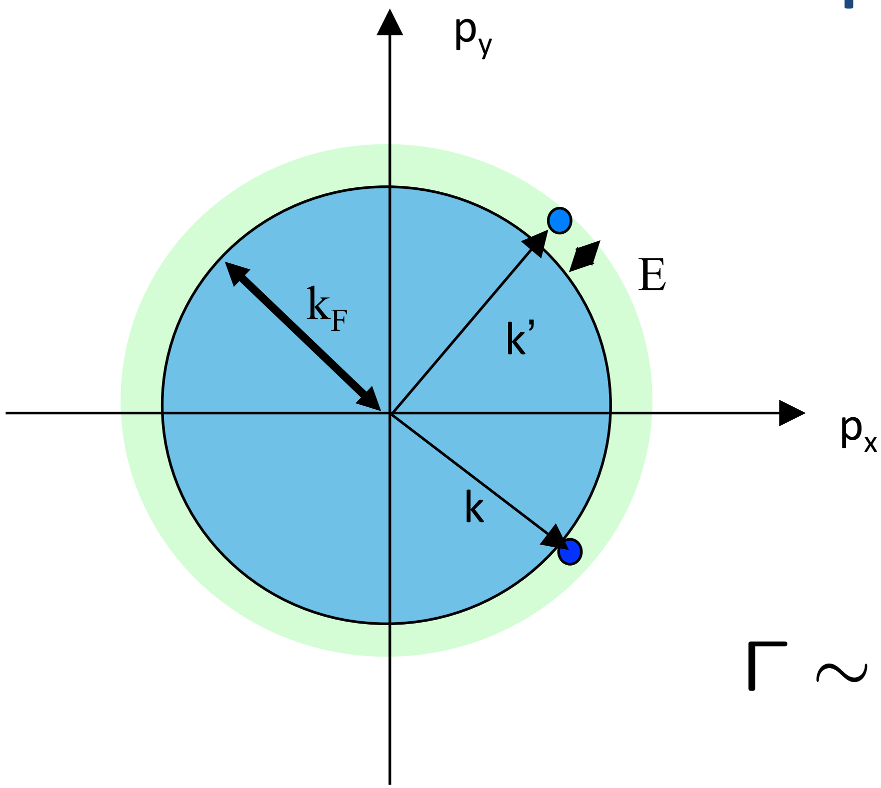
Fermi Liquids

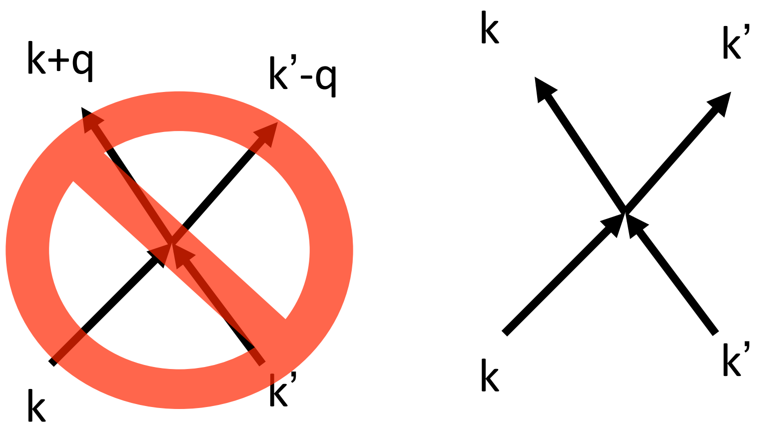
- In certain metals, the low temperature properties of interacting fermions are remarkably similar to those of the non-interacting Fermi gas
  - Specific heat : C ~ T
  - Magnetic susceptibility : χ ~ const.
- Landau postulated that low energy eigenstates of the interacting fermions are still labeled in the same way the non-interacting eigenstates are labeled $|n_{k_1,\sigma_1}, n_{k_2,\sigma_2}, \ldots >^{'}$
- The total energy has non-linear terms :

$$H = \sum_{k,\sigma} \epsilon_k n_{k,\sigma} + \frac{1}{V} \sum_{k,k',\sigma,\sigma'} F_{\sigma,\sigma'}(k,k^{'}) n_{k,\sigma} n_{k',\sigma'}$$

Microscopic justification of Landau Fermi Liquid theory

[Shankar,Polchinski]





$$\Gamma \sim E^2$$

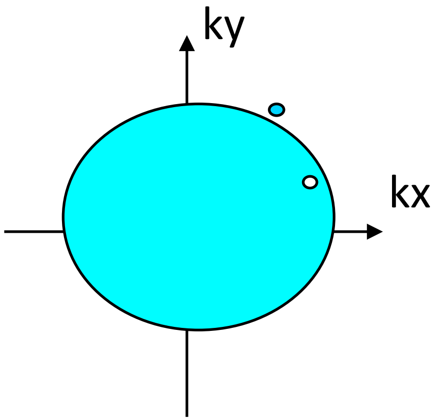
At low energies, the phase space for non-forward scatterings is small : only forward scatterings are important

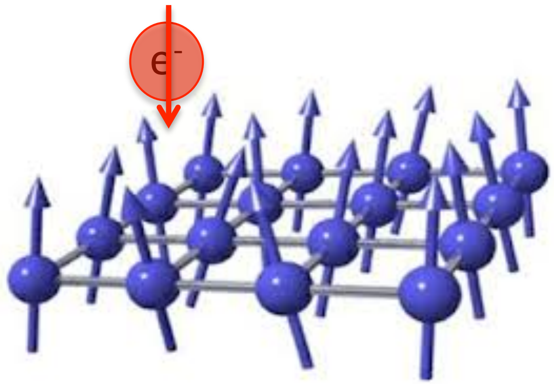
→ particles created near FS have long life time

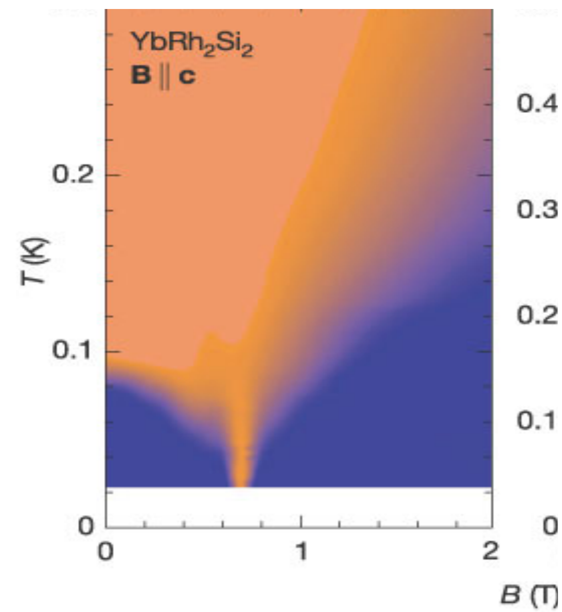
Low energy eigenstates are still labeled by occupation numbers of quasiprticle

Non-Fermi liquids

Strongly Correlated Metals



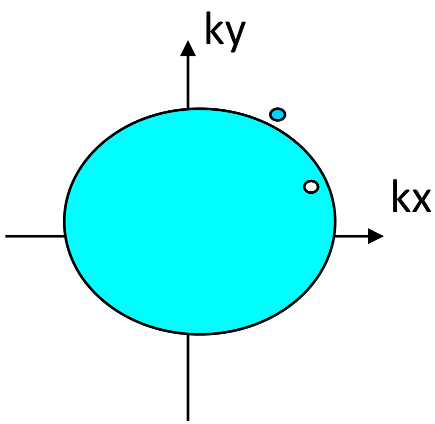




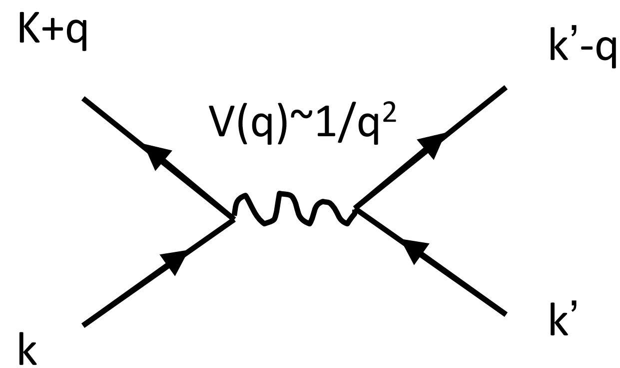
[Custers et al.(2003)]

- Soft collective modes in the system (such as order parameter fluctuations at quantum critical point) can cause strong quantum fluctuations of FS

A route to non-Fermi liquid : long-range force



Fermi surface
+ gapless boson

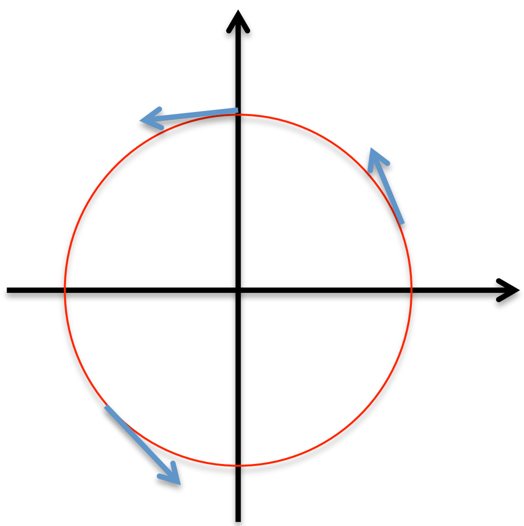


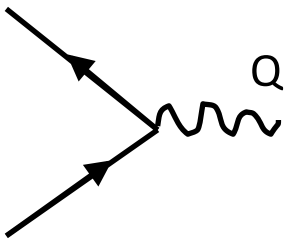
- Non-forward scatterings are enhanced by long-range interactions mediated by collective modes
- Bare fermion quickly decays into a complicated superposition of states
- Single particle is no longer a good basis to understand low energy properties

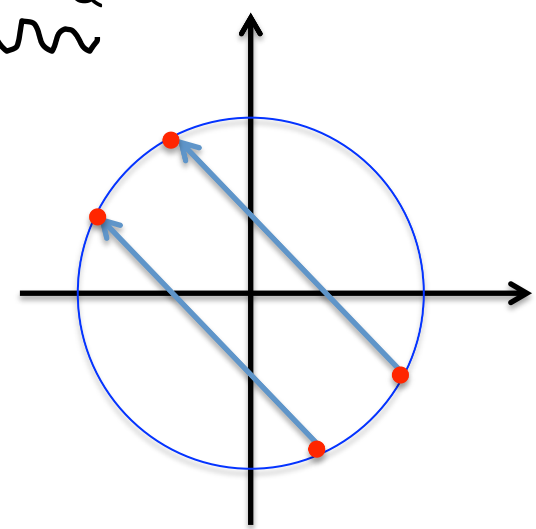
Examples of non-Fermi liquids

	Collective mode	Momentum of collective mode
Spin liquid	(Emergent) Gauge field	0
Nematic criticality	Nematic order	0
Charge density wave criticality	Charge density order	$Q \neq 0$
Spin density wave criticality	Spin density order	$Q \neq 0$

In d=2







Q=0
Nematic, ferromagnetic QCP
Spin liquids with emergent gauge boson

Q≠0
Spin & CDW QCP

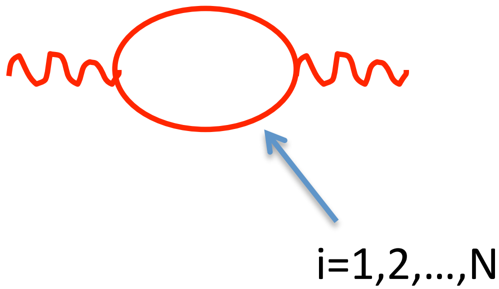
Theoretical Status of Non-Fermi liquids in 2+1D

- Coupling between fermion and boson become strong even though bare coupling is weak (characteristic of low dimensionality)
- In chiral non-Fermi liquids, exact critical exponents are known [Sur, SL (2013)]
- In general, a small parameter is needed to study the system in a controlled way

Different routes to tame quantum fluctuations

	Pro	Con
Large N	Most benign modification (symmetry, locality, finite DOS)	Not controlled
Dynamical tuning	Easy to keep symmetry	Breaks locality
Tune dimension	Keep symmetry, finite DOS	UV/IR mixing (Spurious scale)
Tune co-dimension	Keep locality, no spurious scale introduced	Break some symmetry

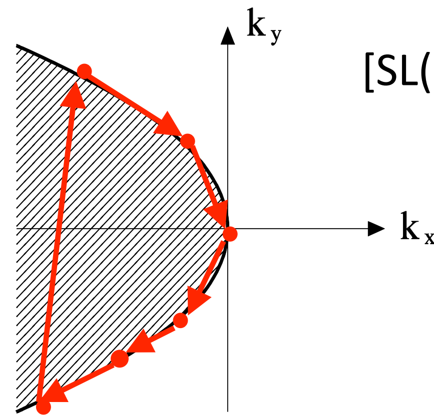
Large N



For large N, collective modes
gets dressed heavily with fermion clouds

- This appears to suggest that effect of fluctuating boson on fermion is small therefore processes which involve excitations of multiple bosons are systematically suppressed for a large N

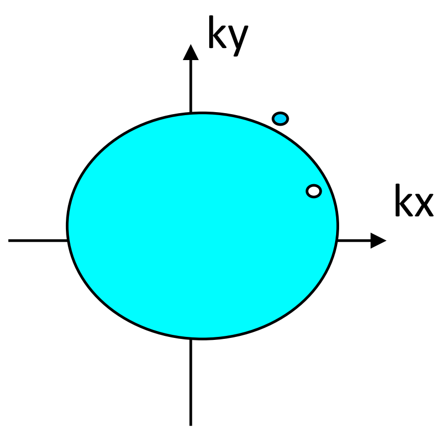
- However, small interaction is amplified when fermions are scattered along the Fermi surface



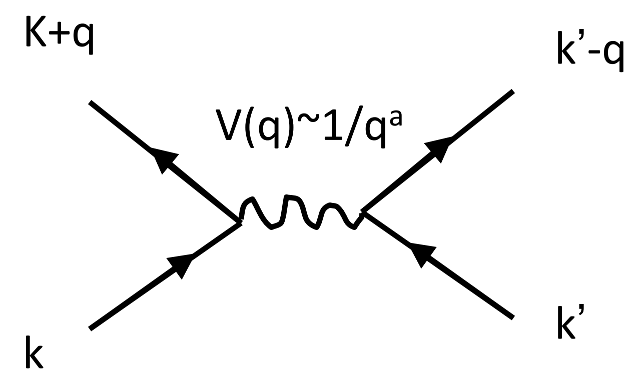
[SL(09)]

- All planar graphs are important

Dynamical tuning

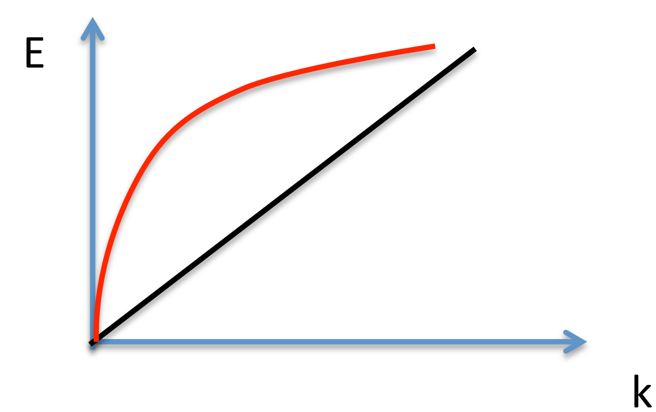


Fermi surface
+ gapless boson

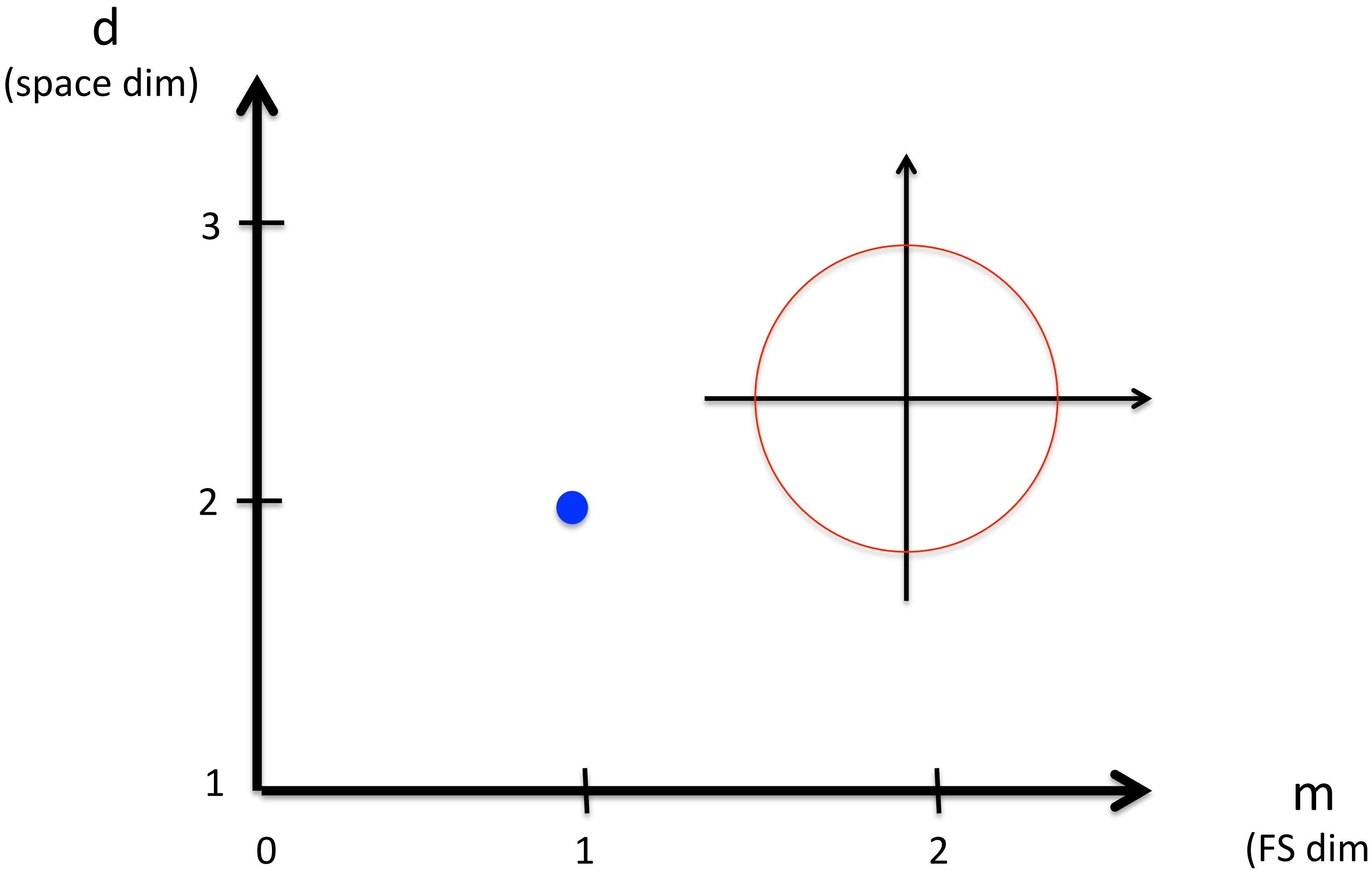


- Tune a from 2 to 1+ε, which makes the boson stiffer
- Breaks locality of the theory

[Nayak, Wilczek(94); Mross, McGreevy, Liu, Senthil(10)]

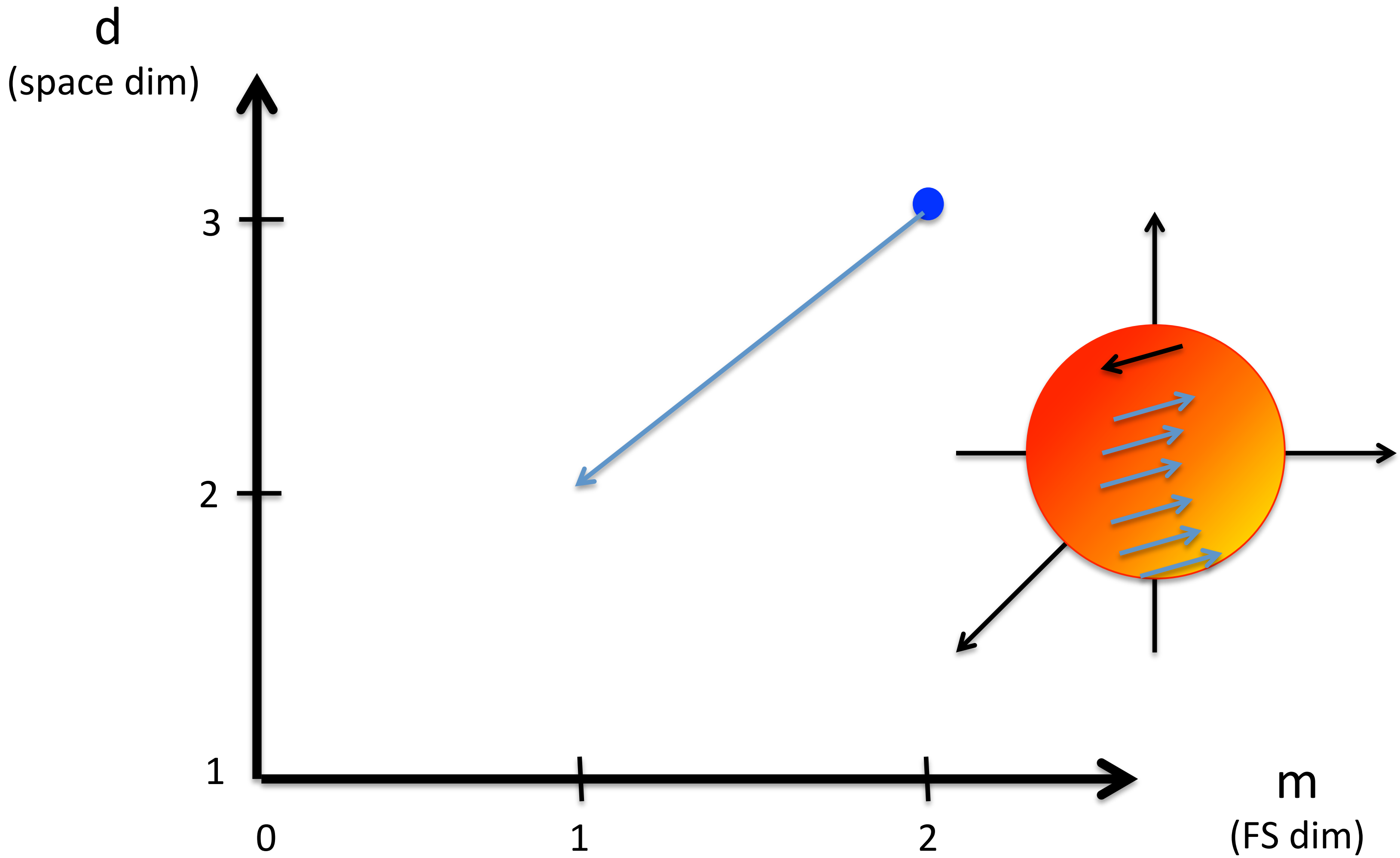


Dimensional Regularization



Tuning d with fixed d-m

[Chakravarty, Norton, Syljuasen(95), Fitzpatrick, Kachru, Kaplan, Raghu (13)]

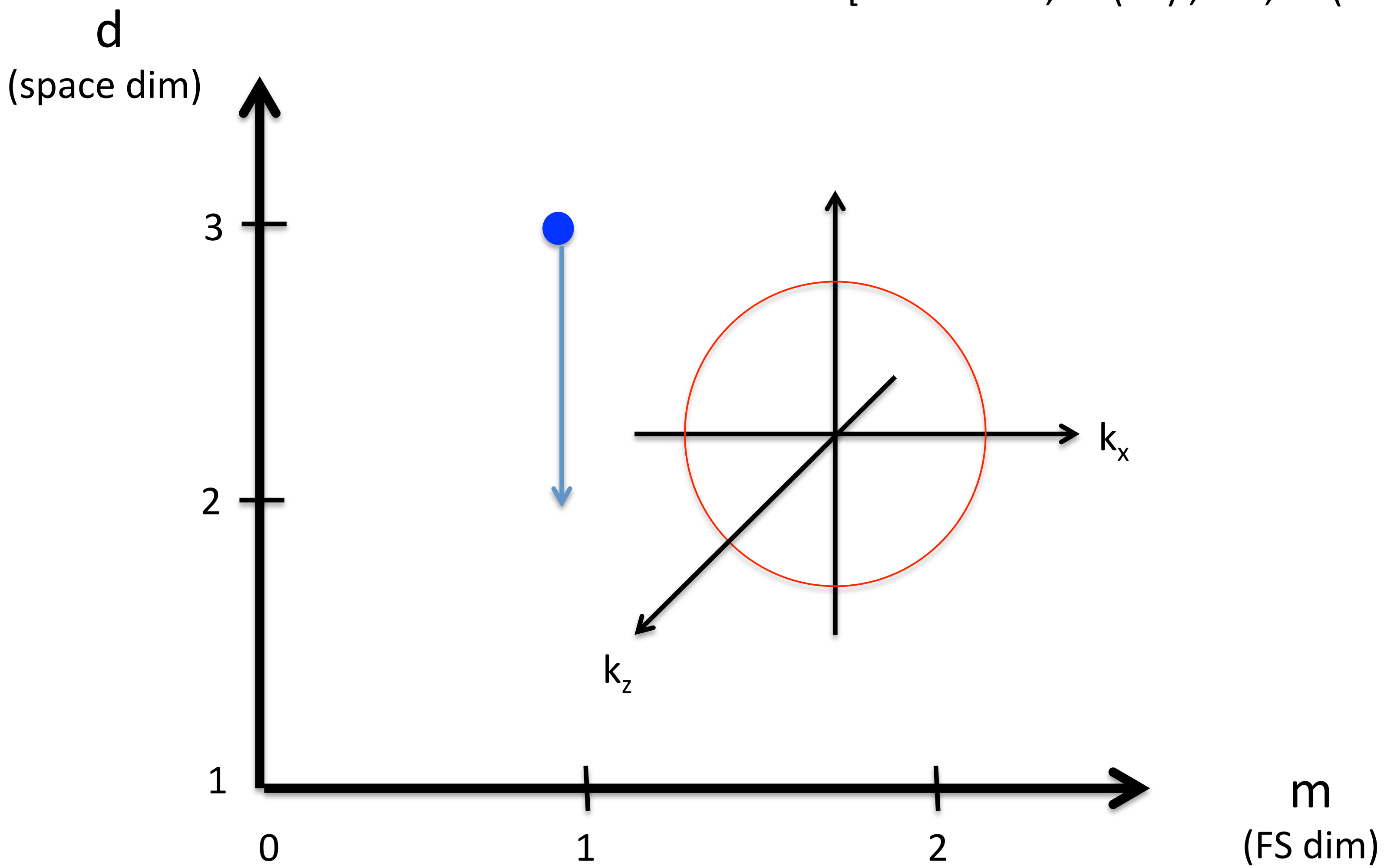


- Size of FS enters as a spurious scale (UV/IR mixing)

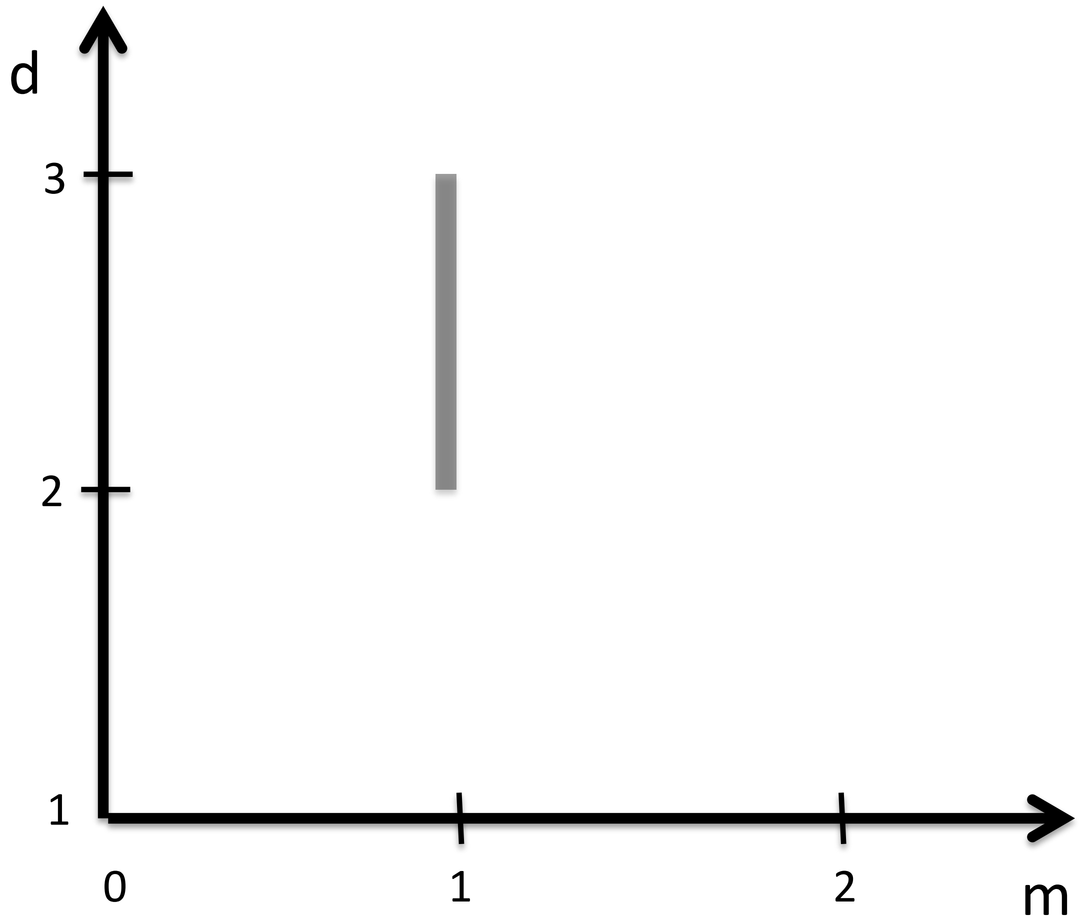
[Mandal, SL (15)]

Tuning d with fixed m

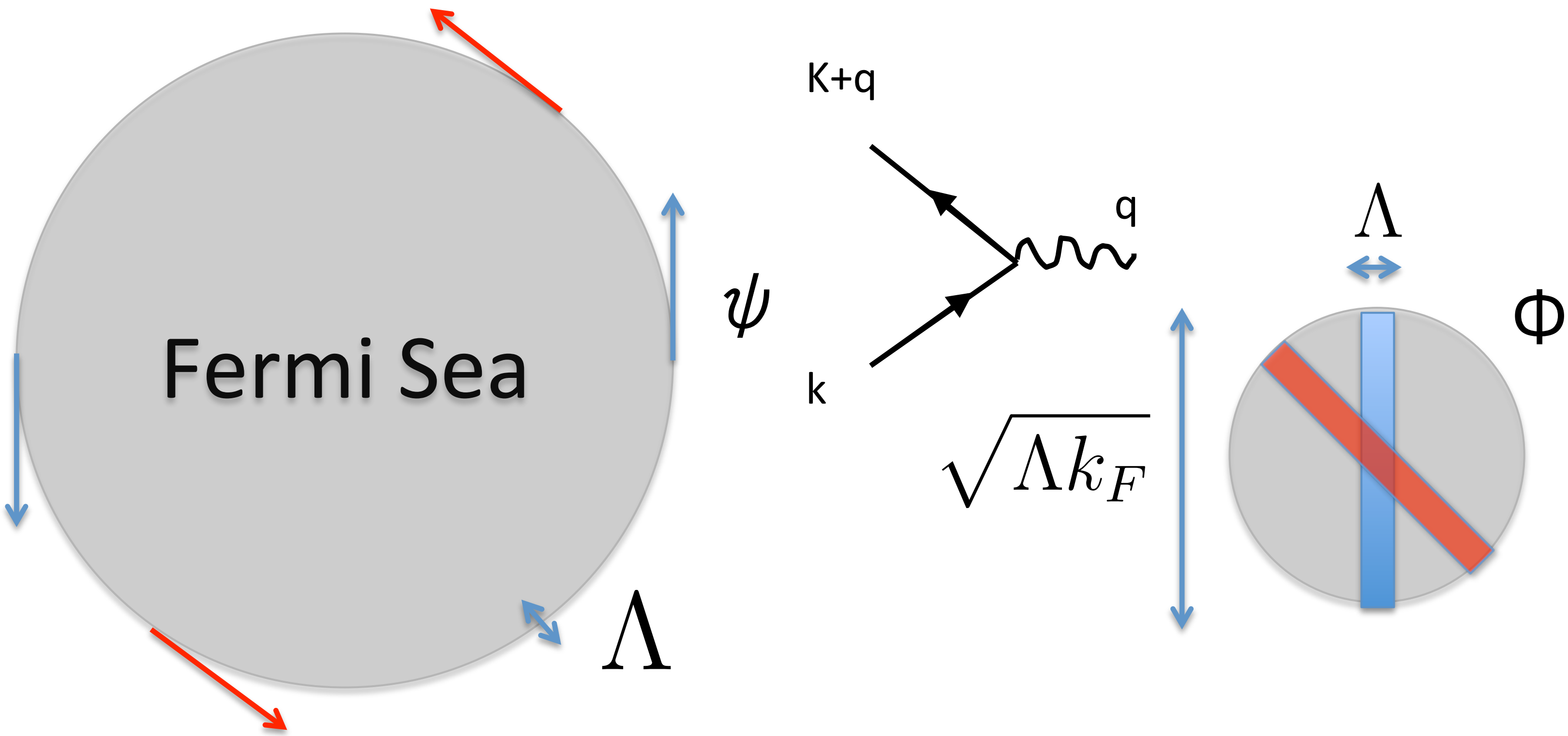
[Dalidovich, SL (13) ; Sur, SL (14)]



m=1, Q=0

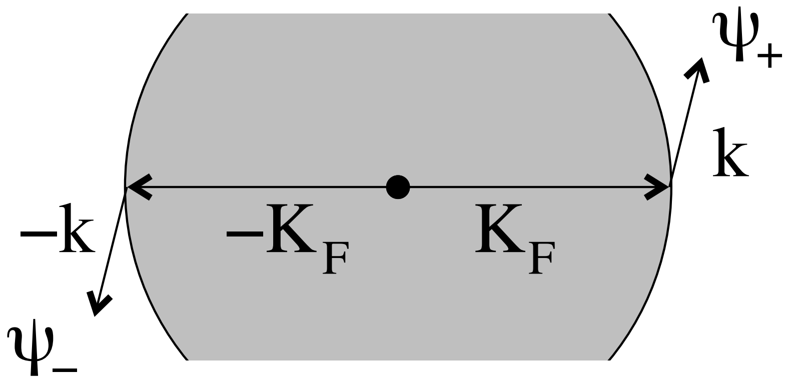


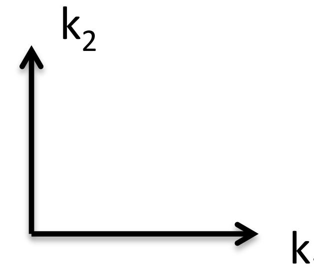
Emergent locality in momentum space for m=1 in general d



- Fermions are primarily scattered along the directions tangential to FS
- At low energies, fermions with different tangential vectors are decoupled from each other in the $\Lambda \to 0$ limit

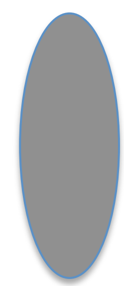
Two-patch theory

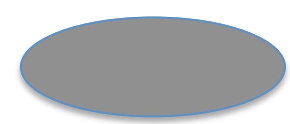




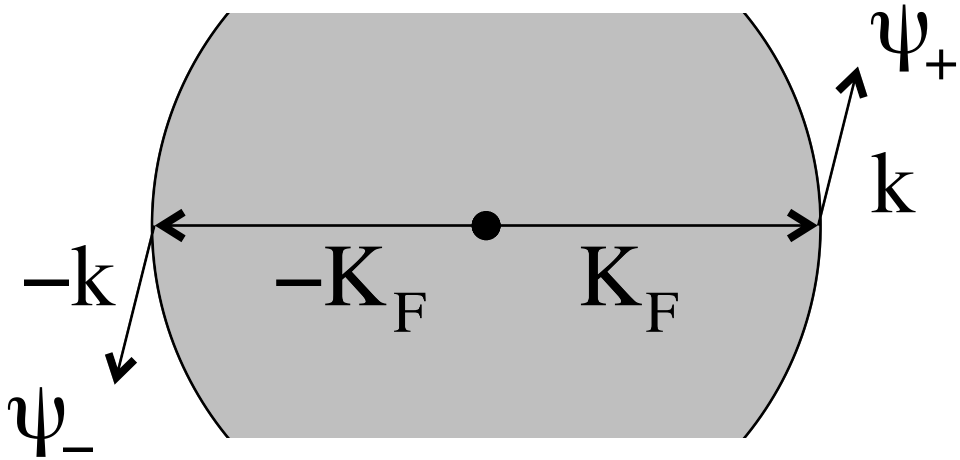
Ising-nematic critical point

$$S = \sum_{s=\pm}\sum_{j=1}^{N}\int \frac{d^3k}{(2\pi)^3}\psi^{\dagger}_{s,j}(k)\left[ik_0 + sk_1 + k_2^2\right]\psi_{s,j}(k)$$
$$+ \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\left[k_0^2 + k_1^2 + k_2^2\right]\phi(-k)\phi(k)$$
$$+ \frac{e}{\sqrt{N}}\sum_{s=\pm}\sum_{j=1}^{N}\int \frac{d^3k d^3q}{(2\pi)^6}\ \phi(q)\ \psi^{\dagger}_{s,j}(k+q)\psi_{s,j}(k).$$





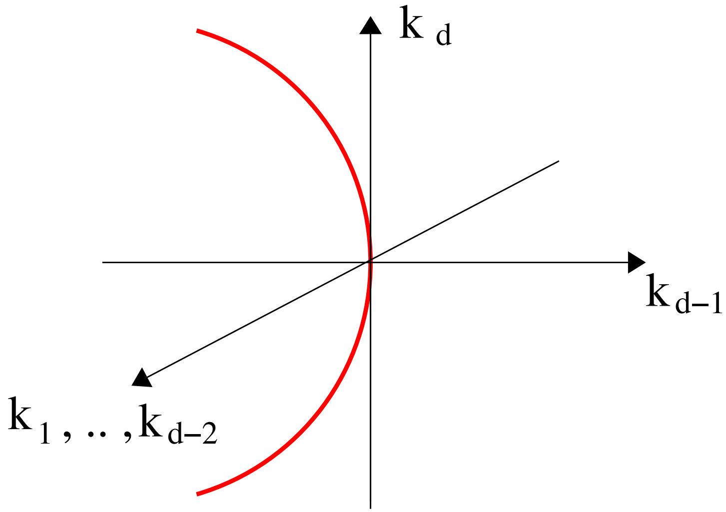
Two-patch theory as (1+1)-dim Dirac fermion with a continuous flavour

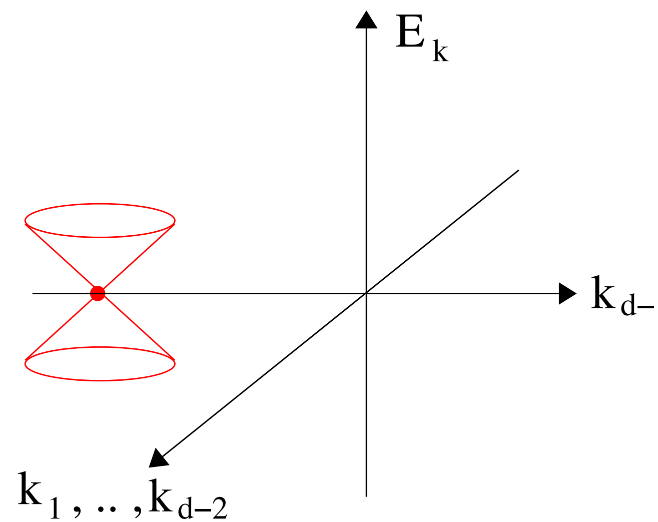


$$\Psi_j(k) = \begin{pmatrix} \psi_{+,j}(k) \\ \psi^\dagger_{-,j}(-k) \end{pmatrix}$$

$$\begin{aligned} S \;=\; & \sum_j \int \frac{d^3k}{(2\pi)^3} \bar{\Psi}_j(k) \Big[ik_0\gamma_0 + i(k_1 + k_2^2)\gamma_1 \Big] \Psi_j(k) \\ + \; & \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \left[k_0^2 + k_1^2 + k_2^2 \right] \phi(-k)\phi(k) \\ + \; & \frac{e}{\sqrt{N}} \sum_j \int \frac{d^3k\,dq}{(2\pi)^6} \phi(q) \bar{\Psi}_j(k+q)\gamma_1 \Psi_j(k) \end{aligned}$$

d-dimensional Dirac fermion with a continuous flavour





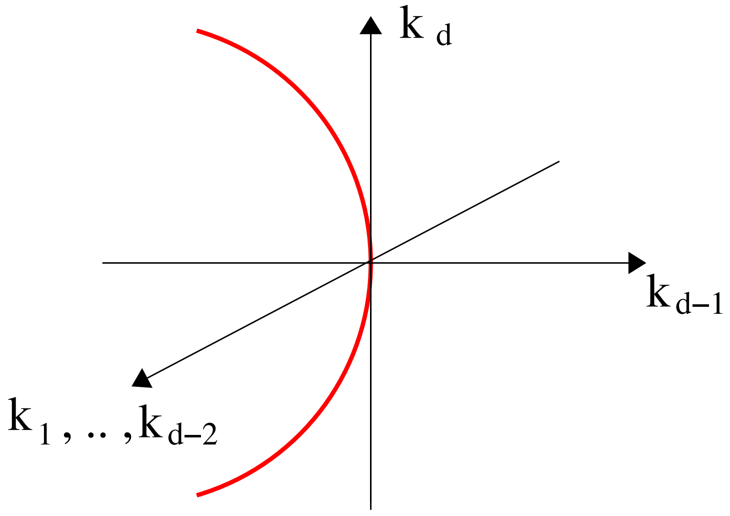
$$E_k = \pm\sqrt{k_1^2 + k_2^2 + .. + k_{d-2}^2 + \delta_k^2}$$

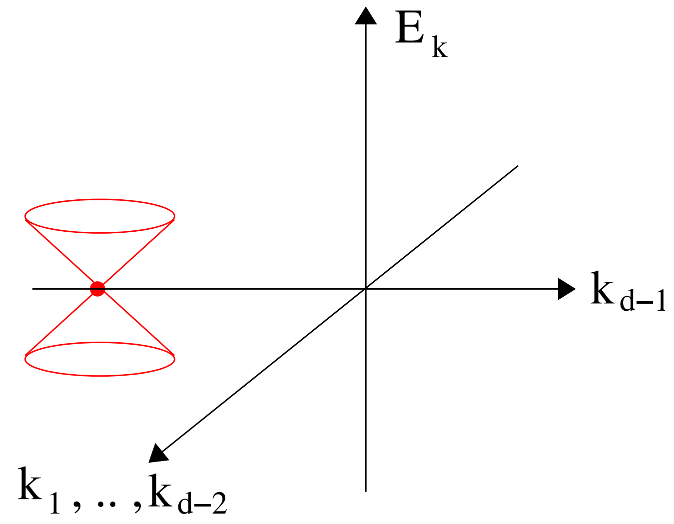
$$\begin{aligned} S &= \sum_j \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \bar{\Psi}_j(k) \left[i\vec{\Gamma}\cdot\vec{K} + i\gamma_{d-1}\delta_k \right] \Psi_j(k) \\ &+ \frac{1}{2}\int \frac{d^{d+1}k}{(2\pi)^{d+1}} \left[|\vec{K}|^2 + k_{d-1}^2 + k_d^2 \right] \phi(-k)\phi(k) \\ &+ \frac{ie}{\sqrt{N}}\sqrt{d-1}\sum_j \int \frac{d^{d+1}k d^{d+1}q}{(2\pi)^{2d+2}} \phi(q)\bar{\Psi}_j(k+q)\gamma_{d-1}\Psi_j(k) \end{aligned}$$

$$\vec{K} \equiv (k_0, k_1, \ldots, k_{d-2})$$

$$\delta_k = k_{d-1} + \sqrt{d-1}k_d^2$$

The theory at d = 3 describes a spin triplet p-wave SC

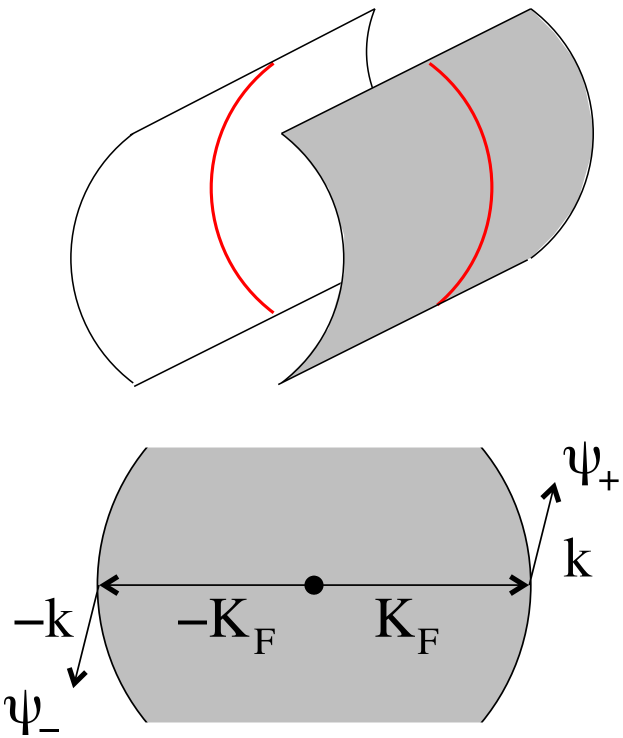


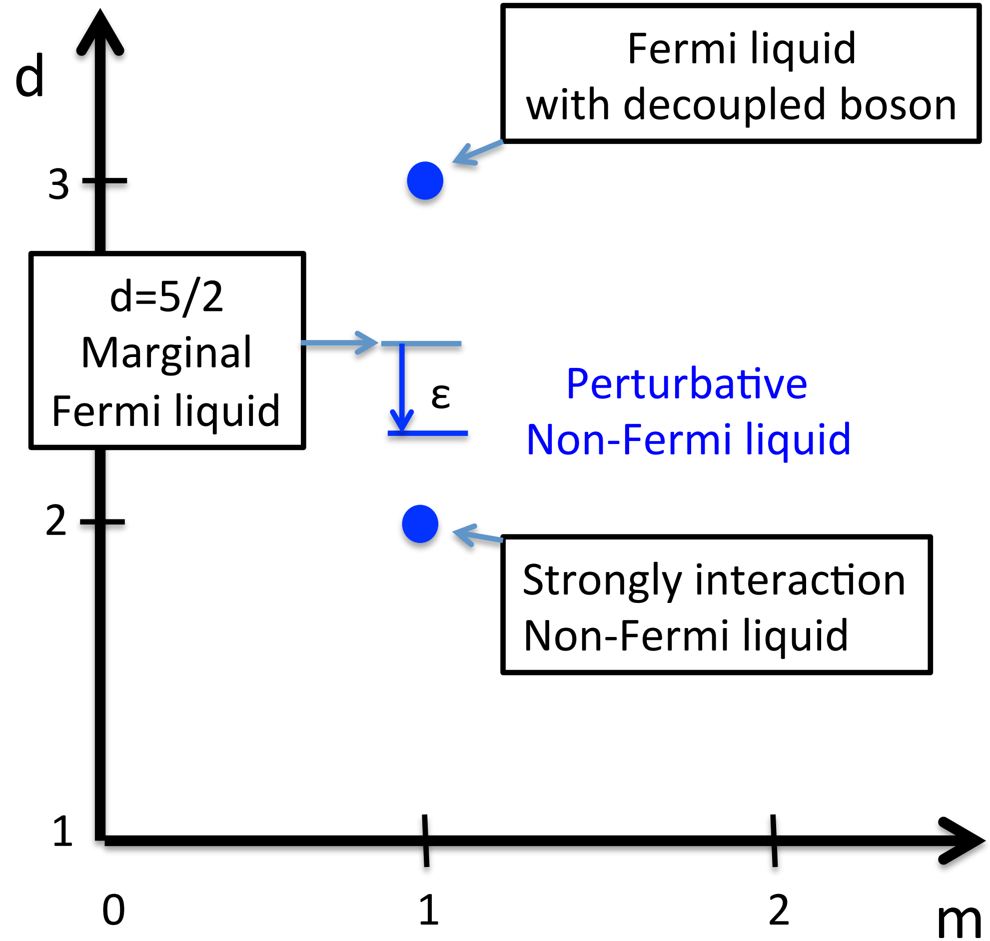


$$S = \int \frac{d^4k}{(2\pi)^3} \Bigg\{ \sum_{s=\pm} \sum_{j=\uparrow,\downarrow} \psi^\dagger_{s,j}(k) \left(ik_0 + sk_2 + k_3^2 \right) \psi_{s,j}(k)$$

$$-k_1 \left(\psi^\dagger_{+,\uparrow}(k) \psi^\dagger_{-,\uparrow}(-k) + \psi^\dagger_{+,\downarrow}(k) \psi^\dagger_{-,\downarrow}(-k) + h.c. \right) \Bigg\}$$

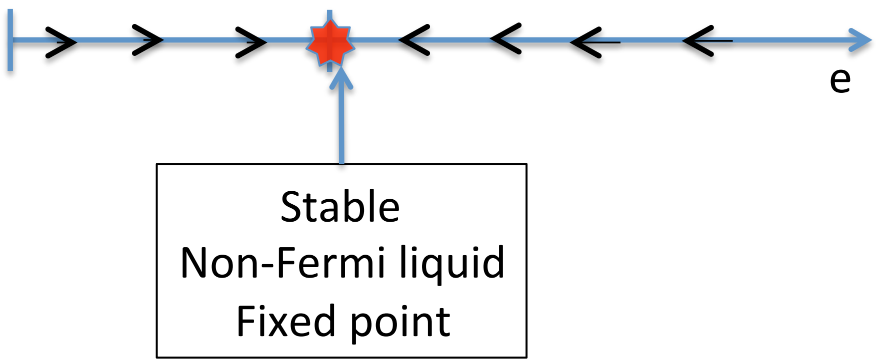
A continuous interpolation between 2d Fermi surface to 3d p-wave SC





Two-loop results

$$\frac{de}{dl} = \frac{\epsilon}{2}e - 0.02920\left(\frac{3}{2}-\epsilon\right)\frac{e^{7/3}}{N} + 0.01073\left(\frac{3}{2}-\epsilon\right)\frac{e^{11/3}}{N^2}$$



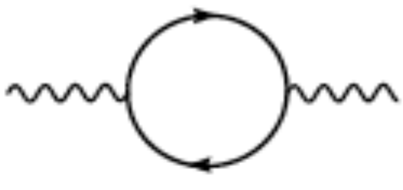
$$\frac{e^{*4/3}}{N} = 11.417\epsilon + 55.498\epsilon^2$$

$$z = \frac{3}{3-2\epsilon}$$

[D. Dalidovich, SL (13)]

Expansion in $e^{4/3}$ instead of e^2

$$\frac{de}{dl} = \frac{\epsilon}{2}e - 0.02920\left(\frac{3}{2}-\epsilon\right)\frac{e^{7/3}}{N} + 0.01073\left(\frac{3}{2}-\epsilon\right)\frac{e^{11/3}}{N^2}$$



$$D_1(k) = \frac{1}{\boxed{|\vec{K}|^2 + k_x^2} + k_y^2 + \beta_d e^2 \frac{|\vec{K}|^{d-1}}{|k_y|}}$$

irrelevant

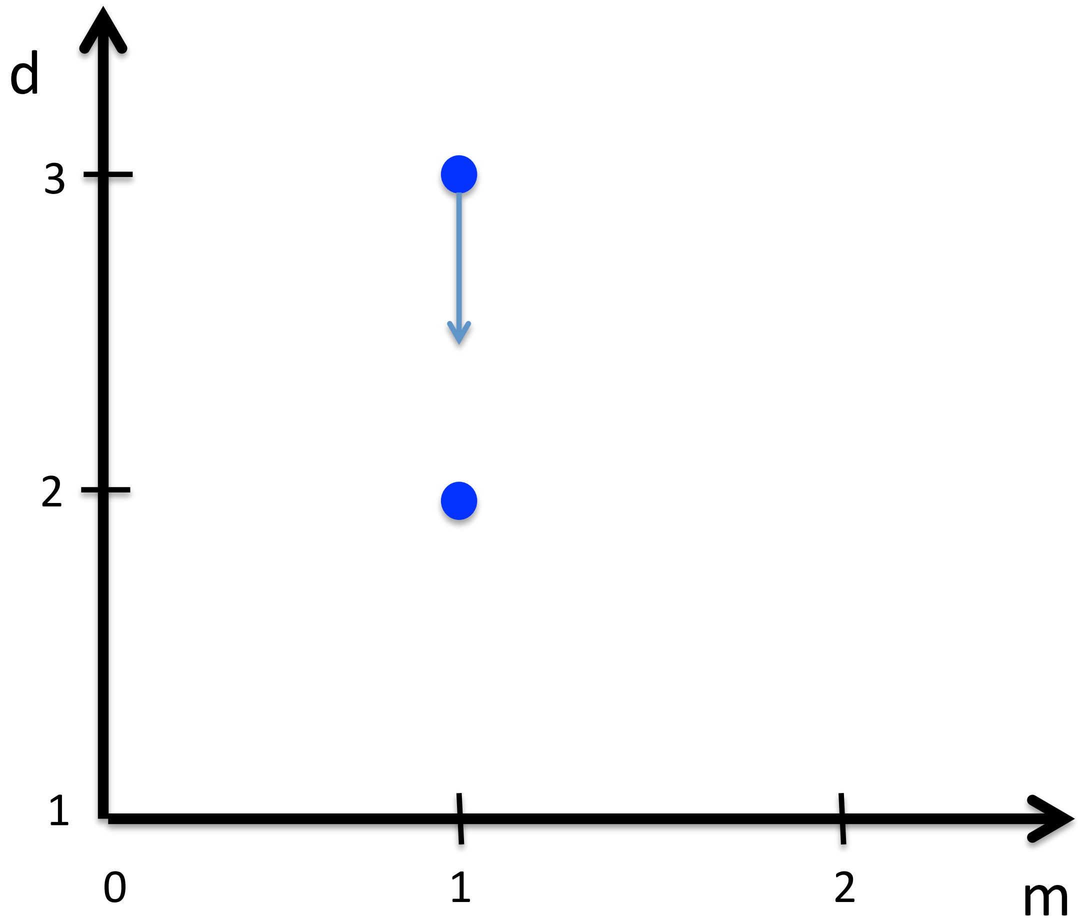
$$\vec{K} = k_0, k_1, .., k_{d-2}$$

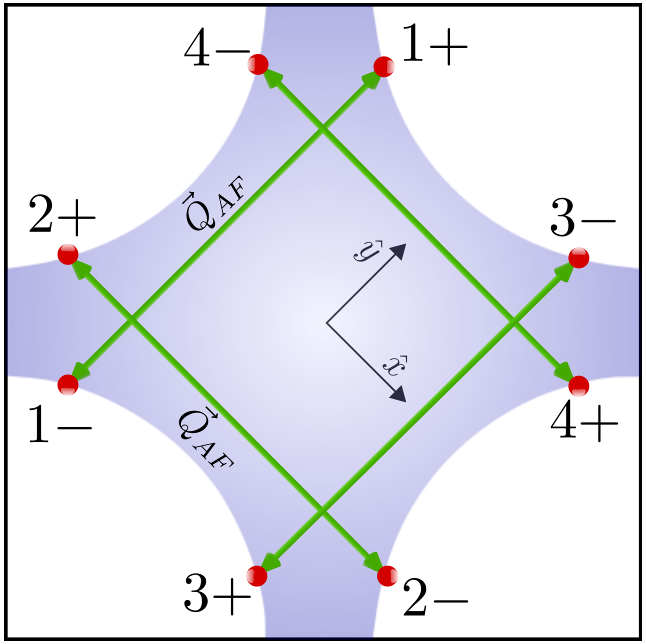
Landau damping, which is generated by interaction, dominates over the bare kinetic term

Physical properties

- Fermion Green fnt : $$G(k) \;=\; \frac{1}{|\delta_k|^{1-0.1508\epsilon^2}} \;\; g\left(\frac{|\vec{K}|^{1/z}}{\delta_k}\right)$$
- Boson Green fnt : $$D(k) \;=\; \frac{1}{k_d^2} \;\; f\left(\frac{|\vec{K}|^{1/z}}{k_d^2}\right)$$
- Specific heat : $$c \;\sim\; T^{(d-2)+\frac{1}{z}}$$

m=1, Q≠0





Minimal Theory for SDW in 2d

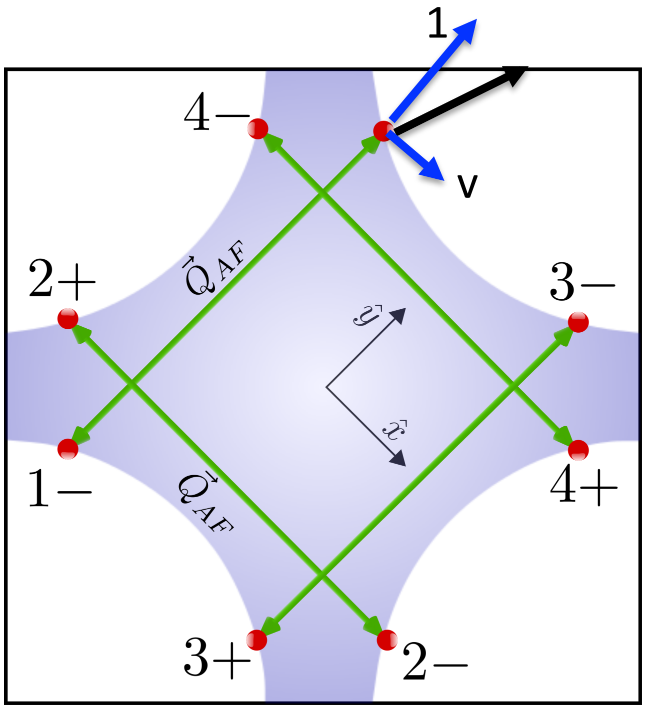
[Abanov, Chubukov]

$$e_1^{\pm}(\vec{k}) = -e_3^{\pm}(\vec{k}) = vk_x \pm k_y$$

$$e_2^{\pm}(\vec{k}) = -e_4^{\pm}(\vec{k}) = \mp k_x + vk_y$$

$$\begin{aligned}\mathcal{S} = & \sum_{l=1}^{4} \sum_{m=\pm} \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3}\, \psi_{l,\sigma}^{(m)*}(k) \left[ik_0 + e_l^m(\vec{k})\right] \psi_{l,\sigma}^{(m)}(k) \\ & + \frac{1}{2}\int \frac{d^3q}{(2\pi)^3} \left[q_0^2 + c^2|\vec{q}|^2\right] \vec{\Phi}(-q)\cdot\vec{\Phi}(q) \\ & + g_0 \sum_{l=1}^{4} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3} \left[\vec{\Phi}(q)\cdot\psi_{l,\sigma}^{(+)*}(k+q)\vec{\tau}_{\sigma,\sigma'}\psi_{l,\sigma'}^{(-)}(k) + c.c.\right] \\ & + \frac{u_0}{4!}\int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3q}{(2\pi)^3} \left[\vec{\Phi}(k_1+q)\cdot\vec{\Phi}(k_2-q)\right]\left[\vec{\Phi}(k_1)\cdot\vec{\Phi}(k_2)\right]\end{aligned}$$

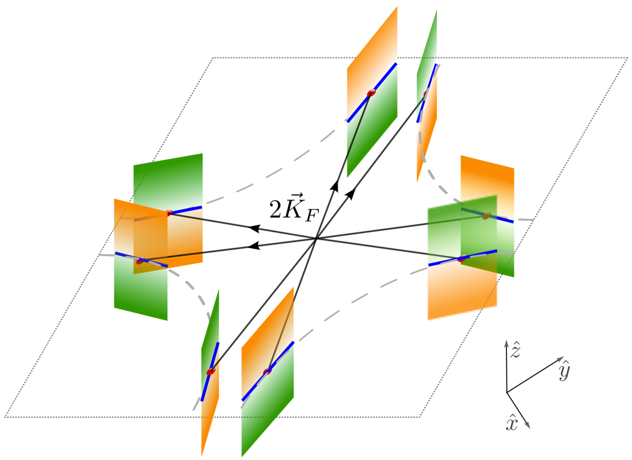
Parameters of the theory

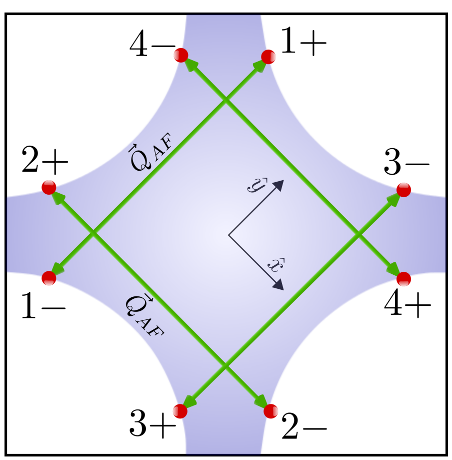


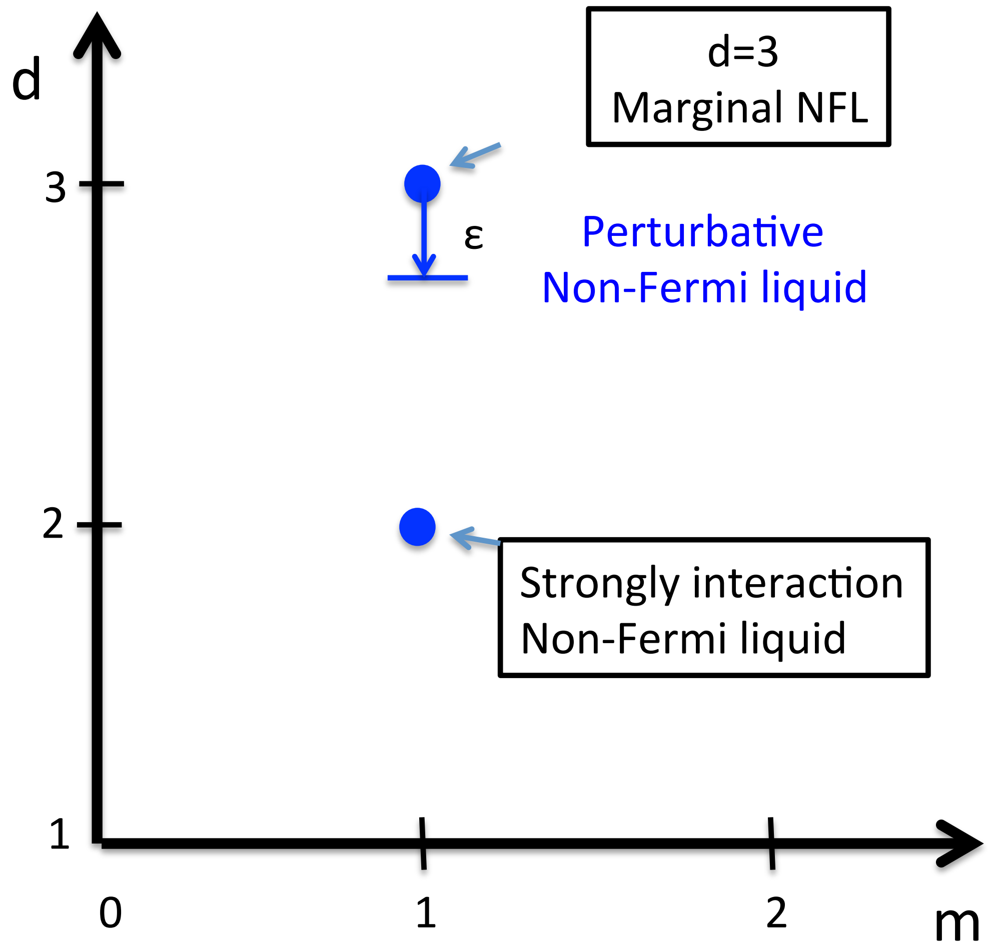
- v : Fermi velocity perpendicular to Q_{AF}
- c : boson velocity
- g : Yukawa coupling
- u : quartic boson coupling

- If v=0, hot spots connected by Q_{AF} are nested
- The four parameters can not be scaled away

A continuous interpolation between 2d Fermi surface and 3d metal with line nodes

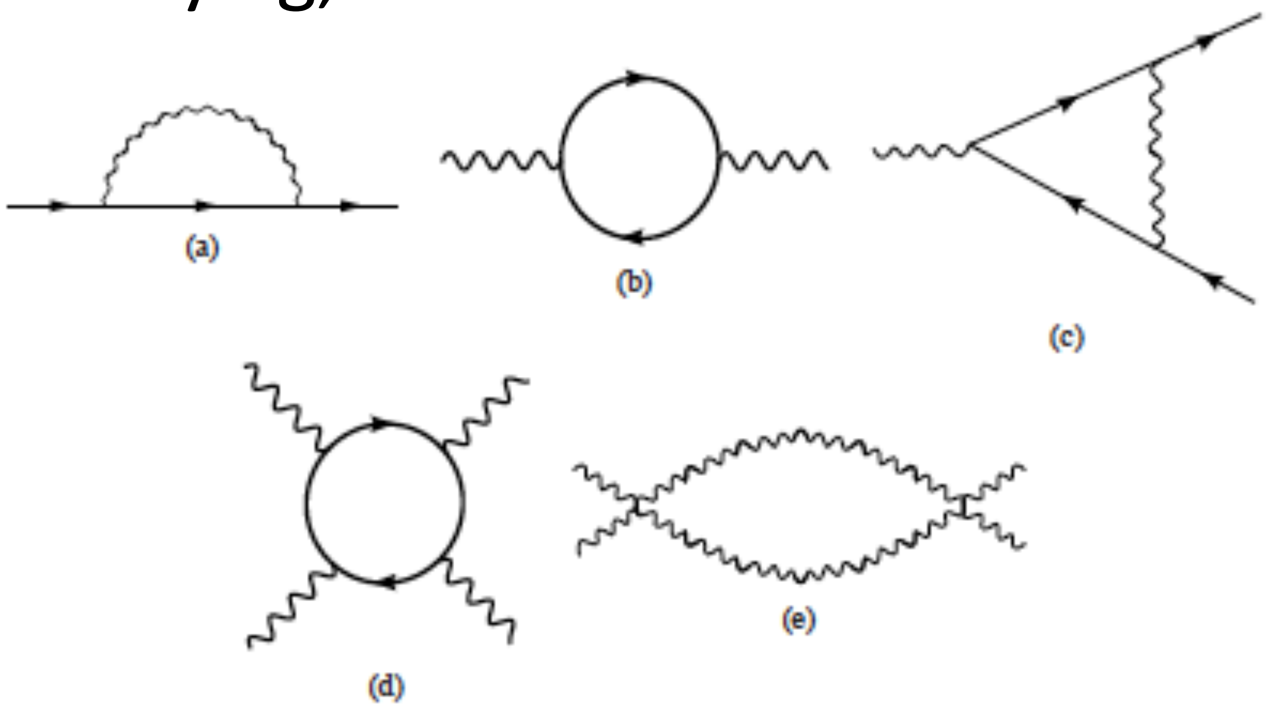




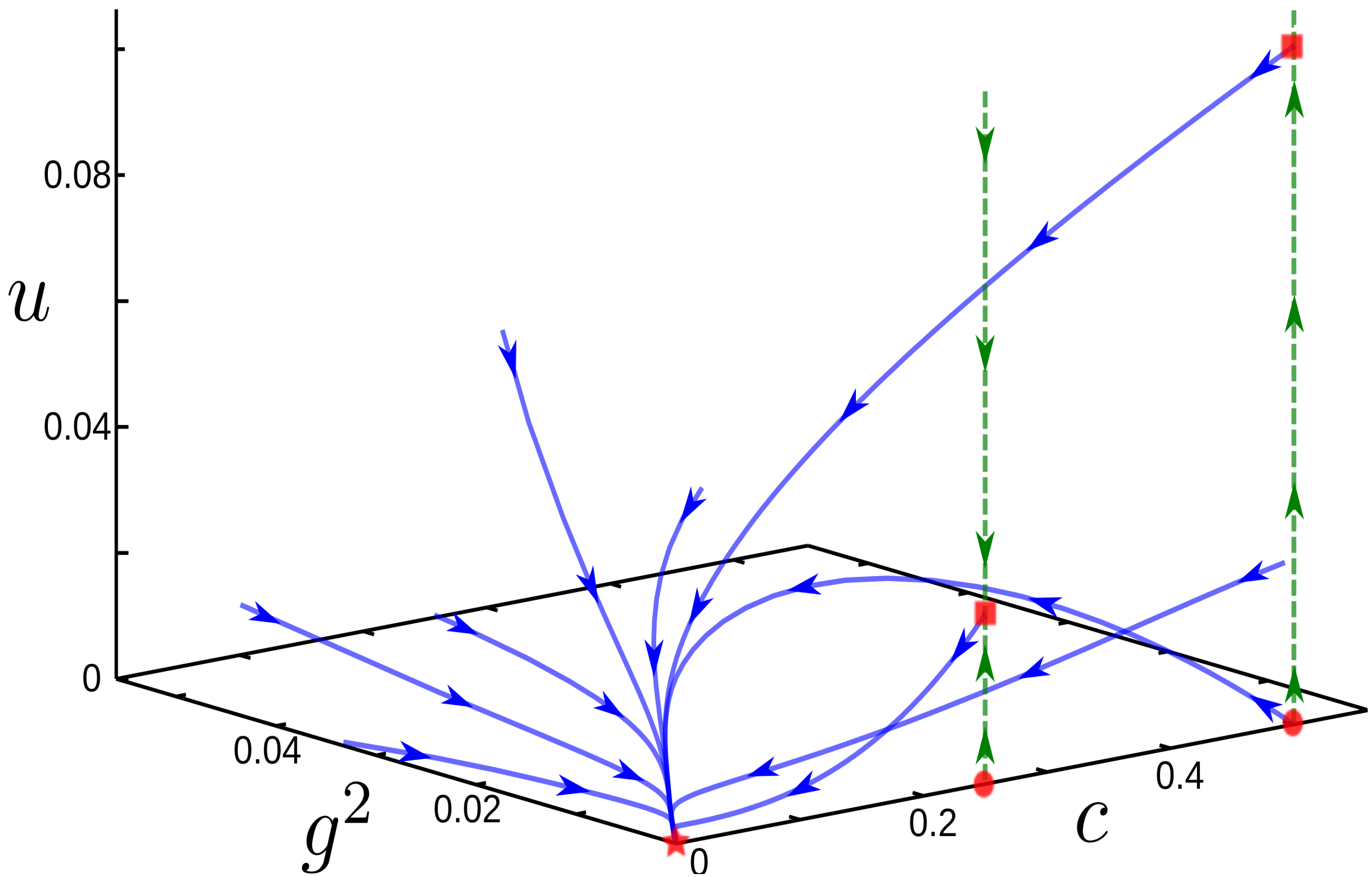


One-loop RG flow

- Yukawa coupling induces nesting : v ➘
- Nesting makes boson slower : c ➘
- Nested FS and slow boson screen more efficiently : g, u ➘

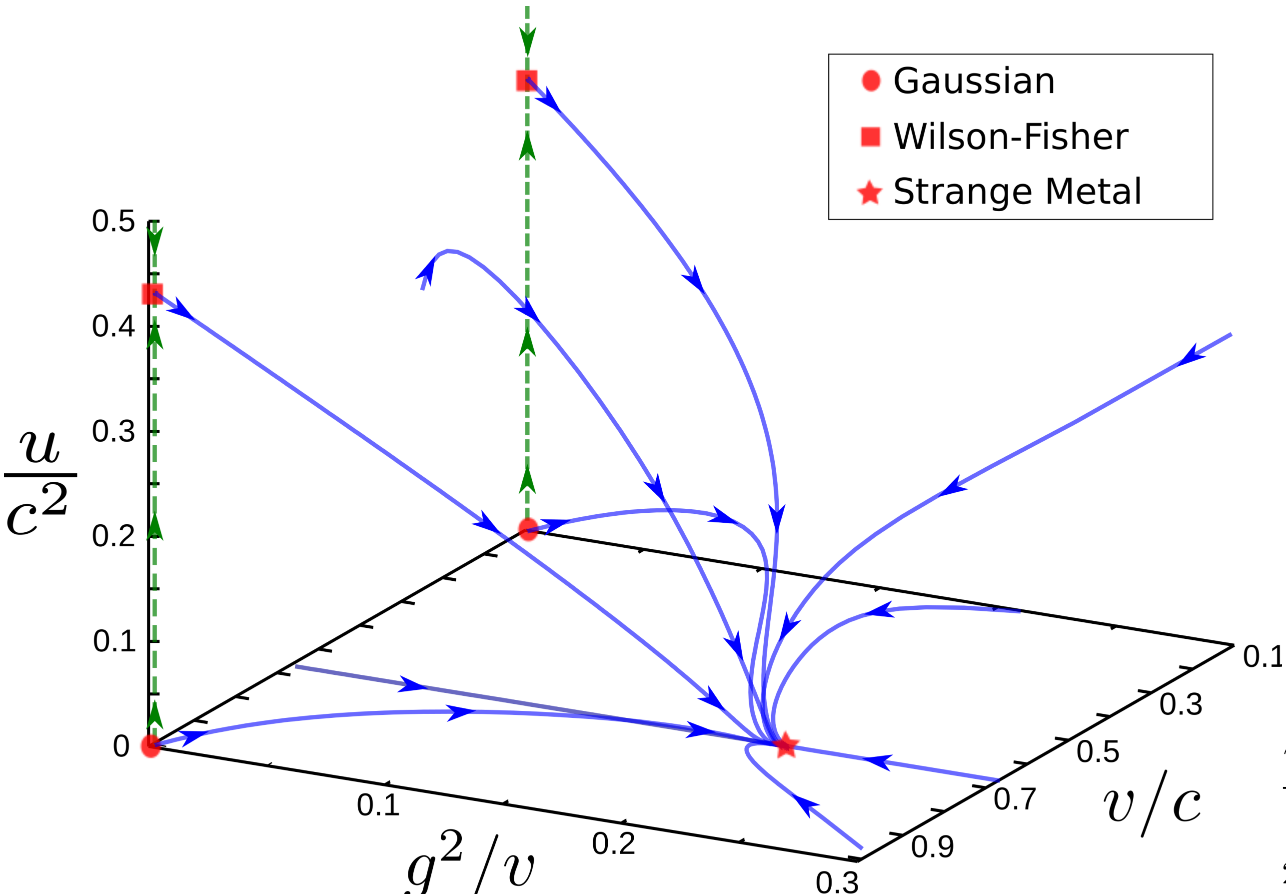


Cycle of negative feedback between (v,c) and (g,u) make them all flow to zero!



One-loop is asymptotically exact in the low energy limit in three dimensions

The kinetic energy & interactions maintain balance as they die



$$\left(\frac{v}{c}\right)^* = \frac{N_c N_f}{N_c^2 - 1}$$

$$\left(\frac{g^2}{v}\right)^* = \frac{4\pi(N_c^2 + N_c N_f - 1)}{N_c^2 + N_c N_f - 3}\epsilon$$

$$\left(\frac{u}{c^2}\right)^* = 0$$

Properties of the IR fixed point

- Interactionless
- Nested FS + dispersionless boson (quasi-local)
  - v,c flow to zero 1/log(L) for d<3
  - v,c flow to zero 1/log(log(L)) at d=3
- Breakdown of Fermi liquid (strange metal)
  - Non-Fermi liquid for d<3
  - Marginal Fermi liquid at d=3
- New form of stable metallic state :

Quasi-Local Strange Metal

[S. Sur, SL (14)]

Spectral functions in QLSM

$$\mathcal{G}(k) \;=\; \frac{1}{|k_y|^{1-2\tilde{\eta}_\psi}}\tilde{G}\left(\frac{\mathbf{K}}{|k_y|^z}\right)$$

For hot spots 1,3

$$\mathcal{D}(k) \;=\; \frac{C}{|\mathbf{K}|^{\frac{2-2\tilde{\eta}_\phi}{z}}}$$

$$z = 1 + \frac{(N_c^2 + N_c N_f - 1)}{2(N_c^2 + N_c N_f - 3)}\epsilon$$

$$\tilde{\eta}_\psi, \tilde{\eta}_\phi \sim O(\epsilon^2)$$

Summary

- Controlled low energy effective field theory for non-Fermi liquids needed

- Dim. Reg. which tunes co-dimension of Fermi surface provides a controlled expansion while avoiding non-locality and UV/IR mixing