

AdS space and its dual theory

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@ Holography and Topology of Quantum Matter

- Brief review on the AdS/CFT correspondence
- holographic two-point correlation function at zero temperature
- holographic two-point correlation function at finite temperature (linear response theory)
- Conclusion

*dH*² AdS geometry⁼ *^r*² *dH*² where *fk*(*r*) is given by

 O - constantly curved space with a negative cosmological constant *^d*¹ ⁼ *du*² + sinh² *u d*⌦² *^d*² (36) *^d*¹ implies the metric of a (*^d* 1)-dimensional hyperbolic space with a unit radius. These and *k* is either or *zumed space* with a negative cosmological constant (*^d* 1)-dimensional flat space, ^R*d*1, while *^d*⌃² *^k* is the metric of a unit sphere, S*d*1, for *k* = 1 or that

 $A = \frac{1}{2}$ cosmological constant Action for AdS_{d+1} \mathbf{C}

$$
S = \frac{1}{16\pi G} \int d^{d+1}x \sqrt{-g} \, (\mathcal{R} - 2\Lambda)
$$

with a negative cosmological constant 56 *x* with a negative cosmo *r ^Rx^µ* (*^µ* = 0*,* ¹*, ··· , d* 1)*.* (29) smological constant

$$
\Lambda = -\frac{d(d-1)}{2R^2}
$$

Its equation of motion (Einstein equation), $0 = \kappa_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\kappa + \Lambda g_{\mu\nu}$, allows an Ads geometry and the state of the s Its equation of motion (Einstein equation), $0 = \mathcal{R}_{\mu\nu} - \frac{1}{2} q_{\mu\nu} \mathcal{R} + \Lambda q_{\mu\nu}$, dation of Korea funded by the Ministry of Education (NRF-2013R1 α) and also by the Ministry of Education (NRF-2013R1A142A10057490) and also by the Ministry of Education (NRF-2013R1A142A10057490) and also by the Ministry $\frac{1}{2}$ R_{max} (Finstein equation) $0 - P$ $\frac{1}{2}$ P_{max} $\frac{1}{2}$ denotes a $\frac{1}{2}$ denotes the metric. Substituting the metric. Substituting the metric of metric of $\frac{1}{2}$ denotes the metric of metric of $\frac{1}{2}$ denotes the metric of $\frac{1}{2}$ denotes the metric of \frac Its equation of motion (Einstein equation), $0=\mathcal{R}_{\mu\nu}-\frac{1}{2}q_{\mu\nu}\mathcal{R}+\Lambda q_{\mu\nu}$, $0 = \mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} + \Lambda g_{\mu\nu}$, $1 - \frac{1}{2}$

$$
ds^{2} = -\frac{r^{2}}{R^{2}}dt^{2} + r^{2} (du^{2} + u^{2} d\Omega_{d-2}^{2}) + \frac{R^{2}}{r^{2}} dr^{2} \qquad \text{where} \quad R = -\frac{d(d+1)}{R^{2}}
$$

AdS(1,d) geometry $\overline{}$

boundary topology.

0 - can be is defined as <u>a hypersurface in an one-dimensional higher flat</u> $space$ denoted by $\mathbf{R}^{2,d}$ The AdS*d*+1 geometry, which is a negatively curved spacetime, is defined as a hypersurface in an s defined as a hupersurface in an one-dimensional higher flat $2^{\log n}$ discussion 1 disc

Proper distance in the (d+2)-dimensional ambient flat space one-dimensional higher flat spacetime denoted by R2*,d*. The proper distance in the ambient flat space is described by $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ cional anabi

$$
ds^2 = -dy_{-1}^2 - dy_0^2 + dy_1^2 + \dots + dy_d^2
$$

shows the same problems discussed in the previous section although the origin of them are di↵erent.

In this section, we first try to clarify the similar problems of the AdS black hole with a non-trivial ω

which is invariant under the $SO(2,d)$ Lorentz transformation *a* (1)

 \cdot Then, the AdS_{d+1} geometry appears as the hypersurface satisfying $-y_{-1}^2 - y_0^2 + y_1^2 + \cdots + y_d^2 = -R^2$ $\begin{array}{ccc} & & \mathcal{A} \\ 2 & & \mathcal{D}^2 \end{array}$

and (*d* + 1)-dimensional flat space is called a maximally symmetric spacetime which does not break

2 Discussion 1 Discussion Since this constraint does not break the Lorentz symmetry of the ambient space, the resulting geometry also preserves the SO(2, d) symmetry which is nothing but the isometry of the AdS_{d+1} space.

a (1)

Rewriting the metric of the ambient space in terms of coordinates Onlisfying the constraint $1 - 1 - 1$ ontisfuing the constraint ambient space in terminates the result space in terms of coordinates the south is not the *S* Sacisfying the constraint *y*² ¹ *^y*² ¹ ⁺ *···* ⁺ *^y*² *^d* ⁼ *R*² where *R* indicates the AdS radius. Since this constraint does not break the Lorentz symmetry of the In order to get the AdS*d*+1 metric, we need to rewrite the metric of the ambient space in terms ambient spacetime, the resuting geometry also preserves the *SO*(2*, d*) symmetry which is nothing but the isometry of the AdS*d*+1 spacetime. Due to this reason, the AdS*d*+1 geometry together with dS*d*+1 and α 1) α 1) α 1) α is called a maximally symmetric space is α not be a maximally symmetric space α Rewriting the metric of the ambient space in terms of coordinates antiching the resuding spacetime, the *SO(2², d)* symmetry which is not the *SO(2^{<i>, d)*} symmetry</sub> which is not his nothing but it is not his nothing \sim satisfying the constraint ambient spacetime, the resuting geometry also preserves the *SO*(2*, d*) symmetry which is nothing but the isometry of the AdS*d*+1 spacetime. Due to this reason, the AdS*d*+1 geometry together with dS*d*+1 the isometry of the AdS*d*+1 spacetime. Due to this reason, the AdS*d*+1 geometry together with dS*d*+1 and (*d* + 1)-dimensional flat space is called a maximally symmetric spacetime which does not break *d* $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ *y*²

where R indicates the AdS radius. Since this constraint does not break the Lorentz symmetry of the L

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where R indicates the AdS radius. Since this constraint does not break the Lorentz symmetry of the L

appears as the hypersurface satisfying the following the following constraints $\mathcal{L}_\mathbf{r}$

which is invariant under the *SO*(2*, d*) Lorentz symmetry. In the ambient space, the AdS*d*+1 geometry

$$
y_{-1} + y_d = R^2 r
$$
 and $y^{\mu} = \frac{r}{R} x^{\mu}$ $(\mu = 0, 1, \dots, d - 1)$

one can reproduce the AdS_{d+1} metric *<u><i>R*₂</sub> $\frac{1}{2}$ $\$ *R*² α (α ²) α (α ²) α I one can reproduce the AdS_{d+1} metric I one can reproduce che one can reproduce the AdS_{d+1} metric $\sum_{i=1}^n \frac{1}{\sigma_i}$ the isometric of the AdS_{d+1} metric spacetime. Due to this reason, the AdS_{d+1} geometric with discrete with

(*^d* 1)-dimensional flat space, ^R*d*1, while *^d*⌃²

appears as the hypersurface satisfying the following the following constraints $\mathcal{L}_{\mathcal{A}}$

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¹ ⁺ *···* ⁺ *^y*²

where R indicates the AdS radius. Since this constraint does not break the Lorentz symmetry of the L

of a hyperbolic space denoted by ^H*d*¹ for *^k* ⁼ 1.

$$
ds^2 = \frac{r^2}{R^2} \left(-dt^2 + d\vec{x}^2 \right) + \frac{R^2}{r^2} dr^2
$$

For more concreteness, let us further consider the explicit representation of AdS*d*+1. For *k* = 0

Using different coordinate, different AdS metrics can be obtained where the different coordinate different ads metrics can be obtained Using different coordinate, different AdS metrics co Using different coordinate, different Ads metrics can boundary topologies. One of the well-known AdS*d*+1 metric is given as *R*² *r*2*fk*(*r*) *metrics can be obtained* where $\frac{1}{2}$ is given by $\frac{1}{2}$ boundary topologies. One of the well-known and the well-known and the well-known and the well-known as α metric is α metric in α metric is α metric in α metric is a metric in α metric in α metric is a In order to get the AdS*d*+1 metric, we need to rewrite the metric of the ambient space in terms Osing different coordinate, different AdS metrics can be obtained
^{2 d}die - ^{R2} ⁼ *r*2*fk*(*r*) *R*2 rdinate, different Ads metric rate, different *^R*² *dt*² ⁺ *^r*2*d*⌃² *^k* + *r*2*fk*(*r*) *dr*2 ferent coordinate. different AdS metrics can be obtained boundary topologies. One of the well-known and well-known as a second well-known as \mathcal{L} metric is given as \mathcal{L}

$$
ds^2 = -\frac{r^2 f_k(r)}{R^2} dt^2 + r^2 d\Sigma_k^2 + \frac{R^2}{r^2 f_k(r)} dr^2
$$

f α ^{*r*} α ^{*r*} α of a hyperbolic space denoted by ^H*d*¹ for *^k* ⁼ 1. \mathbf{w} *here*

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appears as the hypersurface satisfying the following constraints $\mathcal{L}_{\mathcal{A}}$

the isometry of the AdS*d*+1 spacetime. Due to this reason, the AdS*d*+1 geometry together with dS*d*+1

⁰ + *y*²

and (*d* + 1)-dimensional flat space is called a maximally symmetric spacetime which does not break

where *u* is dimensionless and the bulk spacetime is foliated with slices corresponding to the flat *d*-C. Park was supported by Basic Science Research Program through the National Research Foun $d_k(r) = 1 + k \frac{1}{r^2}$ For $k = 1$, $d\Sigma_i^2$ implies the metric of S^{d-1} by a *d*-dimensional Minkowki metric up to the conformal factor, *r*2*/R*2. For *k* = 1, the constraint can ϵ_{cm} l_2 $d\nabla^2$ insuling blue minimize ϵ Ω ⁻¹ Korea Ministry of Education, Science and Technology, Gyeongsangbuk-Do and Pohang City. C_{max} $l = dN^2$ incuring the normal R^{-1} dation of Korea funds by the Ministry of Education (NRF-2013R1 $\,$ *for* $k = 1$, $d\Sigma^2$ implies the metric of S^{d-1} *R*2 and *^k* is either 0 or *[±]*1 relying on the boundary topology. For *^k* = 0, *^d*⌃² *^k* represents the metric of a **For** $k = -1$, $d\Sigma_k^2$ implies the metric of \mathbf{H}^{a-1} $f_k(r) = 1 + k$ *R*2 $\frac{r}{r^2}$ **For** $k = 1$, $d\Sigma_i^2$ implies and *^k* is either 0 or *[±]*1 relying on the boundary topology. For *^k* = 0, *^d*⌃² *k* represents the metric of a **For** $k = -1$ or $d\Sigma_k^2$ implies where For , implies the metric of For more concreteness, let us further consider the explicit representation of AdS*d*+1. For *k* = 0 *^Rx^µ* (*^µ* = 0*,* ¹*, ··· , d* 1)*.* (29) where P^2 for $k = 0$, $d\Sigma_k^2$ implies the metric of \mathbf{R}^{d-1} $f_k(r) = 1 + k \frac{R}{r^2}$ *^k* is the metric of a unit sphere, S*d*1, for *k* = 1 or that of a hyperbolic space denoted by ^H*d*¹ for *^k* ⁼ 1. For $k=-1$, $d\Sigma_i^2$ implies the metric of \mathbf{H}^d of a hyperbolic space denoted by ^H*d*¹ for *^k* ⁼ 1. \sum_{k} implies the metric of S^{a-1} . $\sqrt{N^2}$ in using the eighting $\int \mathbf{H}d-1$ *^y*¹ ⁺ *^y^d* ⁼ *^R*2*^r* and *^y^µ* ⁼ *^r* \mathcal{F}^2 for $k=1$, $d\Sigma_k^2$ implies the metric of \mathbf{S}^d *fk*(*r*)=1+ *k* r^2 , r^2 $\sigma_{\text{max}} = 1 - d\overline{\Sigma^2}$ is the metric of $\overline{\text{rad}}$ For $k=-1$, $d\Sigma_{\rm\scriptscriptstyle L}^2$ implies the metric of *^R*² *dt*² ⁺ *^r*2*d*⌃² $\mathbf{r} \cdot \mathbf{k} = 0$ $\epsilon = 0$. $d\Sigma_i^2$ implies the metric of \mathbf{R}^d *fk*(*r*)=1+ *k R*2 $\kappa = 1$, $a \angle_k$ impries the metric of S^2 \blacktriangleleft For $k=-1$ \bigcirc $d\Sigma_k^2$ implies the metric of \mathbf{H}^d *^k* is the metric of a unit sphere, S*d*1, for *k* = 1 or that $f_k(r) = 1 + k^{\frac{R^2}{r}}$ **k** represents the metric of a representation of \mathbf{r} r^2 **For** $k = 1$, $d\Sigma_k^2$ For more concreteness, let us further consider the explicit representation of AdS*d*+1. For *k* = 0 $f(x) = 0$ $d\Omega^2$ implies the metric of R^{d-1} $f_k(r) = 1 + k \frac{R^2}{r}$ 1 rely is the metric of a point of $\int^{2\pi}$ for $k=1$, $d\Sigma_k^2$ implies the metric of S^{d-1} \Box For $k = -1$, For more concreteness, let us further consider the explicit representation of AdS*d*+1. For *k* = 0 Σ^2_k *^r*² *,* (28) and r^2 for $k=1$, $d\Sigma_k^2$ implies the metric of \mathbf{S}^{d-1} (*^d* 1)-dimensional flat space, ^R*d*1, while *^d*⌃² *^k* is the metric of a unit sphere, S*d*1, for *k* = 1 or that \leftarrow For $k = -1$. and $d\Sigma^2_k$ implies the metric of S^{d-1} (*^d* 1)-dimensional flat space, ^R*d*1, while *^d*⌃² 1 $\bigcup d \Sigma^2_k$ implies the metric of $\| {\bf H}^{d-1} \|_F$

For more concreteness, let us further consider the explicit representation of AdS*d*+1. For *k* = 0

where R indicates the AdS radius. Since this constraint does not break the Lorentz symmetry of the L

ambient spacetime, the resuting geometry also preserves the *SO*(2*, d*) symmetry which is nothing but

which is invariant under the *SO*(2*, d*) Lorentz symmetry. In the ambient space, the AdS*d*+1 geometry

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^k is the metric of a unit sphere, S*d*1, for *k* = 1 or that

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For more concreteness, let us further consider the explicit representation of AdS*d*+1. For *k* = 0

^k is the metric of a unit sphere, S*d*1, for *k* = 1 or that

where *R* indicates the AdS radius. Since this constraint does not break the Lorentz symmetry of the

which is invariant under the *SO*(2*, d*) Lorentz symmetry. In the ambient space, the ambient space, the AdS_d

ambient spacetime, the resuting geometry also preserves the *SO*(2*, d*) symmetry which is nothing but

¹ ⁺ *···* ⁺ *^y*²

, (26)

Ads/CFT correspondence (strong/weak duality) 1 International Section 1980 and 1980

AdS/CFT correspondence

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$$
Z_{gravity} \approx e^{-S_{on-shell}}
$$

- this SUGRA solution is valid in the $\ket{gN \to \infty}$ limit (classical limit, $\mathcal{R} \ll 1$)

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C. Park was supported by Basic Science Research Program through the National Research Foun-

C. Park was supported by Basic Science Research Program through the National Research Foun-

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Korea Ministry of Education, Science and Technology, Gyeongsangbuk-Do and Pohang City.

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 \mathbb{R}^n Ministery of Education, Science and Technology, Gyeongsangbuk, Gyeongsangbuk-Do and Pohang City.

<u>- Large t'Hooft coupling</u> $\lambda = g_{YM}^2 N = gN$ \blacksquare \blacktriangleright - large $\bm{t'}$ Hooft coupling $\lambda=g_{YM}^2 N=gN$. Monperturbatvie gauge theory

Applying the AdS/CFT correspondence to a deformed CFT 1 International Communication 1 International Communication 1 International Communication 1 International Comm z Applying the AdS/CFT correspondence to a deformed CFT Applying the Ads/CFT correspondence to a deformed CFT τ and Ω

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Deform the Ads geometry with a massive scalar field which breaks the conformal symmetry 16⇡*G dar field which bree* 20 Deform the Ads geometry with a massive scalar field which breaks the 2 Pierre Hart 10 and 20 July 2014 and 20 and 20 feet to the boundary Har 1 Introduction 1 1 International Control of the Control of which Dreaks the

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$$
S = \frac{1}{16\pi G} \int d^5 x \sqrt{-g} \left(\mathcal{R} - 2\Lambda - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right)
$$

If $-\frac{4}{R^2} < m^2 < 0$, ϕ corresponds to a relevant operator deforming the dual CFT. $\frac{4}{\pi}$ < *m* **R** 2</u> 2 1 $\frac{1}{2}$ @*M*@*^N* ¹ $\overline{ }$ (1) 16⇡*G* 2 2 16⇡*G* 2 2 and the free energy, *F* = *E THSBH*, is given by $P = \frac{F}{F}$ $P = \frac{F}{F}$ $F(x) = \frac{1}{2}$ *Sbd* ⇠

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 ϕ_0 discussion $\mathcal C$ $\phi=$ $\frac{\phi_0}{r^{4-\Delta}} \ (1+\cdots) + \frac{\mathcal{O}}{r^{\Delta}} \ (1+\cdots) \qquad \text{with} \quad \ 2<\Delta<4 \quad \ (m^2=\Delta)$ \cdot) + $\frac{\mathcal{O}}{r\Delta}(1+\cdots)$ with $2 < \Delta < 4$ ($m^2 = \Delta(\Delta-4)$) $\frac{\mathcal{O}}{r^{\Delta}}$ $(1 + \cdots)$ with $2 < \Delta < 4$ $(m^2 = \Delta(\Delta - 4))$ Near the boundary

when $r \rightarrow 0$, $\phi \rightarrow 0$ which does not affect the UV physics of the dual QFT (relevant). when $r\to 0$, physics of the allat or

C. Park was supported by Basic Science Research Program through the National Research Foun-

dation of Korea funded by the Ministry of Education (NRF-2013R1A1A2A10057490) and also by the

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and the free energy, *F* = *E THSBH*, is given by

- ϕ_0 $\frac{\varphi_0}{\sqrt{2}}$: source of the dual operator : source of the dual operator *^r*4 (1 + *···*) + *^O* 2 Discussion 1972 – 2002 – 2002 – 2003 – 2003 – 2003 – 2003 – 2003 – 2003 – 2003 – 2003 – 2003 – 2003 – 2003 –
2003 – 2003 – 2003 – 2003 – 2003 – 2003 – 2003 – 2003 – 2003 – 2003 – 2003 – 2003 – 2003 – 2003 – 2003 – 2003 Contents
- *^r*4 (1 + *···*) + *^O* 1 Introduction 1 introdu : dual operator
	- *^r* (1 + *···*) (2) Δ Δ : conformal dimension of the dual operator

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2

*m*2²

^r (1 + *···*) (2)

@*M*@*^N* ¹

<u>The holographic renormalization</u> |
|
|-16⇡*G d*5*x* p*^g* <mark>rmalizatio</mark> *^gµ*⌫@*µ*@⌫ ¹ 1 Introduction

The boundary term of the scalar field reduces to d redi 16⇡*G* 60

$$
S_{bd} \sim \int d^4x \sqrt{-g} \ g^{rr} \phi \ \partial_r \phi
$$

$$
\approx \int d^4x \left(\frac{\phi_0^2}{r^{4-2\Delta}} + \phi_0 \mathcal{O} + \frac{\mathcal{O}^2}{r^{2\Delta - 4}} + \cdots \right)
$$

near the boundary $(r \to 0)$ divergence finite term

near the boundary ($r \to 0$) divergence finite term and \bigcap *m* $\frac{1}{2}$ divergence finite term

 $\frac{1}{\sqrt{1-\frac{1}{$ In order to obtain a finite generating functional, we need to add an addition counter term. $2^{\frac{1}{2} \cdot \cdot \cdot \cdot}$ In order to obtain a finite generating functional, we need to add an

After the appropriate holographic renormalization, and the free energy, *F* = *E THSBH*, is given by After the appropriate holographic renormalization,

$$
Z = e^{-S_{on-shell}}
$$

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1 International Production Control in the United States of the United States of the United States of the Unite

Z

16⇡*G*

^R ²⇤ ¹

p*^g*

*d*5*x*

*d*5*x*

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with

$$
S_{on-shell} = S_{CFT} - \int d^4x \phi_0 \mathcal{O}
$$

which corresponds to the generating functional of the dual QFT. (*m*² ⁼ (4)) which corresponds to the generating functional of the dual QFT. ا correspond: **2** *g* to the genero 2 ting ncti \cdot \overline{a} *d*5*x* p*^g* QFT.

One-point correlation function from the generating functional
\n
$$
\frac{\partial Z}{\partial \phi_0} = \langle \mathcal{O} \rangle|_{\phi_0=0}
$$

Acknowledgement C. Park was supported by Basic Science Research Program through the National Research Foundation of Korea fund by the Ministry of Albert of Albert of Albert of Albert (NRF-2013) and also by the also by the Albert of Albert n-point correlation function can be obtained by applying n-derivatives with respect to the source to the above generating functional 2 Grandina por bought the delivered by *S*
S
SCCCC CAN \overline{z} *d*200*obtained by applying n-derivatives*

$$
\frac{\partial^n Z}{\partial \phi_0^n} = \langle \mathcal{O}^n \rangle|_{\phi_0 = 0} \qquad \qquad \bigcirc
$$

Two-point correlation function <u>-point</u> *<u>xo-point</u>* **correlation function</u>** 2 Discussion Concenting to the continuation of the continuatio

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For convenience, introduce a new coordinate, $\,z=1/r\,$ 16⇡*G* ¹ p*^g* -or convenience, introduce a no 2 discussion 1 discussion 2 discussion 2 discussion 2 discussion 2 discussion 2 discussion 2 onvenience, introduce a new coordinate, $z = 1/r$

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Then, the AdS metric becomes Then. the Ads metric becomes

$$
ds^2=\frac{dz^2-dt^2+d\vec{x}^2}{z^2}
$$

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Sonshell = *SCFT*

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and the massive scalar field is governed by ² ssive *d*5*x* p*^g* eld is gover:
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 $0 = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \phi(z,t,\vec{x}) \right) - m^2 \phi(z,t,\vec{x}) \quad ,$ dation of \mathcal{N} and \mathcal{N} $\omega \omega \mathcal{L}_{\phi}(z, \omega, 0, \omega)$ y $\varphi_0(\omega, \omega)$ References \mathcal{L} $\phi_0(0, x')$ with Bulk-to-boundary propagator with the National Research Propagator dation of α funding α funding α funding α $(x^2 + (x - x')^2)$ AdS *^S* ⁼ ¹ 1 $\frac{1}{\sqrt{-g}}\partial_{\mu}\left(\sqrt{-g}g^{\mu\nu}\partial_{\nu}\phi(z,t,\vec{x})\right)-m^{2}\phi(z,t,\vec{x})$ *z* = 1*/r* $\phi(z, x)$ Acknowledgement C. Park was supported by Basic Science Research Program through the National Research Foundation of Korea funded by the Ministry of Education (NRF-2013R1A1A2A10057490) and also by the Solution $m^2\phi(z,t,\vec{x})$ 0(0*, x <u>DUCCOR</u> z, x*; 0*, x* $(2, 0, 1)$ and the free energy, *F* = *E THSBH*, is given by \mathcal{Y} $\phi(z,x)$ $\phi(z, r) = \int d^4r' \mathcal{D}_\perp(z, r; 0, r') d\phi(0, r)$ $\phi_0(0, x')$ 2 Discussion \mathbf{v}_{\perp} *R* 2 2 1 \mathbf{r} *^gµ*⌫@*µ*@⌫ ¹ with <u>Bulk-to-</u> oundary propagator and \overline{O} ν_{ϕ} $(z, x; 0, x')$ \sim ✓ *z* $z^2 + (x - x')^2$ $D_{\phi}\left(z, x; 0, x'\right) \sim \left(\frac{z}{z-1}\right)^{\Delta}$ $\overline{}$ *^gµ*⌫@*µ*@⌫ ¹ $\overline{1}$ 16⇡*G* and the free energy, $\frac{1}{2}$ $\phi(z,x) = \int d^4x' \mathcal{D}_{\phi}$ $(z, x; 0, x') \phi_0(0, x')$ $)$ <u>a sa salah sahi</u> <u>Indary propagator</u>

Near the boundary ($z \to 0$), 1 introduction 1 introdu

 $\mathcal{D}_1(z, x: 0, x')$ the bulk-to-boundary propagator reduces to *d*4*x* 0 *D z, x*; 0*, x* 0 0(0*, x* ν_ϕ $(z, x; 0, x')$ \approx z^{Δ} $\mathcal{D}_{\phi}\left(z, x; 0, x^{\prime}\right) \approx \frac{1}{(x-x^{\prime})^{2\Delta}}$ $\sqrt{2\Delta}$ b *k bul* k-to-boundary propagato r red \mathbf{r} $\overline{\mathcal{L}}$ es 16⇡*G* $\mathbf{z}^{\mathbf{\Delta}}$ of the bulk-to-boundary propagator reduces to 1 Introduction

(*z, x*) *z* ! 0 which leads to

and the free energy, *F* = *E THSBH*, is given by

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Z

16⇡*G*

$$
\lim_{z \to 0} \phi(z, x) = z^{\Delta} \int d^4 x' \frac{\phi_0(0, x')}{(x - x')^{2\Delta}}
$$

^gµ⌫@*µ*@⌫ ¹

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C. Park was supported by Basic Science Research Program through the National Research Foun-

che asymptotic behavior of $\varphi(z,x)$ is given by $\phi(z, x) \approx \phi_0(x) z^{4-\Delta} (1 + \cdots) + \mathcal{O}(x) z^{\Delta} (1 + \cdots)$ $\sqrt{2}$ $\mathcal{L}(\mathcal{L})$ (*z, x*) *z* ! 0 Noting that the asymptotic behavior of $\phi(z,x)$ is given by and the free energy, *F* = *E THSBH*, is given by and the asymptotic $\varphi(z, x) \sim \varphi_0(x)$ λ $(1 + \cdots) + \mathcal{O}(x)$ λ $(1 + \cdots)$

we can obtain \mathbb{R}^2

$$
\mathcal{O}(x) \approx \int d^4x' \frac{\phi_0(0, x')}{(x - x')^{2\Delta}}
$$

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 $F⁺$ $F⁺$ $S_{bd}\sim$ z
Z d^4x Z $d^4x' \frac{\phi_0(0,x)\phi_0(0,x')}{\sqrt{2\Delta}}$ $\frac{4}{x}x^{\prime} \frac{\varphi_{0}(0, x)\varphi_{0}(0, x)}{(x-x^{\prime})^{2\Delta}}$ Then, the boundary (on-shell) action of matter becomes *Sbd* ⇠ *d*4*x d*4*x* ⁰ 0(0*, x*)0(0*, x*⁰) f matter becomes
 $\sqrt{2}$ *O*(*x*) ⇡ $(x - x')^{2\Delta}$

S =

*d*5*x*

 \cup

✓

R 2

and the free energy, *F* = *E THSBH*, is given by

As a consequence, the two-point correlation function reads ac correlation
0 $\begin{array}{c}\n\alpha \\
\alpha\n\end{array}$ (alation function reads

$$
\langle \mathcal{O}(x)\mathcal{O}(x')\rangle = \frac{\partial}{\partial \phi_0(0, x')}\frac{\partial}{\partial \phi_0(0, x)}Z = \frac{1}{(x - x')^{2\Delta}}
$$

^R ²⇤ ¹

^gµ⌫@*µ*@⌫ ¹

(*z, x*) *z* ! 0 which is exactly the two-point function expected in the CFT.

dation of $\mathcal{O}(\mathcal{A})$ fund by the Ministry of Education (NRF-2013R1 $\mathcal{O}(\mathcal{A})$

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dation of Korea funded by the Ministry of \mathbb{R}^2

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and *and plack hole geometry* symmetric space is called a maximally symmetric space is \mathcal{A} Ads black hole geometry It is the that there exist and there exist and the there is a matter localized solution. Suppose that the support of the support of

Equation of motion for a black hole \circ $\qquad \qquad$ $\qquad \qquad$ $\qquad \qquad$ $\qquad \qquad$ Equation of motion for by a di↵erent vacuum solution known as a black hole solution. The black hole metric has the same Equation of motion for a black hole space. and (*d* + 1)-dimensional flat space is called a maximally symmetric spacetime which does not break

It is worth noting that there exist and there exist and there is a matter localized solution. Suppose that the \mathcal{L}_{max}

$$
0 = \mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} + \Lambda g_{\mu\nu}
$$

 2 discussion 1 discussio

ambient spacetime, the resuting geometry also preserves the *SO*(2*, d*) symmetry which is nothing but

the isometry of the AdS*d*+1 spacetime. Due to this reason, the AdS*d*+1 geometry together with dS*d*+1

Black hole geometry
\n
$$
ds^2 = -\frac{r^2 f_k(r)}{R^2} dt^2 + r^2 d\Sigma_k^2 + \frac{R^2}{r^2 f_k(r)} dr^2
$$

with du₂ + <u>i</u> *^k*(*u*) *d*⌦²

where R indicates the AdS radius. Since this constraint does not break the Lorentz symmetry of the L

ambient spacetime, the resuting geometry also preserves the *SO*(2*, d*) symmetry which is nothing but

 3.2 Black hole topologies with distribution \sim

 \bigcup

$$
f_k(r)=1+k\frac{R^2}{r^2}-\frac{m}{r^d}
$$

Acknowledgement C. Park was supported by Basic Science Research Program through the National Research Foundational Research Foundational Research Foundational Research Foundational Research Foundational Research Foundational Research Foun where m is the black hole mass. and *k* is the black had boundary to be a second topology. For $\frac{1}{2}$ relying $\frac{1}{2}$ (*d* \overline{u} is the space more *k* is the metric of a unit sphere, Sd1, for a unit *fk*(*r*)=1+ *k* $\frac{1}{2}$ *r*_{*d*} *<i>r***d** *<i>r***d** *<i><i>r <i>d* *****<i><i>d <i><i>d <i><i>d <i><i>d* *****<i>d <i>d <i>d***** *<i>d <i>d <i>d***** and *k is the blank* had a represent to be a second topology. For *k k a*^{*z*} *k d z k k d z k k d z k k d z k k d z k k d z k k d z k k d z k k d z* (*^d* 1)-dimensional flat space, ^R*d*1, while *^d*⌃² *k* is the metric of a unit sphere, $\frac{1}{2}$ or the sphere, $\frac{1}{2}$ o

From now on, we focus on the case with $k=0$ for simplicity. of a hyperbolic space denoted by ^H*d*¹ for *^k* ⁼ 1. From now on, we focus on the case with $k=0$ for simplicity.

^Rx^µ (*^µ* = 0*,* ¹*, ··· , d* 1)*.* (29)

Contents

- Black hole horizon, r_h , where g_{tt} vanishes 2 Discussion 1982 – 200 personality in the control of the control of the control of the control of the control of
- Hawking radiation (Hawking temperature) $2 \cos \theta$ derriperature 1 international control in the control of the control of

$$
T_H = \frac{1}{4\pi} \left. \frac{\partial g_{tt}}{\partial r} \right|_{r=r_h}
$$

- Bekenstein-Hawking entropy (area law) 1 Introduction

$$
S_{BH} = \frac{A}{4G}
$$

dation of \mathcal{A} funding by the Ministry of Education (NRF-2013R1A2A10057490) and also by the Ministry of Education (NRF-2013R1A10057490) and also by the Ministry of Education (NRF-2013R1A2A10057490) and also by the Min

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s succsfies one enermologicismic caw (macroscopic) and also by the Ministers of T - Black hole physics satisfies the thermodynamic law (macroscopic)

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Mystery of black hole

- 1. Area law of the Bekenstein-Hawking entropy
- 2, What is the microscopic origin of the Bekenstein-Hawking entropy?

't Hooft proposed the holographic principle. The AdS/CFT correspondence is the realization of the holography

- 1. the area can be matched to the volume of the dual QFT (extensive).
- 2. the Bekenstein-Hawking entropy then counts the degrees of freedom of the dual QFT

AdS/CFT correspondence of a black hole geometry

Pure Ads space \longleftrightarrow groud state of dual QFT at zero temperature

Ads black hole \longleftrightarrow dual QFT at finite temperature

Black hole thermodynamics
2 Discussion 1 *^k* is the metric of a unit sphere, S*d*1, for *k* = 1 or that Contents

*^r*² *,* (28)

contents to the contents of the

 k represents the metric of k

 $\overline{}$

For
$$
k = 0
$$
 and $d = 4$,
 $f_0(r) = 1 - \frac{m}{r^4}$

- *r* Black hole horizon : r_h - Black hole horizon : $r_h = m^{1/4}$ Contents of the contents of th
	- Hawking temperature : $T_H = \frac{r_h}{\pi D^2}$ $\frac{h}{\pi R^2}$ $\overline{10}$
- where ⌘*µ*⌫ denotes a *d*-dimensional Mikowski metric. Substituting these relations into the metric of - Bekenstein-Hawking entropy : $S_{BH} = \frac{V}{4C} r_h^3$ with enstein-Hawking entropy : $S_{BH} = \frac{V}{4G} r_h^3$ with $V = \int d^3x$ $24G$ 2 Discussion 1

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dation of Korea funded by the Ministry of Education (NRF-2013R1A1A2A10057490) and also by the Ministry of Education (NRF-2013R1A1A2A10057490) and also by the Ministry of Education (NRF-2013R1A142A10057490) and also by the

- The first law of thermodynamics : $dE=T_H\ dS_{BH}$ 2 Discussion - The first law of thermodynamics : $dE = T_H \ dS_{BH}$ I'm I'm just the 1 International Control of the United States

2 Discussion 1

 10 Introduction 1 introd

*^r*⁴ (1)

1 Introduction 1

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dation of K \mathcal{A}

- internal energy :
$$
E = \frac{3V}{16\pi GR^2} m = \frac{3V}{16\pi GR^2} T_H^4
$$

by a *d*-dimensional Minkowki metric up to the conformal factor, *r*2*/R*2. For *k* = 1, the constraint can \mathbf{M} K_{max} \sim Park was supported by Basic Science V_{max} dation of K^2 funded by the Ministry of $16\pi GR^2$ ¹¹ free energy : C. Park was supported by Basic Science Research Program through the National Research Foun- $\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2}}$ $\frac{16\pi}{4}$ \circ (2) \circ - free energy : $F=E-T_HS_{BH}=-\frac{V}{16\pi GR^2}T_H^4$ *^H.* (2)

Holographic renormalization 2 Discussion 12 Discussion 12 District 2000 and 2000 and

How can Black hole thermodynamics be identified with that of the dual QFT?

 1 introduction 1 introdu

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From the AdS/CFT correspondence $Z=e^{-S_{on-shell}}=\left\langle e^{-S_{CFT}}\right\rangle$

one can evaluate the nonperturbative partition function (or generating
C *F* $\frac{1}{2}$ $\frac{1}{2}$ ¹⁶⇡*GR*² *^T*⁴ functional) of the dual QFT and then extract many physical information from it

Now, let us calculate the on-shell gravity action. When evaluating the on-shell gravity action, we encounter several problems.

- \sum - not well-defined variation
- divergence

*d*1 bons-Hawking term $\overline{\mathcal{A}}$ metrics with di $\overline{\mathcal{A}}$ and a vacuum solution of a gravity theory with a negative theory wi

 \circ and constant In the gravity action that is a problem of the gravity action

 $\left($

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Contents of the Contents of t

$$
S = \frac{1}{16\pi G} \int d^{d+1}x \sqrt{-g} \, (\mathcal{R} - 2\Lambda)
$$

the curvature scalar involves the control of the c
1 Introduction of the control of th
 the Euclidean scalar involved

$$
\mathcal{R} = (\partial^{\mu}g_{\mu\nu})^2 + \cdots + \partial^{\mu}\partial^{\nu}g_{\mu\nu}
$$

Although this boundary term does not affect the bulk equation of motion, it requises problem for defining the variation of the boundary action.
 A causes problem for defining the variation of the boundary action.

*^d*² (36)

$$
\delta S = \delta S_{bulk} + \delta S_{bd}
$$

metric form in (27) with a black hole factor. If we denote the mass denote the mass density of the localized ma
If we denote the localized matter as we denote the localized matter as we denote the localized matter as we de and the free energy, \overline{P} = \over equation of motion In general, the variation of the variation of the gravity action is not well defined at the boundary, so that the well-

m, the get rid of such a problematic term, one shoul 2 Discussion of the contract o *F* a problematic term, one one snould add a Glivons-Hawking cerm
 σ To get rid of such a problematic term, one should add a Gibbons-Hawking term

*R*2

*dr*² (39)

$$
S_{GB} = \frac{1}{8\pi G} \int_{\partial M} d^4 x \sqrt{\gamma} \mathcal{K} \tag{1}
$$

where μ is an induced metric on the boundary and μ indicates the trace of the extrinsic curvature, the extrinsic cur

and the free energy, *F* = *E THSBH*, is given by Counter terms

Although the Gibbons-Hawking term leads to a well-defined action variation, the on-shell gravity action usually diverges at the boundary due to the $\frac{1}{2}$ integration over r . *^F* ⁼ *^E ^THSBH* ⁼ *^V* 1 variation_,
0 ^Fles *^H.* (2) unh the τ ibbons-Hauleinn term leads to a well-defined action variation 2 Discussion over 1.1

 $\overline{}$ $\overline{\$

² ⁺ *···* ⁺ @*µ*@⌫*gµ*⌫ (1)

$$
S = \frac{1}{16\pi G} \int d^4x \int_0^\infty dr \sqrt{-g} \frac{(\mathcal{R} - 2\Lambda)}{\sqrt{\Lambda}}
$$

$$
= \frac{1}{16\pi G} \int d^4x \int_0^\infty dr \, r^3 \left(-\frac{8}{R^2}\right)
$$

On the dual QFT, the previous divergence corresponds to the UV divergence which should be removed by adding appropriate counter terms similar to the \odot renormalization procedure of an ordinary QFT. H<u>olographic renormalization</u>

boundary terms called the counterterms, which get rid of the divergences of the divergences of the on-shell gravity α $S_{ct} =$ 1 $8\pi G$ Z @*M* $d^4x\sqrt{\gamma}$ $\sqrt{3}$ $\frac{6}{R}$ + $\left(\frac{R}{4}\mathcal{R}^{(4)}\right)$ Counter terms for the AdS geometry *d*4*x* <u>unter terms for the AdS ge</u> 16⇡*G* Counter terms for the AdS geometry $S_{ct} = \frac{1}{8\pi}$ boundary terms called the counterterms, which get rid of the divergences of the divergences of the on-shell gravity Ω $\overline{1}$ $J \partial f$ Z $\sqrt{\gamma} \left(\frac{\sigma}{R} \right)$ $_{+}$ $\frac{R}{-}$ $\frac{1}{4}$ ^{κ} $\int_{\mathcal{M}} d^4x \sqrt{\gamma} \left(\frac{3}{R} + \frac{R}{4} \mathcal{R}^{(4)} \right)$ \mathcal{S}_{S} 8⇡*G* @*M d*4*x* p \ddot{a} *^R* ⁺

 t hell gravity action is identified with the generating functional of the dual quantum field theory. The dual quantum field the

 \mathcal{S} since the on-shell gravity action usually diverges at the asymptotic boundary, we need a diverges at the asymptotic boundary, we need a symptotic boundary, we need a symptotic boundary, we need a symptotic bounda

 \mathcal{S} since the on-shell gravity action usually diverges at the asymptotic boundary, we need a diverges at the asymptotic boundary, we need a symptotic boundary, we need a symptotic boundary, we need a symptotic bounda

 $\mathcal{R}^{(4)}~$: intrinsic curvature of the boundary space R^4 merusic curvature of the boundary space

² (r*µn*⌫ + r⌫*nµ*), with a unit normal vector *nµ*. According to the AdS/CFT correspondence,

 $t_{\rm{max}}$ and gravity action is identified with the dual quantum functional of the dual quantum field theory. The dual quantum f

^p00*T*00*,* (61)

^p*iiTii,* (62)

^p00*T*00*,* (61)

^p00*T*00*,* (61)

^p*iiTii,* (62)

Shell Shell States and States α the renormalized action *Sre* is given by $S_{re} \equiv \beta F = S_G + S_{GH} + S_{ct}$ The renormalized on-shell gravity action $F = S_G + S_{GH} + S_H$

Energy-momentum tensor of the dual QFT

dation of \mathcal{O} funding \mathcal{O}

Then, the internal energy and pressure in an *i*-direction are defined as

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1 Introduction

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Z

^Kµ⌫ ⁼ ¹

$$
T_{\mu\nu} \equiv -\frac{2}{\sqrt{\gamma}} \frac{\delta F}{\delta \gamma^{\mu\nu}} = \frac{1}{8\pi G} \left[-\mathcal{K}_{\mu\nu} + \gamma_{\mu\nu} \mathcal{K} + \frac{3}{R} \gamma_{\mu\nu} - \frac{R}{2} \left(\mathcal{R}_{\mu\nu}^{(4)} - \frac{1}{2} \gamma_{\mu\nu} \mathcal{R}^{(4)} \right) \right]
$$

Dual QFT of the Ads black hole for k=0 2 Discussion 1 discussio

¹ International thermodynamic quantities derived from the holographic renormalization

 \sim

 $\overline{}$

 \mathcal{L}

$$
F = \frac{S_{re}}{\beta} = -\frac{V}{16\pi GR^2}m
$$

\n
$$
E = \int d^3x \sqrt{\gamma} \gamma^{00} T_{00} = \frac{3V}{16\pi GR^2}m
$$

\n
$$
P^i = -\int d^3x \sqrt{\gamma} \gamma^{ii} T_{ii} = \frac{V}{16\pi GR^2}m
$$

1 International Communication 1 international control of the communication 1 international control of the communication 1 international control of the communication of the communication of the communication of the communic

which perfectly matches with the black hole thermodynamics $(m=T^4_H)$.

- *R*(*k*) (3) *R*(*k*) (3) *R*(*k*) *R*(*k*) *R*(*k*) *R(k) <i>R(x)* (3) *R(x) R(x)* (3) *R(x)* (3) (3) (3) (3) (3) (3) (3) (3) (3) (- the above results consistent with the black hole thermodynamic law
- the thermal entropy exactly coincides with the Bekenstein-Hawking entropy

linear response theory

In the low energy limit,

after applying a time varying source, we can investigate the linear response of the medium

 $\langle \mathcal{O} \rangle_{\text{OFT}} = -\chi \partial_t \phi_0(t)$

Transport coefficients

- typical parameters of an effective low energy description
- Once they are specified, they completely determine the macroscopic behavior of the medium

Ex) Transport coefficient of the electromagnetic theory

applying a time dependent vector potential in x-direction to conductor

$$
J_x = \sigma \underbrace{\partial_t A_x(t)}_{(= E_x)} \qquad \sigma : \text{conductivity}
$$

In general, the exact microscopic theory of the AdS/CFT correspondence is not known except several cases (N=4 SYM, ABJM, and their deformations).

In such a situation, macroscopic properties governed by thermodynamics and linear response theory do not rely on the microscopic details and become helpful to understand the dual QFT.

Two-point function at finite temperature τ compaignt function of finite temperature 3 Discussion 1

We can also apply the holographic technique to obtain a two-point function at zero temperature. In this case, it should be notee that the existence of a black hole requires a different boundary condition (incomng boundary condition) unlike the zero temperature case. 1 Introduction

$$
\Phi = \phi^{(in)} + \phi^{(out)}
$$

 2 Contents 1 Contents

 \overline{a} Contents 1 Con

with
\n
$$
\phi^{(out)} = (\phi^{(in)})^* \qquad \text{(uniform)}
$$

In-coming boundary condition

$$
\phi^{(out)}\quad =\quad 0
$$

⌘⇤

⇣

2 Contents 1982 - Contents 198
2 Contents 1982 - Contents 19

A.1 Longitudinal modes: *At*(*z*) and *Ay*(*z*) 1

(*in*) breaks the unitarity (leads to dissipation)

 $G_{unitary} = G_{advanced} + G_{retarded}$ Chroom function at the outgoing-wave condition at the condition of the advanced Green's Conditions of the advanced Gr $G_{unitary} = G_{advanced} + G_{retarded}$ the retarded and advanced \sim Green function 3 Discussion 1 $G_{unitary} = G_{advanced} + G_{retarded}$

1 Introduction 1

 G

$$
G^{R}(\omega, \mathbf{q}) = -i \int d^{4}x \, e^{-iq \cdot x} \, \theta(t) \langle [\hat{\mathcal{O}}(x), \hat{\mathcal{O}}(0)] \rangle \,,
$$

$$
G^{A}(\omega, \mathbf{q}) = i \int d^{4}x \, e^{-iq \cdot x} \, \theta(-t) \langle [\hat{\mathcal{O}}(x), \hat{\mathcal{O}}(0)] \rangle \,.
$$

In reference that the three steps outlined above index on the three steps of the correct outlined above the correct outlined above in the correct outlined above in the correct outlined above in the correct outline of the c Transport coefficient

Green function

 $\overline{}$

$$
\chi = \frac{\text{Im}\left[G_{retarded}\right]}{\omega} = \frac{1}{\omega} \left\langle \mathcal{O}\mathcal{O}\right\rangle
$$

with

with
$$
\langle \mathcal{O} \mathcal{O} \rangle = \frac{\partial^2 S_{on}}{\partial \phi_0 \partial \phi_0}
$$

prescription at hand is sufficient for the purpose of this paper.

 1 introduction 1 introdu

Transport coefficients of the dual QFT

Turn on the U(1) vector fluctuations on the AdS black hole background

$$
S_M=-\frac{1}{4g_4^2}\int d^4x\sqrt{-g}F^{\mu\nu}F_{\mu\nu}
$$

After taking the $A_r = 0$ gauge and the following Fourier mode expansions

$$
A_i(t, \mathbf{x}, r) = \int \frac{d\omega d^2q}{(2\pi)^3} e^{-i(\omega t - \mathbf{q} \cdot \mathbf{x})} A_i(\omega, \mathbf{q}, r)
$$

the momentum only along y direction like $q = (0, q)$ with

the equations of motion can be divided into two parts: the longitudinal and transverse one

Longitudinal modes A_t and A_y

$$
0 = b2 \omega A'_{t} + g q A'_{y}
$$

\n
$$
0 = b2 A''_{t} + 2bb' A'_{t} - \frac{1}{g} (q \omega A_{y} + q^{2} A_{t})
$$

\n
$$
0 = g A''_{y} + g' A'_{y} + \frac{1}{g} (\omega q A_{t} + \omega^{2} A_{y}),
$$

\n
$$
0 = A''_{t} + \frac{F'(z)}{F(z)} A''_{t} + \frac{\tilde{\Lambda}^{2}}{F^{2}(z)} [\tilde{\omega}^{2} - F(z)\tilde{q}^{2}] A'_{t}
$$

- Transverse mode A_x

$$
0 = A''_x + \frac{g'}{g}A'_x + \frac{1}{g^2} \left[\omega^2 - q^2 \frac{g}{b^2} \right] A_x
$$

In the <u>hydrodynamic limit</u> ($\tilde{\omega} << 1$ and $\tilde{q}^2 << 1$), we can solve these equations perturbatively

$$
A'_t(z) = (1 - z^d)^{\nu} G(z),
$$

$$
G(z) = G_0(z) + \tilde{\omega} G_1(z) + \tilde{q}^2 G_2(z) + \tilde{\omega} \tilde{q}^2 G_3(z) + \cdots
$$

Boundary conditions

- Incoming BC at the horizon, which breaks the unitarity of the dual QFT.
- Dirichlet BC at the asymptotic boundary, which fixes the source of dual QFT.

The retarded Green function of longitudinal modes

in the low frequency and low momentum limit (hydrodynamic limit), the retarded Green functions become

The longitudinal Green function has a

charge diffusive pole governed by the

following dispersion relation

$$
\omega = -iDk^2, \qquad k = k_z
$$

with $D = \frac{\tilde{\Lambda}}{T_H} = \frac{(-\Lambda)}{16\pi T_H} \frac{(4+\eta^2)}{4-\eta^2}$

the charge diffusion constant

- The charge diffusion constant implies that the quasi normal mode (charge current of the dual QFT) eventually diffuses away back into the thermal equilibrium with a half-life time

$$
t_{1/2} = \tfrac{1}{D-q^2}
$$

The retarded Green function of transverse mode

$$
\mathcal{G}_{xx} = \frac{1}{g_4^2} \left[i\omega - \left(\frac{\tilde{\Lambda}}{T_H} \right) q^2 \right]
$$

- There is no pole.
- The DC conductivity is given by

$$
\sigma = \lim_{\omega \to 0} \text{Re}\left(\frac{\mathcal{G}_{xx}}{i\omega}\right) = \frac{1}{g_4^2}
$$

Important holographic results

In RHIC experiment, quark-gluon plasma showed a very small ratio between the shear viscosity and thermal entropy.

Intriguingly, the holographic calculation leads to such a small ratio

- The AdS/CFT correspondence provides a new way to understand a strongly interacting QFT.

- Although the microscopic details of the dual QFT are not known, many macroscopic properties can be understood by the holographic methods.

- It would be helpful to understand various macroscopic properties of the real physical systems in nuclear and condensed matter physics.

THANK YOU FOR YOUR ATTENTION !