## Bosonization and Mirror Symmetry



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(based on arXiv:1608.05077 with Mulligan, Torroba, Wang; see also our earlier paper arXiv:1506.01376)

## I. Introduction

Duality in quantum field theory has been a powerful tool in many contexts.

Since the mid-1990s, studies of duality in particle theory focused on supersymmetric QFTs.

\* mirror symmetry of 2d sigma models



A Calabi-Yau and its mirror



# $*$  Seiberg duality of 4d N=1 gauge theories

 $SU(N_c)$ ,  $N_f$  flavors  $\leftrightarrow SU(N_f - N_c) N_f$  flavors + meson

However, the subject predates fancy constructions in string theory and supersymmetric QFT.

Kramers-Wannier duality of the Ising model and many other examples arise in condensed matter physics. (More relevant to talk: bosonization/Luttinger liquid).

Today, I'll give a very brief description of a derivation of a duality of interest in condensed matter, starting from a classic example of mid 1990s supersymmetric dualities.

The duality I'll be focusing on relates a theory of free fermions to a scalar QED3 theory.

One place where such dualities may be relevant is in understanding the fractional quantum Hall effect.



Electrons moving in a magnetic field can be dressed by a certain number of flux quanta each. The resulting quasiparticle sees different flux density and in general has different statistics, which can be helpful.

# \* Can map FQHE to IQHE \* Can map 1/2 - filled LL to (non)-Fermi liquid (?)

In fact our work was motivated by duality conjectures made roughly in that context, by D.T. Son and by Metlitski, Senthil, Vishwanath, and Wang. c.f. Senthil, Seiberg, Witten, Wang; Karch, Tong

II. Mirror symmetry of 3d N=4 gauge theories

We will start with a well studied and (fairly) rigorously understood duality from high-energy physics, the mirror symmetry of 3d N=4 gauge theories.

These can be thought of as the dimensional reductions of (perhaps more familiar) 4d N=2 theories.

Hypermultiplet :  $(q, \psi_q), (\tilde{q}, \psi_{\tilde{q}})$ The two types of supermultiplets that arise are: Vectormultiplet :  $(A_\mu, \sigma, \phi), (\lambda, \psi_\phi)$ 

## N=4 theories have moduli spaces of vacua whose basic structure is:



Higgs and Coulomb branches, which are hyperKahler manifolds. We will not have mixed branches today.

# Mirror symmetry is a symmetry of pairs of 3d N=4 gauge theories A and B, where

 $Higgs(A) = Coulomb(B)$  $Higgs(B) = Coulomb(A)$ 

Intriligator, Seiberg

# Example:

We will be satisfied today with using just the simplest, prototypical example of 3d mirror symmetry.

> Theory A: Free hypermultiplet Theory B: QED with one charged hyper

## The Lagrangian of theory A is  $T_{\text{max}} = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=$

) 1 21

*,* (2.2)

( +*,* ⇤

$$
L^{(\mathrm{A})} = \sum_{\pm} \left( |\partial_{\mu} v_{\pm}|^2 + i \bar{\Psi}_{\pm} \not{\partial} \Psi_{\pm} \right)
$$

The +/- labels the charge under an important symmetry. A table of the fields and their transformations under the **various symmetries is:**  $\Box$  The  $\Box$  Ishele the ghoupe waden on imposition to where  $\Box$ convenient to consider the following linear combination of symmetries: *U*(1)*<sup>R</sup>* ⌘ *U*(1)*<sup>L</sup>* and *A* table of the fields and their transformations under



### The Lagrangian and table of symmetries of theory B: interactions between the charged hypermultiplet and the emergent vector multiplet and the emergent vector mult<br>Interactions between the emergent vector multiplet: which is a strategic multiplet: which is a strategic multip *e*<sup>2</sup>⇡*i/g*<sup>2</sup>

interactions between the charged hypermultiplet and the emergent vector multiplet:

 $L^{(\mathrm{B})} = L_V(\mathcal{V}) + L_H(\mathcal{Q}, \mathcal{V}),$  $\begin{array}{ccc} \n\mathbf{r} & \mathbf{r} & \$  $g^2$  (4<sup>- $\mu$ </sup> 2<sup>-1</sup>, 3) (2<sup>-2</sup>)  $L_V(\mathcal{V}) = \frac{1}{a^2}$  $g^2$  $\left(-\frac{1}{4}\right)$ 4  $f_{\mu\nu}^2$  + 1 2  $(\partial_{\mu}\phi_{ij})^2 + i\bar{\lambda}_{ia}\ \partial_{\lambda_{ia}} +$ 1 2  $D^2_{(ab)}$  $\begin{bmatrix} - & -\nu & \nu & \nu + \nu & -\nu & \nu & \nu \end{bmatrix}$ *g*2  $\begin{array}{c} \n\overline{\phantom{a}} \\
1\n\end{array}$  $\overline{c}2$ *f* 2 *<sup>µ</sup>*⌫ +  $\frac{1}{2}$  $\mathbf{c}$ (@*µij* ) <sup>2</sup> <sup>+</sup> *<sup>i</sup>*¯*ia* <sup>6</sup>@*ia* <sup>+</sup> *D*<sup>2</sup>  $\mathcal{Y}$  and the Lagrangian reads of the Lagrangian reads  $\mathcal{Y}$  $I_{\alpha}^{(B)} = I_{\alpha}^{(D)}(V)$ *ia* ⌘ (*,* ) 2 20  $L_V(V) = \frac{1}{a^2} \left( -\frac{1}{4} f_{\mu\nu}^2 + \frac{1}{2} (\partial_\mu \phi_\nu) \right)$ ) 2 10

(2.8)

 $\varphi_H(z;\nu) = |\nu\mu\omega_a| + \varphi\varphi_t \varphi_t \varphi_t - \varphi_{ij}|\omega_a| + \varphi_{ij} \varphi_t \varphi_t$ *LH*(*Q, V*) = *|Dµua|*  $\mu_1$  +  $i\psi_i$   $\psi\psi_i$  –  $\psi_{ij}$  |  $a_{a}$  | –  $\psi_{ij}\psi_i\psi_j$  = *ij |ua|* 2(*iiau*⇤ *<sup>a</sup> <sup>i</sup>* + h*.*c*.*) + *D*(*ab*)*u*⇤ *<sup>a</sup>u<sup>b</sup> .* (2.9)  $L_H(Q, V) = |D_\mu u_a|^2 + i \bar{\psi}_i \, D \psi_i - \phi_{ij}^2 |u_a|^2 - \phi_{ij} \bar{\psi}_i \psi_j + \sqrt{2} (i \lambda_{ia} u_a^* \psi_i + \text{h.c.}) + D_{(ab)} u_a^* u_b.$ *U*(1)*<sup>a</sup>* symmetries are given in (2.17).



 $\alpha$ 

Slogan: following  $U(1)$ *J* charge, particles of theory A are vortices of theory B.

The moduli spaces are quite simple:

The tree level moduli space in theory A is  $\mathbb{R}^4$ . As the theory is free, there are no corrections.

On the other hand, theory B has a Coulomb branch parametrized by  $\sigma, \phi$  and the dual photon. Its geometry receives quantum corrections.

In the IR limit where  $g\to\infty$  , there is a symmetry exchanging these two moduli spaces of vacua.

#### <sup>+</sup> -1 1 0 1 an *N* = 2 vector multiplet *V* and chiral multiplet . The hypermultiplet lagrangian for a More formal formulation: *v*<sup>+</sup> 1 -1 1 *v* 1 -1 -1

*u* 0 1 0 -1

#### mote the topological U background vector multiplet.  $W$ **e** can promote the top **olc**  $\overline{a}$ We can promote the topological U(1) to a full

$$
\mathcal{L}^{(A)}(\mathcal{Q},\hat{\mathcal{V}}_J) = \mathcal{L}^{\mathcal{H}}(\mathcal{Q},\hat{\mathcal{V}}_J) = \int d^4\theta \left( V_+^\dagger e^{2\hat{V}_J} V_+ + V_-^\dagger e^{-2\hat{V}_J} V_- \right) + \int d^2\theta \sqrt{2} i \hat{\Phi}_J V_+ V_- + \text{h.c.}
$$

#### In theory B, it enters subtly through a BF term:  $P$ <sub>*r*</sub> *r heor* 4*g*<sup>2</sup>  $\frac{1}{2}$  it onto  $\epsilon$ *g*2 *d*4 ✓ *†* (2.9) ✓ ◆  $\blacksquare$ [*V*ˆ*<sup>J</sup>* ] = <sup>Z</sup> *<sup>D</sup><sup>Q</sup>* exp ✓ Z ◆ In theory B, it enters subtly through a BF term:

$$
\mathcal{L}^{(B)}(\mathcal{U},\mathcal{V},\hat{\mathcal{V}}_J)=\mathcal{L}^{\mathcal{V}}(\mathcal{V})+\mathcal{L}^{\mathcal{H}}(\mathcal{U},\mathcal{V})-\mathcal{L}^{\mathcal{N}=4}_{BF}(\mathcal{V},\hat{\mathcal{V}}_J)\,.
$$

$$
\mathcal{L}_{BF}^{\mathcal{N}=4}(\mathcal{V}^{(1)},\mathcal{V}^{(2)}) = \frac{1}{2\pi} \int d^4\theta \, V^{(1)} \, \Sigma^{(2)} - \frac{1}{2\pi} \int d^2\theta \, \Phi^{(1)} \Phi^{(2)} + \text{h.c.}
$$

$$
\mathcal{L}_{BF}^{\mathcal{N}=2}(V^{(1)}, V^{(2)}) = \frac{1}{2\pi} \int d^4\theta \, V^{(1)} \, \Sigma^{(2)}
$$
\n
$$
= \frac{1}{2\pi} \left( \epsilon^{\mu\nu\rho} A_{\mu}^{(1)} \partial_{\nu} A_{\rho}^{(2)} + D^{(1)} \sigma^{(2)} + D^{(2)} \sigma^{(1)} + \frac{1}{2} (\bar{\lambda}^{(1)} \lambda^{(2)} + \bar{\lambda}^{(2)} \lambda^{(1)}) \right)
$$
\n
$$
\text{Max. August 22. 16}
$$

where  $22, 16$ <br> $\blacksquare$ Monday, August 22, 16

In this fancier formulation, the formal statement of mirror symmetry is that  $formulation, the$  $\int$  formal statement of mirror Mirror symmetry states that the partition functions of theory A and B are the same:

> $Z^{(A)}[\hat{\mathcal{V}}_J] = Z^{(B)}[\hat{\mathcal{V}}]$  $J$   $J$   $.$

 $\bigcap_{x \in \mathbb{R}}$  and  $\bigcap_{x \in \mathbb{R}}$  is to consider what happens when we branch parametrized by *v±*) maps to the moduli space of theory B – the Coulomb branch parametrized by the scalars *, ,* . Our plan now is to consider what happens when we similarly promote the other U(1) symmetries.

 $\mathsf{values}$  of their  $\hat{\sigma}, \hat{D}$  fields. We can then consider perturbations by background  $\hat{\mathcal{D}}$ 

We now consider an extension of mirror symmetry that includes *U*(1)*<sup>A</sup>* and *U*(1)*<sup>R</sup>* background This will lead to supersymmetry breaking, and allow us to infer a non-supersymmetric duality.

is simple: both sides of the mirror pair have a conserved *U*(1)*<sup>A</sup>* current, so the partition

function should agree also in the presence of a background gauge field that couples to the

III. Perturbations to the basic N=4 duality  $2\frac{1}{2}$  Deptemberiane to the bacic

We now consider an extension of mirror symmetry that includes *U*(1)*<sup>A</sup>* and *U*(1)*<sup>R</sup>* background

A. Promoting other global symmetries  $\blacksquare$  first discuss the simpler case of global non-R-symmetries. The basic observation observation observation

is simple: both sides of the mirror pair have a conserved *U*(1)*<sup>A</sup>* current, so the partition

function should agree also in the presence of a background gauge field that couples to the

Using the tables of charges, it is straightforward to see what happens when we promote the  $U(1)_A$  symmetry to  $\; \Big|$ a full background multiplet  $\hat{V}_A$ :  $\hat{V}$ *A* U(1)<br>A current thange of the fact of the straightforward to see avec the superfield.  $a$  full background multiplet  $\hat{V}_A$ :  $\mathbf{v}$  superfield.  $\frac{1}{2}$  che cables of charges, it is suraightforward to see  $\lambda$ at happens when we promote the  $U(1)_A$  symmetry to

$$
\mathcal{L}^{(A)}(\mathcal{Q},\hat{V}_A) = \int d^4\theta \left( V_+^\dagger e^{2(V-\hat{V}_A)}V_+ + V_-^\dagger e^{-2(\hat{V}+\hat{V}_A)}V_- \right)
$$

*.* (2.21)

✓ *V* ⌃ˆ *.* (2.22)

$$
\mathcal{L}^{(B)}(\mathcal{U}, \mathcal{V}, \hat{V}_A) = \frac{1}{4g^2} \int d^2 \theta \, W_\alpha^2 + \text{h.c.} + \frac{1}{g^2} \int d^4 \theta \Phi^\dagger e^{-4\hat{V}_A} \Phi + \int d^4 \theta \left( U_+^\dagger e^{2(V + \hat{V}_A)} U_+ + U_-^\dagger e^{-2(V - \hat{V}_A)} U_- \right) - \frac{1}{2\pi} \int d^4 \theta \, V \, \hat{\Sigma} \, .
$$

*U*<sup>+</sup> + *U†*

*e*2(*<sup>V</sup> V*ˆ*A*)

*U*

s<br>S

1

z<br>Z

*d*4

 $\overline{a}$ 

z<br>Z

*d*4 ✓ ⇣

*U†*

+*e*2(*<sup>V</sup>* <sup>+</sup>*V*ˆ*A*)

parametrized by the scalars *, ,* .

Promoting the R-symmetry is a bit harder, because it does not commute with supersymmetry. However, the basic elements of the map are easy to infer:

 $*$   $\hat{A}_R$  couples to  $j_R$  on both sides of the duality



 $^*$  The coupling to  $\hat{\sigma}_R$  is more subtle. Think of the 3d theory as a dimensional reduction of a 4d theory. Then this field is the 4th component of the gauge field, and couples to the 4th component of the appropriate current. With this understanding, the mirror duality is promoted to:

$$
Z^{(A)}[\hat{V}_J, \hat{V}_A, \hat{V}_R] = Z^{(B)}[\hat{V}_J, \hat{V}_A, \hat{V}_R].
$$

#### It is important to stress that (2.24) holds as long as the mass scales associated to the **b.** First step: a simpler N=2 theory **b** B. First step: a simpler N=2 theory Because theory A is free, these symmetries are exact. (The R-symmetries do not commute **SUSY SUSY SUSY SUP.** A SIMPLE IN FLATHEORY

interacting fixed point description. Let us note one immediate consequence of (2.24) that will be important below. Consider a point in the phase of background coupling where some of the some of the fermionic some of the fermionic field single chiral multiplet. Looking at the charges. As the first step on our road to a non-supersymmetric duality, lets break the N=4 to an N=2 theory with a single chiral multiplet. Looking at the charges:  $\Delta$ c the first step on our road to a non-supersymmetric convenies to component to consider the following lines of symmetric  $U$ **UUAIILY, IELS DI EAR LIIE INTT LO AII INTZ LIIEOI Y WILII A**<br>Alialah suka compilation in the commute with SUSY of the communist with SUSY of the communist of the commute of single chiral multiplet. Looking at the charges:



(2.12)

2.4 General mirror duality

grounds,

### we see it will be interesting to consider a perturbation of the form We see it will be interest 3.1 Chiral theory A

$$
|\hat{\sigma}_A - \hat{\sigma}_J| \ll \hat{\sigma}_A \sim \hat{\sigma}_J.
$$

*|*ˆ*<sup>A</sup>* ˆ*<sup>J</sup> |* ⌧ ˆ*<sup>A</sup>* ⇠ ˆ*<sup>J</sup> .* (3.1)

*<sup>A</sup>* + ˆ*<sup>J</sup> ,* (3.2)

*<sup>J</sup> ,* (3.2)

*A*.

 $\mathcal{L}(\mathcal{S})$ 

Since we know that  $\sigma$  fields coupled to charged scalars via:

 $-2 \times 21 \times 12$  $\sigma^2q^2|\phi|^2$  $\overline{O}$  **g**  $|\varphi|$ 

*A*. Our goal is the state is the control is the contribution of the state is the control of the contr *A*<br>its superpartner) The remaining light fields & charges: type couplings, this will give a large mass to  $\,v_{-}\,$  (and its superpartner). The remaining light fields & charges: Within the e↵ective theory, we will denote ˆ*A,J* = ˆ*A,J* for notational simplicity. v *y* pure coupling will streated by the service of the s



*V*<sup>+</sup> 1 -1 1

with *|*ˆ*A,J |* ⌧ ˆ<sup>0</sup>

*A,J |* ⌧

## The resulting theory A Lagrangian for light fields is: **A**, the e $\frac{1}{2}$  and the e $\frac{1}{2}$  and the e $\frac{1}{2}$

$$
\mathcal{L}_{\text{chiral}}^{(A)} = |D_{\hat{A}_J - \hat{A}_A + \hat{A}_R} v_+|^2 - \left( (\hat{\sigma}_J - \hat{\sigma}_A + \hat{\sigma}_R)^2 + \hat{D}_J - \hat{D}_A \right) |v_+|^2 + i \bar{\Psi}_+ \mathcal{D}_{\hat{A}_J - \hat{A}_A} \Psi_+ - (\hat{\sigma}_J - \hat{\sigma}_A) \bar{\Psi}_+ \Psi_+ + \frac{1}{8\pi} k_{MN}^{(A)} \hat{A}_M d\hat{A}_N,
$$

### Here,  $\overline{\phantom{a}}$ *L*(*A*) chiral = *|DA*ˆ*JA*ˆ*A*+*A*ˆ*<sup>R</sup>* 2 (ˆ*<sup>J</sup>* ˆ*<sup>A</sup>* + ˆ*R*)

where *A*ˆ*<sup>M</sup>* = (*A*ˆ*<sup>J</sup> , A*ˆ*A, A*ˆ*R*) and the Chern-Simons "K-matrix,"

comes from integrating out the component of the superfield *V*.

$$
\hat{A}_M = (\hat{A}_J, \hat{A}_A, \hat{A}_R)
$$

$$
k_{MN}^{(A)} = \text{sgn}(\hat{\sigma}_A^0) \begin{pmatrix} -1 & -1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}
$$

CA *,* (3.5)

break SUSY. The chiral theory in (3.4) is stable as long as scalar *v*<sup>+</sup> mass-squared is non-

comes from integrating out the component of the superfield *V*.

<sup>+</sup> (ˆ*<sup>J</sup>* ˆ*A*) ¯ <sup>+</sup> <sup>+</sup> <sup>+</sup>

<sup>9</sup> Non-zero *D*ˆ*<sup>J</sup>* or *D*ˆ*<sup>A</sup>*

<sup>9</sup> Non-zero *D*ˆ*<sup>J</sup>* or *D*ˆ*<sup>A</sup>*

1

CA *,* (3.5)

(ˆ*<sup>J</sup>* ˆ*<sup>A</sup>* + ˆ*R*)

*MN <sup>A</sup>*ˆ*MdA*ˆ*<sup>N</sup> ,* (3.4)

negative:

The k-matrix arises from integrating out the fermion  $\Psi_-$ ; in general integrating out a fermion induces a change in the k-matrix  $\sqrt{2}$  $d$ *Urix dr*i *k* **Example 20 A**  $\boldsymbol{\mu}$  and  $\boldsymbol{\mu}$  are the set of  $\boldsymbol{\mu}$ *<sup>i</sup>* under *U*(1)*<sup>i</sup>* produces a *N* = 2 Chern-Simons term:  $\frac{1}{2}$  motive origan from *<sup>A</sup>*) ¯ <sup>+</sup> <sup>+</sup> +  $\overline{9}$ *MN A <sup>M</sup>dA <sup>N</sup> ,* (3.4) where *A*  $\Phi$  =  $\Phi$  =  $\Phi$  =  $\Phi$  =  $\Phi$  =  $\Phi$  $\Psi_{-}$ ; in general integrating out a fer  $\mathbb{R}$ -Thau IX

$$
k_{ij} = \frac{1}{2} \sum_f q_i^f q_j^f \operatorname{sgn}(m_f)
$$

CA *,* (3.5)

ˆ

Our convention for the fermion mass sign is *<sup>L</sup> m<sup>f</sup>* ¯*<sup>f</sup> <sup>f</sup>* .  $\mathbf{F}$ out a single fermion. This will also clarify the statement of the dualities in  $(1.4)$ Under further perturbations, vacuum stability of this theory will require comes from integrating out the component of the superfield *V*. <sup>9</sup> Non-zero *D* bridt furtier perturbations, vacuum stabinty of this stable as stable as scalar *v* 

$$
m_{v_{+}}^{2} = (\hat{\sigma}_{J} - \hat{\sigma}_{A} + \hat{\sigma}_{R})^{2} + \hat{D}_{J} - \hat{D}_{A} \ge 0.
$$

### eta-invariant and *A* and *g* are the gauge field and metric to which the fermion couples. We Weilch wise,  $v_{+}$  will condense and flat away to the space **g**  $GULOII$ . Otherwise,  $v_+$  will condense and run away to the cutoff.

3.2 Chiral theory B

<sup>2</sup> ⌘(*A,* 2*,*1) = <sup>1</sup>

negative:

unstable.

a SUSY Chern-Simons lagrangian at level *k* corresponds to

We can map over the perturbation to theory B as well. The analysis is a bit more involved, but in the end the light fields are the  $u_-, \psi_- \hspace{0.1in}$  multiplet along with a (shifted) sigma field. The effective Lagrangian is:

$$
\mathcal{L}_{\text{chiral}}^{(B)} = \frac{1}{2g_{\text{eff}}^2} \left( (\partial \tilde{\sigma})^2 + D^2 \right) + |D_{-a+\hat{A}_A} u_-|^2 + \bar{\psi}_- i \not{D}_{-a+\hat{A}_A - \hat{A}_R} \psi_- - ((\tilde{\sigma} - \hat{\sigma}_A)^2 - D + \hat{D}_A) |u_-|^2 \n- 8\pi \bar{\psi}_- \psi_- |u_-|^2 - (-\tilde{\sigma} + \hat{\sigma}_A - \hat{\sigma}_R) \bar{\psi}_- \psi_- + \frac{1}{8\pi} (a + \hat{A}_A - \hat{A}_R) d(a + \hat{A}_A - \hat{A}_R) \n+ \frac{1}{4\pi} (\tilde{\sigma} + \hat{\sigma}_A) (D + \hat{D}_A) - \frac{1}{8\pi} \hat{A}_R d\hat{A}_R - \frac{1}{8\pi} (2\hat{A}_A - \hat{A}_R) d(2\hat{A}_A - \hat{A}_R) \n- \frac{1}{2\pi} (\hat{A}_J da + \hat{D}_J \tilde{\sigma} + D \hat{\sigma}_J).
$$
\n(3.16)

*<sup>A</sup>* that were turned on in the UV. The last terms of the second line The duality we've exhibited here is one between a  $\frac{1}{2}$  out of  $\frac{1}{2}$  the gaugino,  $\frac{1}{2}$  and  $\frac{1}{2}$   $\frac{1}{2$ to the background *U*(1)*<sup>J</sup>* fields. It is now straightforward to integrate out *D*, yielding the a charged chiral multiplet. single free chiral N=2 superfield, and N=2 QED with

*|u|*

<sup>2</sup> +

 $\frac{1}{4}$ 

(˜ + ˆ*<sup>A</sup>* 2ˆ*<sup>J</sup>* )

*.* (3.17)

<sup>2</sup> <sup>+</sup> *<sup>g</sup>*<sup>2</sup>

2

*V* chiral

<sup>e</sup>↵ = (˜<sup>2</sup> <sup>+</sup> *<sup>D</sup>*ˆ*A*)*|u<sup>|</sup>*

### C. Breaking supersymmetry The demonstration of the highest process by a supersymmetry

#### The final step is to break supersymmetry. We do this by turning on a  $\hat{D}_J$  term. *J* (ˆ*<sup>A</sup>* ˆ*<sup>J</sup>* ) From the state of the state remains as a light field. <sup>+</sup> is massless at the critical point and obtains a mass *m* <sup>+</sup> break supersymmetry. We do this From the quadratic lagrangian in (3.4), *v*<sup>+</sup> is heavy and may be integrated out, but <sup>+</sup>

<sup>2</sup> *.* (4.1)

= ˆ*J*ˆ*<sup>A</sup>*

In theory A, as  $v_+$  carries positive topological charge, this results in a massless Fermi theory: *In theory A. as*  $v_1$  *carries positive topological charge.* symmetry. The critical theory has the e $\sim$ and the critical point  $\mathbf{A}$   $\mathbf{A}$  In theory A, as  $v_+$  carries positive topological charge,

away from the critical point. We refer to these two massive phases as the ˆ*<sup>J</sup>* ˆ*<sup>A</sup> >* 0 and

$$
\hat{\mathcal{L}}_{\text{Dirac}}^{(A)} = \bar{\Psi}_{+} i \not{D}_{\hat{A}_{J} - \hat{A}_{A}} \Psi_{+} - m_{\Psi_{+}} \bar{\Psi}_{+} \Psi_{+} + \frac{k_{MN}^{\text{crit}}}{8\pi} \hat{A}_{M} d\hat{A}_{N}
$$

$$
k_{MN}^{\text{crit}} = \begin{pmatrix} -1 & -1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
$$

 $\mathbb{R}^n$ 

 $\mathcal{A}^{\mathcal{A}}_{\mathcal{A}}$  ,  $\mathcal{A}^{\mathcal{A}}_{\mathcal{A}}$ 

We can consider varying away from the critical point by slightly detuning  $\hat{\sigma}_A - \hat{\sigma}_J$ , giving a mass to the fermion. The resulting k-matrices are: Setting *A*ˆ*<sup>A</sup>* = *A*ˆ*<sup>R</sup>* = 0 and renaming <sup>+</sup> = and *A*ˆ*<sup>J</sup>* = *A*ˆ, we find the left-hand side of (1.4) e can consiger **v** htly datuning  $\hat{\sigma}_A - \hat{\sigma}_I$  giving a mass to  $\tau$ ン<br>つ  $k$ *-matrices are:* 

$$
k_{MN}^{(A)} = \begin{pmatrix} -1 & -1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + sgn(\hat{\sigma}_J - \hat{\sigma}_A) \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
$$



Now, we can consider the same perturbation to theory B. This theory is strongly coupled. But we can use macroscopic considerations: combination *<sup>a</sup>* <sup>+</sup> *<sup>A</sup>*ˆ*<sup>A</sup> <sup>A</sup>*ˆ*R*. Adding these to the topological terms in (3.16), we obtain **P** and the same perturbation to the same perturbation to thus matching the K-matrix in Eq. (3.25) for ˆ*<sup>J</sup> >* ˆ*<sup>A</sup> >* 0.

becomes massive and integrating it out produces a level-1/2 Chern-Simons term for the simons term for the simon

- \* Duality implies that there is a single critical point as we vary  $\hat{\sigma}_A - \hat{\sigma}_J$  in the small range we consider. *L*<br>Arv  $\overline{\phantom{a}}$  ${\mathcal T} _{\not \perp}$  $\hat{\sigma}$ <sup>*T*</sup> in the small range we conside
- \* A simple calculation tells one that the k-matrix is: where the term proportional to the step function with ⇥(*x >* 0) = 1 and ⇥(*x <* 0) = 0 is

$$
\mathcal{L}_{CS}^{(B)} = \frac{1}{8\pi} \text{sgn}(m_{\psi_{-}})(-a + \hat{A}_{A} - \hat{A}_{R})d(-a + \hat{A}_{A} - \hat{A}_{R}) - \Theta(-m_{u_{-}}^{2})(-a + \hat{A}_{A})^{2} + \frac{1}{8\pi} \left[ (a + \hat{A}_{A} - \hat{A}_{R})d(a + \hat{A}_{A} - \hat{A}_{R}) - 4\hat{A}_{A}d\hat{A}_{A} + 4\hat{A}_{A}d\hat{A}_{R} - 2\hat{A}_{R}d\hat{A}_{R} - 4\hat{A}_{J}da \right]
$$

## \* Dual theories must have matching k-matrices!

## The only reasonably parsimonious phase diagram for theory B is then



Figure 2: Phase diagram of theory B. Phases I-III are separated by second order critical

points (indicated by the solid blue line). Setting *A*ˆ*<sup>A</sup>* = 0, the transition at ˆ*<sup>A</sup>* = ˆ*<sup>J</sup>* represents

the point across which the Chern-Simons level for *A*ˆ*<sup>J</sup>* changes by unity. The horizontal axis

### The resulting critical theory has:  $\begin{aligned} \mathcal{T}_{\mathsf{A}} \mathsf{C} \mathsf{C} \end{aligned}$  and the moduli spaces of the moduli space  $\mathcal{T}_{\mathsf{A}} \mathsf{C} \mathsf{C}$ B which are (partially) parameterized by h*|v*+*|* <sup>2</sup>i and h˜i. Because there is no breaking of

 $2$  and h $2$  and h $2$  and  $\bar{B}$  and  $\bar$ 

\$ N<sup>f</sup> scalars coupled to *U*(*N*)*k,k*; (1.1)

\$ N<sup>f</sup> scalars coupled to *U*(*N*)*k,k±<sup>N</sup> .* (1.3)

; (1.2)

B which are (partially) parameterized by h*|v*+*|*

the *U*(1)*<sup>R</sup>* ⇥ *U*(1)*<sup>A</sup>* ⇥ *U*(1)*<sup>J</sup>* global symmetry in either phase – h*|v*+*|*

The last remaining field to consider is  $\mathcal{R}$  . Recall that is  $\mathcal{R}$  that is  $\$ 

*N*+

2

 $A$ s required by duality, the phase diagram in  $\mathbb{R}^n$  for the phase diagram in  $\mathbb{R}^n$  matches that of theory  $\mathbb{R}^n$ 

 $\frac{4}{\sqrt{2}}$ 

description for an integer quantum Hall plateau transition: the point across which the Chern-

<sup>2</sup> *<sup>|</sup><sup>|</sup>*

$$
\mathcal{L}^{(B)}_{\text{sQED3}} = |D_{-a + A_A} u_-|^2 - m_{u_-}^2 |u_-|^2 - \lambda_{u_-} |u_-|^4 + \frac{1}{4\pi} a da - \frac{1}{2\pi} \hat{A}_J da - \frac{1}{4\pi} \hat{A}_A d\hat{A}_A.
$$

#### obtains from integrating out massive fields. Setting *<sup>A</sup>*ˆ*<sup>A</sup>* <sup>=</sup> *<sup>A</sup>*ˆ*<sup>R</sup>* = 0 and renaming *<sup>u</sup>* <sup>=</sup> ' Setting some of the background fields <u>፡</u> *d* f the 2⇡ *<sup>A</sup>*ˆ*<sup>J</sup> da* <sup>1</sup> 4⇡ **A** A *A A* Setting some of the background fields

consistency checks wherein conjectured dual pairs have matching phase structure [9] or may

*U*(*N*)*k,l* ⌘ (*SU*(*N*)*<sup>k</sup>* ⇥ *U*(1)*N l*)*/*Z*<sup>N</sup>* . (1.1) - (1.3) have been validated in the large *N* 't Hooft

$$
\hat{A}_A = \hat{A}_R = 0
$$

and doing some re-naming, we have shown:  $A_{\rm 12,2}$  has shown, if  $(12,29)$  is assumed, various additional dualities additional dualit In this paper, we derive the *N<sup>f</sup>* = *N* = *k* = 1 versions of (1.1) and (1.2) and find that  $t_{\text{all}}$  and doing some re-naming, w

$$
\bar{\Psi}i\rlap{\,/}D_{\hat{A}}\Psi-\frac{1}{8\pi}\hat{A}d\hat{A}\leftrightarrow|D_{-a}\varphi|^2-|\varphi|^4+\frac{1}{4\pi}ada-\frac{1}{2\pi}\hat{A}da
$$

\$ ¯*iD/ <sup>a</sup>* <sup>1</sup>

8⇡

*ada* <sup>1</sup>

2⇡

the scalar in the QED3 theory B.

and *A*ˆ*<sup>J</sup>* = *A*ˆ, we recover the right-hand side of (1.4).

N<sup>f</sup> fermions coupled to *U*(*k*)

description near the critical point,

sQED3 =*|Da*+*A<sup>A</sup> u|*

The e↵ective mass-squared *m*<sup>2</sup>

#### **This is the promised duality between a free Fermi theory** and scalar QED3. consistency checks where  $\sim$  material pairs  $\sim$  matrix  $\sim$  matr  $\mathbf{u}_0$  obtained upon definition of better-understood supersymmetric (SUSY) parameters  $\mathbf{v}_1$ This is the promised duality between a free Fermi theory I strong dynamics of the theory B description. The theory B description of the theory B description of the theory B description. The theory B description of the theory B description of the theory B description. The theory B of (1.4). Our arguments are rather general and help us temperature and help us temperature interesting, but subtle, but subtle strong dynamics of the theory B description.

We note that this approach is purely 2+1D in nature. We do not consider theories arising

*U*(*N*)*k,l* ⌘ (*SU*(*N*)*<sup>k</sup>* ⇥ *U*(1)*N l*)*/*Z*<sup>N</sup>* . (1.1) - (1.3) have been validated in the large *N* 't Hooft

at the critical point in the critical point in the theory B description and allow us to deduce the right-hand side  $\mu$ 

massive phases.<sup>3</sup> These two requirements uniquely constrain what field must become light

and a matching of the e $\alpha$ ective actions for various background gauge fields in the nearby fields in the near

As argued by Senthil, Seiberg, Witten and Wang, starting from this duality, and using SL(2,Z) arguments, one can derive several further dualities: In this paper, we derive the *N<sup>f</sup>* = *N* = *k* = 1 versions of (1.1) and (1.2) and find that  $\sim$  20 Star citing in OIII critical duality, a ¯ *iD/ <sup>A</sup>*<sup>ˆ</sup> <sup>1</sup> *Ad*<sup>ˆ</sup> *<sup>A</sup>*<sup>ˆ</sup> \$ *<sup>|</sup>Da*'*<sup>|</sup>* <sup>2</sup> *<sup>|</sup>*'*<sup>|</sup>* <sup>4</sup> + l Turtne<br>*. Address.*  $\rho$  is proper regularization is proper regular in the set of the starting from this duality and using SI (2, arguments one can derive several further dualities.  $\mathcal{I}_1$  , and if  $\mathcal{I}_2$  is assumed, a large web of dualities can be found a large web of dualities can be found assumed, a large web of dualities can be found assumed, a large web of dualities can be found assumed, a  $\mathbf{A}$  is purely 2-1D in nature. We do not consider the nature. We do not consider the nature. We do not consider the  $\mathbf{A}$ As argued by senthil, selberg, vvitten and vvang, a gaments, one can genre several it (an eller gaancies.

$$
|D_{\hat{A}}\phi|^2 - |\phi|^4 + \frac{1}{4\pi} \hat{A} d\hat{A} \leftrightarrow \bar{\psi} i \not{D}_a \psi - \frac{1}{8\pi} a da - \frac{1}{2\pi} \hat{A} da.
$$
  

$$
|D_{\hat{A}}\phi|^2 - |\phi|^4 \leftrightarrow |D_a\phi|^2 - |\phi|^4 - \frac{1}{2\pi} \hat{A} da,
$$

$$
\bar{\Psi}i\rlap{\,/}D_{\hat{A}}\Psi - \frac{1}{8\pi}\hat{A}d\hat{A} \leftrightarrow \bar{\Psi}i\rlap{\,/}D_{-a}\Psi + \frac{1}{8\pi}ada + \frac{1}{2\pi}bda + \frac{2}{4\pi}bdb - \frac{1}{2\pi}\hat{A}db.
$$

#### and a level-1/2 Chern-Simons term in the derivation of the dynamic therefore the dynamical gauge field. In the derivation of the dynamical gauge of the dynamical gauge of the dynamical gauge of the dynamical gauge of the d are in the simon of and BF couplings that ensure the simon coupling to the simon coupling to  $\frac{1}{2}$ Prior work studying proposals closely related to (1.4) and (1.5) includes [15–18].  $\overline{8}$ *Ad*<sup>ˆ</sup> *<sup>A</sup>*<sup>ˆ</sup> \$ ¯ *iD/ <sup>a</sup>* <sup>+</sup> ion of one one is the  $\overline{\mathcal{S}}$  and the division requires that the di $\overline{\mathcal{S}}$  equality actions the phase transition must match. In this way, regularization-dependent counterterms cancel out. Understanding the derivation of any one is therefore 3 sufficient.

*Adb .* ˆ (1.7)

[13, 14].