On the AdS/dS CFT Correspondence

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Outline

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1.1. Motivation: physics of dS QFT

Quantum theories on curved spacetime are of immense physical importance. e.g.predictions of inflation or semiclasscial physics of blackholes; studies of phase transitions;

1. The definition of Hamiltonian and particles for a QFT is generally difficult in a generic curved spacetime. In some case like deSitter, this can be solved partially use perturbation theory. But one may expect new phenomena in the strongly coupled regime.
2. For example, for dS space, there are infrared divergences $(\log a(t))^n$, leading to large secular effects which break down the perturbative theory at late times. The understanding of the late time secular effects in de Sitter space is an important problem.
   Polyakov speculated that the IR effect to be the key to resolve the cosmological constant problem. Polyakov (1982)
   This calls for a treatment beyond the usual perturbation scheme.

Holographic correspondence may help, especially for the strongly coupled non-perturbative regime of dS QFT.

1.1. Motivation: dS CFT/QFT as hologram

- It is known that AdS space can be sliced in different ways, with the geometry on each slices described by a Minkowski space, de Sitter space, or anti-de Sitter space. This has been used in brane world construction.

  Park, Pope, Sadrzadeh 2001

- In the standard AdS/CFT correspondence, Poincare coordination is used and the boundary is given by Minkowski space. The hologram is a conformal field theory on Minkowski space.
- If we use different slicing, e.g. de Sitter slicing, presumably we would get conformal field theory on dS space as the hologram. This is interesting.

- Gauge/gravity dual for de Sitter space has been studied in the literature,
  e.g. Entanglement of entropy for strongly coupled field theories on de Sitter space with a gravity dual was computed; Maldacena 2012
  e.g. Evidence of dynamical phase transition for confining gauge theory on de Sitter space was found in the strongly coupled regime of some QFT on dS Marolf, Rangamani, Van Raamsdonk 2012
- Neverteless, in all previous studies, the gauge/gravity correspondence was only needed to be considered in the *generic sense* without having to precisely spell out the involved string theory and boundary field theory.
- It would be interesting to have a concrete duality so that one can ask other more precise dynamical questions.

1.1. Motivation: dS superconformal Yang-Mills theory

- The construction of global supersymmetric field theory in four dimensional de Sitter spacetime is impossible due to a lack of Majorana Killing spinor on de Sitter spacetime; and the lack of unitary representation for the de Sitter superalgebra.
- Nevertheless, it is possible if we use conformal Killing spinor (CKS)

$$\left(D_\mu - \frac{1}{4}\gamma_\mu \not{D} \right)\epsilon = 0$$

  instead of Killing spinor, since CKS is is compatible with the Majorana condition on spinor, and a Yang-Mills theory with $\mathcal{N} = 4$ superconformal symmetries on dS_4 has been constructed.

  Anous, Freedman, Maloney 2014

- The studies of the properties of this maximal superconformal Yang-Mills theory should be interesting. e.g. is it exact superconformal? strong-weak duality? integrability?

1.2. The statement

Combining these observations, we propose a holographic duality with dS hologram:

Type IIB string theory on $AdS_5 \times S^5$ is equivalent to the $\mathcal{N} = 4$ superconformal Yang-Mills theory on dS_4.

Outline

2.1. dS slicing of AdS

- AdS_{d+1} is a maximally symmetric space with negative cosmological constant defined by the embedding

$$-X_0^2 + X_1^2 + \ldots + X_d^2 - X_{d+1}^2 = -L^2 \ ,$$

in flat $\mathbf{R}^{d+2}$ with the metric $\eta_{\underline{MN}} = \mathrm{diag}(-1, 1_\mathrm{d}, -1)$. Here L is the radius of the AdS space and the cosmological constant is

$$\Lambda_0 = -\frac{d(d-1)}{2L^2} \ .$$

AdS_{d+1} is invariant under the group $SO(2,d)$.

- de Sitter space dS is a maximally symmetric space with positive cosmological constant defined by

$$-Y_0^2 + Y_1^2 + \ldots + Y_d^2 = L^2 \ ,$$

in the flat space $\mathbf{R}^{d+1}$ with the metric $\eta_{\underline{MN}} = \mathrm{diag}(-1, 1_\mathrm{d})$. The cosmological constant is

$$\Lambda_1 = \frac{(d-1)(d-2)}{2L^2} \ ,$$

and dS_d has the symmetry group $SO(1,d)$.

- In the standard application of AdS/CFT correspondence, one uses the Poincare coordinates

$$X_0 = \frac{r}{2}\left[1 + \frac{x^2 - t^2 + L^2}{r^2}\right], \qquad X_i = \frac{Lx_i}{r}, \quad i = 1, \cdots d-1,$$
$$X_d = \frac{r}{2}\left[1 + \frac{x^2 - t^2 - L^2}{r^2}\right], \qquad X_{d+1} = \frac{Lt}{r},$$

  in which case the AdS metric takes the form

$$ds^2 = \frac{L^2}{r^2}(dr^2 - dt^2 + dx_i^2), \quad r \geq 0.$$

  It is clear that each constant r-slice describes a copy of Minkowski space.

- Note that a boundary Minkowski space has been created at $r = 0$ with this choice of coordination of the AdS space.
  This fact has been of crucial importance in the prescription of GKPW for the realization of the holography of gravity in AdS space.

- The $(d+1)$-dimensional Anti-de Sitter space AdS_{d+1} also admit a coordinate patch with d-dimensional de Sitter space dS_d slicing:

$$X_{d+1} = L \cosh \frac{z}{L} \ , \qquad X_\mu = Y_\mu \sinh \frac{z}{L}, \quad \mu = 0, 1, \ldots, d,$$

with Y_μ describes de Sitter space dS_d. In this coordinate patch, the AdS metric takes the form

$$ds^2 = dz^2 + \sinh^2(\frac{z}{L})\, ds^2_{dS} \ , \quad z \geq 0.$$

- This metric describes a portion of the AdS space with boundary consisting of a copy of the de Sitter space dS_d at $z = \infty$, together with a single point at $z = 0$. An explicit description of the dS part is given by

$$Y_0 = \frac{\sinh Ht}{H} - \frac{1}{2} H x_i^2 e^{-Ht}, \quad Y_i = x_i e^{-Ht}, \quad Y_d = \frac{\cosh Ht}{H} - \frac{1}{2} H x_i^2 e^{-Ht},$$

with $H = 1/L$ and $i = 1, \cdots, d-1$.

- This gives the dS_d metric in terms of the planar coordinates (t, x^i):

$$ds^2_{dS} = -dt^2 + e^{-2Ht} dx_i^2 \ .$$

- Geodesic distance $\mathcal{D}$ between any two points in the AdS or dS can be obtained in terms of the choral distance between two points X and X' in the ambient space

$$P(X, X') := \frac{\eta_{\underline{MN}} X^{\underline{M}} X'^{\underline{N}}}{L^2} ,$$

- For AdS_{d+1}, P is given by $P_{\mathrm{AdS}}(X, X') = -\frac{1}{\xi}$, where

$$\xi^{-1} = \frac{r^2 + r'^2 + (x_i - x_i')^2 - (t - t')^2}{2rr'}, \quad \text{in the Poincare coordinates,}$$

$$\xi^{-1} = \cosh Hz \cosh Hz' - \sinh Hz \sinh Hz' \times P_{dS}(x^\mu, x'^\mu),$$

in the dS planar coordinates,

Here

$$P_{\mathrm{dS}} := \cosh H(t - t') - \frac{e^{-H(t+t')}}{2} H^2 (x_i - x_i')^2$$

- Note that $P_{\rm dS} = 1$ for coincident points. More convenient to use the geodesic distance of dS_d:

$$\sigma^2(x, x') := e^{-H(t+t')}(x_i - x_i')^2 - \frac{\cosh H(t - t') - 1}{H^2/2}$$

In general it is

$$\frac{P_{\rm dS} - 1}{H^2} = -\frac{1}{2}\sigma^2.$$

- σ^2 approaches its Minkowski limit when $H \to 0$,

$$\sigma^2 \to |x - x'|^2 := -(t - t')^2 + (x - x')^2.$$

- In terms of the conformal time $x^0 = H^{-1}\exp(Ht)$, the dS metric can be written as

$$ds^2 = \frac{1}{H^2 x_0^2}\left(-dx_0^2 + dx_i^2\right)$$

and it is

$$\sigma^2 = \frac{(x_\mu - x_\mu')^2}{H^2 x_0 x_0'} \ ,$$

2.2. $\mathcal{N} = 4$ superconformal Yang-Mills theory

- Killing spinor equation

$$D_\mu \epsilon = \frac{i}{2L} \gamma_\mu \epsilon$$

  has no real solution.

- However, the conformal Killing spinor (CKS) defined by the equation

$$\left(D_\mu - \frac{1}{4} \gamma_\mu \not{D} \right) \epsilon = 0$$

  is compatible with the Majorana condition on spinor. This can be solved and one obtain the conformal Killing spinors on dS_4

$$\epsilon(x) = \frac{1}{\sqrt{Hx^0}} (\eta_0 + x^\mu \gamma_\mu \eta_1) \ ,$$

  where η_0, η_1 are arbitrary Majorana spinors. This gives $\mathcal{N} = 1$ superconformal symmetry in dS_4 and corresponds to a basis of 8 real supercharges.

- The $\mathcal{N}=4$ maximal superconformal Yang-Mills theory on dS_4 contains the gauge potentials A^a_μ, four Majorana gauginos λ^a_α and six real scalars X^a_i, where the indices a is in the adjoint of the gauge group $SU(N)$. Anous, Freedman, Maloney 2014
- The Lagrangian is $\mathcal{L}=\mathcal{L}_2+\mathcal{L}_3+\mathcal{L}_4$

$$\mathcal{L}_2 = -\Big[\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a}+\overline{\lambda}^{a\alpha}\gamma^\mu D_\mu P_L\lambda^a_\alpha+\frac{1}{2}D_\mu X^a_i D^\mu X^a_i+H^2X^a_iX^a_i\Big] ,$$

$$\mathcal{L}_3 = -\frac{1}{2}f^{abc}X^a_i\Big[C_i^{\alpha\beta}\overline{\lambda}^b_\alpha P_L\lambda^c_\beta+C_{i\alpha\beta}\overline{\lambda}^{b\alpha}P_R\lambda^{c\beta}\Big] ,$$

$$\mathcal{L}_4 = -\frac{1}{4}f^{abc}f^{a'b'c'}X^b_iX^c_jX^{b'}_iX^{c'}_j .$$

Here $P_{L,R}=$ chiral projectors, $C_i=$ ‘t Hooft instanton matrices:

$$C_1 = \begin{pmatrix}0 & \sigma_1\\ -\sigma_1 & 0\end{pmatrix} , \quad C_2=\begin{pmatrix}0 & -\sigma_3\\ \sigma_3 & 0\end{pmatrix} , \quad C_3=\begin{pmatrix}i\sigma_2 & 0\\ 0 & i\sigma_2\end{pmatrix} ,$$

$$C_4 = -i\begin{pmatrix}0 & i\sigma_2\\ i\sigma_2 & 0\end{pmatrix} , \quad C_5=-i\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix} , \quad C_6=-i\begin{pmatrix}-i\sigma_2 & 0\\ 0 & i\sigma_2\end{pmatrix}$$

and $\sigma_i=$ Pauli matrices. Note $C_{1,2,3}$ are real, $C_{4,5,6}$ are imaginary.

-

- The action admits an $SU(4)$ R-symmetry and the superconformal symmetry:

$$
\begin{aligned}
\delta A_\mu^a &= -\bar{\epsilon}^\alpha \gamma_\mu P_L \lambda_\alpha^a - \bar{\epsilon}_\alpha \gamma_\mu P_R \lambda^{a\alpha} \ , \\
\delta X_i^a &= -\bar{\epsilon}_\alpha P_L C_i^{\alpha\beta} \lambda_\beta - \epsilon^\alpha P_R C_{i\alpha\beta} \lambda^{a\beta} \ , \\
\delta \lambda_\alpha^a &= \frac{1}{2}\gamma^{\mu\nu} F_{\mu\nu}^a \epsilon_\alpha - \gamma^\mu D_\mu X_i^a (P_L C_i^{\alpha\beta} \epsilon_\beta + P_R C_{i\alpha\beta} \epsilon^\beta) \\
&\quad - \frac{1}{2} X_i^a (P_R C_i^{\alpha\beta} \not{D} \epsilon_\beta + P_L C_{i\alpha\beta} \not{D} \epsilon_\beta) \\
&\quad - \frac{1}{2} f^{abc} X_i^b X_j^c [(C_i C_j)^\alpha{}_\beta P_R \epsilon^\beta + (C_i C_j)_\alpha{}^\beta P_L \epsilon_\beta] \ ,
\end{aligned}
$$

  where $P_L\epsilon_\alpha, P_R\epsilon^\alpha$ are an $SU(4)$ quartet of Majorana CKS. Here $(C_iC_j)^\alpha{}_\beta := C_i^{\alpha\gamma} C_{j\gamma\beta}$ and $(C_iC_j)_\alpha{}^\beta := C_{i\alpha\gamma} C_j^{\gamma\beta}$.
- Due to its large amount of superconformal symmetry, the theory is expected to be UV and IR finite. Hence the theory is expected to enjoy exact $SU(2,2|4)$ supersymmetry.

  (work in progress with Yoji Koyama)
- Adding a θ-term, one also expect the theory to enjoy exact $SL(2,Z)$ strong-weak duality, just as the type IIB superstring theory does.

2.3 Proposal of AdS/dS CFT correspondence

We propose that:

Type IIB string theory on $AdS_5 \times S^5$ is equivalent to the $\mathcal{N} = 4$ superconformal Yang-Mills theory on dS_4.

with the relation of parameters:

$$
\begin{aligned}
g_{YM}^2 &= g_s, \\
4\pi\lambda &= \frac{L^4}{(\alpha')^2}, \quad \lambda := g_s N, \\
\Lambda &= 3/L^2
\end{aligned}
$$

Outline

3.1. Conformal field theory in dS space: general

- The $SO(1,d)$ isometries of dS_d is generated by the generators:

$$L_{AB} = Y^A \frac{\partial}{\partial Y^B} - Y^B \frac{\partial}{\partial Y^A}, \quad A, B = 0, 1, \cdots, d,$$

which acts linearly on the de Sitter hyperboloid. In terms of the dS conformal coordinates, they acts as:

$$\begin{aligned} \text{Spatial Rotation } J_{ij} &= -iL_{ij} = -i(x_i\partial_j - x_j\partial_i), \quad i = 1, \cdots, d-1 \\ \text{Dilation } D &= -iL_{0d} = -ix^\mu \partial_\mu, \\ \text{Spatial Translation } P_i &= -i(L_{id} + L_{0i}) = -iH^{-1}\partial_i, \\ \text{Special Conformal Transf } K_i &= i(L_{id} - L_{0i}) = -2iHx_i\, x^\mu \partial_\mu - iHx^2\partial_i. \end{aligned}$$

The corresponding finite transformations are:

$$\begin{aligned} x_i' &= \Lambda_i^j x_j, \qquad \Lambda \in SO(d-1) \text{ rotations}, \\ x'^\mu &= \lambda x^\mu, \\ x_i' &= x_i + a_i, \\ x'^\mu &= \frac{x^\mu + b^\mu x^2}{1 + 2b_\mu x^\mu + b^2 x^2}, \quad b^\mu = (0, b^i). \end{aligned}$$

- The conformal symmetry of dS_d is obtained by adding to the dS isometries the generators:
  3 Lorentz boosts J_{0i}, 1 time translation P^0 and 1 special conformal transformation K^0.
- The corresponding finite transformations are:

$$
\begin{aligned}
\text{3 Lorentz boosts } J_{0i},\ x'^{\mu} &= \Lambda^{\mu}_{\nu} x^{\nu}, \qquad \Lambda^0_i, \Lambda^i_0 = \text{Lorentz boost},\\
\text{1 time translation } P^0,\ x'^0 &= x^0 + a^0,\\
\text{1 special conformal transf } K^0,\ x'^{\mu} &= \frac{x^{\mu} + b^{\mu} x^2}{1 + 2b_{\mu} x^{\mu} + b^2 x^2}, \quad b^{\mu} = (b^0, 0).
\end{aligned}
$$

The metric transforms as

$$
ds^2 \rightarrow ds'^2 = \Lambda^2(x) ds^2,
$$

where, respectively,

$$
\Lambda(x)^2 = \left(\frac{x_0}{\Lambda^0_{\mu} x^{\mu}}\right)^2, \qquad \left(\frac{x_0}{x_0 + a_0}\right)^2, \qquad 1.
$$

- The 15 isometries of AdS_5 acts as conformal symmetries on the boundary.
  For Minkowski boundary:

$$\underbrace{6 \text{ Lorentz} + 4 \text{ translations}}_{\textit{Poincare symmetries}} + \underbrace{4 \text{ special conformal} + 1 \text{ dilation}}_{\textit{conformal symmetries}}$$

  For de Sitter boundary:

$$\underbrace{3 \text{ rot} + 3 \text{ spatial transl} + 3 \text{ special conformal} + 1 \text{ dilation}}_{\textit{dS symmetries}}$$

$$+ \underbrace{3 \text{ boost} + 1 \text{ special conformal} + 1 \text{ time transl}}_{\textit{conformal symmetries}}$$

- As usual, the special conformal transformations can be constructed out of translations and the inversion

$$x^\mu \to x'^\mu = \frac{x^\mu}{x^2} := \left(\frac{1}{x}\right)^\mu .$$

- Inversion is an isometry and induces an $SO(1, d-1)$ rotation on vector

$$\frac{\partial}{\partial x'^\mu} = x^2 M_\mu^\nu(x) \frac{\partial}{\partial x^\nu},$$

where

$$M_\mu^\nu(x) := \delta_\mu{}^\nu - 2\frac{x_\mu x^\nu}{x^2}$$

and satisfies $M_\mu^\alpha M_\nu^\beta \eta_{\alpha\beta} = \eta_{\mu\nu}$.

- For a spinor in fundamental representation, inversion induces the transformation

$$\psi'(x) = S_\alpha^\beta(x)\psi_\beta\left(\frac{1}{x}\right),$$

where

$$S(x) = \frac{\Gamma_\mu x^\mu}{|x|}$$

and satisfies

$$S^\dagger(x)\Gamma^\mu S(x) = M_\nu^\mu \Gamma^\nu.$$

- In a CFT, scalar operator $\mathcal{O}$ of conformal dimension Δ satisfies under conformal transformation $x \to x'$ as

$$\mathcal{O}'(x') = \frac{1}{|\Lambda(x)|^{\Delta}} \mathcal{O}(x).$$

  dS invariance implies that their 2 point function must be function of the geodesic distance $\sigma(x, y)^2$. Conformal invariance further implies:

$$\boxed{\langle \mathcal{O}_1(x) \mathcal{O}_2(y) \rangle = \frac{C_{12}}{\sigma(x, y)^{2\Delta}} \delta_{\Delta_1, \Delta_2 = \Delta}.}$$

- As for operator $\mathcal{O}^{\alpha}(x)$ in spin 1/2 representation, it satisfies:

$$\begin{aligned} \mathcal{O}'^{\alpha}(x') &= \frac{1}{|\Lambda(x)|^{\Delta}} S^{\alpha}_{\beta}(x) \mathcal{O}^{\beta}(x^{\mu}), \quad \text{under translation in } x^0, \textit{ and} \\ \mathcal{O}'^{\alpha}(x') &= S^{\alpha}_{\beta}(x) \mathcal{O}^{\beta}(x^{\mu}), \quad \text{under inversion} \end{aligned}$$

  These implies that

$$\boxed{\langle \mathcal{O}^{\dagger}_{1\,\alpha}(x) \mathcal{O}^{\beta}_{2}(y) \rangle = \frac{D_{\alpha}{}^{\beta}(x, y)}{\sigma(x, y)^{2\Delta}} \delta_{\Delta_1, \Delta_2 = \Delta}.}$$

- Here D is given by

$$D_{\alpha}{}^{\beta}(x,y) = \frac{(x-y)_{\mu}\Gamma^{\mu}}{|x-y|}$$

satisfies the relation

$$D(x,y) = S^{\dagger}(x) D\left(\frac{1}{x}, \frac{1}{y}\right) S(y).$$

- Constraints on 3 and higher point functions of dS CFT can be similarly worked out.

3.2. Bulk to Boundary Formalism: general

Consider a $(d+1)$-dimensional manifold $\mathcal{M}$ with boundary and the metric

$$ds^2 = g_{MN} dy^M dy^N \ .$$

Without loss of generality, assume near the boundary $(z = a)$:

$$ds^2 = dz^2 + \gamma_{\mu\nu}(z,x) dx^\mu dx^\nu \ , \qquad \gamma_{\mu\nu}(z,x) = p^2(z) h_{\mu\nu}(x) \ ,$$

for some function $p(z)$ and $h_{\mu\nu}$ is the boundary metric.

Scalar Case:

- Real scalar field with action

$$\begin{aligned} I(\phi) &= -\frac{1}{2}\int d^{d+1}y\sqrt{g}(g^{MN}\partial_M\varphi\partial_N\varphi + m^2\varphi^2) \\ &= -\frac{1}{2}\int_{\partial M} d^d x\sqrt{\gamma}\varphi n^M\partial_M\varphi + \frac{1}{2}\int d^{d+1}y\sqrt{g}\varphi(\Box - m^2)\varphi, \end{aligned}$$

Get EOM and the boundary action.

- Introduce bulk to boundary propagator:

$$\varphi(z,x) = \int d^d x' \sqrt{h(x')} K(z,x,x') \varphi_0(x') \ ,$$

where

$$\Box K(z,x,x') = 0 \ , \quad \lim_{z \to a} K(z,x,x') \sim f(z) \frac{\delta^{(d)}(x-x')}{\sqrt{h}} \ ,$$

Here $f(z)$ specifies the asymptotic behavior of φ near the boundary,

$$\varphi \sim f(z)\varphi_0(x), \quad z \sim a.$$

- And we obtain the two point function:

$$I(\phi) = -\frac{1}{2} \int d^d x \sqrt{h(x)} d^d x' \sqrt{h(x')} \, \varphi_0(x) \mathcal{G}(x,x') \varphi_0(x') \ ,$$

with $\mathcal{G}(x,x')$ defined as

$$\boxed{\mathcal{G}(x,x') := \lim_{z \to a} f(z) p^d(z) \partial_z K(z,x,x').}$$

Spin 1/2 fermion

- Consider massive free spin 1/2 fermion

$$I_0 = \int d^{d+1}x \sqrt{g}\,\overline{\psi}(\not{D} - \mu)\psi,$$

- The action vanishes on-shell and one needs to supplement it with the boundary action Henningson, Sfetsos 98; Henneaux 99

$$I_b := \lim_{z\to a} \frac{1}{2}\int_{\partial\mathcal{M}} d^d x \sqrt{\gamma}\,\overline{\psi}\psi.$$

- The fermionic bulk-to-boundary propgator S is defined by

$$\psi(z,x) = \int d^d x' \sqrt{h(x')}\, S(z,x,x')\psi_0(x') \ ,$$

where $\quad (\not{D} - \mu)S = 0, \quad \lim_{z\to a} S(z,x,x') \sim f(z)\dfrac{\delta^{(d)}(x-x')}{\sqrt{h}}\mathbf{1}.$

and $f(z)$ specifies the boundary behaviour:

$$\psi \sim f(z)\psi_0(x), \quad z \sim a.$$

- It is easy to see that, for $\mu > 0$, the non-normalizable mode is obtain from ψ_0 of negative chirality: $\Gamma^z\psi_0 = -\psi_0$.

- We obtain

$$I_b = \int d^d x' \sqrt{h(x')} d^d x'' \sqrt{h(x'')}\, \overline{\psi}_0(x') \mathcal{G}(x', x'') \psi_0(x'') \ ,$$

where

$$\mathcal{G}(x', x'') := \lim_{z \to a} \frac{1}{2} \int d^d x \sqrt{\gamma(z, x)} S^\dagger(z, x, x') S(z, x, x'') \ .$$

- As S behaves like a delta function near the boundary, the integral picks up its contribution from the two regions: $x \sim x'$ and $x \sim x''$ and we obtain

$$\boxed{\mathcal{G}(x', x'') = \lim_{z \to a} \frac{1}{2} f(z) p^d(z) \big(S^\dagger(z, x'', x') + S(z, x', x'') \big).}$$

So our job reduces to determining the bulk-to-boundary propagators K and S for the AdS space with dS boundary. However, we don't need the full knowledge. In fact only need them near the boundary. And there is a simple relation

$$\boxed{S = \not{D} K}$$

which holds near the boundary.

- Instead of trying to solve for K directly from its defining DE and BC, one has a more effective way.
  Introduce the Green function for the bulk

$$(-\Box + m^2)G(z,x;z',x') = \frac{1}{\sqrt{g}}\delta^{(d)}(x-x')\delta(z-z') \ .$$

  Using the Green's identity, one can easily obtain the solution of the scalar Klein-Gordon equation in terms of the Green function as

$$\begin{aligned}
\varphi(z,x) &= \int d^{d+1}x\sqrt{g}\,\varphi(-\Box + m^2)G - G(-\Box + m^2)\varphi \\
&= \int d^d x' \varphi_0(x')\sqrt{\gamma(z',x')}\Big(G\partial_{z'}f(z') - f(z')\partial_{z'}G\Big)_{z'\to a} \ .
\end{aligned}$$

  As a result

$$\boxed{K(z,x,x') = \lim_{z'\to a} p^d(z')\Big(G(z,x;z',x')\partial_{z'}f(z') - f(z')\partial_{z'}G(z,x;z',x')\Big)}$$

- This formula displays clearly how the bulk physics, as encoded in the bulk Green function, is translated to the physics on the holographic field theory through the boundary data

Outline

4.1. 2-point function for scalar operators in dS CFT

- For AdS_{d+1} in the Poincare coordinates, we have,

$$L^{d-2} G(X, X') = \frac{C_\Delta}{2\nu} \left(\frac{\xi}{2}\right)^{\Delta} F\left(\frac{\Delta}{2}, \frac{\Delta+1}{2}, \nu+1; \xi^2\right) ,$$

where C_Δ = constant,

$$\Delta = \text{either } \Delta_\pm := \tfrac{d}{2} \pm \nu \ , \quad \nu := \sqrt{\frac{d^2}{4} + m^2 L^2} \ .$$

- Flat case: In the Poincare coordinate:

$$\xi \approx \frac{2rr'}{r^2 + |x - x'|^2} \to 0, \quad \text{near the boundary } r' \to 0 \text{ or } z' \to \infty.$$

- We obtain the bulk to boundary propagator and the 2-point function

$$K = \left(\frac{r}{r^2 + |x - x'|^2}\right)^{\Delta}$$

$$\mathcal{G}(x, x') = \frac{1}{|x - x'|^{2\Delta}},$$

which is expected for CFT in Minkowski space.

dS case

- The AdS_{d+1} metric is

$$ds^2 = dz^2 + \sinh^2(Hz)\, ds^2_{\mathrm{dS}} \ ,$$

and has a dS boundary with metric h, where $\sqrt{g} = \sinh^d(Hz)\sqrt{h}$. Near the boundary $z' \to \infty$, we have

$$\xi = \frac{e^{-H\left(z+z'\right)}}{H^2\rho^2/8} \to 0,$$

where

$$\rho^2 = \sigma^2 + e^{-2Hz} \cdot \frac{1 - H^2\sigma^2/4}{H^2/4}$$

and

$$\sigma^2 = \frac{(x_\mu - x'_\mu)^2}{H^2 x_0 x'_0} \ .$$

- As the Green function is diffeomorphic invariant, we obtain immediately

$$K = \left(\frac{e^{-Hz}}{\rho^2}\right)^{\Delta}, \quad \text{with } \Delta = \Delta_+$$

and the two point function

$$\mathcal{G}(x, x') = \frac{1}{\sigma(x, x')^{2\Delta}}.$$

This is the expected form of the two point function for operators of dimension Δ of a conformal field theory in dS spacetime.

4.2. 2-point function for spin 1/2 operators in dS CFT

- For the metric

$$ds^2 = dz^2 + \sinh^2 Hz\left(-dt^2 + e^{-2Ht} dx_i^2\right).$$

The Dirac operator reads

$$\not{D} = \Gamma^z \partial_z + \frac{1}{\sinh Hz} \not{D}_{\mathrm{dS}} + \frac{dH}{2 \tanh Hz} \Gamma^z.$$

- One can show that the solution of S to the defining DE and BC can be written as

$$S = (\not{D} + \mu - H \coth Hz\ \Gamma^z)K + \delta\ ,$$

where

$$K = \left(\frac{e^{-Hz}}{\rho^2}\right)^{\Delta_+}$$

is the bulk to boundary propagator for an aux. scalar field of mass m

$$m^2 := \mu^2 - d^2/4$$

and δ is some function on AdS which vanishes at the bounary.

- As

$$\mathcal{G}(x',x'') = \lim_{z\to a} \frac{1}{2} f(z) p^d(z) \big(S^\dagger(z,x'',x') + S(z,x',x'') \big)$$

is eventually sandwiched between $\overline{\psi}_0$ and ψ_0 (opposite chirality) inside

$$I_b = \int d^d x' \sqrt{h(x')} d^d x'' \sqrt{h(x'')}\, \overline{\psi}_0(x') \mathcal{G}(x',x'') \psi_0(x'') \ ,$$

one see that the constant and Γ^z term do not contribute

$$\overline{\psi}_0 \psi_0 = 0 \ , \qquad \overline{\psi}_0 \Gamma^z \psi_0 = 0,$$

and

$$S = \not{D} K.$$

- As a result, we obtain the boundary two-point function

$$\mathcal{G}(x,x') = \frac{D}{\sigma^{2\Delta_+}} \ .$$

This agrees with the result for CFT in dS spacetime.

Outline

work in progress with Dimitris Giatagans

- An observer in de Sitter space will observe a bath of thermal particles emitted from the de Sitter horizon at the Hawking temperature

$$T_H = \frac{H}{2\pi}.$$

- For flat space, $Q^2 = H$, thus SUSY invariance of vacuum implies $\langle H \rangle = 0$ for any SUSY inv state. Therefore a temperature would break SUSY.
- For dS space, H is not the square of a supersymmetry generator, and so the temperature does not break SUSY. This finite temperature radiation is present even though the vaccuum state preserves exact supersymmetry.

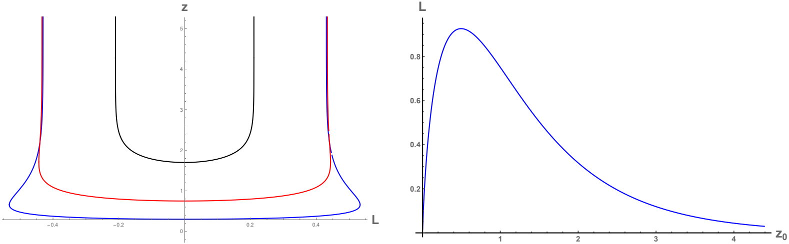
In flat case, $Q\bar{Q}$ potential is fixed by conformal symmetry to be $V = 1/\ell$.
Q. For dS case, how does the temperture the physics?

$$\text{Expect:} \quad V = V(\ell, H) = \frac{1}{\ell} f(H\ell).$$

- Static heavy quarks on the dS boundary of the AdS space follow spacelike geodesics which diverges from each other, with increasing distance between them as time goes.
- A simple case where the invariant distance between the quarks is preserved a constant ℓ is to give the quarks a constant speed with direction pointing to each other

$$v^2 = -\frac{\ell^2 H^2}{4}.$$

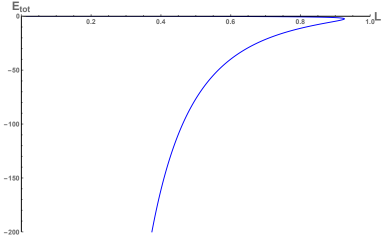
- The profile of the string solution and the inter-quark distance in terms of the tunning point:



- For each value of ℓ, there are 2 solutions corresponding to 2 turning points z_0. The preferred solution (with smaller area) has smaller z_0.
- The on-shell Nambu-Goto action is ∞ due to the ∞ lenght of the string worldsheet, out of which the energy of the 2 single quarks has to be subtracted:

$$E_{tot}(L) = S_{Q\bar{Q}} - 2S_Q \ .$$

- We obtain the energy of the bounded state in terms of the size of the bound state ℓ in units of H.



- There is a maximum value of ℓ beyond which there is no minimal surface with the boundary conditions. The turning point occurs for negative values of energy, The almost flat branch corresponds to the non-stable solutions that are energetically non-favorable.
- The behaviour is quite similar to the thermal $Q\bar{Q}$ potential computed using the AdS-BH background, and is a result of the presence of the cosmological horizon.

Outline

- The thermal behaviour of the Wilson loop signifies the effects of the cosmological horizon.
  Q. In flat space, a finite temperature breaks supersymmetry. However here we have exact SUSY preserved here. could we use SUSY to constraint the form of the potential, or other operators?
- It would be interesting to compute the higher point functions and see if there is any nonrenormalization theorem for chiral operators as in the standard $\mathcal{N} = 4$ SYM.
- The $\mathcal{N} = 4$ SCYM seems to be an interesting QFT with remarkable properties: exact conformality, $SL(2, Z)$ duality, integrability, etc. Further anaysis are called for.

  in progress with Yoji Koyama
- On the $AdS_5 \times S^5$ side, the existence of the Lax pair and an infinite set of classically conserved nonlocal charges are properties of the Green-Schwarz string sigma model. One may speculate that the $\mathcal{N} = 4$ superconformal Yang-Mills on dS_4 may also be integrable in some of its sectors.

Thank you!