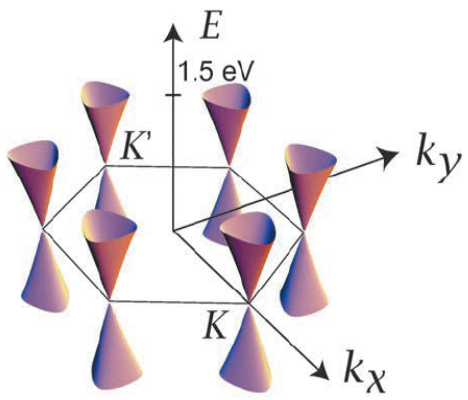
APCTP focus week, 23 Aug, 2016

Edge states and D-branes

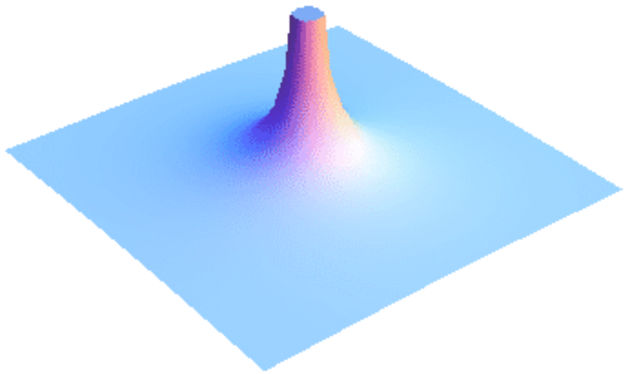
ArXiv:1509.04676(hep-th), 1602.05577(cond-mat),
1608.?????, 1609.?????

Koji Hashimoto (Osaka u) w/ Taro Kimura (Keio u)
Xi Wu (Osaka u)

Band is a D-brane



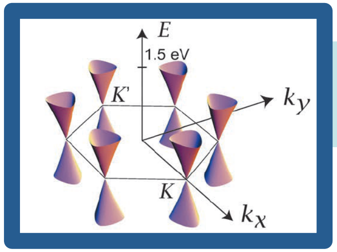
[Ezawa, Buturi 2014-11]



[Callan, Maldacena 9708147]

Band is a D-brane

Review 1	**Band of topological insulator**	4p
Review 2	**BPS brane**	3p
Claim	**Band – Brane correspondence**	1p
Evidence 1	**Spectrum = D-brane**	1p
Evidence 2	**4d topol. insulator = Supertube**	1p
Application	**Edge state = Tilted D1**	3p



Band of topological insulator

Topological insulator

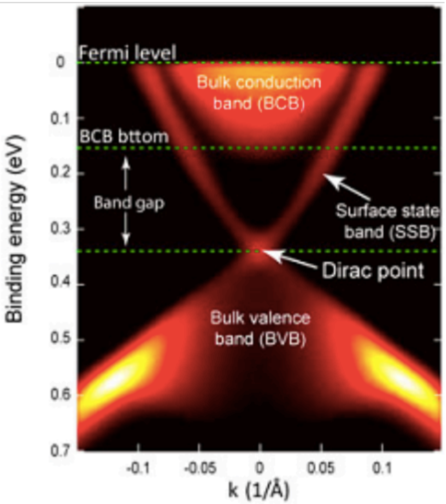
- Bulk is insulating, characterized by a topological number

$$\sigma_{xy}^{\mathrm{Hall}} = \frac{e^2}{h}\nu, \quad \nu \in \mathcal{Z}$$

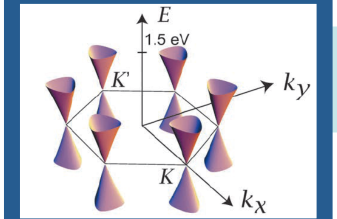
- Surface has a gapless mode

Chiral, Majorana, etc.

$Bi_{2-x}Sb_xTe_{3-y}Se_y$ (Osaka-u, Ando group)



[Chen et al., Science `09]

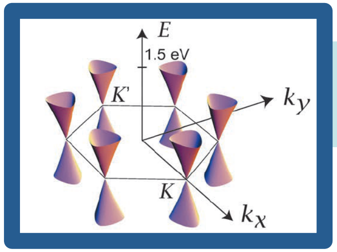


Band of topological insulator

Topological numbers classified

class \ d	0	1	2	3	4	5	6	7	T	C	S
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	0	0	0
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	0	1
AI	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$+$	0	0
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$+$	$+$	1
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	0	$+$	0
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	$-$	$+$	1
AII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$-$	0	0
CII	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$-$	$-$	1
C	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$-$	0
CI	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$+$	$-$	1

[Schnider,Ryu,Furusaki,Ludwig `08] [Kitaev `09]



Band of topological insulator

[Thouless, Kohmoto, Nightingale, den Nijs `82]

Topological insulator characterized by 1st Chern class

1) Hamiltonian of 2d relativistic electron

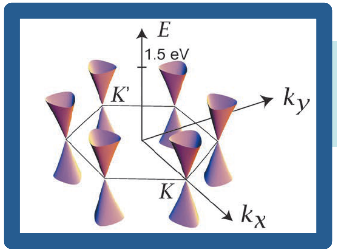
$$\mathcal{H} = \sigma_1 p_1 + \sigma_2 p_2 + \sigma_3 m$$

2) Solve for energy eigenstates $\left[i\frac{\partial}{\partial t} + \mathcal{H} \right] v(t) = 0$

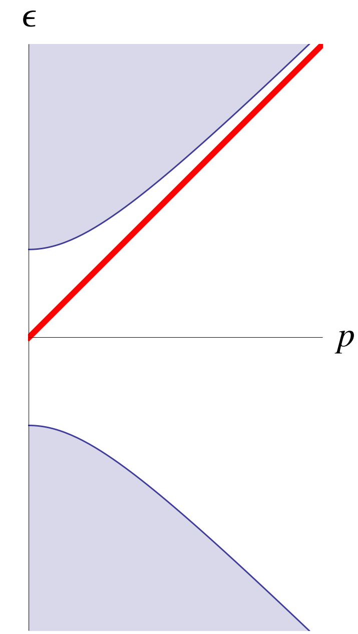
$$v = \frac{e^{i\epsilon t}}{\sqrt{2(\epsilon + m)}} \begin{pmatrix} -p_1 + ip_2 \\ \epsilon + m \end{pmatrix} \qquad \epsilon = \pm\sqrt{p_1^2 + p_2^2 + m^2}$$

3) Obtain Berry's connection $A_\mu^{(\mathrm{B})} = v^\dagger \frac{d}{dp_\mu} v \ (\mu = 1, 2)$

4) 1st Chern class $\frac{1}{2\pi} \int dp_1 dp_2 \, F_{12}^{(\mathrm{B})} = -\frac{1}{2}\mathrm{sign}(m)$



Band of topological insulator



Existence of Chiral edge state

1) Hamiltonian

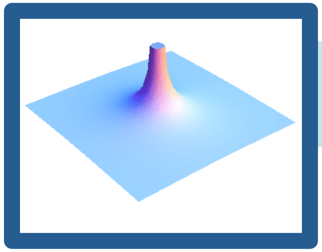
$$\mathcal{H} = \sigma_1 p_1 + \sigma_2 p_2 + \sigma_3 m$$

2) Put a boundary: $(\sigma_1 - 1_2)v(t)\Big|_{x^2=0} = 0$

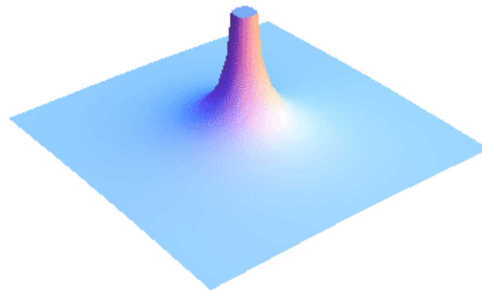
3) Solve for energy eigenstates $\left[i\frac{\partial}{\partial t} + \mathcal{H}\right] v(t) = 0$

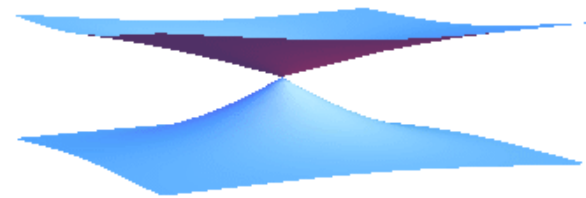
Localized mode: $v(t) = e^{-i\epsilon t}\sqrt{m}\exp[mx^2]\begin{pmatrix}1\\1\end{pmatrix}$

Dispersion is chiral: $\epsilon = p_1$



BPS brane





Original

Spiky [Callan,Maldacena] [A.Hashimoto]

- Dictated by D-brane actions and BPS equations

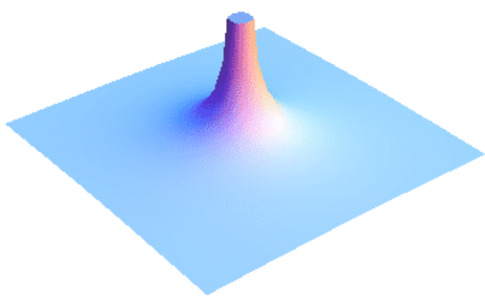
$$S = T_p \int d^{p+1}x \sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab} + (2\pi\alpha')^2 \partial_a\phi\partial_b\phi)}$$

- D-brane charges (= Ramond-Ramond charges)

	D(−1)	D0	D1	D2	D3	D4	D5	D6	D7	D8	D9
type IIB	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$O9^-$ (type I)	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
$O9^+$	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$

[Witten `98], See [Ryu, Takayanagi `10]

BPS brane

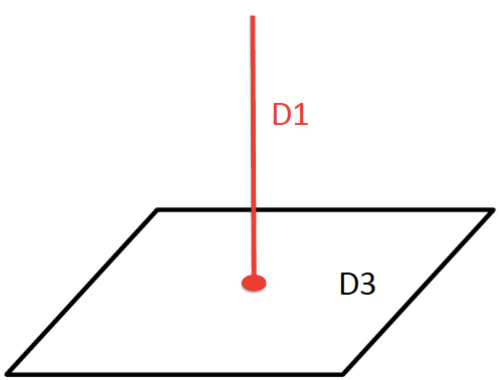


BPS equation solving EoM

$$\partial_i \Phi(x) = -\frac{1}{2}\epsilon_{ijk} F_{jk}(x)$$

Spiky solution (Dirac monopole)

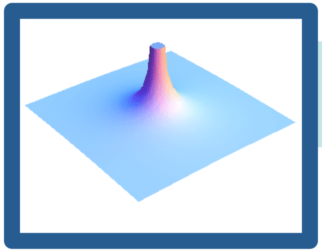
$$\Phi(x) = \frac{-1}{2r}\,, \quad B_i \left(= \frac{1}{2}\epsilon_{ijk} F_{jk} \right) = \frac{x^i}{2r^3}$$



Interpretation :

D1-brane stuck to D3-brane

Evidence: Susy, tension, fluctuation



BPS brane

[Nahm `80][Diaconescu `97]

Nahm construction : all monopole solutions of BPS equation $\partial_i \Phi(x) = -\frac{1}{2}\epsilon_{ijk}F_{jk}(x)$

1) "Tachyon" operator [Terashima, KH, `06]

$$T \equiv \sigma_i x^i$$

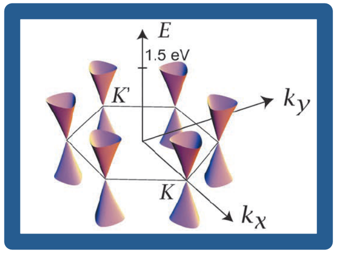
2) Solve for normalized zeromode $\left[\frac{d}{d\xi} + T\right] v(\xi) = 0$

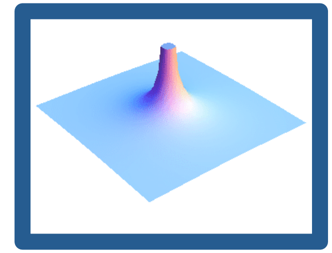
3) Obtain scalar and gauge fields

$$\Phi(x) \equiv \int d\xi \; v^\dagger \xi v \,, \quad A_i(x) \equiv \int d\xi \; v^\dagger \frac{d}{dx^i} v$$

4) Monopole number $\frac{1}{2\pi}\int dS^i \, \frac{1}{2}\epsilon_{ijk}F_{jk} = 1$

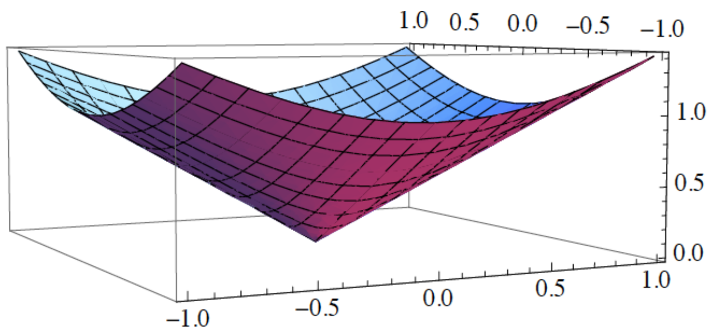
Band-Brane Correspondence

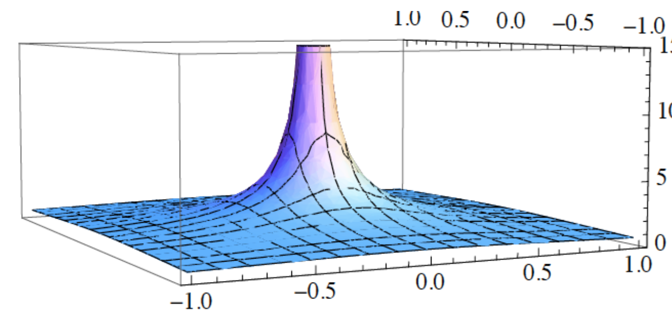




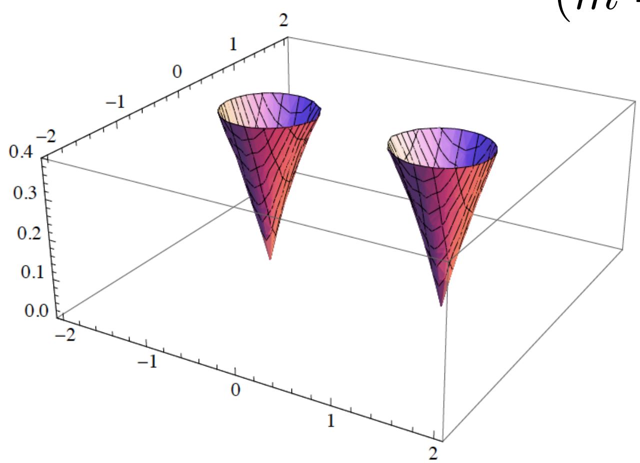
Band	Brane
(p_1, p_2, m)	(x^1, x^2, x^3)
Hamiltonian $(\mathcal{H}, t)$	Tachyon $(T, i\xi)$
Wave fn. $v(p)$	Zero mode $v(x)$
Berry connection $A_\mu^{(\mathrm{B})}(x)$	Monopole gauge field $A_i(x)$
TKNN number ν	RR charge k
Spectrum $\epsilon(p)$	Shape $\Phi(x)$

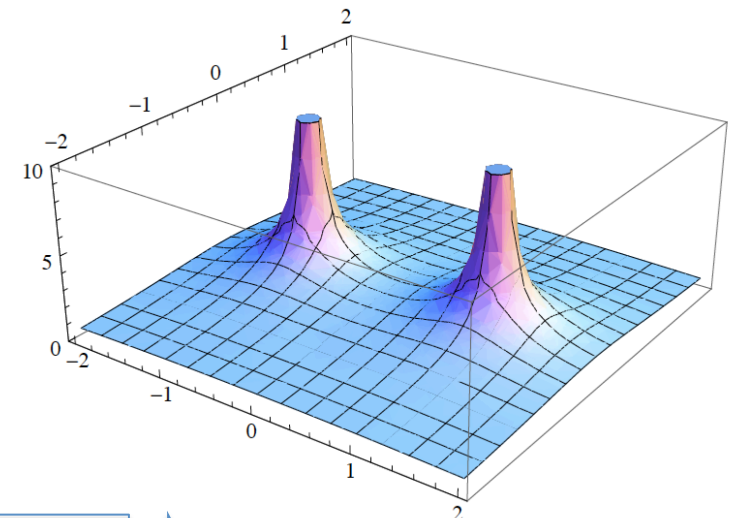
Spectrum = D-brane





$(m = 0)$

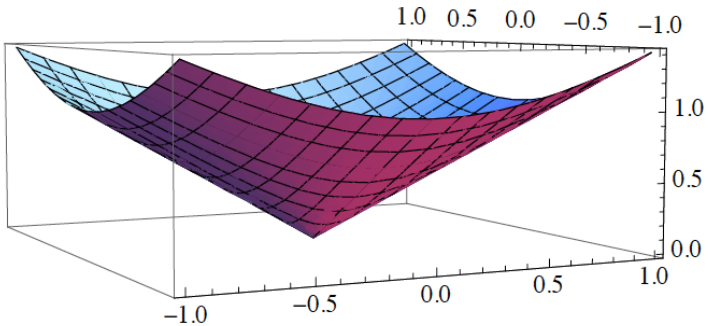




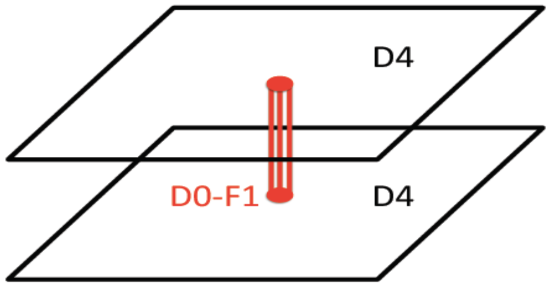
$$\epsilon = -i\langle v|\frac{\partial}{\partial t}|v\rangle \qquad \Phi^{-1} \sim \epsilon \qquad \Phi = \langle v|\xi|v\rangle$$

Spectrum $\epsilon(p)$ Shape $\Phi(x)$

4d topological insulator = Supertube



4d topological insulator
(Class A)



Supertube ending on D4
(Dyonic instanton)

[Kim,Lee `03] [Chen Eto KH `06]

- ADHM construction of instantons

$$\nabla^\dagger \equiv (\rho \mathbf{1}_2, \sigma_\mu x^\mu)$$

Zero mode → ADHM connection → 2nd Chern class

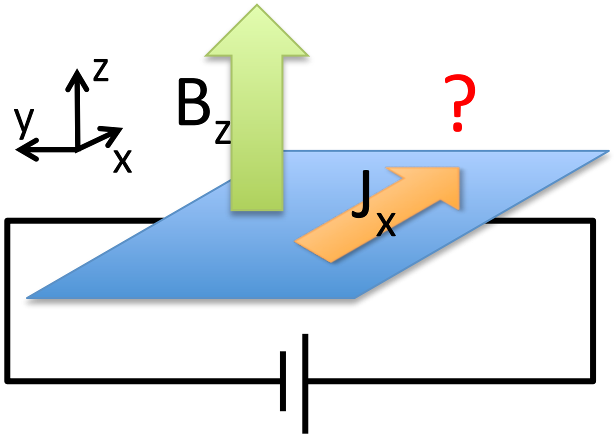
- Hamiltonian of 4d topological insulator

$$\mathcal{H} = p_i \gamma^i + m\gamma_5 \ (i = 1, 2, 3, 4)$$

Eigenstates → Berry connection → 2nd Chern class

Review: Bulk – Edge correspondence

[X.G.Wen `92] [Hatsugai `93]



Integer Hall effect

[Thouless, Kohmoto, Nightingale, den Nijs `82]

- Charge non-conservation in the Bulk

$$\langle J_i \rangle = \sigma_{xy} \sum_j \epsilon_{ij} E_j \quad \left(\sigma_{xy} = \frac{e^2}{2\pi} N_{\mathrm{Ch}} \right) \rightarrow \dot{Q}_{\mathrm{bulk}} = L \frac{e^2}{2\pi} N_{\mathrm{Ch}} E_y$$

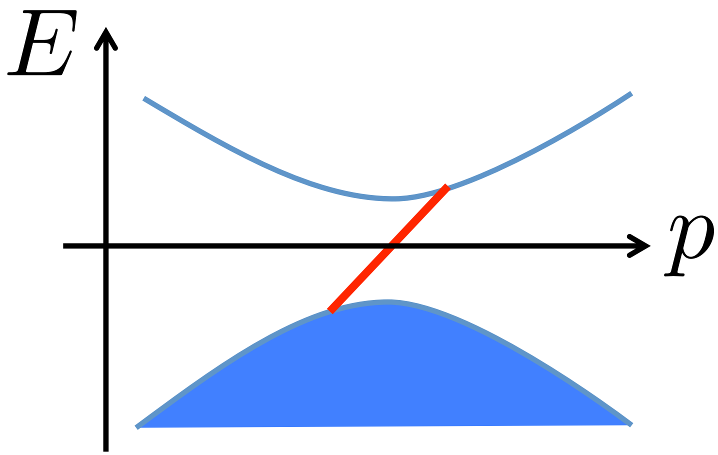
cancel

- Compensated by the Edge state

$$\epsilon_k = v(k_y - eE_y t) \rightarrow \dot{Q}_{\mathrm{edge}} = -e \lim_{\Delta t \to 0} \frac{\Delta N}{\Delta t} = L \frac{e^2}{2\pi} \frac{v}{|v|} E_y$$

Edge state = Tilted D1

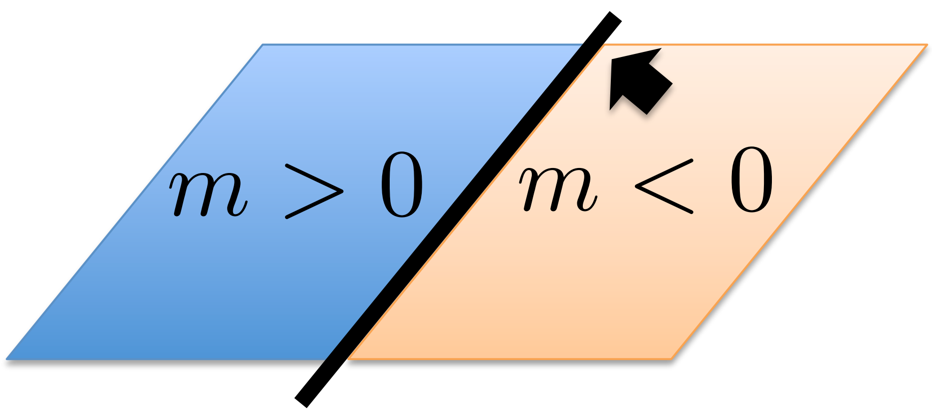
Chiral edge state



Non-commutative monopole

[Gross, Nekrasov] [Hashimoto KH]

Boundary of 2d topol. insulator



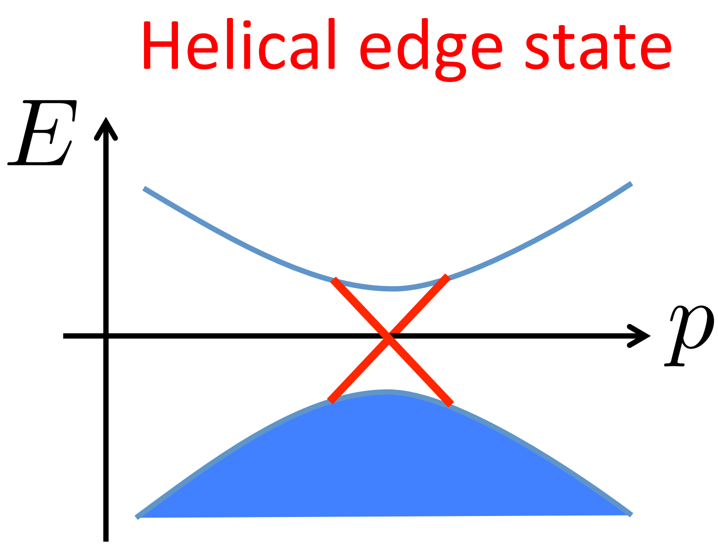
Electron Hamiltonian

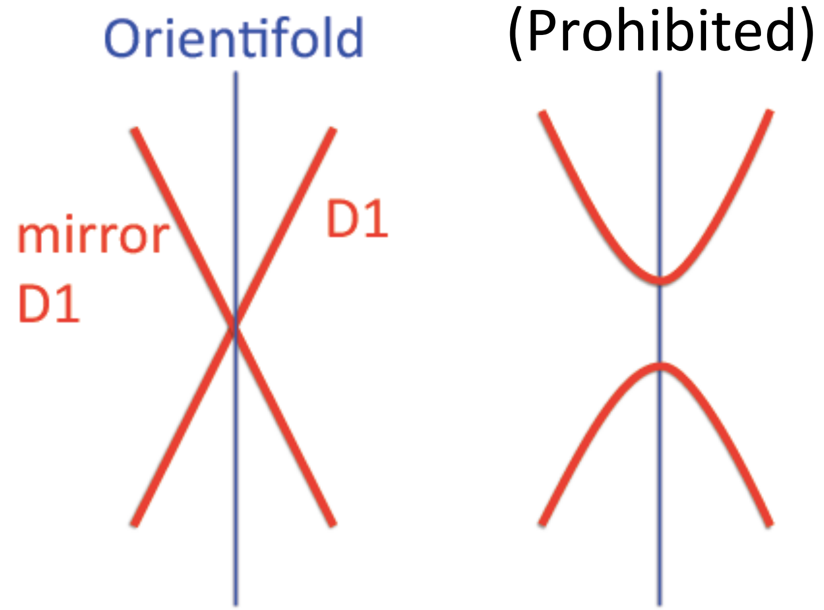
$$\mathcal{H} = p_1\sigma_1 + p_2\sigma_2 + m(x^1)\sigma_3$$

$$m(x^1) \simeq \theta x^1$$

→ non-commutative space!

Edge state = Tilted D1





Electron hamiltonian of 2d topol. insulator of class AII

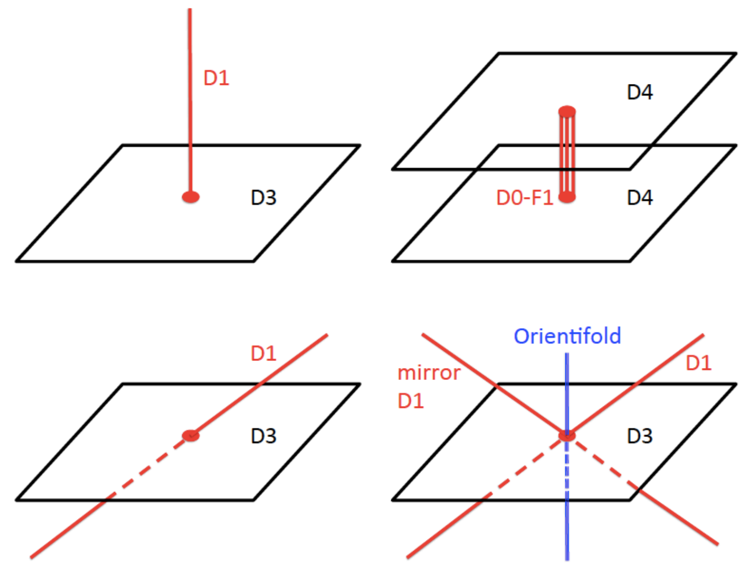
$$\mathcal{H} = (\sigma_1 p_1 + \sigma_2 p_2) \otimes \sigma_2 + m \mathbf{1}_2 \otimes \sigma_3$$

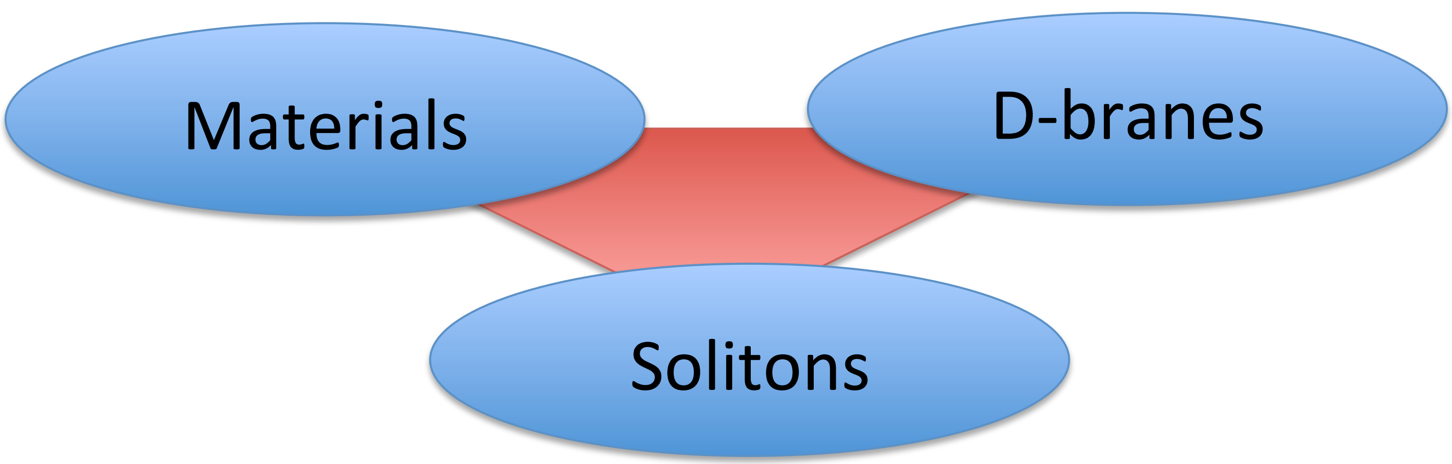
Time reversal = Orientifolding

Fertile applications

Exploration

- Lattice Domain wall fermions
- All classifications mapped?
- Higher winding
- Generic boundary conditions
- Boundary states





2d topological insulator

[Thouless, Kohmoto, Nightingale, den Nijs `82]

Topological insulator characterized by 1st Chern class

1) Hamiltonian of 2d relativistic fermion

$$\mathcal{H} = \sigma_1 p_1 + \sigma_2 p_2 + \sigma_3 m$$

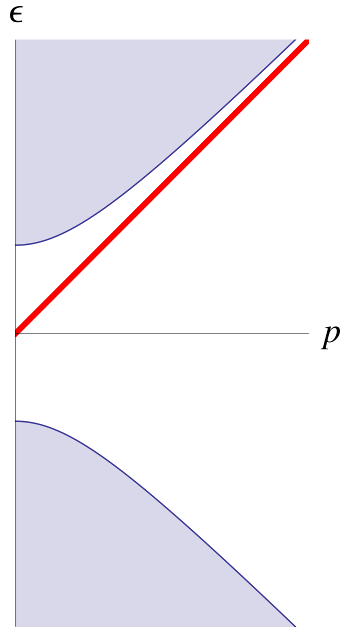
2) Solve for energy eigenstates $\mathcal{H} v = \epsilon v$

$$v = \frac{1}{\sqrt{2(\epsilon + m)}} \begin{pmatrix} -p_1 + i p_2 \\ \epsilon + m \end{pmatrix} \qquad \epsilon = \pm\sqrt{p_1^2 + p_2^2 + m^2}$$

3) Obtain Berry's connection $A_\mu^{(\mathrm{B})} = v^\dagger \frac{d}{dp_\mu} v \; (\mu = 1, 2)$

4) 1st Chern class $\frac{1}{2\pi} \int dp_1 dp_2 \, F_{12}^{(\mathrm{B})} = -\frac{1}{2} \mathrm{sign}(m)$

2d topological insulator



Bulk-Edge correspondence [Hatsugai `93] [X.G.Wen `92]

Existence of Chiral edge state

1) Hamiltonian

$$\mathcal{H} = \sigma_1 p_1 + \sigma_2 p_2 + \sigma_3 m$$

2) Put a boundary: $(\sigma_1 - 1_2)v \Big|_{x^2=0} = 0$

3) Solve for energy eigenstates $\mathcal{H}v = \epsilon v$

Localized mode: $v = \sqrt{m}\exp[mx^2] \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Dispersion is chiral: $\epsilon = p_1$

4d topological insulator

Topological insulator characterized by 2nd Chern class

1) Hamiltonian of 4d relativistic fermion

$$\mathcal{H} = \gamma_\mu p_\mu + \gamma_5 m$$

$$\gamma_\mu \equiv \begin{pmatrix} 0 & \bar{e}_\mu \\ e_\mu & 0 \end{pmatrix}, \quad \gamma_5 \equiv -\gamma_1\gamma_2\gamma_3\gamma_4 = \begin{pmatrix} \mathbf{1}_2 & 0 \\ 0 & -\mathbf{1}_2 \end{pmatrix}$$

2) Solve for energy eigenstates $\mathcal{H}v = \epsilon v$

$$v = \frac{1}{\sqrt{(p_\mu)^2 + m^2}} \begin{pmatrix} \bar{e}_\mu p_\mu \\ (\epsilon - m)\mathbf{1}_2 \end{pmatrix} \qquad \epsilon = \pm\sqrt{(p_\mu)^2 + m^2}$$

3) Obtain Berry's connection $A_\mu^{(\mathrm{B})} = v^\dagger \frac{d}{dp_\mu} v \ (\mu = 1, 2)$

4) 2nd Chern class $\frac{1}{16\pi^2} \int d^4 p \, \mathrm{tr} \left[F_{\mu\nu}^{(\mathrm{B})} * F_{\mu\nu}^{(\mathrm{B})} \right] = -\frac{1}{2} \mathrm{sign}(m)$

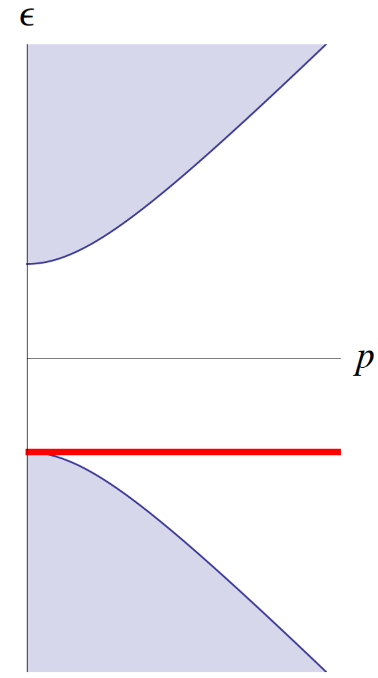
4d topological insulator

Edge state from the Bulk-Edge correspondence

1) Hamiltonian $\mathcal{H} = \gamma_\mu p_\mu + \gamma_5 m$

2) Put a boundary: $(\gamma_5 + \mathbf{1}_4) v \Big|_{x^4=0} = 0$

3) Solve for energy eigenstates $\mathcal{H} v = \epsilon v$



Localized mode: $v = \begin{pmatrix} 0 \\ \eta_0(p) \end{pmatrix}$

$$\eta_0(p_i) \equiv \sqrt{2|\vec{p}|} \exp\left[-|\vec{p}| x^4\right] \tilde{\eta}$$

$$(\vec{p} \cdot \vec{\sigma} - |\vec{p}|)\, \tilde{\eta}(p_i) = 0 \qquad \tilde{\eta} = U(p_i) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Dispersion is flat: $\epsilon = -m$

Dirac monopole at edge

Berry connection of edge state: $A_i = \int_0^\infty dx^4 \; i\eta_0^\dagger \frac{d}{dp_i}\eta_0$

giving a Dirac monopole

$$A_1 + iA_2 = \frac{i(p_1 + ip_2)}{2|\vec{p}|(|\vec{p}| - p_3)}, \quad A_3 = 0$$

Monopole charge is unity: $\frac{1}{2\pi}\int_{S^2} |\vec{p}|^2 d\vec{s}\cdot\vec{B} = 1$

Equivalence to 2d topological insulator :

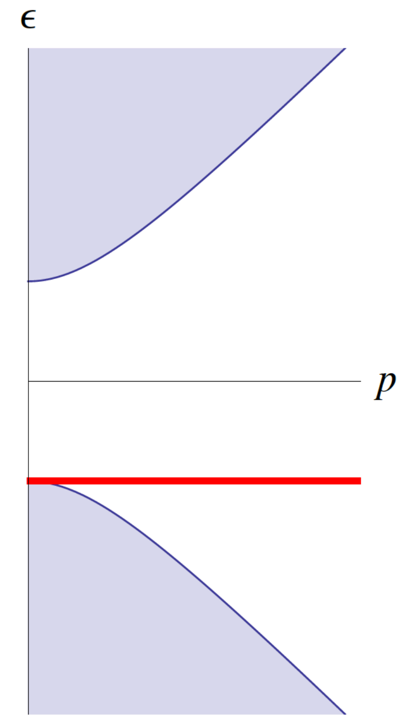
Replacing p_3 by a mass m_3,

$$\nu = \frac{1}{2\pi}\int dp_1 dp_2 \, F_{12} = \frac{1}{2}\mathrm{sign}(m_3)$$

Double boundaries

Two edge states for two boundaries

$$\epsilon = -m$$



1) Hamiltonian $\mathcal{H} = \gamma_\mu p_\mu + \gamma_5 m$

2) Put a boundary: $(\gamma_5 + \mathbf{1}_4) v \Big|_{x^4 = \pm L} = 0$

3) Solve for energy eigenstates $\mathcal{H} v = \epsilon v$

Localized mode: $v = \begin{pmatrix} 0 \\ \eta \end{pmatrix}$

$$\eta = c^+(p_i)\eta^+ + c^-(p_i)\eta^-$$

$$\eta^+ \equiv \sqrt{\frac{|\vec{p}|}{\sinh 2|\vec{p}|L}} \exp[|\vec{p}|x^4]\, U(p_i) \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$$\eta^- \equiv \sqrt{\frac{|\vec{p}|}{\sinh 2|\vec{p}|L}} \exp[-|\vec{p}|x^4]\, U(p_i) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

‘t Hooft Polyakov monopole

Non-Abelian Berry connection of the two edge states:

$$A_i^{ab} \equiv i \int_{-L}^{L} dx^4 \, (\eta^a)^\dagger \frac{d}{dp_i} \eta^b$$

“Depth operator”

$$\Phi_i^{ab} \equiv \int_{-L}^{L} dx^4 \, (\eta^a)^\dagger x^4 \eta^b$$

$$a, b = \pm$$

giving a SU(2) ‘t Hooft Polyakov BPS monopole

$$D_i \Phi = \frac{1}{2} \epsilon_{ijk} F_{jk}$$

Monopole charge is unity: $1 = \frac{1}{4\pi} \int d^3 p \, \frac{1}{2} \epsilon_{ijk} \, \mathrm{tr} \left[D_i \Phi F_{jk} \right]$

‘t Hooft Polyakov monopole

[Nahm `80][Diaconescu `97]

<u>Nahm construction</u> : all monopole solutions of BPS equation $D_i\Phi = \frac{1}{2}\epsilon_{ijk}F_{jk}$

1) “Tachyon” operator [Terashima, KH, `06]

$$T \equiv \sigma_i x^i$$

2) Solve for normalized zeromode $\left[\frac{d}{d\xi} + T\right] v(\xi) = 0$

3) Obtain scalar and gauge fields

$$\Phi(x) \equiv \int d\xi\; v^\dagger \xi v\,, \quad A_i(x) \equiv \int d\xi\; v^\dagger \frac{d}{dx^i} v$$

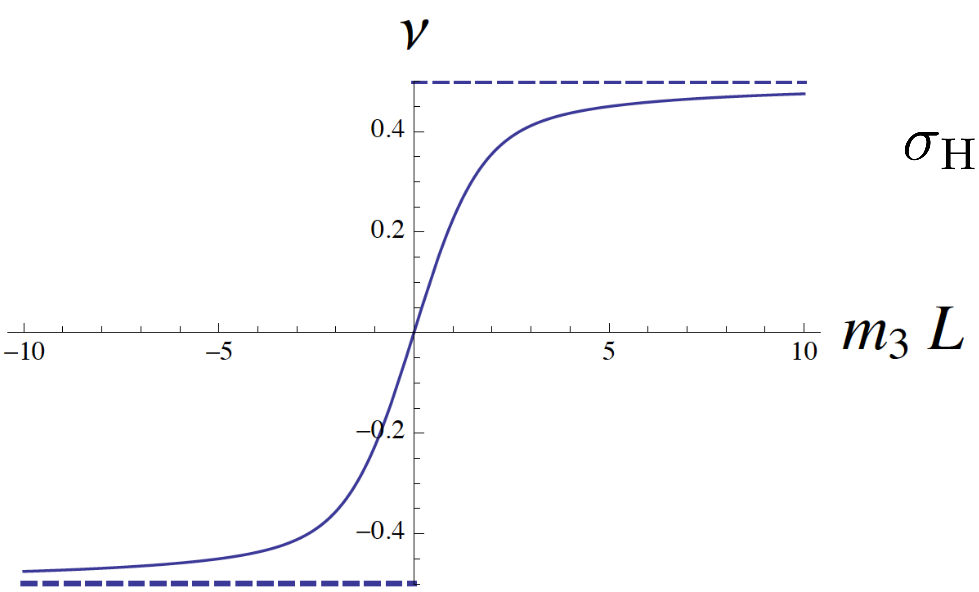
4) Monopole number $1 = \frac{1}{4\pi}\int d^3p\; \frac{1}{2}\epsilon_{ijk}\;\mathrm{tr}\left[D_i\Phi F_{jk}\right]$

't Hooft Polyakov monopole

<u>Relation to TKNN??</u>

Replacing p_3 by a mass m_3, define "topological number"

$$\tilde{\nu} \equiv \frac{1}{4\pi} \int dp_1 dp_2 \, \mathrm{tr}[\Phi F_{12}]$$



$$\sigma_{\mathrm{Hall}} \sim \int_{-L}^{L} \sigma(x^4) f(x^4) dx^4$$