APCTP
Asia Pacific Center for Theoretical Physics

Holography and Topology of Quantum Matter
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Holographic conductivity and Homes' law

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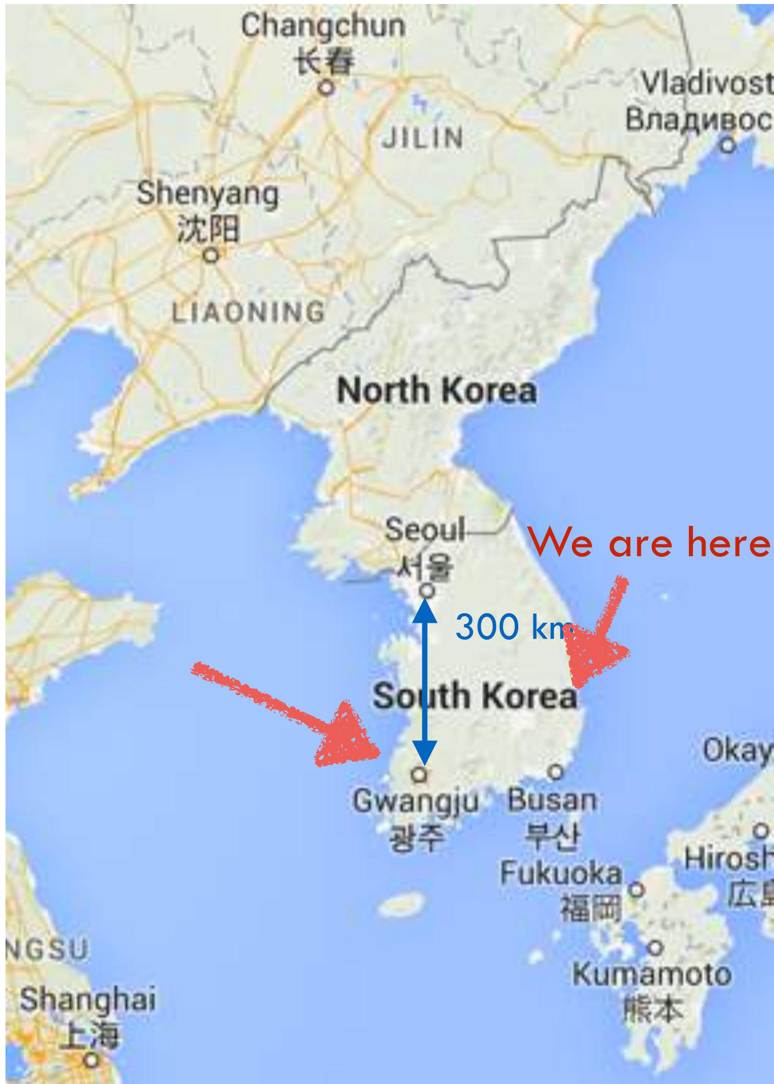


Motivation:

What is Homes’ law?

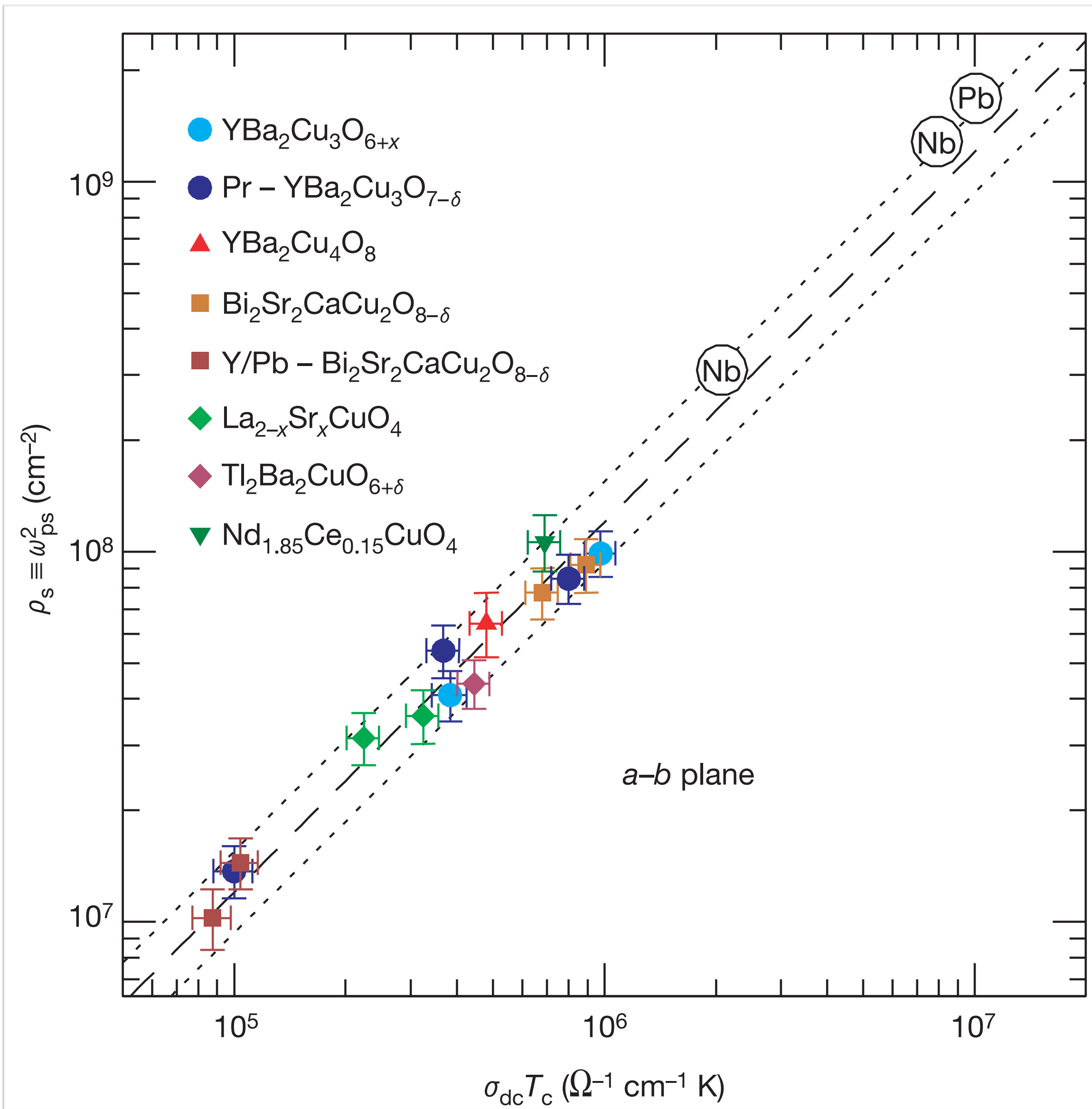
Why is Homes’ law interesting?

Homes law

[Homes, et al. 2004 Nature]

A universal scaling relation in high-temperature superconductors

C. C. Homes[1], S. V. Dordevic[1], M. Strongin[1], D. A. Bonn[2], Ruixing Liang[2], W. N. Hardy[2], Seiki Komiya[3], Yoichi Ando[3], G. Yu[4], N. Kaneko[5*], X. Zhao[5], M. Greven[5,6], D. N. Basov[7] & T. Timusk[8]



Superconducting transition temperature

Electric DC conductivity

Superfluid density

- Homes' law: $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$

C is constant regardless of doping level, nature of dopant, crystal structure and type of disorder.

C=4.4: a-b plane high-Tc superconductor, clean BCS superconductor

C=8.1: c-axis high-Tc superconductor, dirty BCS superconductor

[Erdmenger, Herwerth, Klug, Meyer, Schalm: 1501.07615]

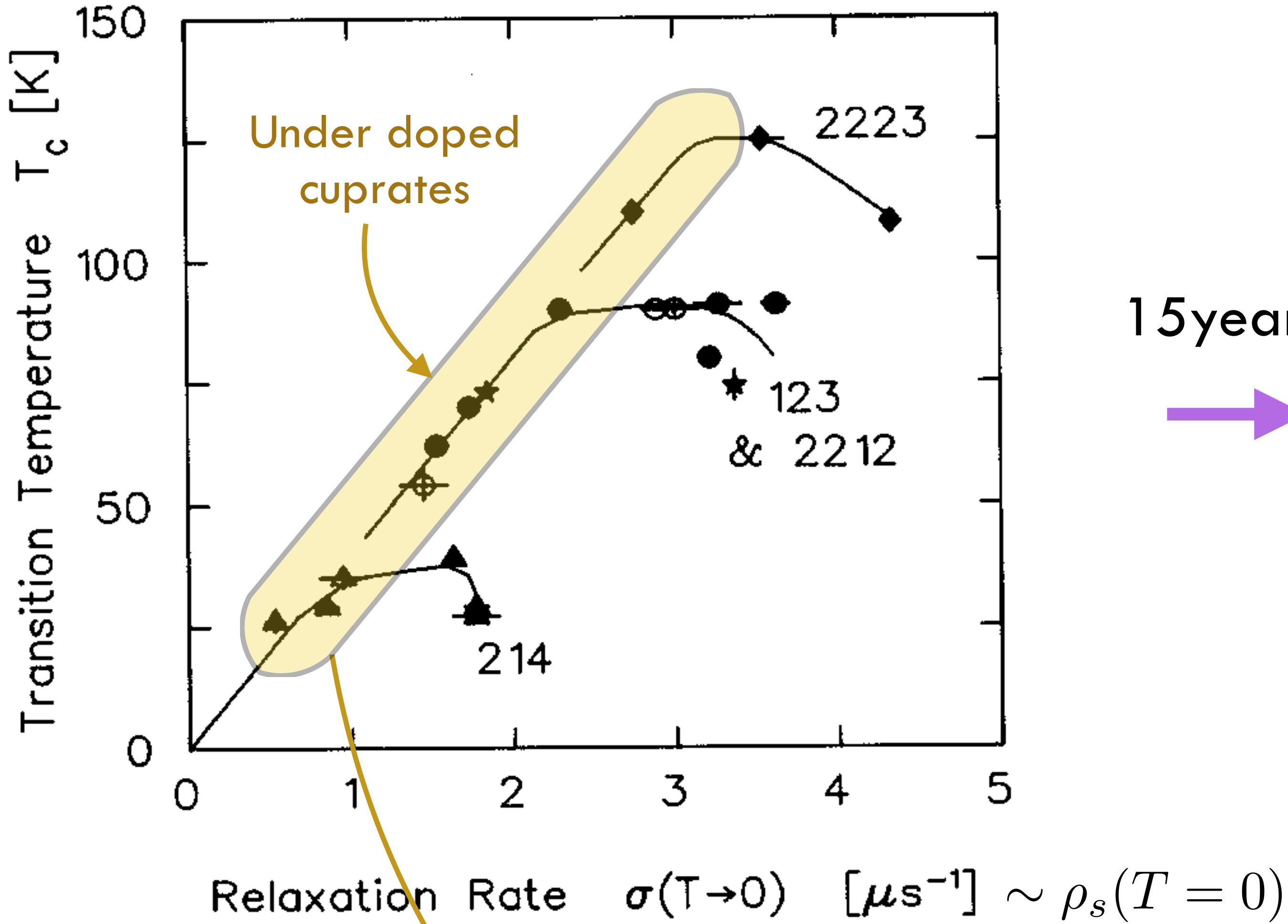
- Understanding high Tc superconductivity?
- Universal property of the hairy black holes?

History for finding universality: Uemura's law

1986: Discovery of cuprate, Bednorz, et al. (Z. Phys. B)

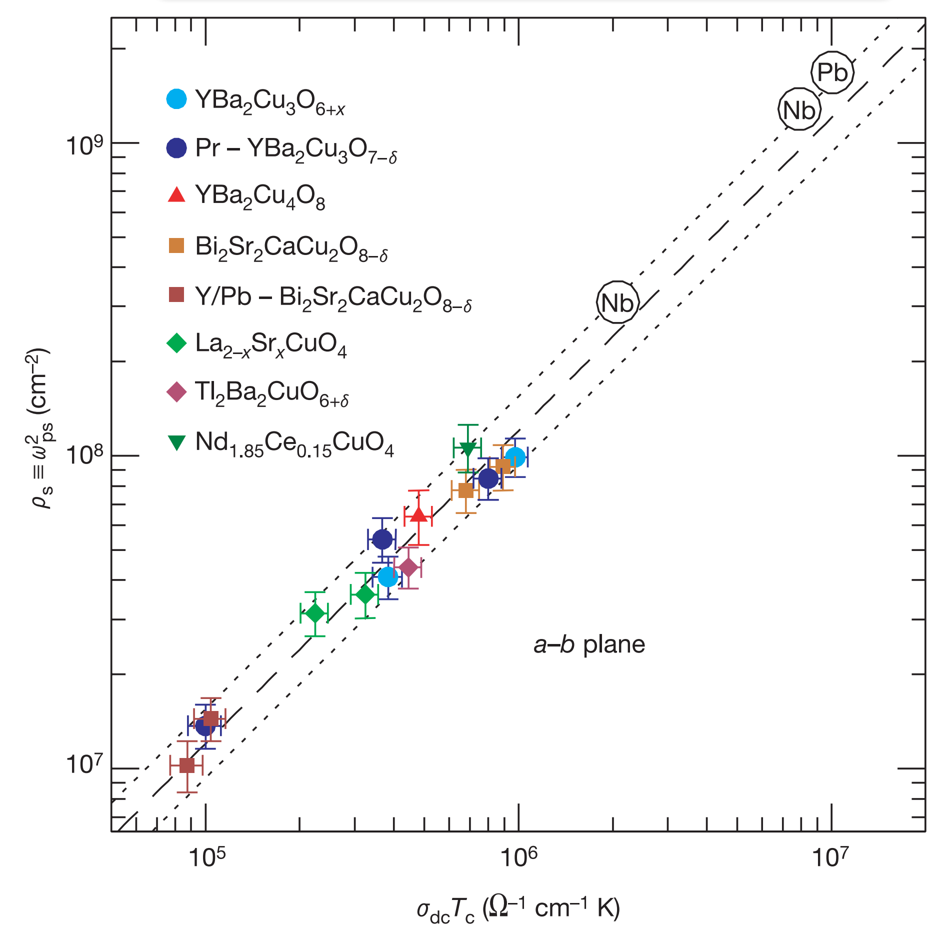
3years

1989: Uemura et al (PRL)



15years

2004: Homes et al (Nature)



Uemura's law

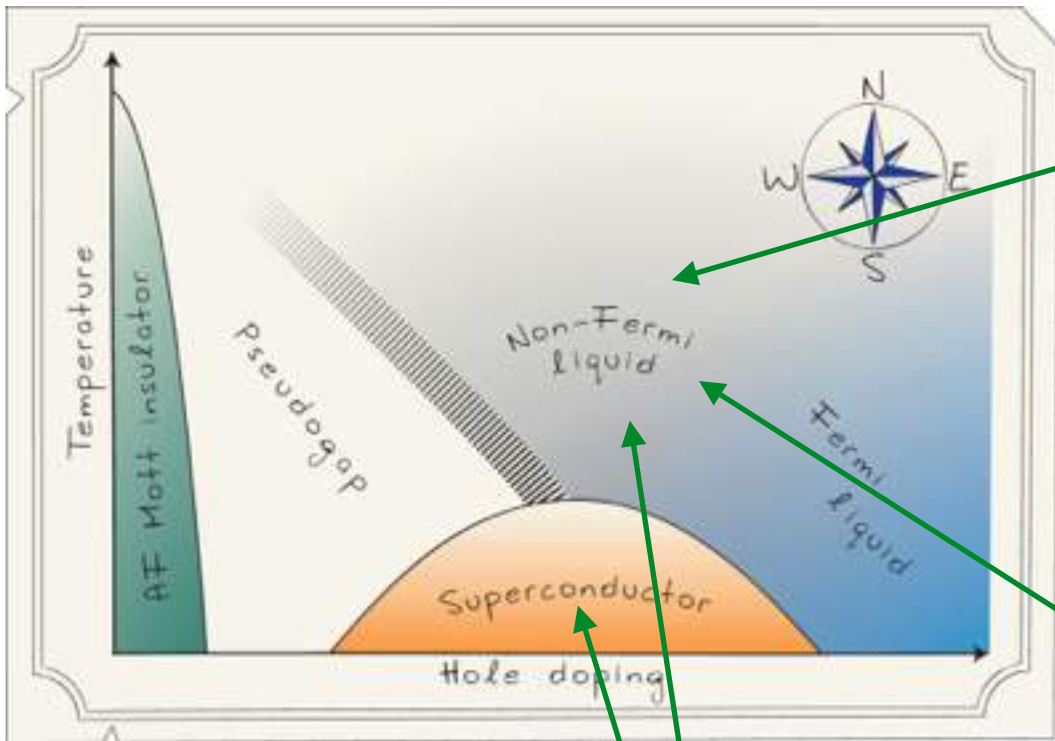
$$\rho_s(T=0) = BT_c$$

Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

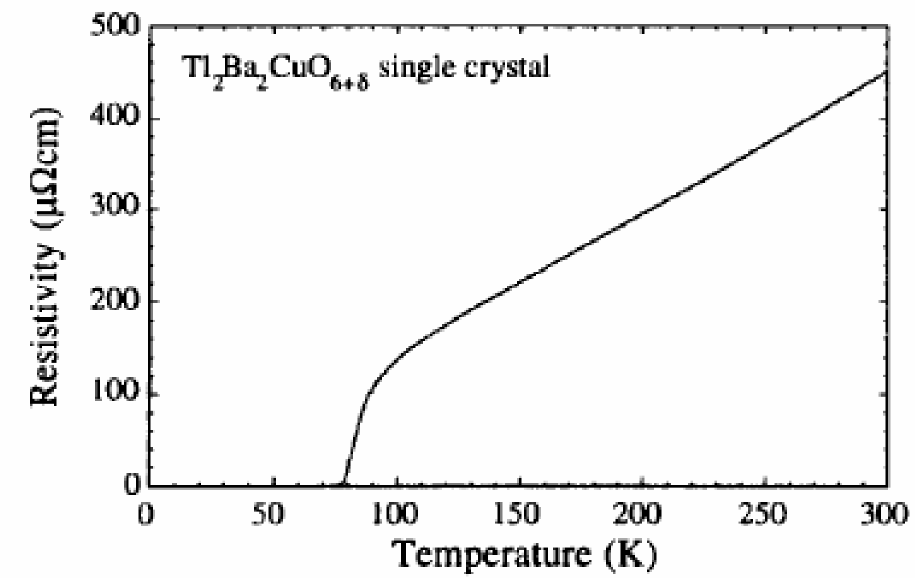
Universal properties in cuprates

Cuprate phase diagram

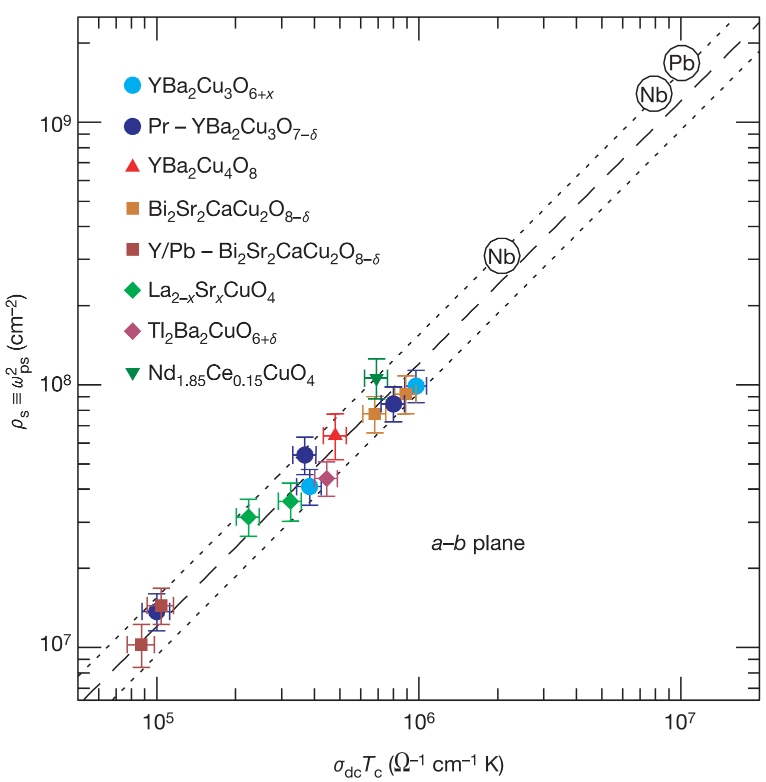


[Peter Wahl: 2012, Nature

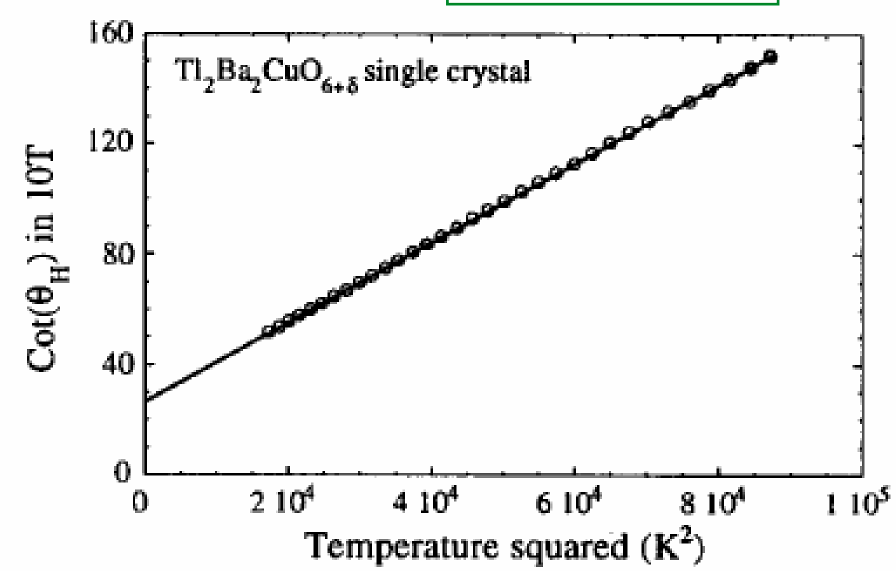
- DC resistivity $\boxed{\rho \sim T}$



- Homes law $\boxed{\rho_s(T=0) = C\sigma_{DC}(T_c)T_c}$



- Hall angle $\boxed{\sigma_{xx}/\sigma_{xy} \sim T^2}$



Mackenzie, 1997

Contents

Homes' law in Holographic context

arXiv.org > hep-th > arXiv:1002.1722

High Energy Physics – Theory

Introduction to Holographic Superconductors

Gary T. Horowitz

8.1 Open problems

We close with a list of open problems[15]. They are roughly ordered in difficulty with the easier problems listed first. (Of course, this is my subjective impression. With the right approach, an apparently difficult problem may become easy!)

1. In the probe limit below the critical temperature, there is an infinite discrete set

.
.
.

10. The high temperature cuprate superconductors satisfy a simple scaling law relating the superfluid density, the normal state (DC) conductivity and the critical temperature [36]. Can this be given a dual gravitational interpretation?

Homes’ law in Holographic context

- Homes’ law: $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$

arXiv.org > hep-th > arXiv:1206.5305

High Energy Physics – Theory

Towards a Holographic Realization of Homes' Law

Johanna Erdmenger, Patrick Kerner, Steffen Muller

arXiv.org > hep-th > arXiv:1501.07615

High Energy Physics – Theory

S-Wave Superconductivity in Anisotropic Holographic Insulators

Johanna Erdmenger, Benedikt Herwerth, Steffen Klug, Rene Meyer, Koenraad Schalm

arXiv.org > hep-th > arXiv:1604.06205

High Energy Physics – Theory

Ward Identity and Homes' Law in a Holographic Superconductor with Momentum Relaxation

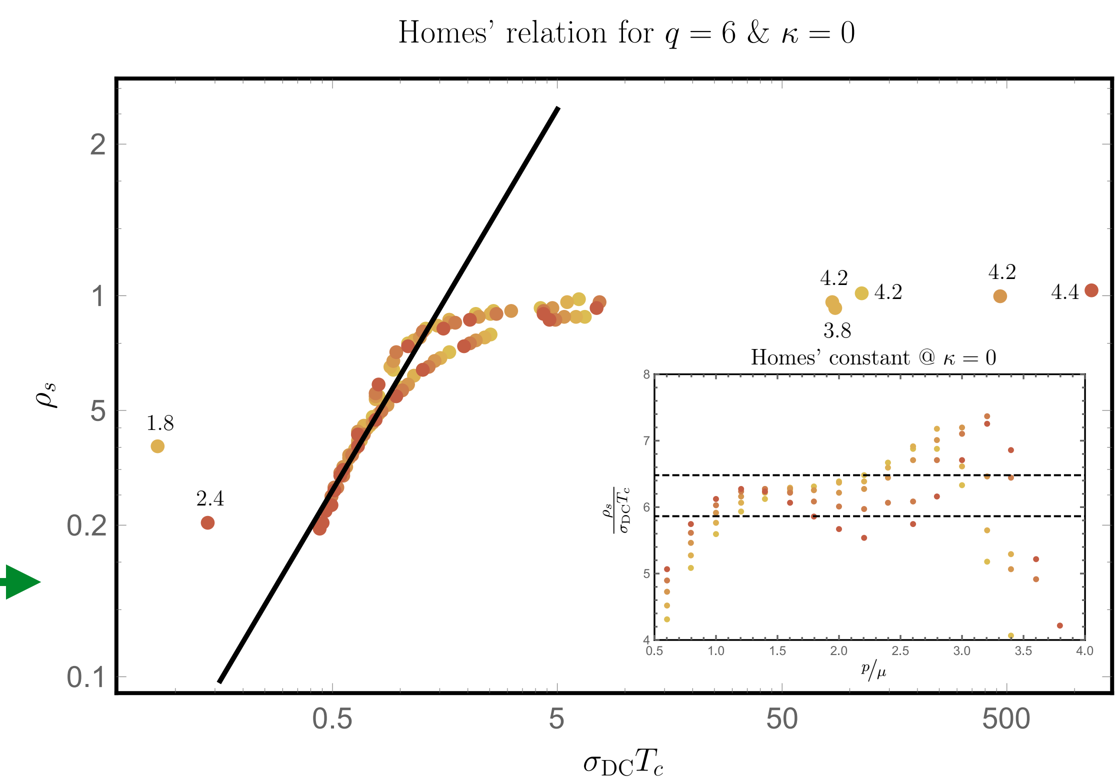
Keun-Young Kim, Kyung Kiu Kim, Miok Park

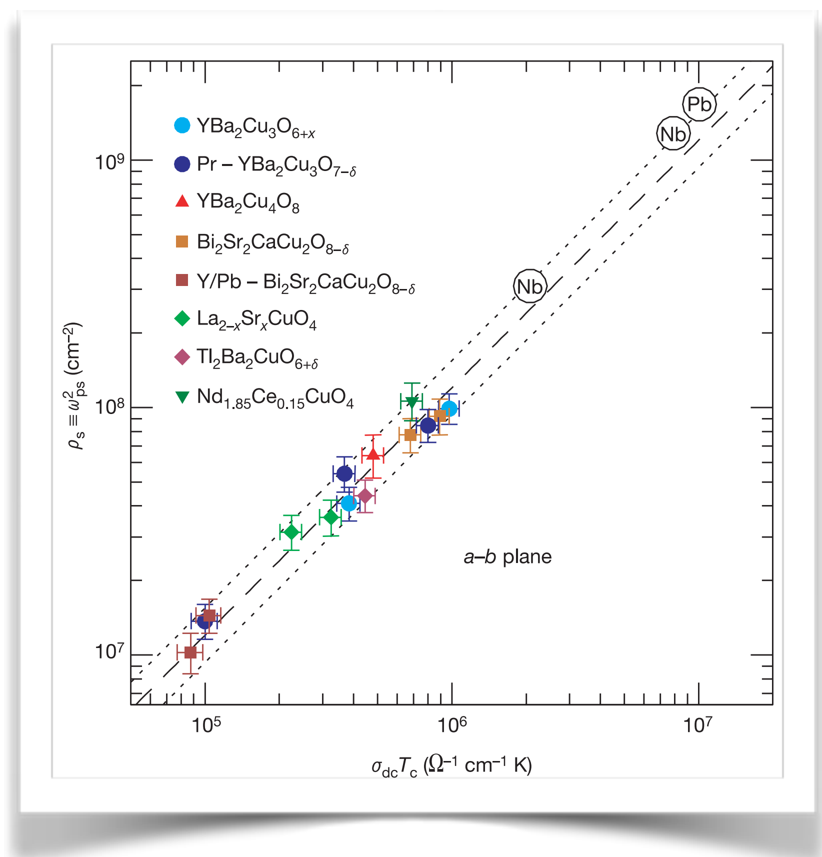
arXiv.org > hep-th > arXiv:1608.04653

High Energy Physics – Theory

Homes' law in Holographic Superconductor with Q-lattices

Keun-Young Kim, Chao Niu





Homes' law in Holographic context

- Homes' law: $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$

arXiv.org > hep-th > arXiv:1206.5305

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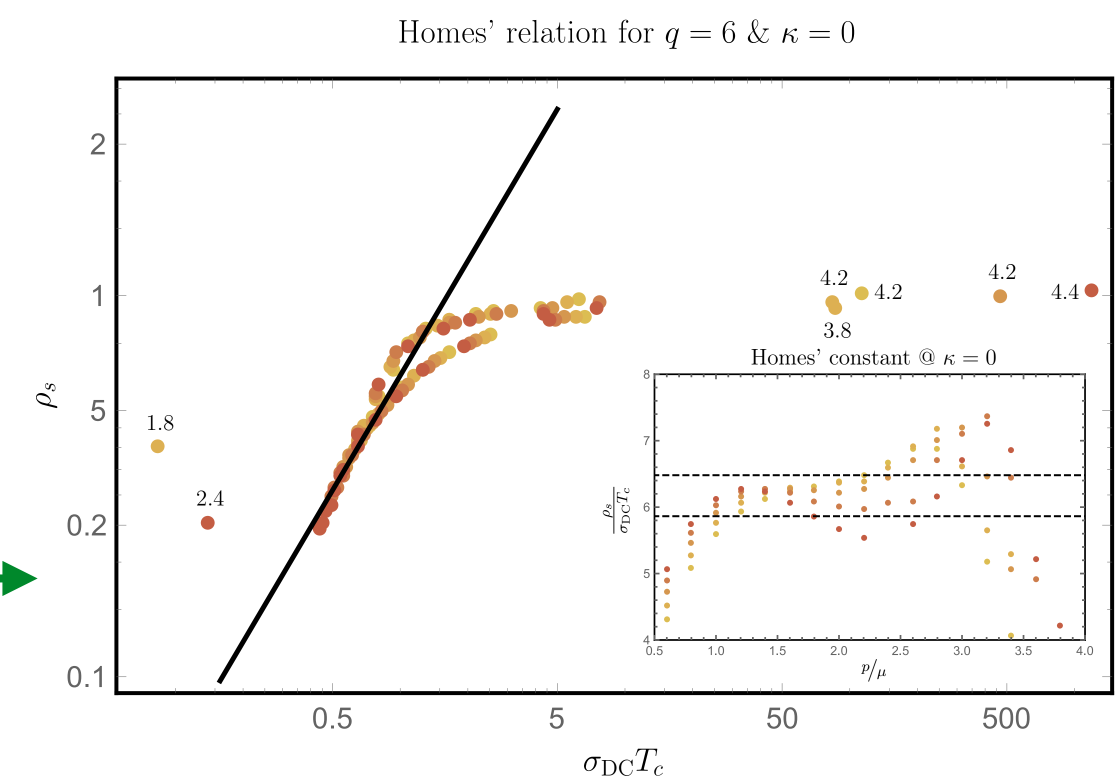

arXiv.org > hep-th > arXiv:1604.06205

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High Energy Physics – Theory

Homes' law in Holographic Superconductor with Q-lattices

Keun-Young Kim, Chao Niu

This talk

- Physical understanding?
- How much model dependent?

arXiv.org > hep-th > arXiv:1409.8346

High Energy Physics – Theory

Coherent/incoherent metal transition in a holographic model

Keun-Young Kim, Kyung Kiu Kim, Yunseok Seo, Sang-Jin Sin

arXiv.org > hep-th > arXiv:1501.00446

High Energy Physics – Theory

A Simple Holographic Superconductor with Momentum Relaxation

Keun-Young Kim, Kyung Kiu Kim, Miok Park

Goals and method

Goals

- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$

Holographer's tool box

1. Need a holographic superconductor ~ hairy black hole (0803.3295: Hartnoll, Herzog, Horowitz)
2. Conductivity?

Linear response theory

$$\boxed{\sigma(\omega) = \frac{G^R_{JJ}(\omega)}{i\omega}}$$

Holography

G^R

Son and Starinets, hep-th/0205051
Herzog and Son, hep-th/0212072
Skenderis and van Rees, 0805.0150

$$\sigma_{DC} = \lim_{\omega\to 0} \mathrm{Re}[\sigma(\omega)]$$

$$\rho_s = \lim_{\omega\to 0} \omega \mathrm{Im}[\sigma(\omega)]$$

The model and method are well established.
Why is the progress slow?

Momentum relaxation matters

Original holographic superconductor: HHH

[Hartnoll, Herzog, Horowitz: 0803.3295]

The first holographic superconductor

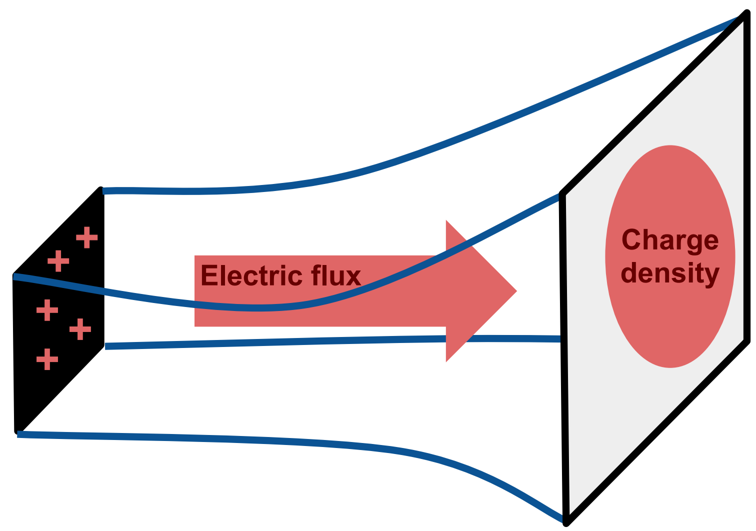
- Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

$$S_{HHH} = \int d^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^*\right]$$

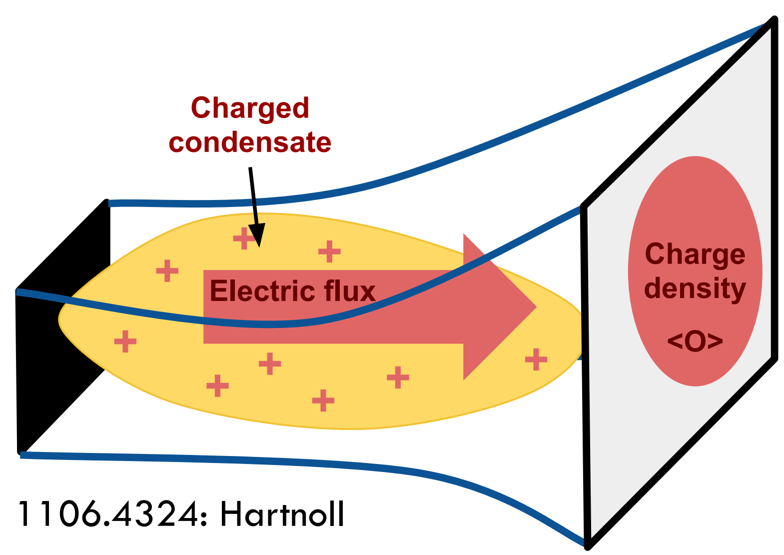
$\Phi = 0$

AdS-RN-black brane



$\Phi \neq 0$

Holographic superconductor



1106.4324: Hartnoll

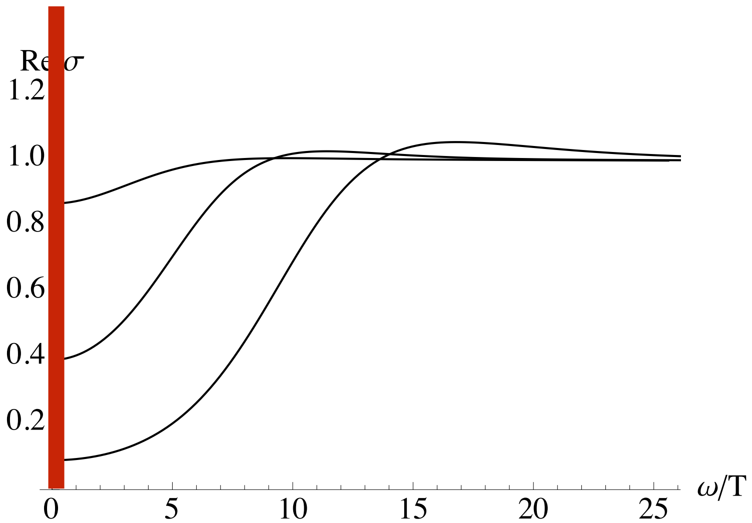
Optical conductivity

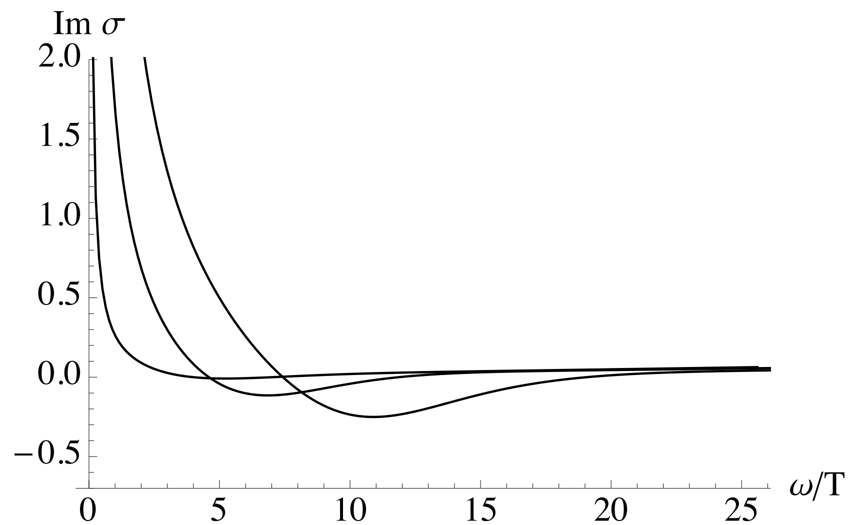
Conductivity: normal phase

[Hartnoll: 0903.3234]

- Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$





$$\mathrm{Im}\,\sigma \sim 1/\omega \quad \Leftrightarrow \quad \mathrm{Re}\;\sigma(\omega) \sim \delta(\omega)$$

Kramers-Kronig relation

$$\chi(\omega) = \chi_R(\omega) + i\chi_I(\omega)$$

$$\chi_R(\omega) = \frac{1}{\pi}\mathcal{P}\int \frac{\chi_I(\omega')}{\omega'-\omega}d\omega' , \qquad \chi_I(\omega) = -\frac{1}{\pi}\mathcal{P}\int \frac{\chi_R(\omega')}{\omega'-\omega}d\omega'$$

Translation invariance + finite density

Helical lattice model

$$S_{\text{total}} = S_{\text{helix}} + \int \mathrm{d}^{4+1}x\,\sqrt{-g}\Big[-|\partial\rho - \mathrm{i}qA\rho|^2 - m_\rho^2|\rho|^2\Big]$$

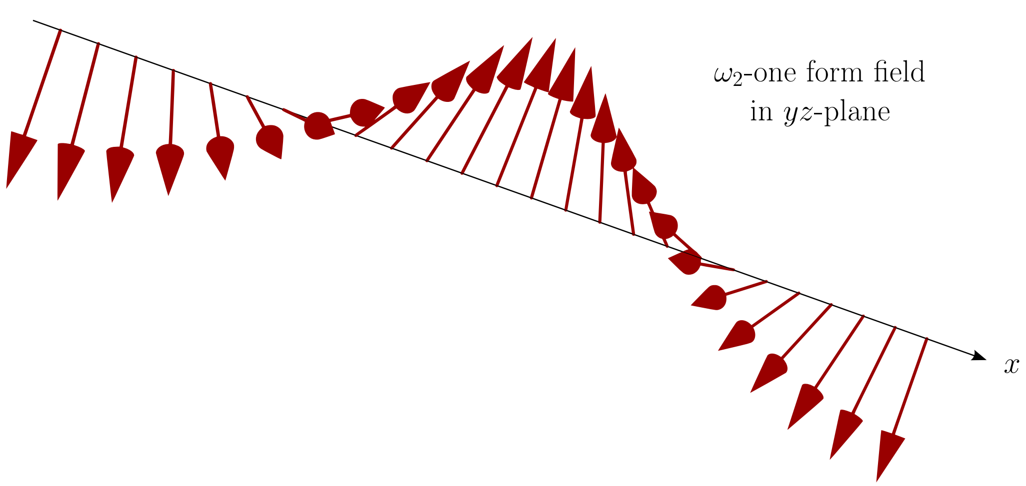
$$S_{\text{helix}} = \int \mathrm{d}^{4+1}x\,\sqrt{-g}\Big[R + 12 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \boxed{- \frac{1}{4}W^{\mu\nu}W_{\mu\nu} - m^2 B_\mu B^\mu}\Big]$$

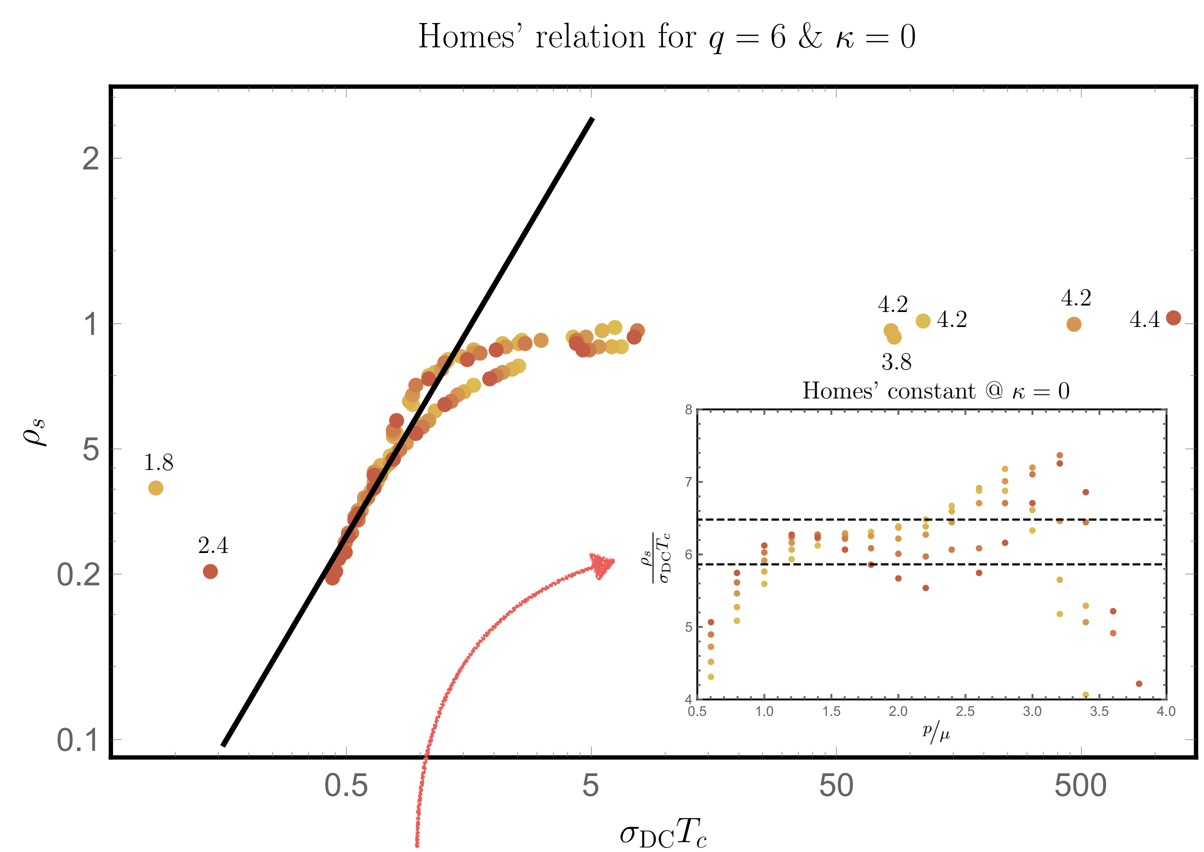
momentum relaxing sector

$$B = w(r)\omega_2$$

$$w(\infty) = \lambda$$

$$\omega_2 = \cos(px)\,\mathrm{d}y - \sin(px)\,\mathrm{d}z$$





$$C = \frac{\rho_s(T=0)}{\sigma_{DC}(T_c)T_c}$$

Holographic superconductor with momentum relaxation

$$S_{HHH} = \int d^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^*\right]$$

Massless scalar [Andrade, Withers: 1311.5157] → [Andrade, Gentle: 1412.6521] [KYK, Kim, Park: 1501.00446]

$$S_{MS} = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\sum_{I=1,2}(\partial\psi_I)^2\right]$$

$\psi_I = (\beta x, \beta y)$

Q-lattice [Donos, Gauntlett: 1311.3292] → [Ling, Liu, Niu, Wu, Xian: 1410.6761] [Andrade, Gentle: 1412.6521]

$$S_Q = \int d^4x\sqrt{-g}\left[-|\partial\Psi|^2 - m_\Psi^2|\Psi|^2\right]$$

$\Psi = e^{ikx}z\psi(z)$

$\psi(0) = \lambda$

- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$

$$C = \frac{\rho_s(T=0)}{\sigma_{DC}(T_c)T_c}$$

$$B = \frac{\rho_s(T=0)}{(T_c)T_c}$$

We want to check if C or B is universal
(independent of momentum relaxation parameters)

Contents

Model

[KYK, Kim, Park: 1501.00446]

Action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^* - \frac{1}{2}\sum_{I=1,2}(\partial\psi_I)^2\right]$$

- Homes' law

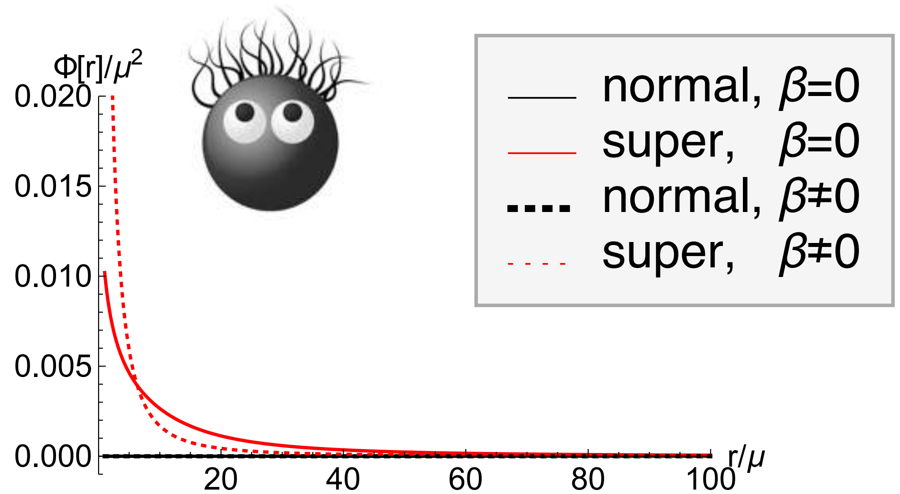
$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

Ansatz

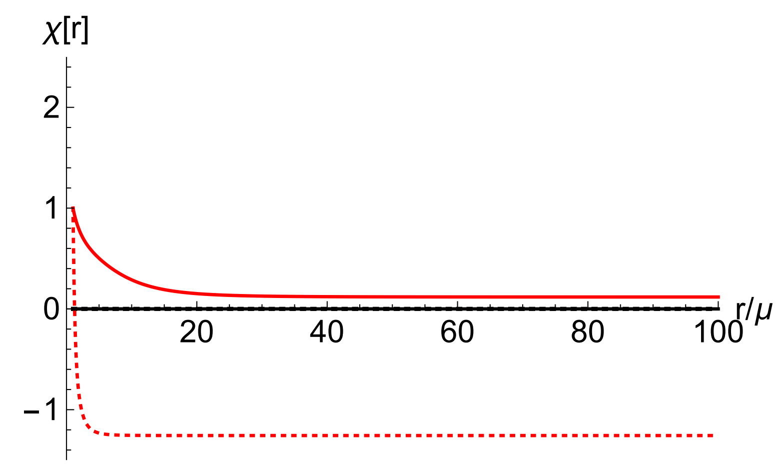
$$A = A_t(r)\mathrm{d}t \qquad \Phi = \Phi(r) \qquad \psi_I = (\beta x, \beta y)$$

$$\mathrm{d}s^2 = -U(r)e^{-\chi(r)}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{U(r)} + r^2(\mathrm{d}x^2 + \mathrm{d}y^2)$$

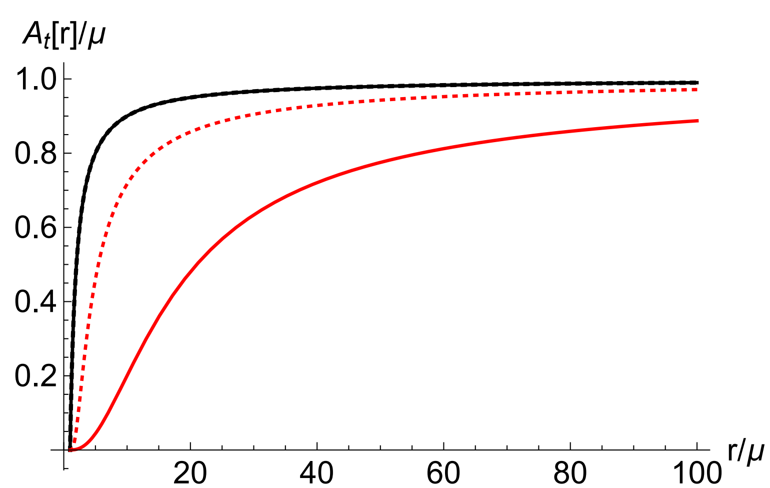
Solutions



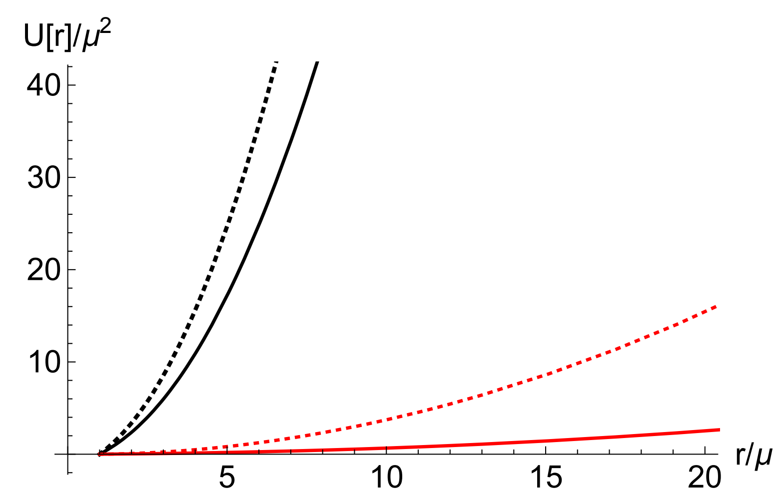
(a) $\Phi(r)/\mu^2$



(b) $\chi(r)$



(c) $A_t(r)/\mu$

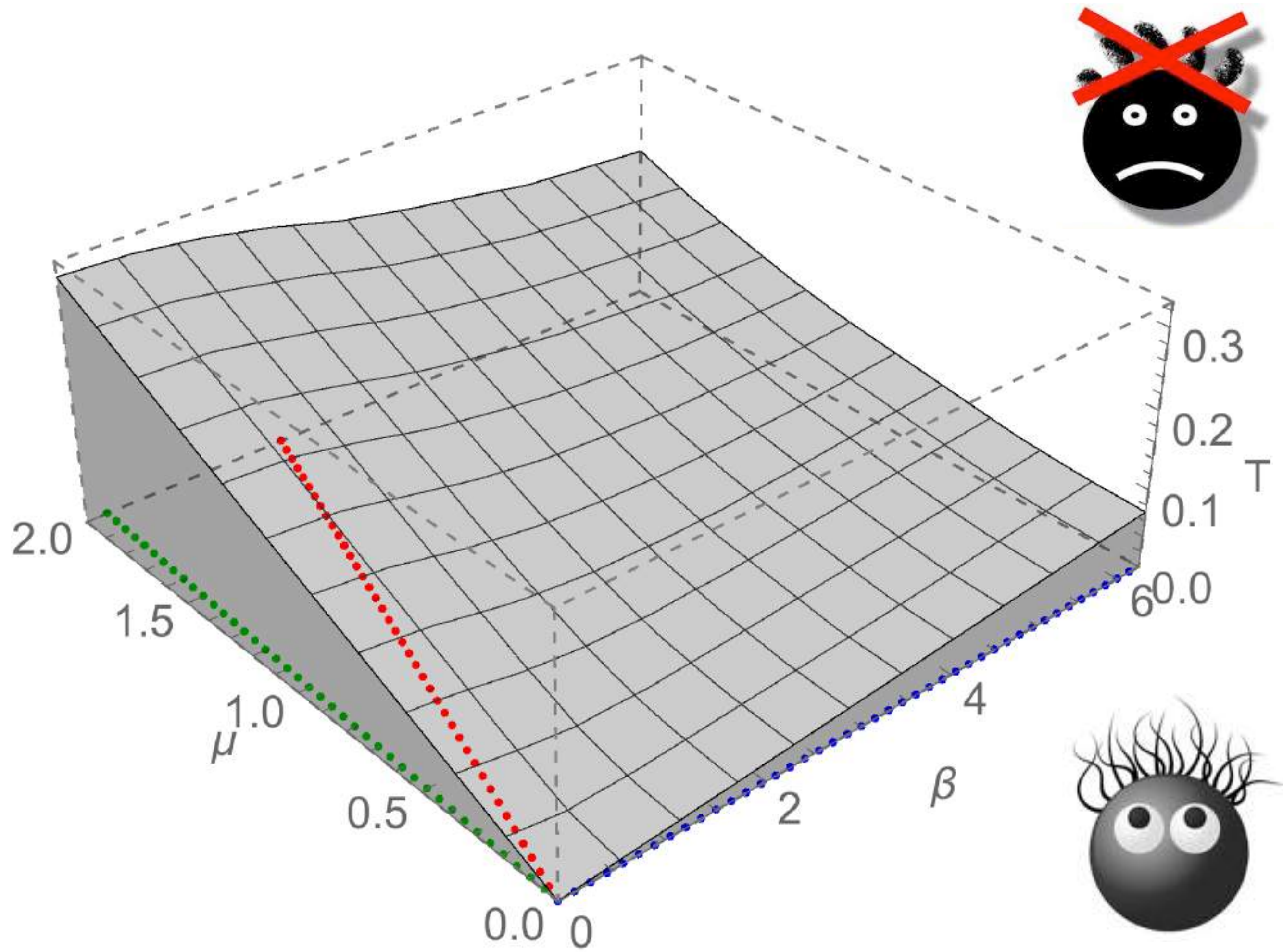


(d) $U(r)/\mu^2$

Critical temperature

[KYK, Kim, Park: 1501.00446]

- **Homes' law** $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$



Conductivities

[KYK, Kim, Park: 1501.00446]

Action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^* - \frac{1}{2}\sum_{I=1,2}(\partial\psi_I)^2\right]$$

Background

$$A = A_t(r)\mathrm{d}t \qquad \Phi = \Phi(r) \qquad \psi_I = (\beta x, \beta y)$$

$$\mathrm{d}s^2 = -U(r)e^{-\chi(r)}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{U(r)} + r^2(\mathrm{d}x^2 + \mathrm{d}y^2)$$

$+$

Fluctuations

$$\delta A_x(t,r) = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}a_x(\omega,r)$$

$$\delta g_{tx}(t,r) = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}r^2 h_{tx}(\omega,r)$$

$$\delta\psi_1(t,r) = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}\xi(\omega,r)$$

$$h_{tx} = h_{tx}^{(0)} + \frac{1}{r^2}h_{tx}^{(2)} + \frac{1}{r^3}h_{tx}^{(3)} + \cdots,$$

$$a_x = a_x^{(0)} + \frac{1}{r}a_x^{(1)} + \cdots,$$

$$\xi = \xi^{(0)} + \frac{1}{r^2}\xi^{(2)} + \frac{1}{r^3}\xi^{(3)} + \cdots,$$

$$S_{\mathrm{ren}}^{(2)} = \frac{V_2}{2}\int_0^{\infty}\frac{\mathrm{d}\omega}{2\pi}\left(-\rho\bar{a}_x^{(0)}h_{tx}^{(0)} - \epsilon\bar{h}_{tx}^{(0)}h_{tx}^{(0)} + \bar{a}_x^{(0)}a_x^{(1)} - 3\bar{h}_{tx}^{(0)}h_{tx}^{(3)} + 3\bar{\xi}^{(0)}\xi^{(3)}\right)$$

$$\frac{1}{2}\int_0^{\infty}\frac{\mathrm{d}\omega}{2\pi}\left[J_{-\omega}^a G_{ab} J_\omega^b\right]$$

$$\begin{pmatrix} a_x^{(1)} \\ h_{tx}^{(3)} \\ \xi^{(3)} \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}\begin{pmatrix} a_x^{(0)} \\ h_{tx}^{(0)} \\ \xi^{(0)} \end{pmatrix},$$

$$R^a = \mathbb{M}^a_b J^b$$

- Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

$$\sigma_{DC} = \sigma(\omega = 0)$$

$$\sigma(\omega) \sim i\frac{\rho_s}{\omega}$$

Numerical method for multi fields

[Kaminski, Landsteiner, Mas, Shock, Tarrio: 2009]
[KYK, Kim, Sin, Seo: 2014)]

How to compute $\mathbb{M}^a_b$

$$R^a = \mathbb{M}^a_b J^b$$

$$\Phi^a_i(r) \to \mathbb{S}^a_i + \cdots + \frac{\mathbb{O}^a_i}{r^{\delta_a}} + \cdots$$

$$\Phi^a(r) = \Phi^a_i(r) c^i \to \mathbb{S}^a_i c^i + \cdots + \frac{\mathbb{O}^a_i c^i}{r^{\delta_a}} + \cdots$$

$$\equiv J^a + \cdots + \frac{R^a}{r^{\delta_a}} + \cdots ,$$

$$c^i = (\mathbb{S}^{-1})^i_a J^a \qquad R^a = \mathbb{O}^a_i c^i = \underbrace{\boxed{\mathbb{O}^a_i (\mathbb{S}^{-1})^i_b}}_{\mathbb{M}^a_b} J^b$$

$$h_{tx} = h^{(0)}_{tx} + \frac{1}{r^2} h^{(2)}_{tx} + \frac{1}{r^3} h^{(3)}_{tx} + \cdots ,$$

$$a_x = a^{(0)}_x + \frac{1}{r} a^{(1)}_x + \cdots ,$$

$$\xi = \xi^{(0)} + \frac{1}{r^2} \xi^{(2)} + \frac{1}{r^3} \xi^{(3)} + \cdots ,$$

ex) one field case: $\dfrac{a^{(1)}_x}{a^{(0)}_x}$

$$\frac{1}{2} \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \left[J^a_{-\omega} G_{ab} J^b_\omega \right]$$

$$\begin{pmatrix} \sigma & \alpha T \\ \bar{\alpha} T & \bar{\kappa} T \end{pmatrix} = \begin{pmatrix} -\frac{iG_{11}}{\omega} & \frac{i(G_{11}\mu - G_{12})}{\omega} \\ \frac{i(G_{11}\mu - G_{21})}{\omega} & -\frac{i(G_{22} + \mu(-G_{12} - G_{21} + G_{11}\mu))}{\omega} \end{pmatrix}$$

[Hartnoll: 0903.3234]

Thermoelectric conductivities and DC limits

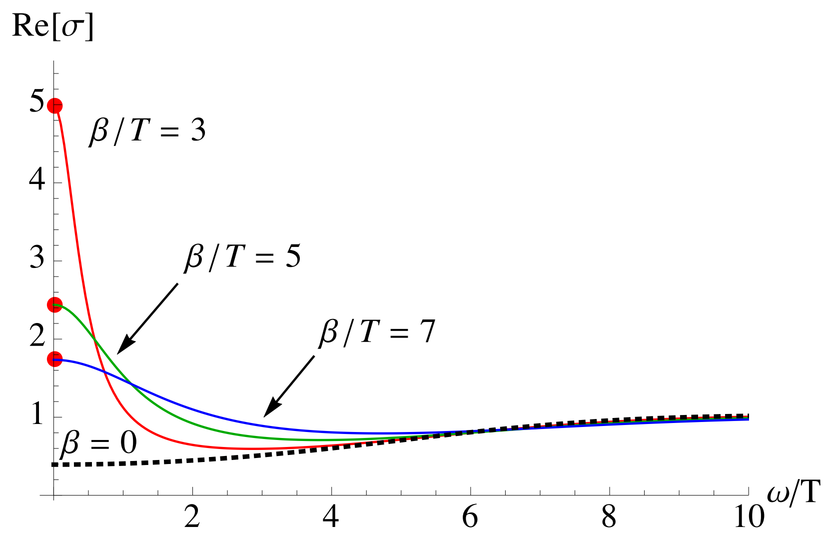
[KYK, Kim, Sin, Seo: 1409.8346]

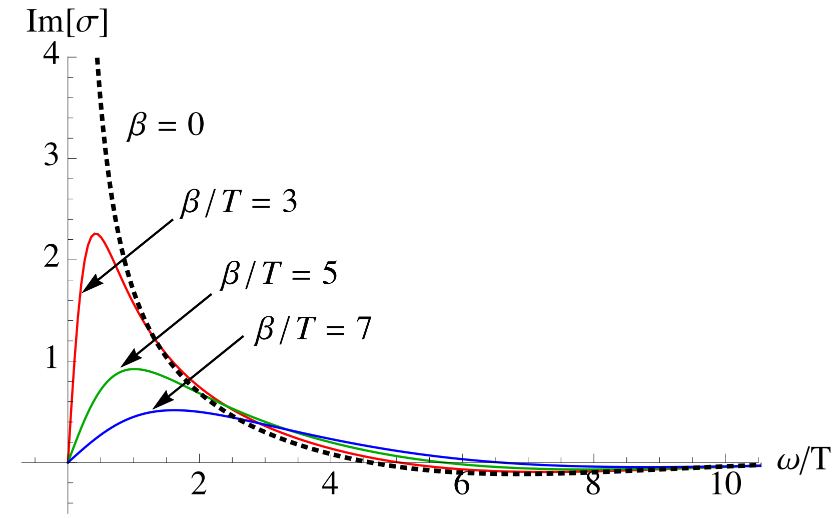
Electric conductivity

$\mu/T = 6$

DC result:
Andrade, Withers
1311.5157

$$\boxed{\sigma = 1 + \frac{\mu^2}{\beta^2}}$$



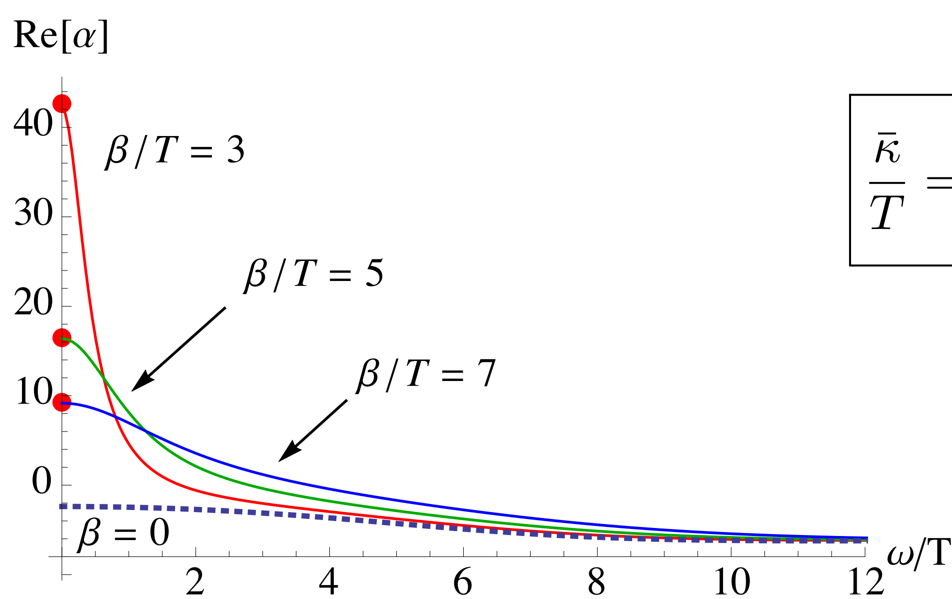


Thermoelectric conductivity

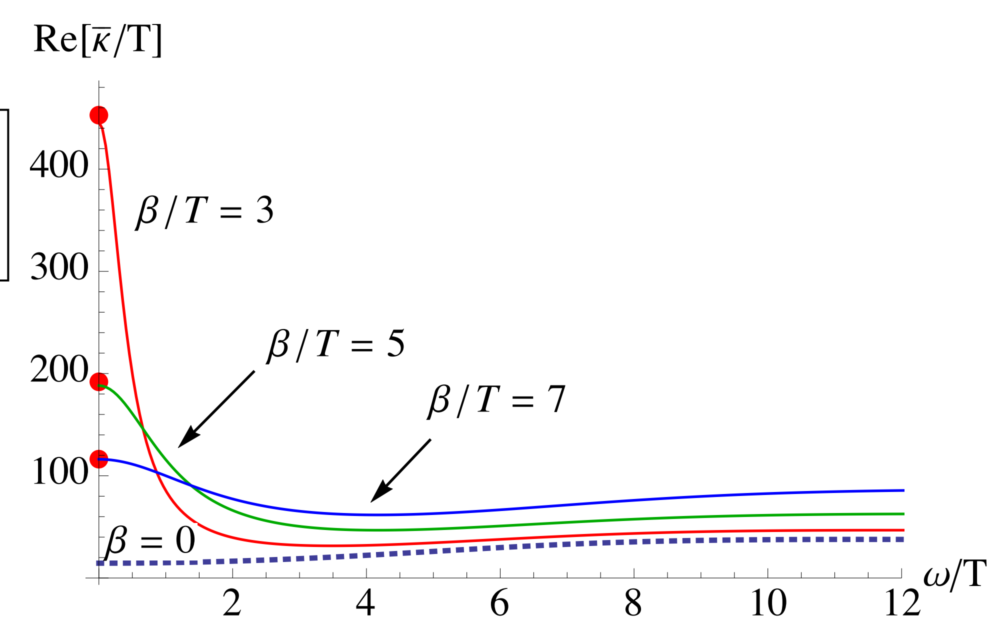
DC results:
Donos and Gauntlett
1406.4742

Thermal conductivity

$$\boxed{\alpha = \frac{4\pi\mu}{\beta^2} r_0}$$



$$\boxed{\frac{\bar{\kappa}}{T} = \frac{(4\pi)^2}{\beta^2} r_0^2}$$



Ward identities

- Generating functional for Euclidean time ordered correlation functions

$$e^{W[g,A,\phi]} = Z[g,A,\phi] = \int D\Phi e^{-S[\Phi,g,A,\phi]}$$

- Diffeomorphisms and gauge transformations

$$x^\mu \to x^\mu + \zeta^\mu \qquad\qquad A_\mu \to A_\mu + \partial_\mu \Lambda$$

$$\delta g_{\mu\nu} = (\mathcal{L}_\zeta g)_{\mu\nu} = \nabla_\mu \zeta_\nu + \nabla_\nu \zeta_\mu \,,$$

$$\delta A_\mu = (\mathcal{L}_\zeta A)_\mu = \zeta^\lambda \nabla_\lambda A_\mu + (\nabla_\mu \zeta^\nu) A_\nu \,,$$

$$\delta \phi_I = (\mathcal{L}_\zeta \phi_I) = \zeta^\lambda \nabla_\lambda \phi_I \,,$$

- Ward identities for diffeomorphisms

$$\int d^3x \left(\frac{\delta W}{\delta g_{\mu\nu}(x)} (\mathcal{L}_\zeta g)_{\mu\nu} + \frac{\delta W}{\delta A_\mu(x)} (\mathcal{L}_\zeta A)_\mu + \frac{\delta W}{\delta \phi_I(x)} (\mathcal{L}_\zeta \phi_I) \right) = 0$$

One-point correlation functions

$$\langle J^\mu(x) \rangle = \frac{\delta W}{\delta A_\mu(x)} \quad , \quad \langle T^{\mu\nu}(x) \rangle = 2 \frac{\delta W}{\delta g_{\mu\nu}(x)} \quad , \quad \langle \mathcal{O}^I(x) \rangle = \frac{\delta W}{\delta \phi_I(x)}$$

- Ward identities for one-point functions

$$\boxed{D_\mu \langle T^{\mu\nu} \rangle + F_\lambda{}^\nu \langle J^\lambda \rangle + \langle \mathcal{O}^I \rangle g^{\nu\lambda} \partial_\lambda \phi_I = 0 \qquad \partial_\mu \langle J^\mu(x) \rangle = 0}$$

Ward identities

KK, Kim, Park(2015)

- Ward identities for one-point functions

$$D_\mu \langle T^{\mu\nu}\rangle + F_\lambda{}^\nu \langle J^\lambda\rangle + \langle \mathcal{O}^I\rangle g^{\nu\lambda}\partial_\lambda \phi_I = 0$$

One-point correlation functions

$$\langle J^\mu(x)\rangle = \frac{\delta W}{\delta A_\mu(x)} \ , \quad \langle T^{\mu\nu}(x)\rangle = 2\frac{\delta W}{\delta g_{\mu\nu}(x)} \ , \quad \langle \mathcal{O}^I(x)\rangle = \frac{\delta W}{\delta \phi_I(x)}$$

- Ward identities for two-point functions

$$\omega \langle JT\rangle + \omega \langle n\rangle - i\beta \langle JS\rangle = 0$$

$$\omega \langle TT\rangle + \omega \langle \epsilon\rangle - i\beta \langle TS\rangle = 0$$

$$\omega < SJ > - i\beta \langle SS\rangle = 0$$

In momentum space (with constant one point function)

$$Q = T - \mu J \ , \ \langle QJ\rangle = i\omega\alpha T \ , \quad \langle QQ\rangle = i\omega\bar{\kappa} T \ , \quad \langle JJ\rangle = i\omega\sigma$$

- Relation between transport coefficients

$$\alpha + \frac{\mu}{T}\sigma - i\frac{\langle n\rangle}{\omega T} - \beta\frac{\langle JS\rangle}{\omega^2 T} = 0$$

$$\frac{\bar{\kappa}}{T} + \frac{2\mu\alpha}{T} + \frac{\mu^2\sigma}{T^2} - i\frac{\langle \epsilon'\rangle}{\omega T^2} - \beta\frac{\langle QS\rangle}{\omega^2 T^2} - \beta\frac{\mu\langle JS\rangle}{\omega^2 T^2} = 0$$

$$\langle SJ\rangle - i\beta\frac{\langle SS\rangle}{\omega} = 0$$

Ward identities

[KYK, Kim, Park: 1604.06205]

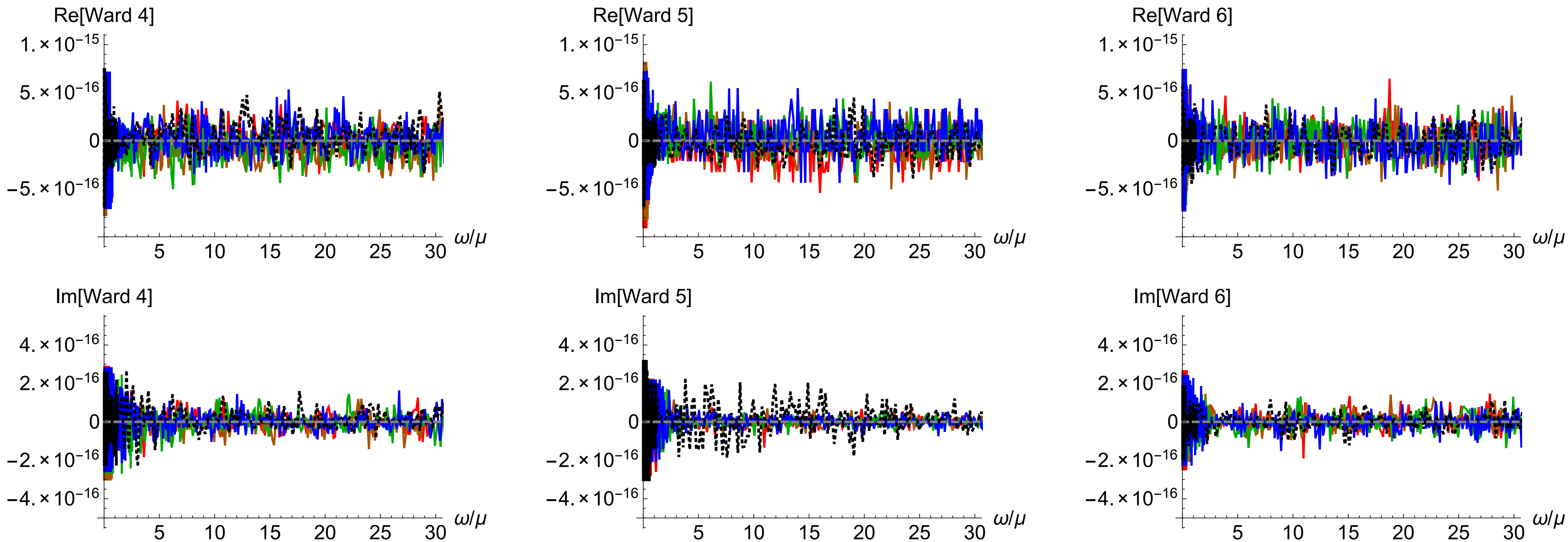
Derivation from field theory

$$\text{Ward 4}: \quad \alpha + \frac{\mu}{T}\sigma - i\frac{n}{\omega T} + \beta\frac{\langle JS\rangle}{\omega^2 T} = 0\,,$$

$$\text{Ward 5}: \quad \bar{\kappa} + 2\mu\alpha + \frac{\mu^2\sigma}{T} - i\frac{\epsilon'}{\omega T} + \beta\frac{\langle QS\rangle}{\omega^2 T} + \beta\frac{\mu\,\langle JS\rangle}{\omega^2 T} = 0$$

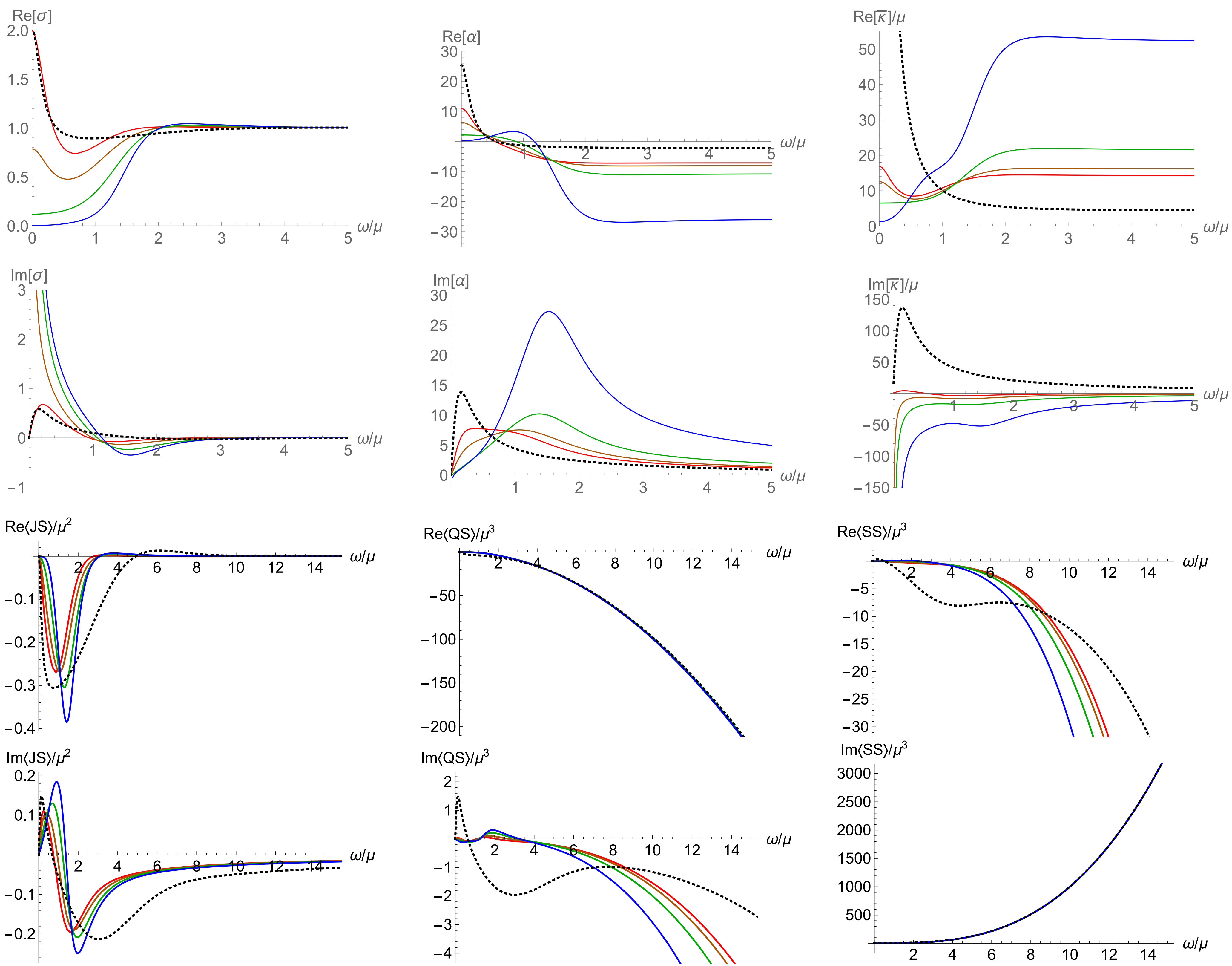
$$\text{Ward 6}: \quad \langle ST\rangle + i\beta\frac{\langle SS\rangle}{\omega} = 0\,,$$

Confirmation by numerical holography



Ward identities

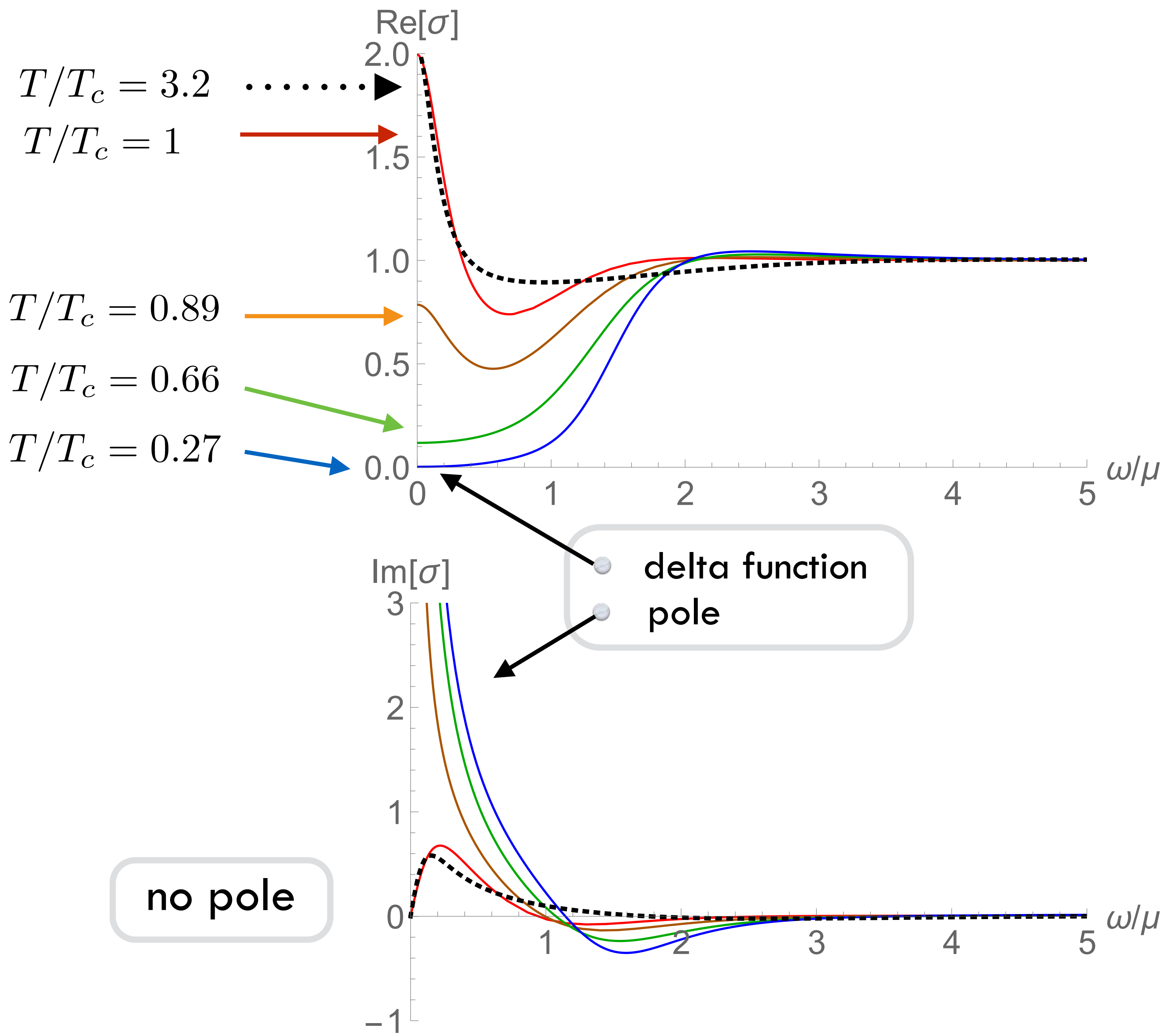
[KYK, Kim, Park: 1604.06205]



Optical electric conductivities: superconducting phase

[KYK, Kim, Park: 1501.00446]

$\beta/\mu = 1$



- Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

$$\sigma_{DC} = \sigma(\omega = 0)$$

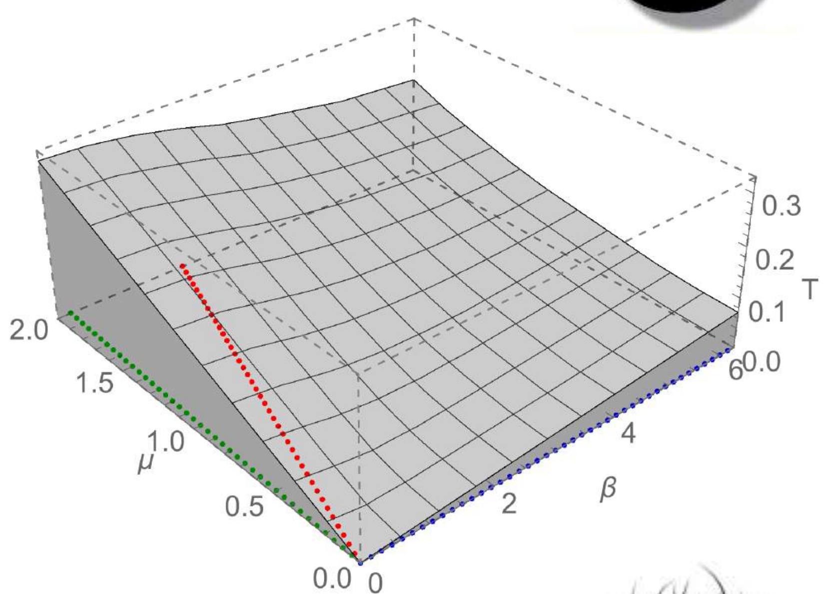
$$\sigma(\omega) \sim i\frac{\rho_s}{\omega}$$

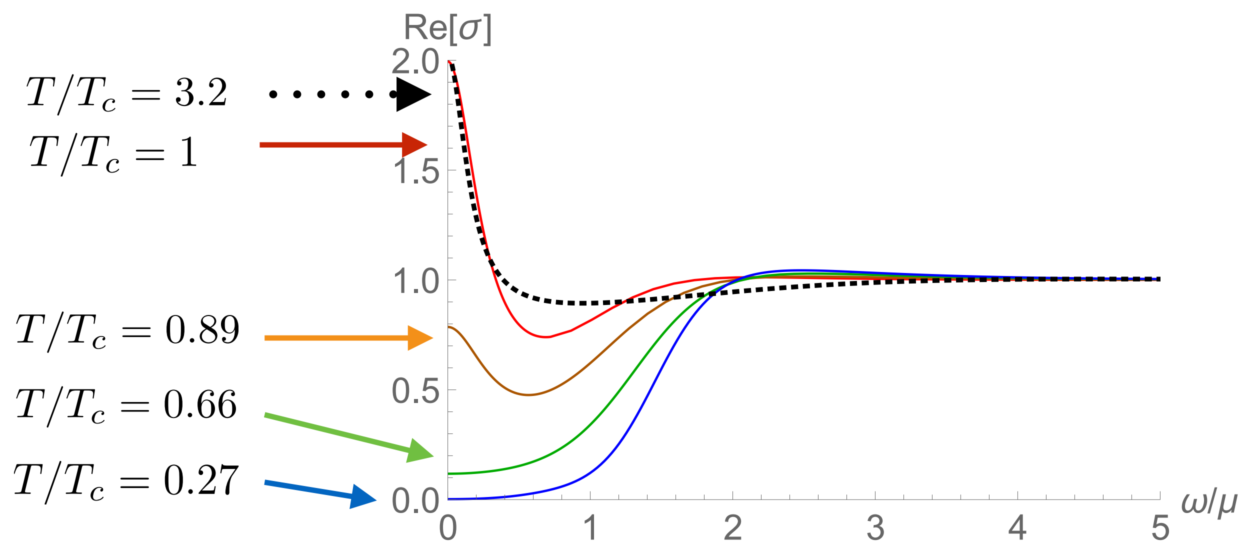
Are we ready?

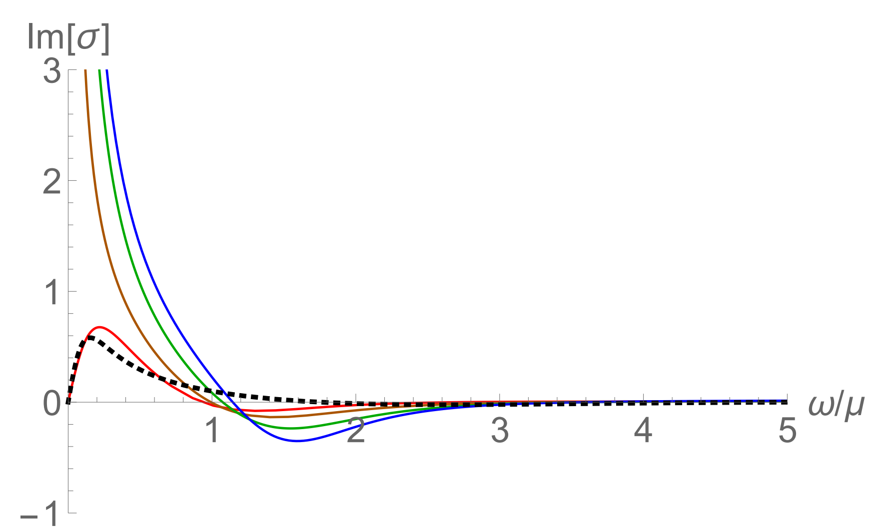
- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$

$$\sigma_{DC} = \sigma(\omega = 0)$$

$$\sigma(\omega) \sim i\frac{\rho_s}{\omega}$$

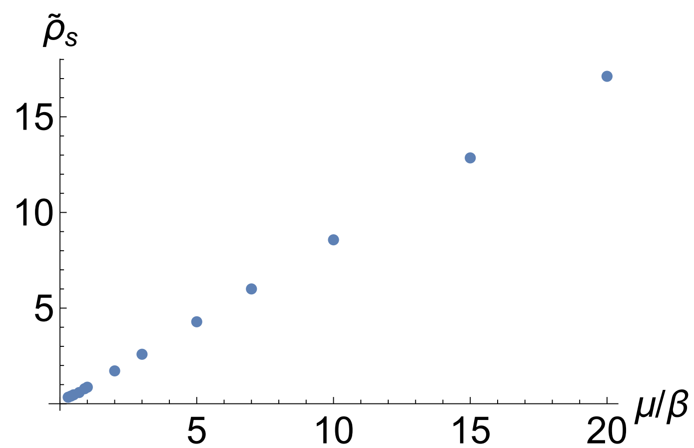


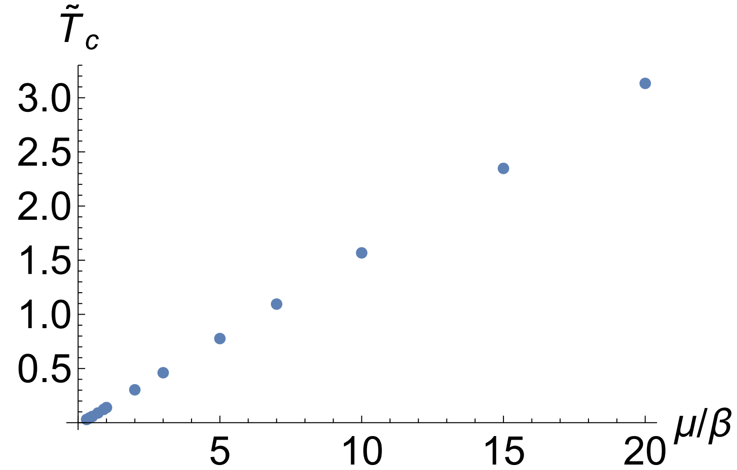


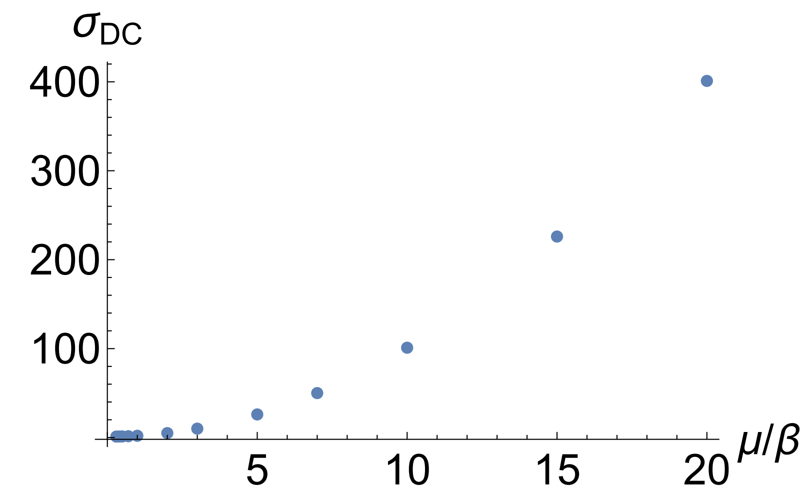


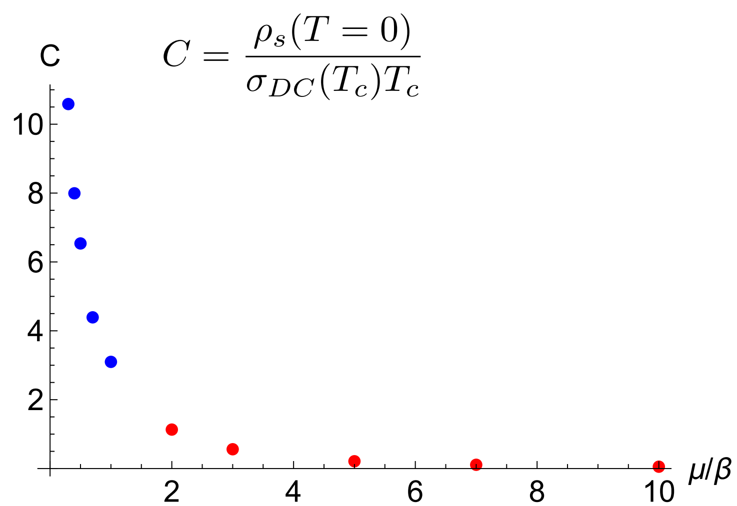
Homes' law and Uemura's law

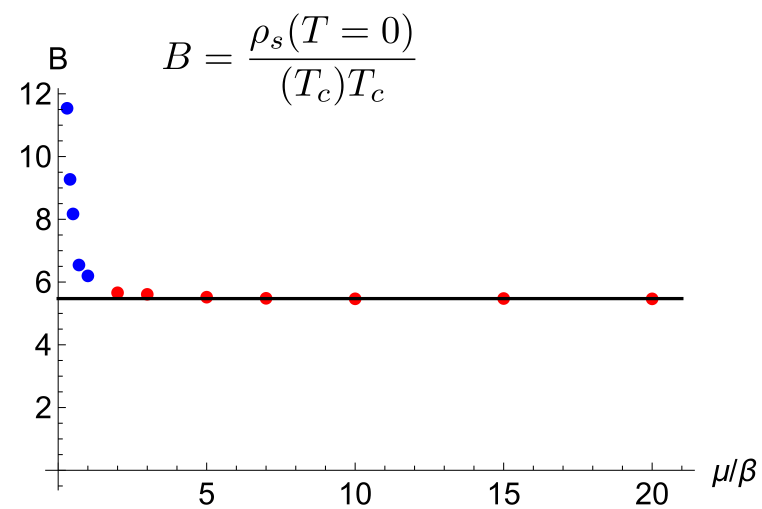
[KYK, Kim, Park: 1604.06205]

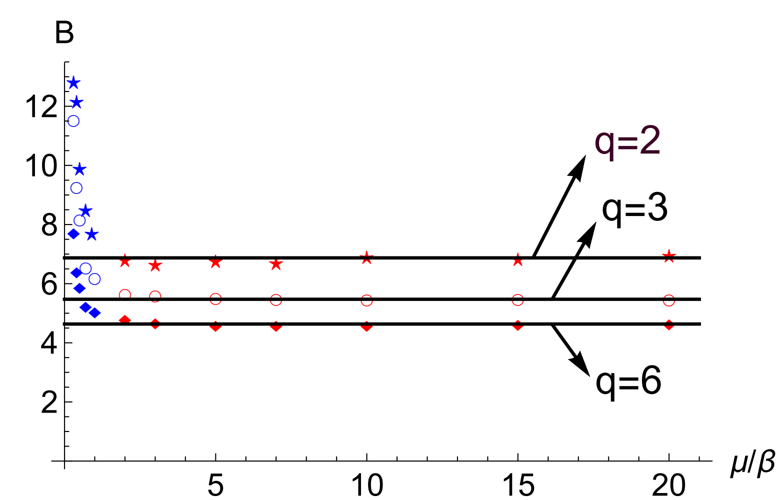












(b) $B(=\tilde{\rho}_s/\tilde{T}_c)$ for $q = 2, 3, 6$

- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$

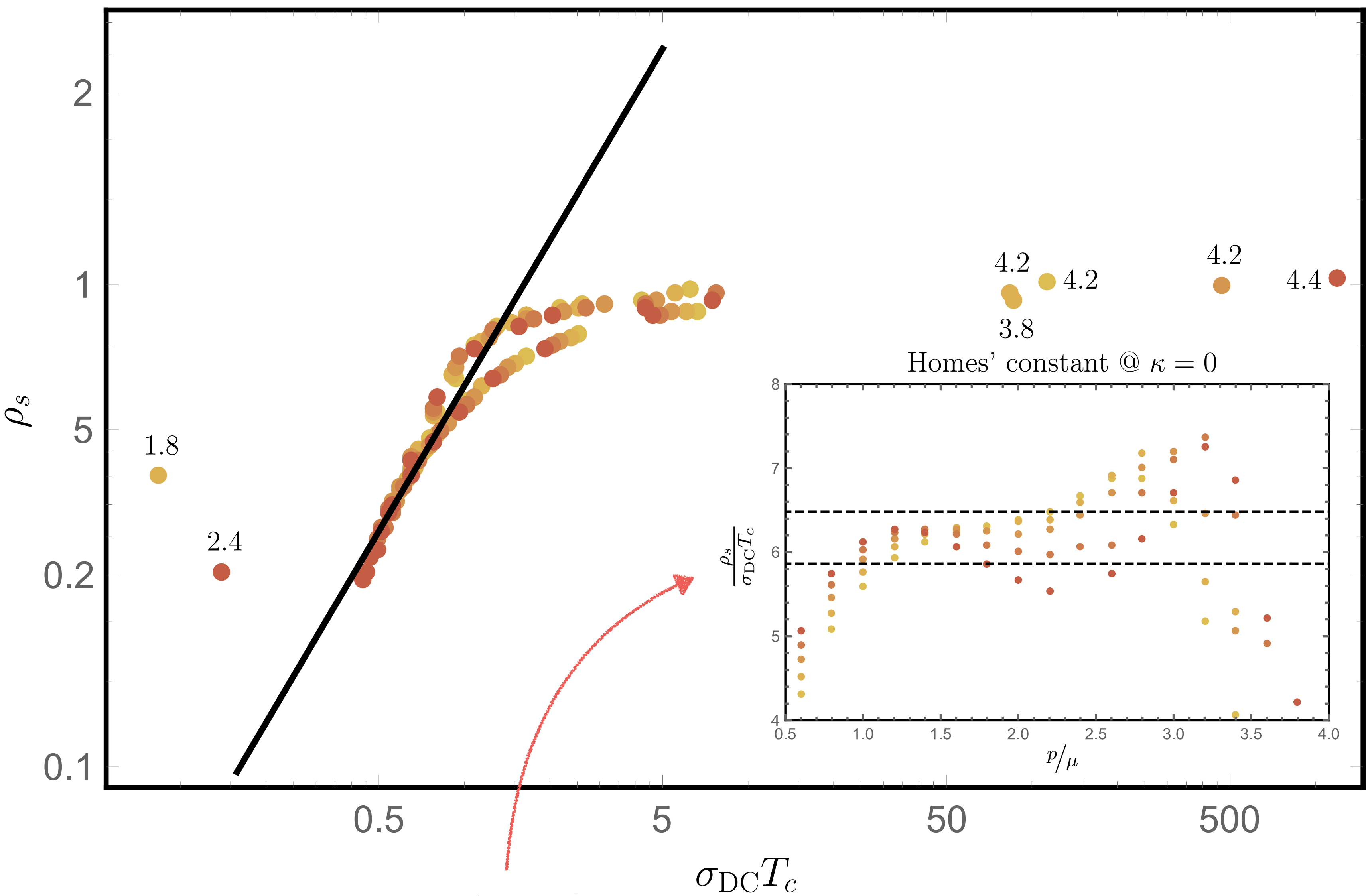
~~Homes' law~~

Helical lattice case

[Erdmenger, Herwerth, Klug, Meyer, Schalm: 1501.07615]

Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$

Homes' relation for $q = 6$ & $\kappa = 0$

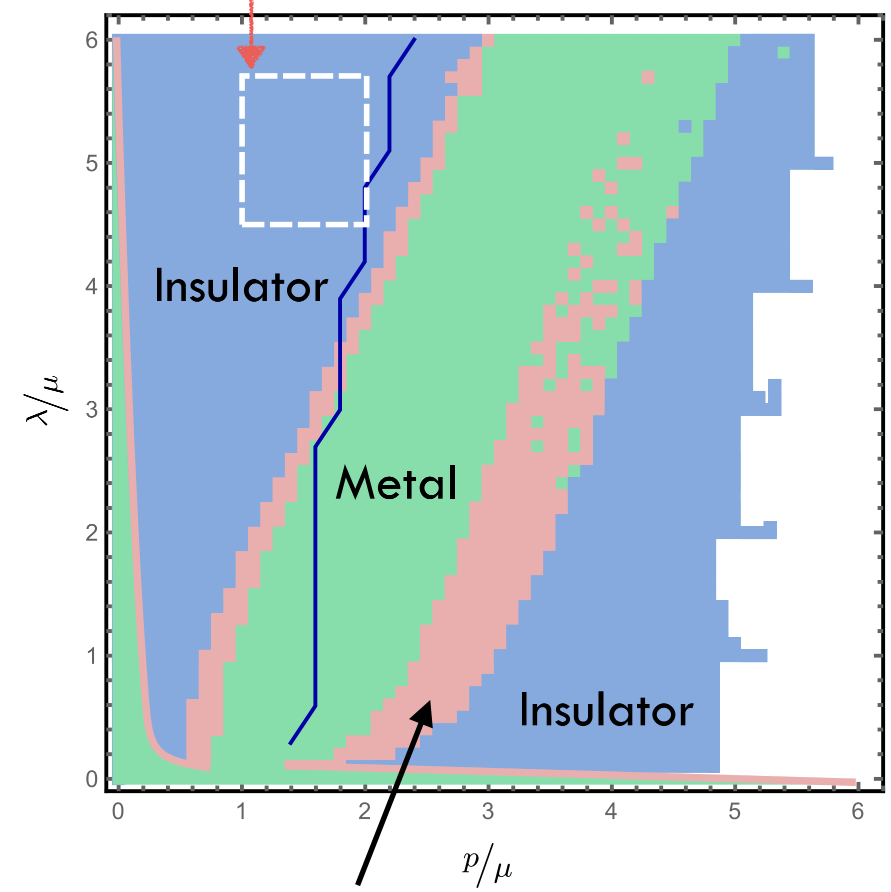


$$C = \frac{\rho_s(T=0)}{\sigma_{DC}(T_c)T_c}$$

$^{\lambda}/_{\mu} = 4.5,\ 4.8,\ 5.1,\ 5.4,\ 5.7$

$p/\mu:\ \ 1 \sim 2$
$\lambda/\mu: 4.5 \sim 5.7$

Phase diagram ($\Phi = 0$)



undetermined

Contents

- Motivations
- Holographic superconductor with momentum relaxation
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- Summary and outlook

Q-lattice model

[KYK, Niu: 1608.04653]

[Donos, Gauntlett: 1311.3292]

[Ling, Liu, Niu, Wu, Xian: 1410.6761]
[Andrade, Gentle: 1412.6521]

Action $$S = \int \mathrm{d}^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^* - |\partial\Psi|^2 - m_\Psi^2|\Psi|^2\right]$$

Ansatz $$\mathrm{d}s^2 = \frac{1}{z^2}\left[-(1-z)U(z)\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{(1-z)U(z)} + V_1(z)\mathrm{d}x^2 + V_2(z)\mathrm{d}y^2\right]$$

$$A = \mu(1-z)a(z)\mathrm{d}t \qquad \Phi = z\phi(z) \qquad \Psi = e^{ikx}z\psi(z) \quad (\psi(0) = \lambda)$$

[Donos, Gauntlett: 1401.5077]

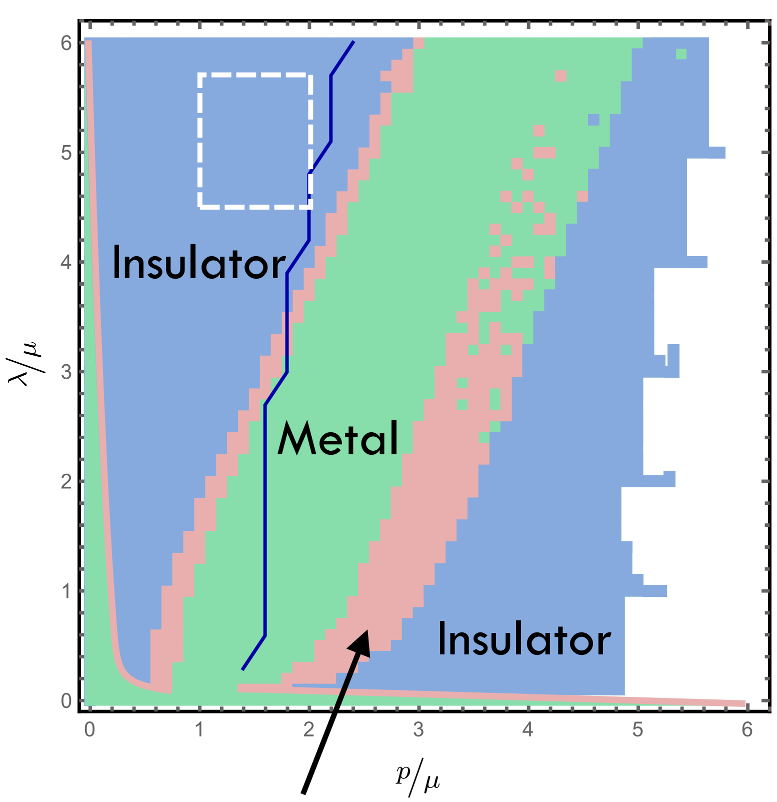
Two parameters k, λ

with $m_\Psi^2 = m_\Phi^2 = -2, \; q = 6$

$$\sigma_{DC} = \left.\left(\sqrt{\frac{V_2}{V_1}} + \frac{\mu^2 a^2\sqrt{V_1 V_2}}{2k^2\psi^2}\right)\right|_{z=1}$$

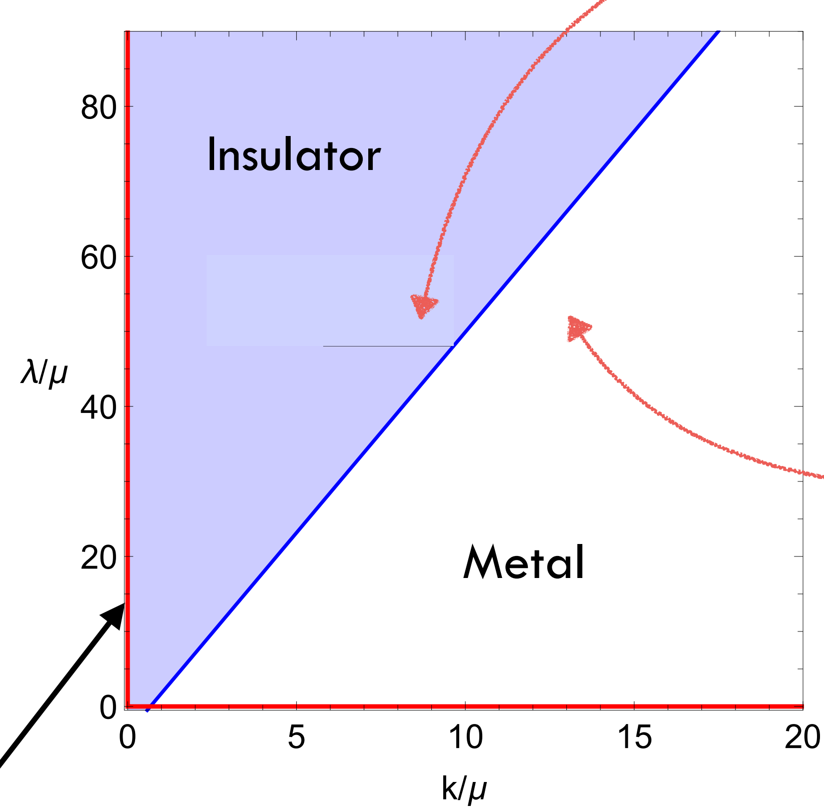
Phase diagram ($\Phi = 0$)

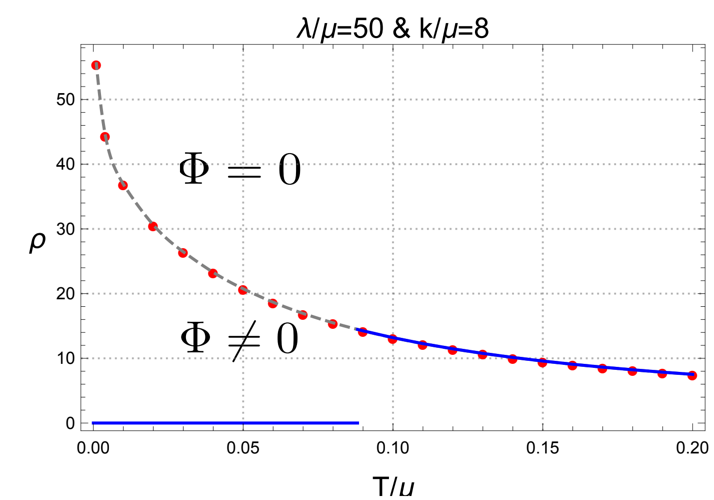
Helical lattice

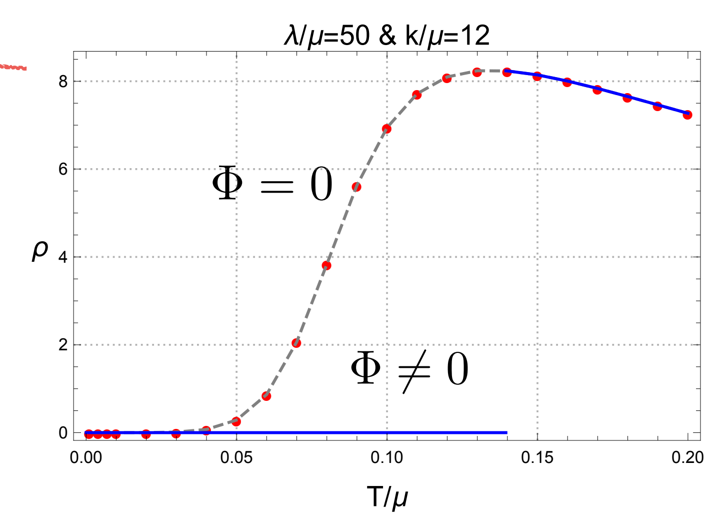


undetermined

Q lattice



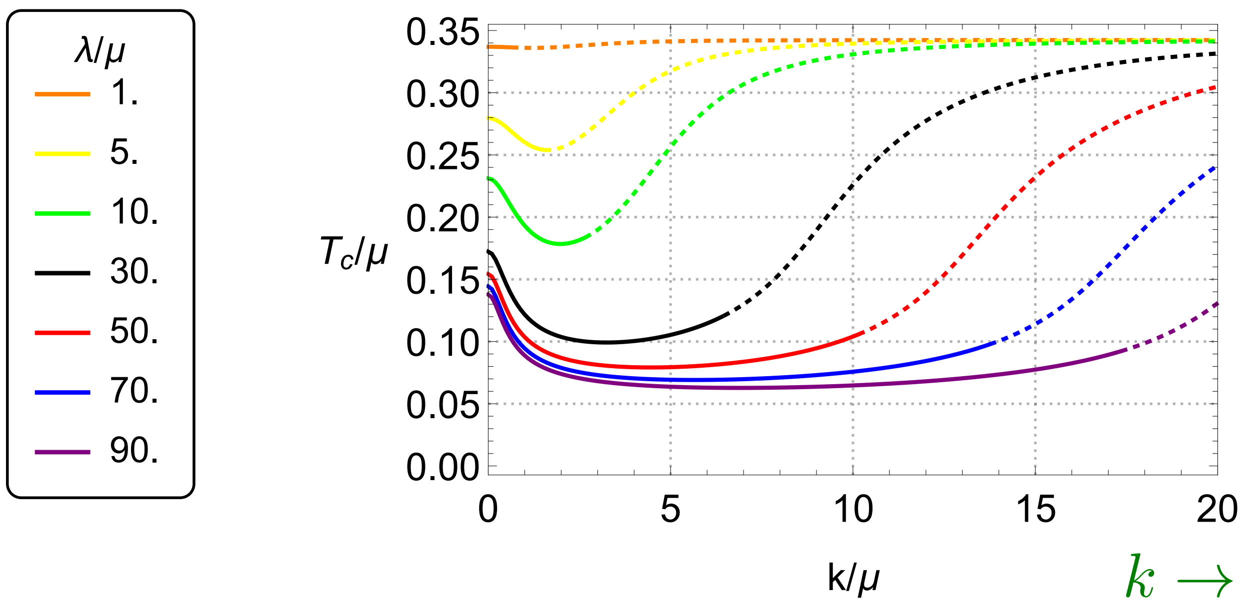




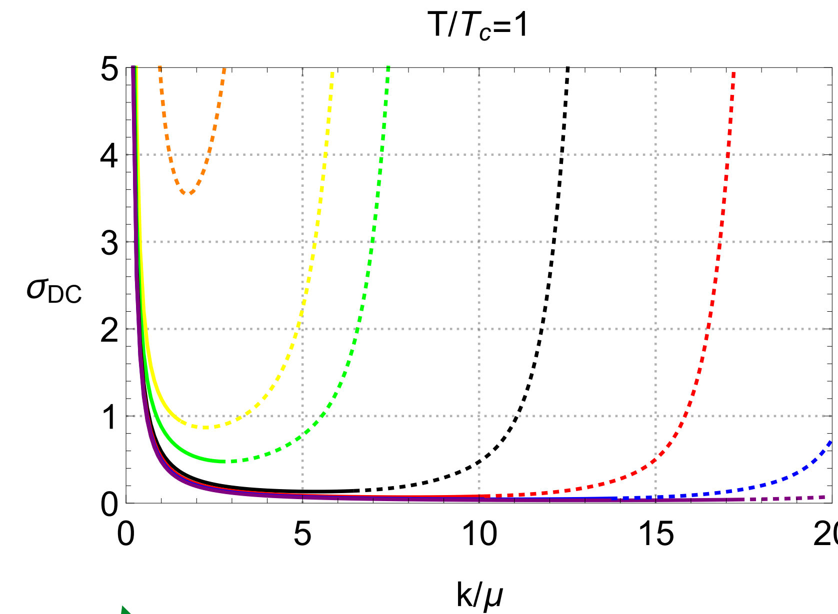
There may be metal regime near k=0: numerical issue

Transition temperature and DC conductivity

[KYK, Niu: 1608.04653]



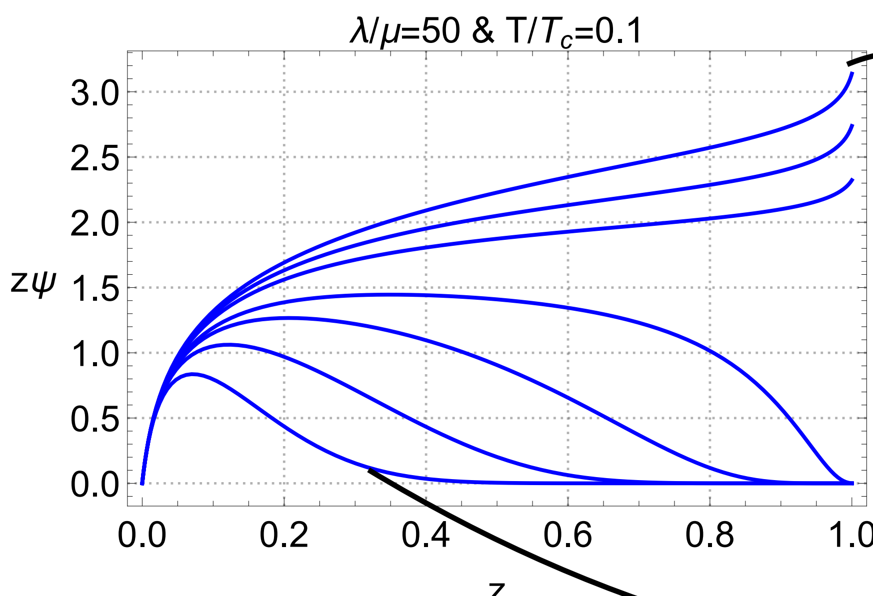
$k \to \infty$
AdS RN limit



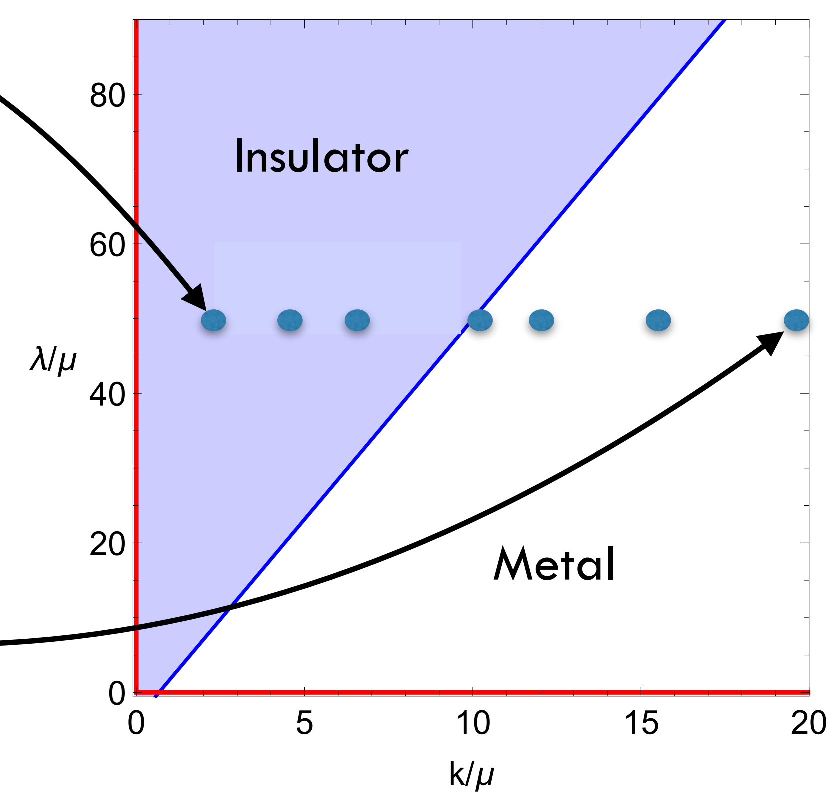
$k \to \infty$
AdS RN limit

No momentum relaxation

$$\Psi = e^{ikx} z\psi(z)$$
$$\psi(0) = \lambda$$



$k/\mu = 2, 4, 6, 10.1, 12, 15, 20.$



Superfluid density

[KYK, Niu: 1608.04653]

London equation

$$J_i(\omega,p) = -K_s A_i(\omega,p)$$

bulk gauge field

$$a_i(z,\omega,p) = a_i^{(0)}(\omega,p) + z a_i^{(1)}(\omega,p) + \cdots$$

$$K_s = -\left.\frac{a_x^{(1)}(\omega,p)}{a_x^{(0)}(\omega,p)}\right|_{\omega,p\to 0}$$

A. In the limit $p=0$ and $\omega \to 0$

$$K_s = -\left.\frac{a_x^{(1)}(\omega,0)}{a_x^{(0)}(\omega,0)}\right|_{\omega\to 0}$$

$$J_i(\omega,0) = \frac{iK_s}{\omega} E_i(\omega,0) \equiv \sigma(\omega) E_i(\omega,0)$$

$$\mathrm{Im}[\sigma(\omega)] = \frac{K_s}{\omega} + \cdots$$

$$\mathrm{Re}[\sigma(\omega)] = \frac{\pi}{2} K_s \delta(\omega)$$

Infinite DC conductivity

B. In the limit $\omega=0$ and $p \to 0$

$$\tilde{K}_s = -\left.\frac{a_x^{(1)}(0,p)}{a_x^{(0)}(0,p)}\right|_{p\to 0}$$

$$\nabla \times \vec{J} = -K_s \vec{B}$$

$$-\nabla^2 \vec{B} = \nabla \times (\nabla \times \vec{B})$$
$$= 4\pi \nabla \times \vec{J} = -4\pi K_s \vec{B} \equiv -\frac{1}{\lambda^2}\vec{B}$$

Meissner effect:
Magnetic penetration depth

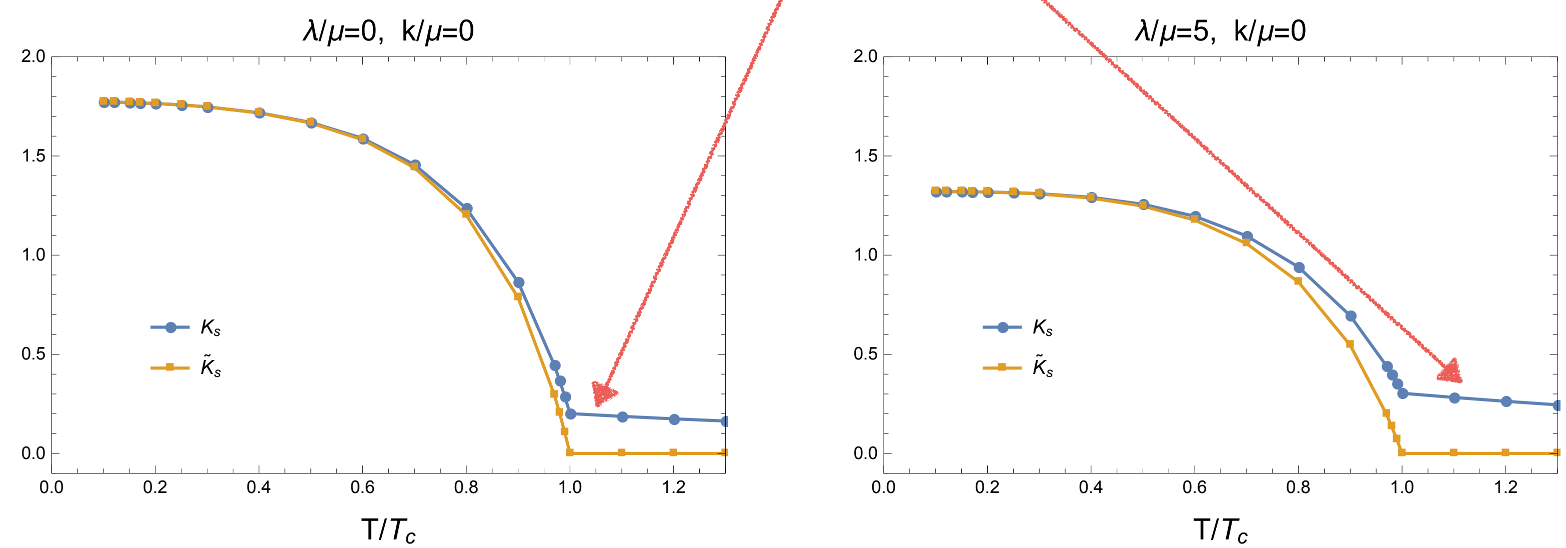
Superfluid density

[KYK, Niu: 1608.04653]

$$K_s = -\frac{a_x^{(1)}(\omega,0)}{a_x^{(0)}(\omega,0)}\bigg|_{\omega\to 0}$$

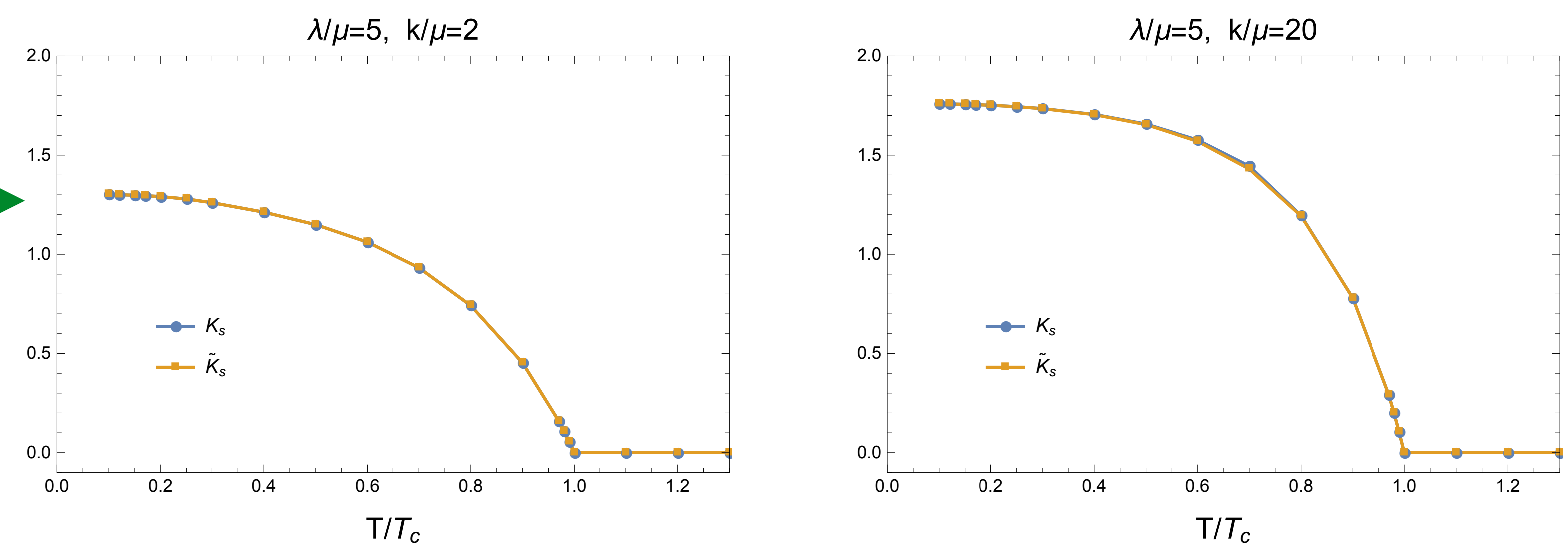
$$\tilde{K}_s = -\frac{a_x^{(1)}(0,p)}{a_x^{(0)}(0,p)}\bigg|_{p\to 0}$$

Perfect conductor: spurious contribution
from translational invariance

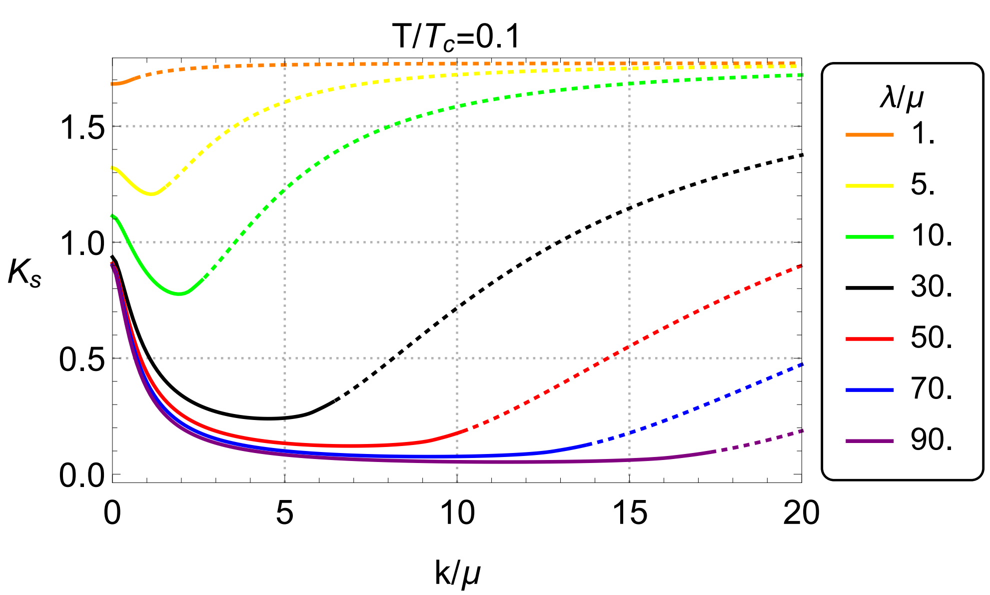


(a) no momentum relaxation (b) no momentum relaxation

extrapolate



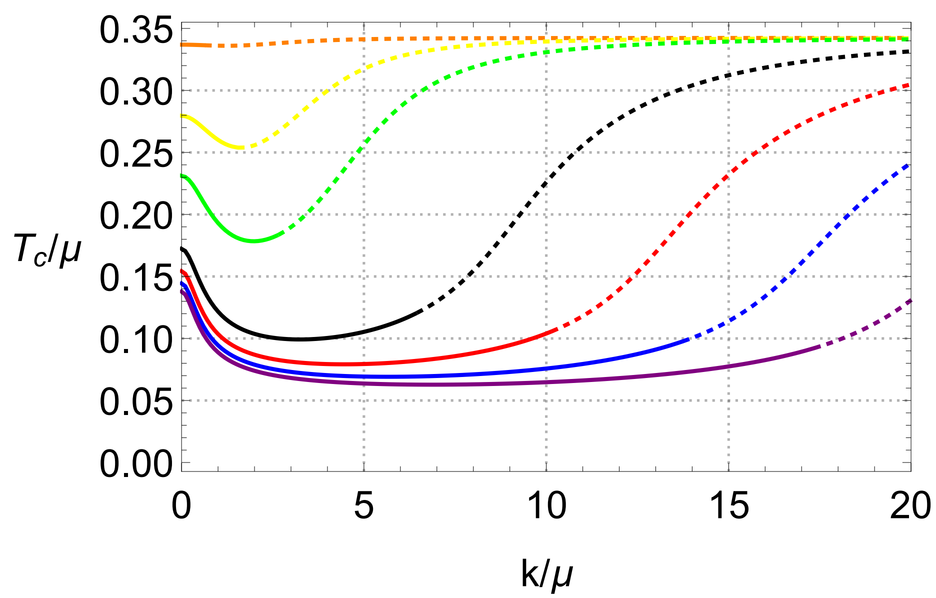
(c) large momentum relaxation (d) small momentum relaxation

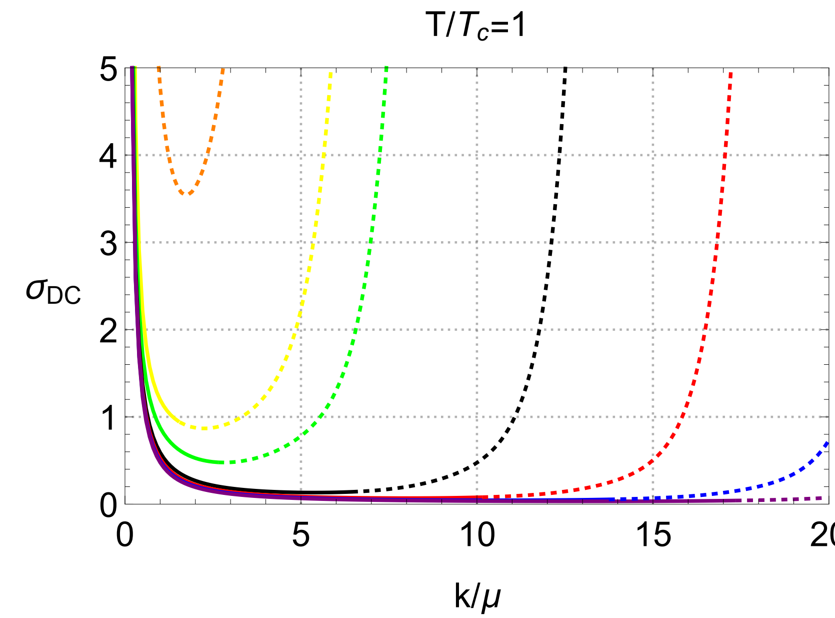


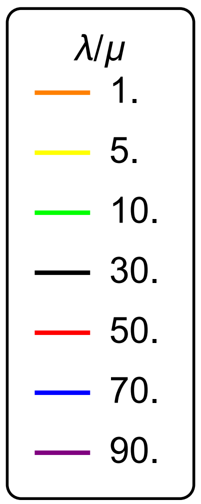
Homes' law?

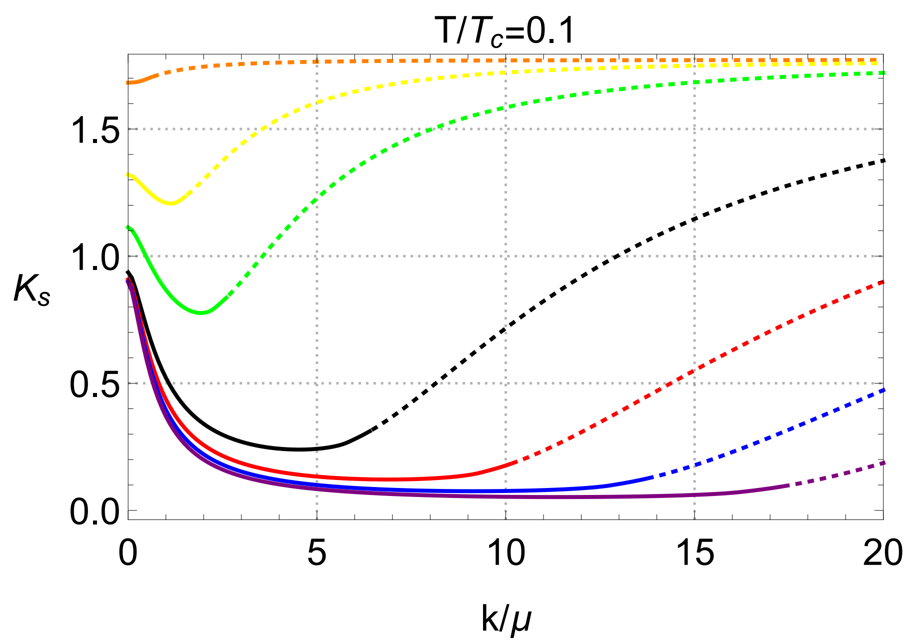
[KYK, Niu: 1608.04653]

Homes' law $K_s(T=0) = C\sigma_{DC}(T_c)T_c$



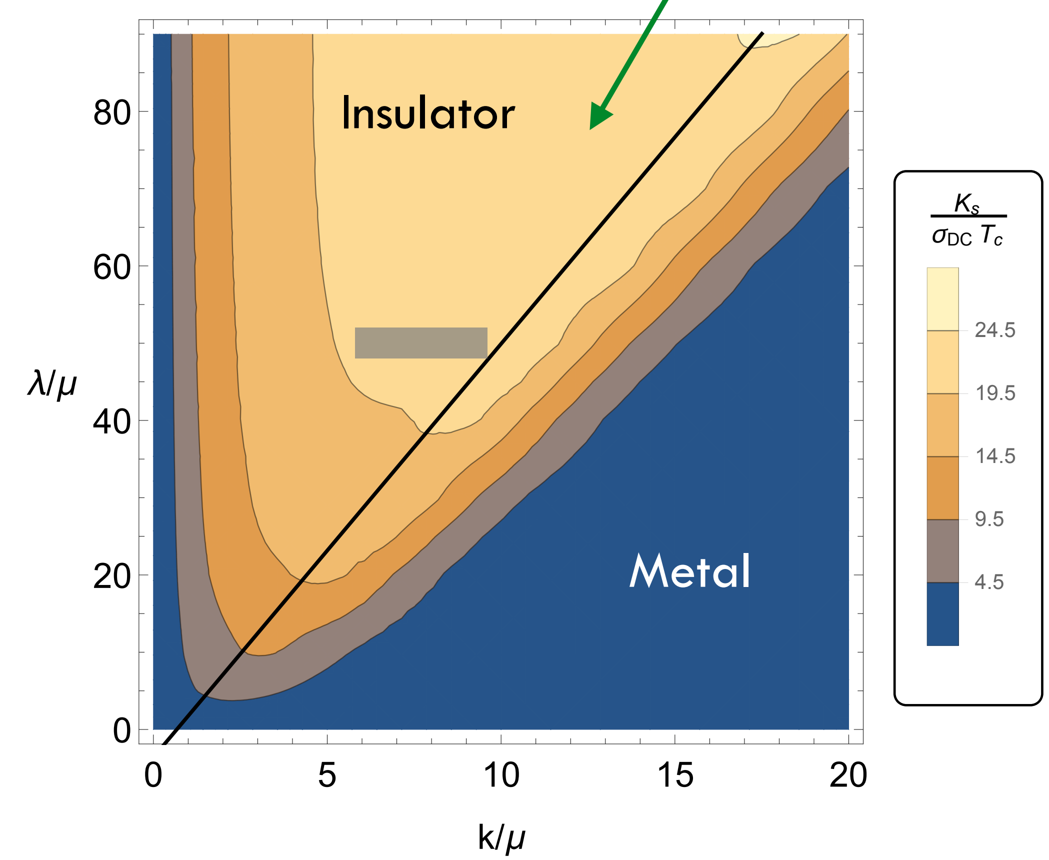






At fixed k, plateau for large λ

Small variation of C



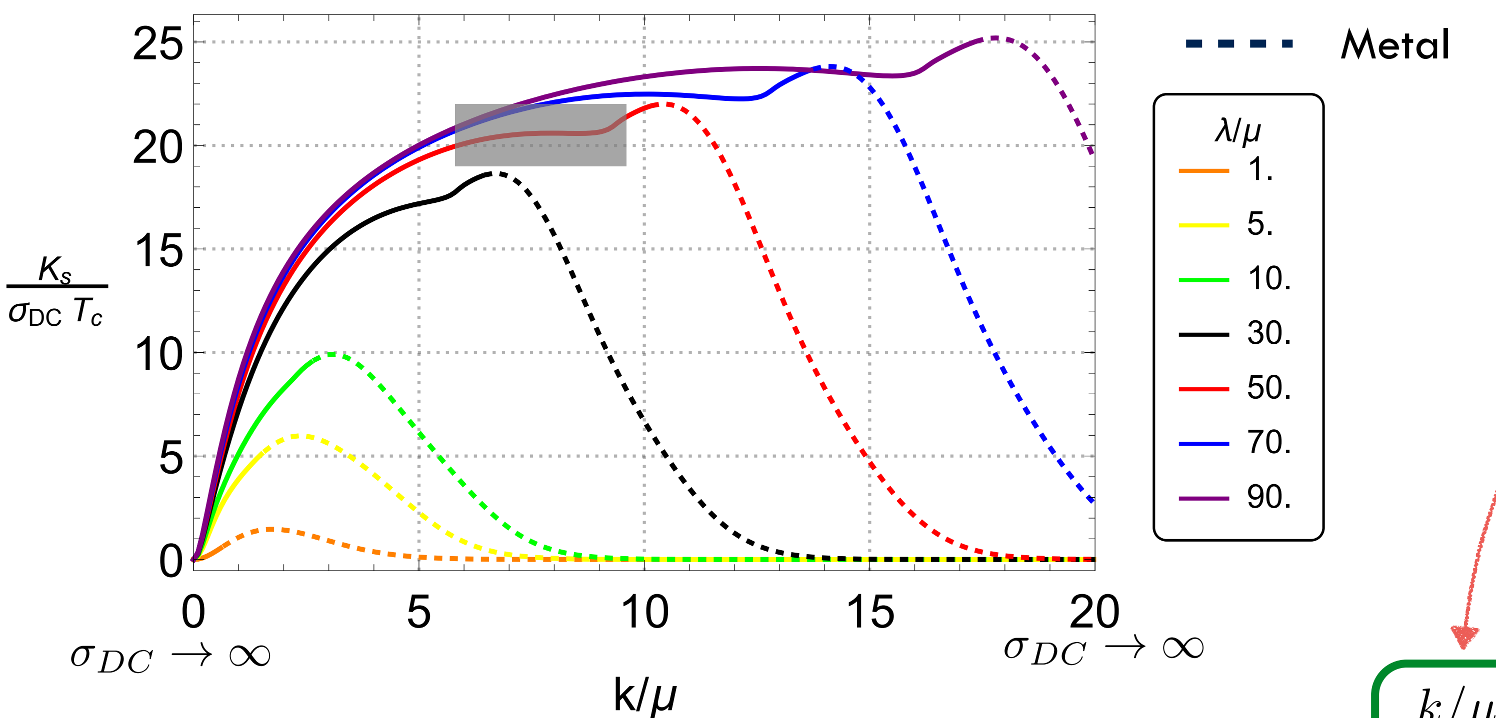
$$C = \frac{\rho_s(T=0)}{\sigma_{DC}(T_c)T_c}$$

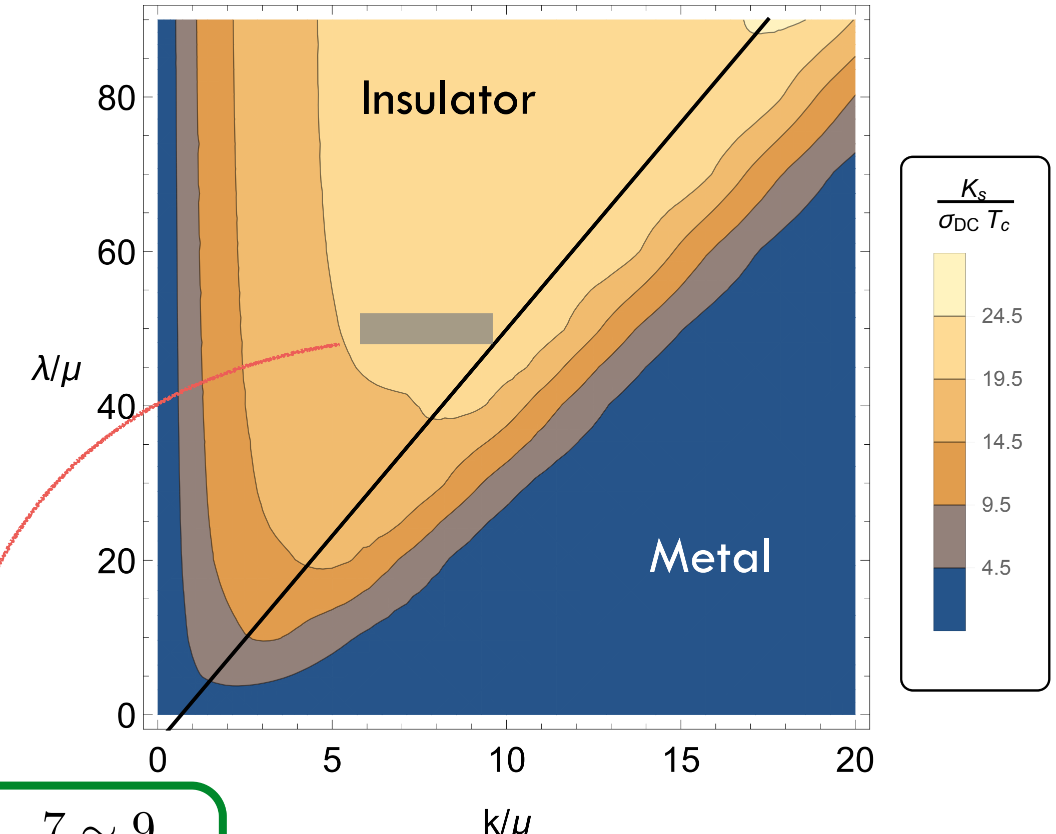
Homes' law: Insulating phase near transition ?

[KYK, Niu: 1608.04653]

Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$

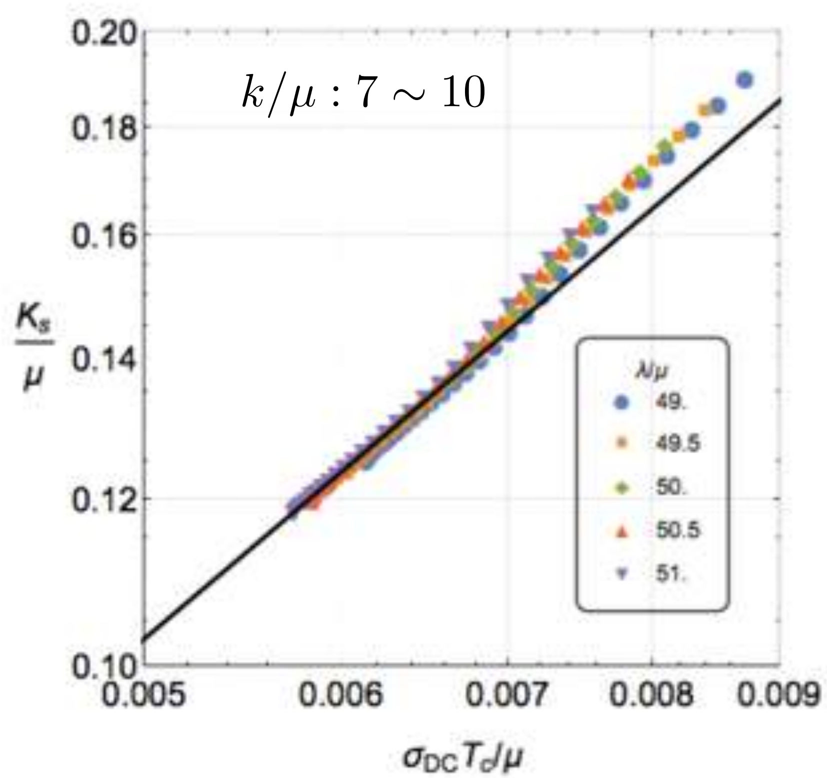
Plateau for large $\lambda \geq 50$

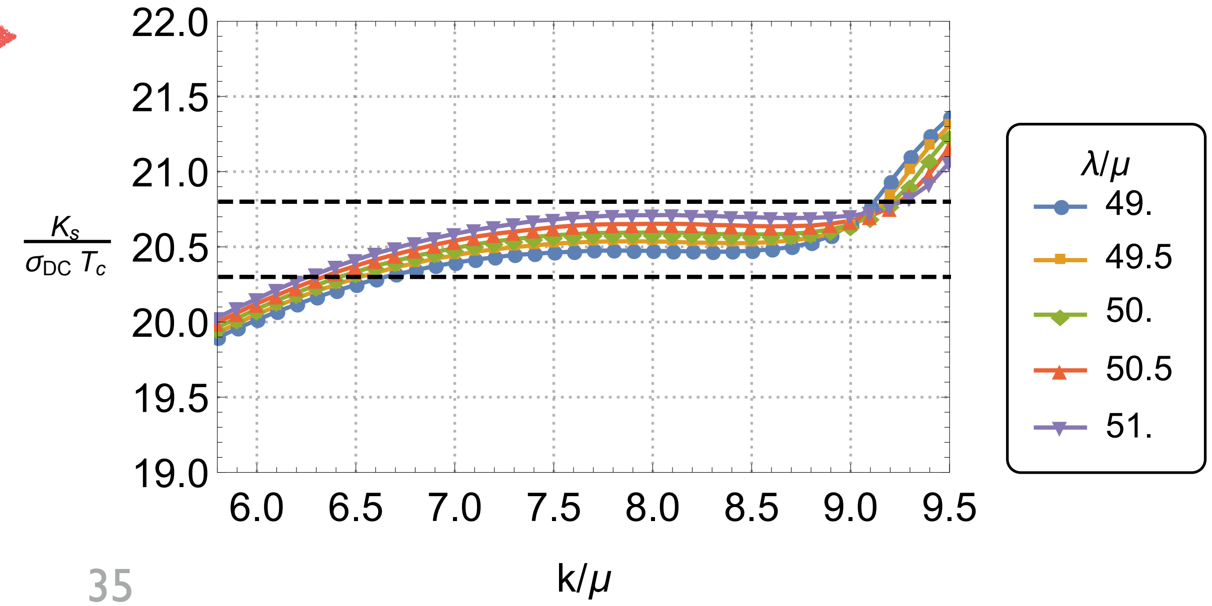




$k/\mu: \;\; 7 \sim 9$

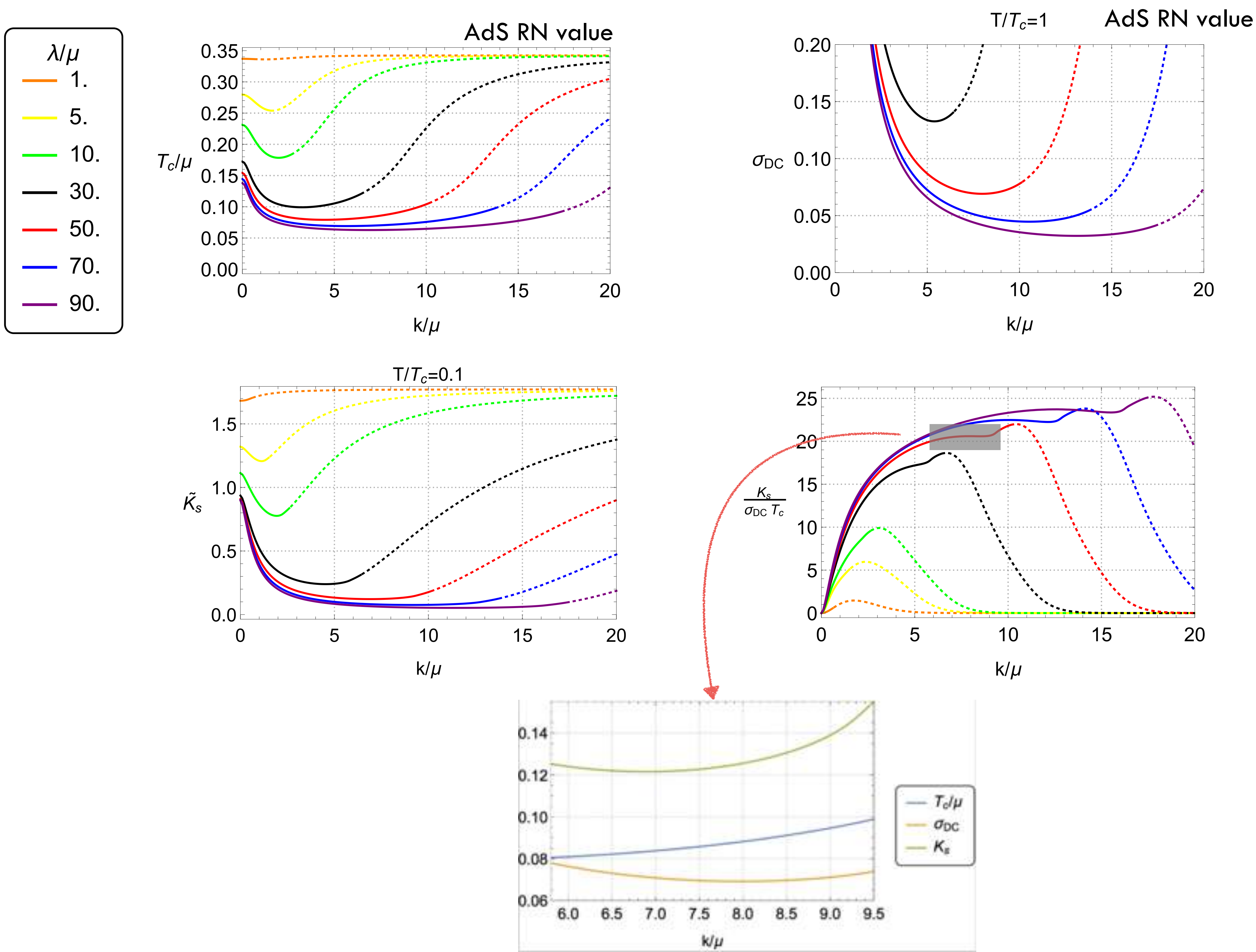
$\lambda/\mu: 49 \sim 51$





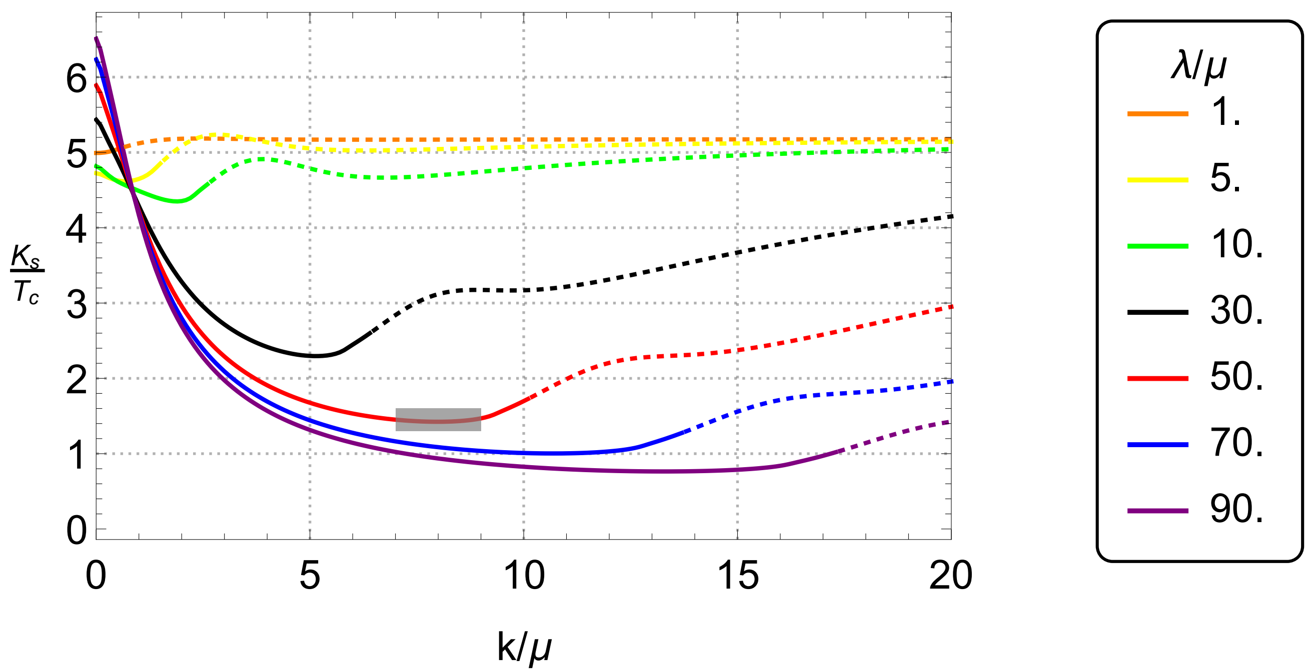
Homes' law: Insulating phase near transition ?

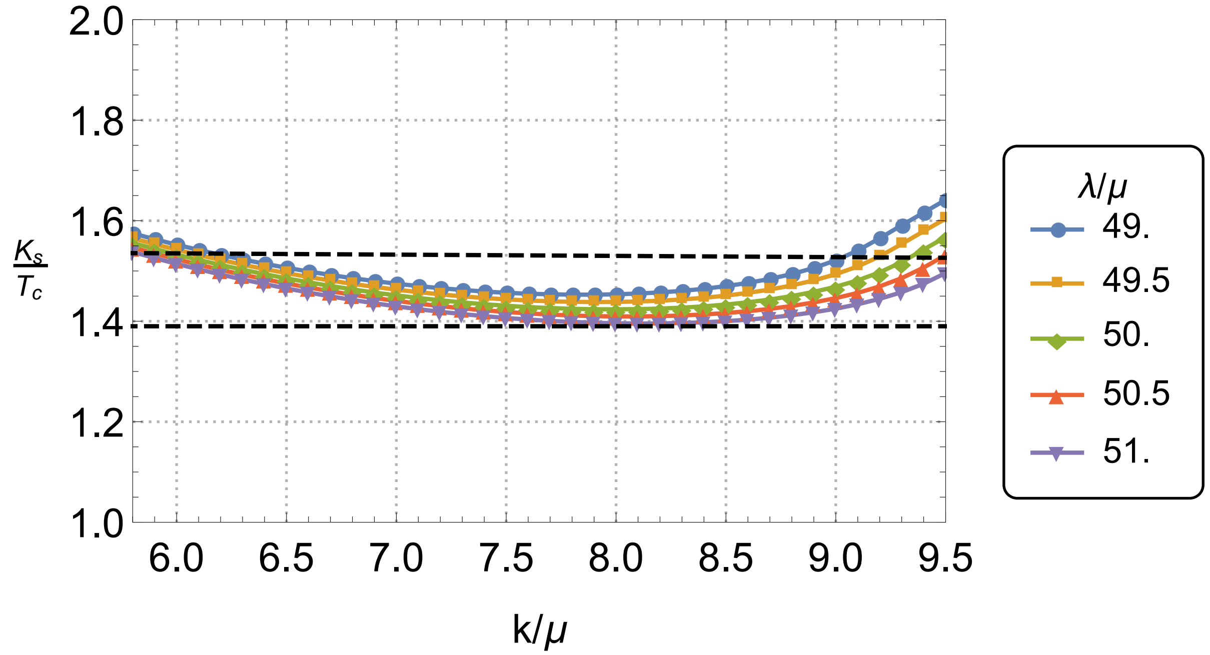
[KYK, Niu: 1608.04653]



Uemura's law?

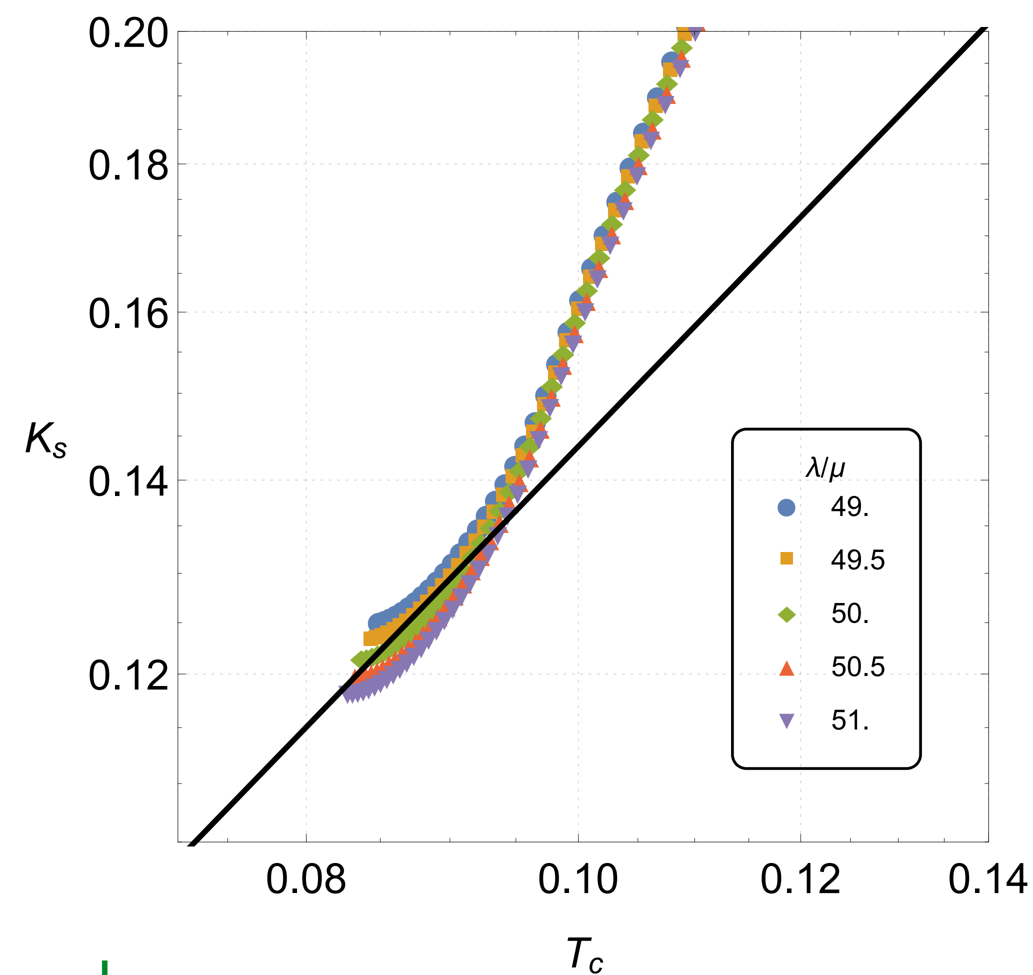
[KYK, Niu: 1608.04653]





- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$

~~Uemura's law~~



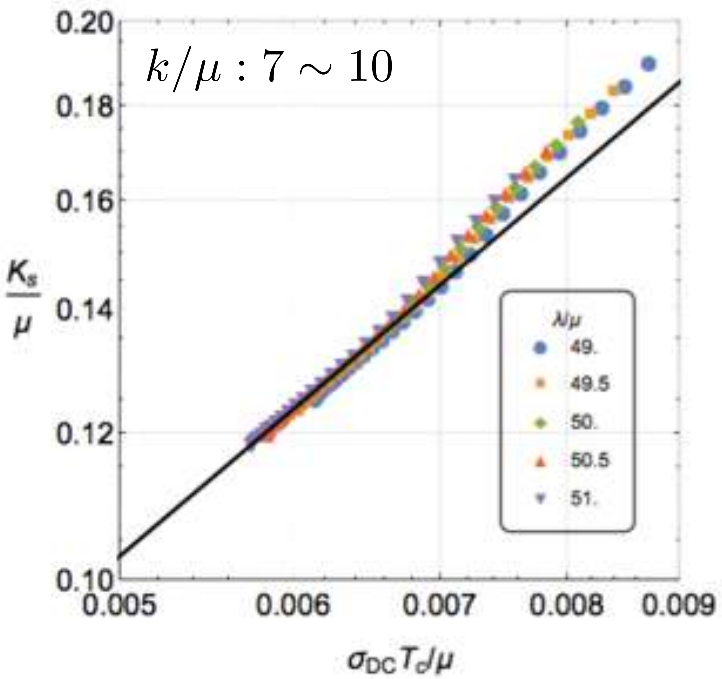
Uemura's law does not hold: DC conductivity plays a role

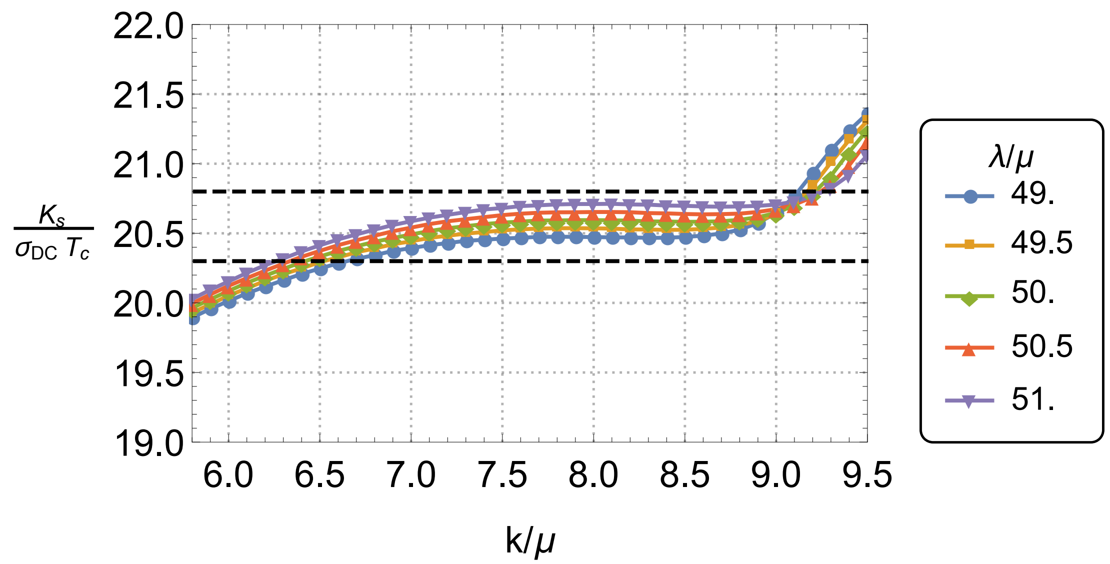
Comparison: Helical lattice and Q-lattice

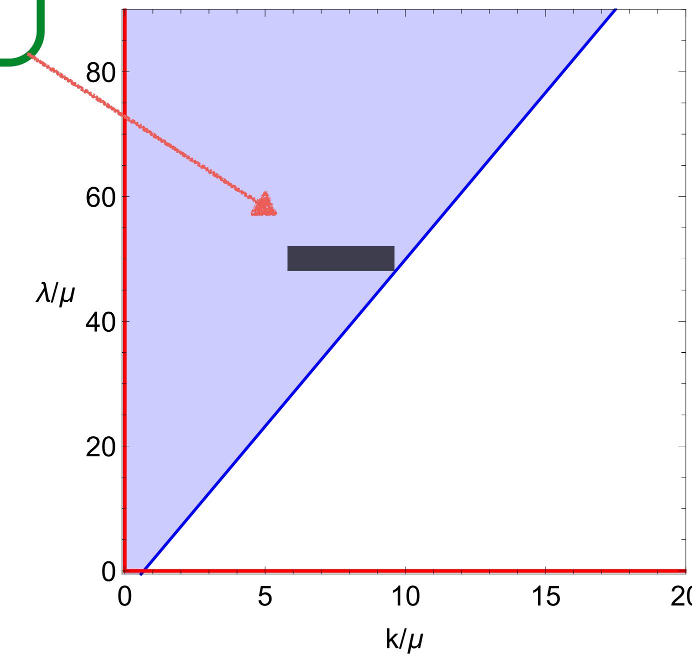
[KYK, Niu: 1608.04653]

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

$k/\mu : \ 7 \sim 9$
$\lambda/\mu : 49 \sim 51$

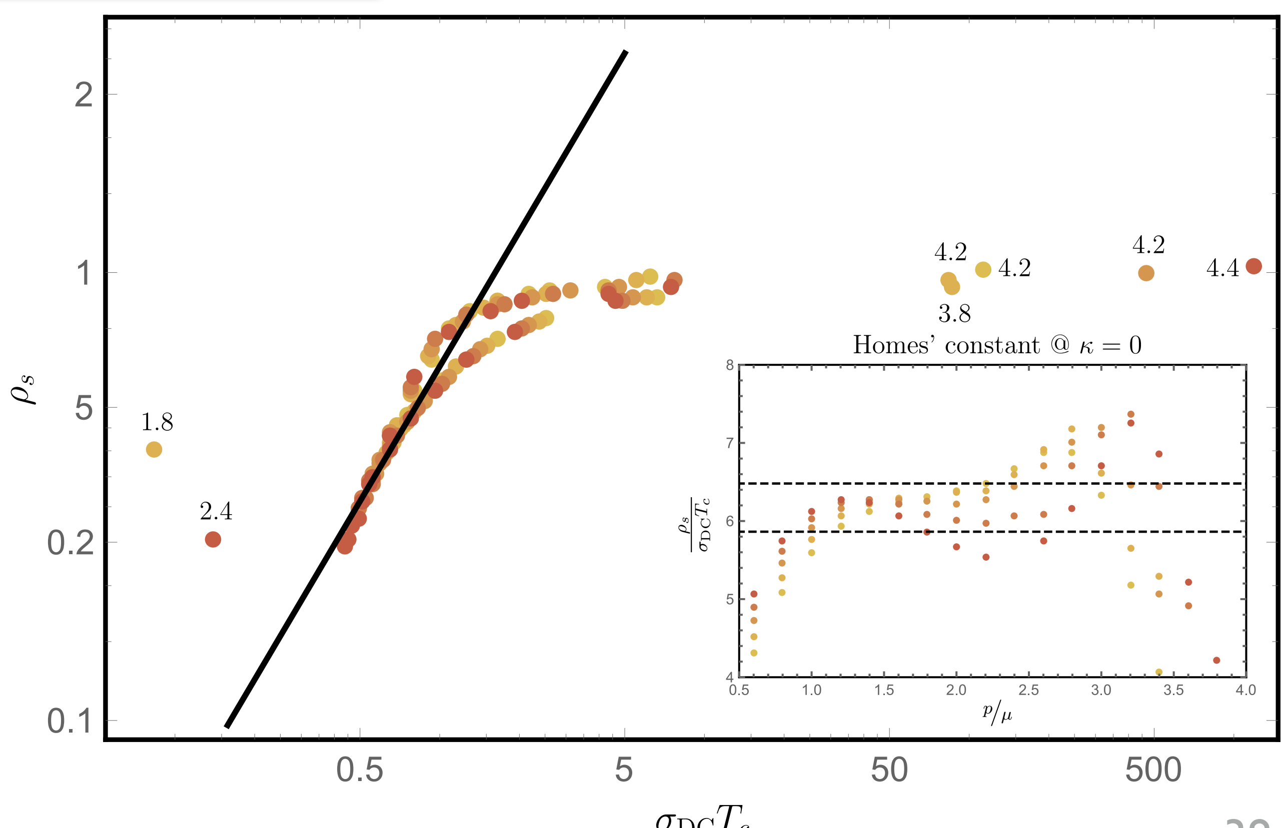






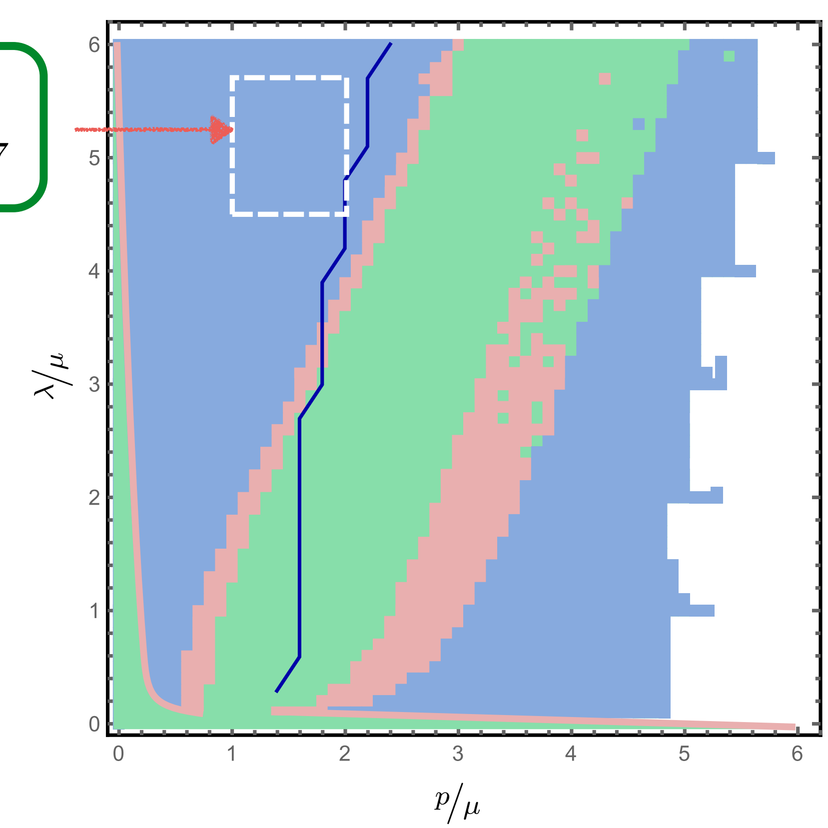
Helical lattice

Homes' relation for $q = 6$ & $\kappa = 0$



$p/\mu : \ 1 \sim 2$
$\lambda/\mu : 4.5 \sim 5.7$

$\kappa = 0$



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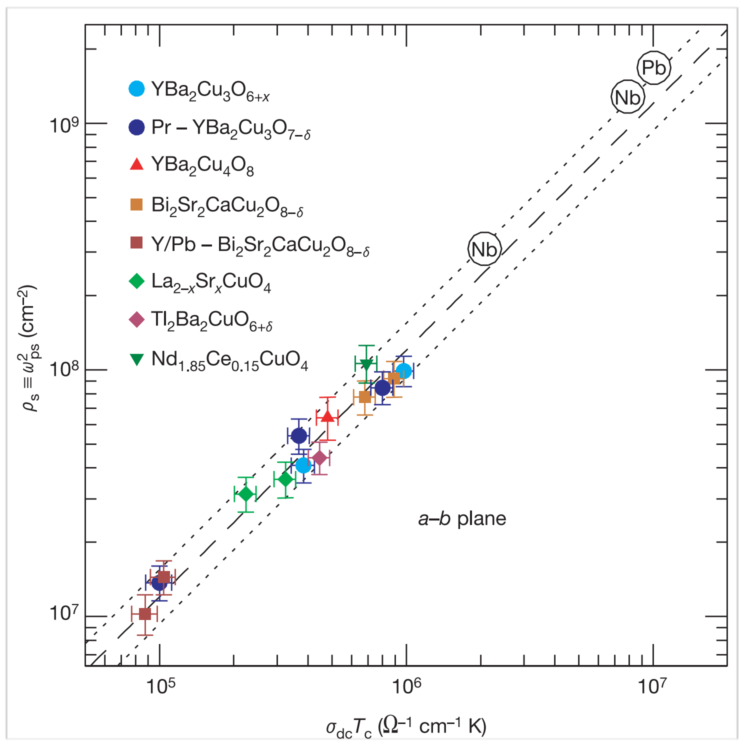
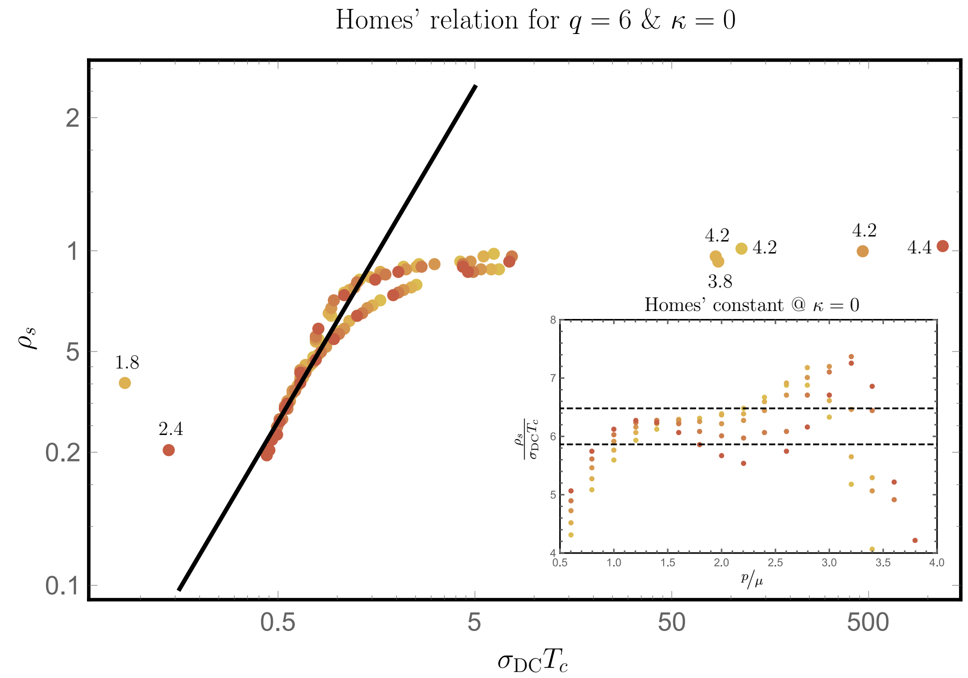
Summary and outlook



- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$



$$S_{HHH} = \int d^4x\sqrt{-g}\left[R+6-\frac{1}{4}F^2-|(\partial-iqA)\Phi|^2-m_\Phi^2\Phi\Phi^*\right]$$

$$S_{MS} = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\sum_{I=1,2}(\partial\psi_I)^2\right]$$

$\psi_I = (\beta x, \beta y)$

$$S_Q = \int d^4x\sqrt{-g}\left[-|\partial\Psi|^2-m_\Psi^2|\Psi|^2\right]$$

$\Psi = e^{ikx}z\psi(z) \quad \psi(0)=\lambda$

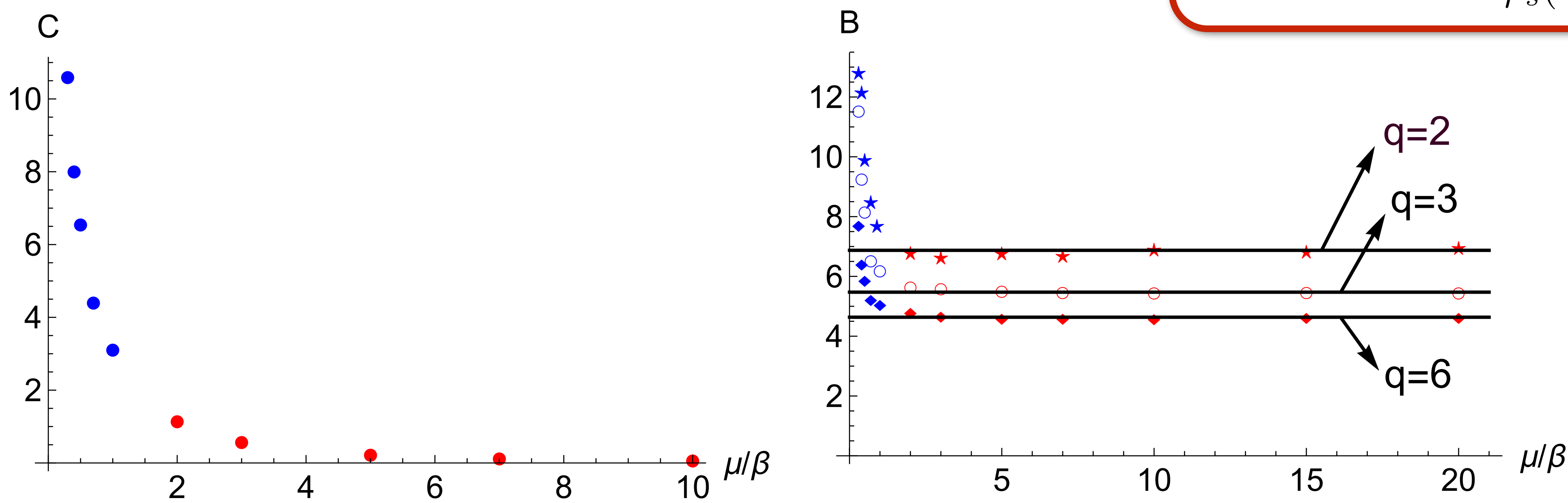
$$S_{\text{helix}} = \int \mathrm{d}^{4+1}x\,\sqrt{-g}\left[R+12-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-\frac{1}{4}W^{\mu\nu}W_{\mu\nu}-m^2B_\mu B^\mu\right]$$

$$B = w(r)\omega_2, \qquad w(\infty)=\lambda,$$
$$\omega_2 = \cos(px)\,\mathrm{d}y - \sin(px)\,\mathrm{d}z$$

Homes' law ~~Uemura's law~~

Summary and outlook

- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$



(a) $C(=\tilde{\rho}_s/(\sigma_{DC}\tilde{T}_c))$, $q=3$

(b) $B(=\tilde{\rho}_s/\tilde{T}_c)$ for $q=2,3,6$

$$S_{HHH} = \int d^4x\sqrt{-g}\left[R+6-\frac{1}{4}F^2-|(\partial-iqA)\Phi|^2-m_\Phi^2\Phi\Phi^*\right]$$

$$\boxed{S_{MS} = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\sum_{I=1,2}(\partial\psi_I)^2\right]}$$

$\psi_I = (\beta x, \beta y)$

~~Homes' law~~ Uemura's law

$$S_Q = \int d^4x\sqrt{-g}\left[-|\partial\Psi|^2-m_\Psi^2|\Psi|^2\right]$$

$\Psi = e^{ikx}z\psi(z) \quad \psi(0)=\lambda$

$$S_{\mathrm{helix}} = \int \mathrm{d}^{4+1}x\,\sqrt{-g}\left[R+12-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-\frac{1}{4}W^{\mu\nu}W_{\mu\nu}-m^2B_\mu B^\mu\right]$$

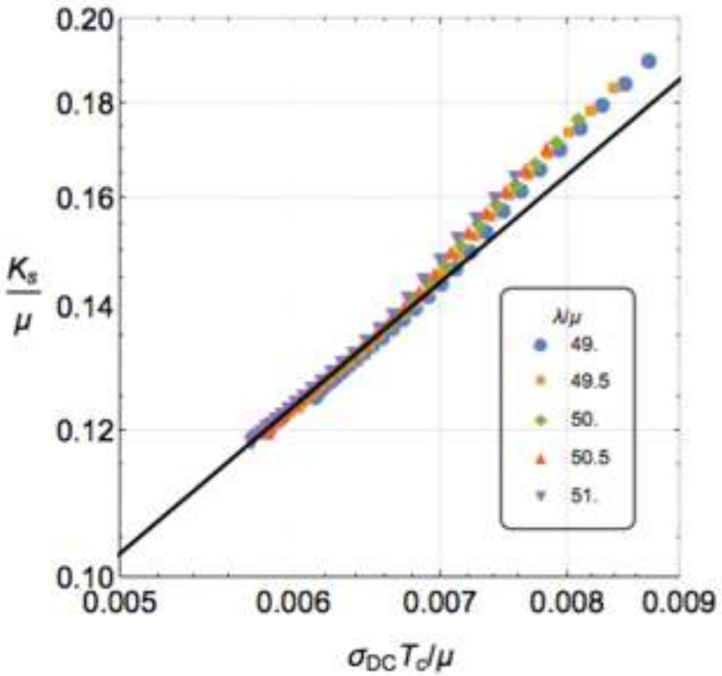
$B = w(r)\omega_2, \qquad w(\infty)=\lambda,$

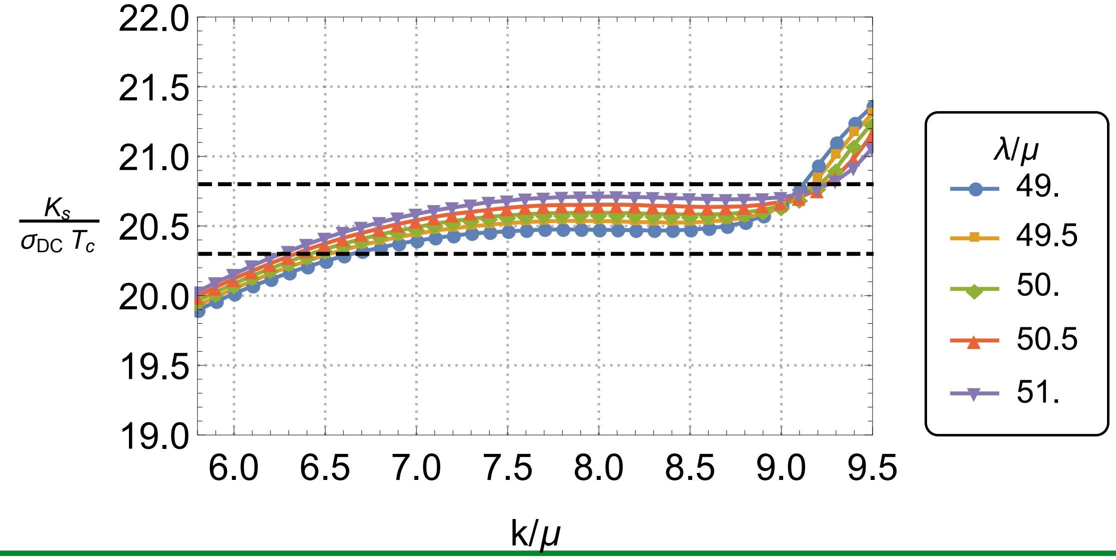
$\omega_2 = \cos(px)\,\mathrm{d}y - \sin(px)\,\mathrm{d}z$

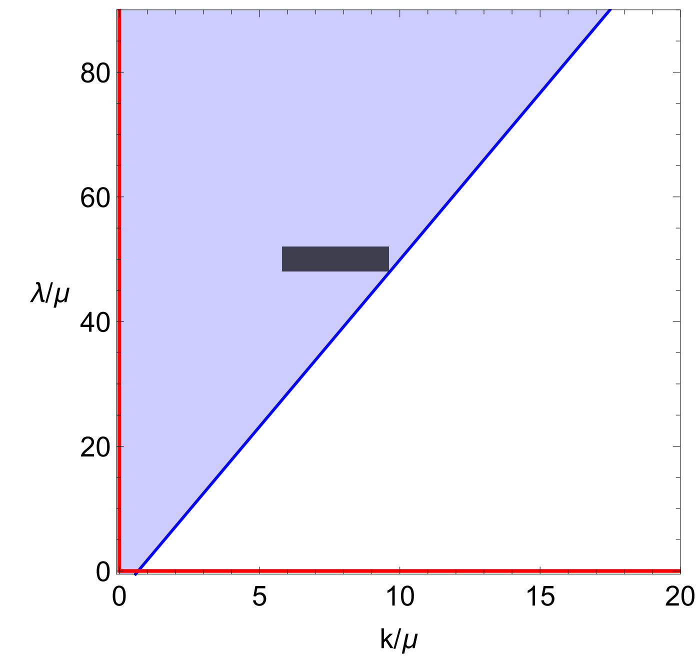
Homes' law ~~Uemura's law~~

Summary and outlook

- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$







$$S_{HHH} = \int d^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^*\right]$$

$$S_{MS} = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\sum_{I=1,2}(\partial\psi_I)^2\right] \quad \psi_I = (\beta x, \beta y)$$

Homes' law Uemura's law

$$\boxed{S_Q = \int d^4x\sqrt{-g}\,[-|\partial\Psi|^2 - m_\Psi^2|\Psi|^2] \quad \Psi = e^{ikx}z\psi(z) \quad \psi(0) = \lambda}$$

Homes' law Uemura's law

$$S_{\text{helix}} = \int \mathrm{d}^{4+1}x\,\sqrt{-g}\left[R + 12 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}W^{\mu\nu}W_{\mu\nu} - m^2B_\mu B^\mu\right]$$

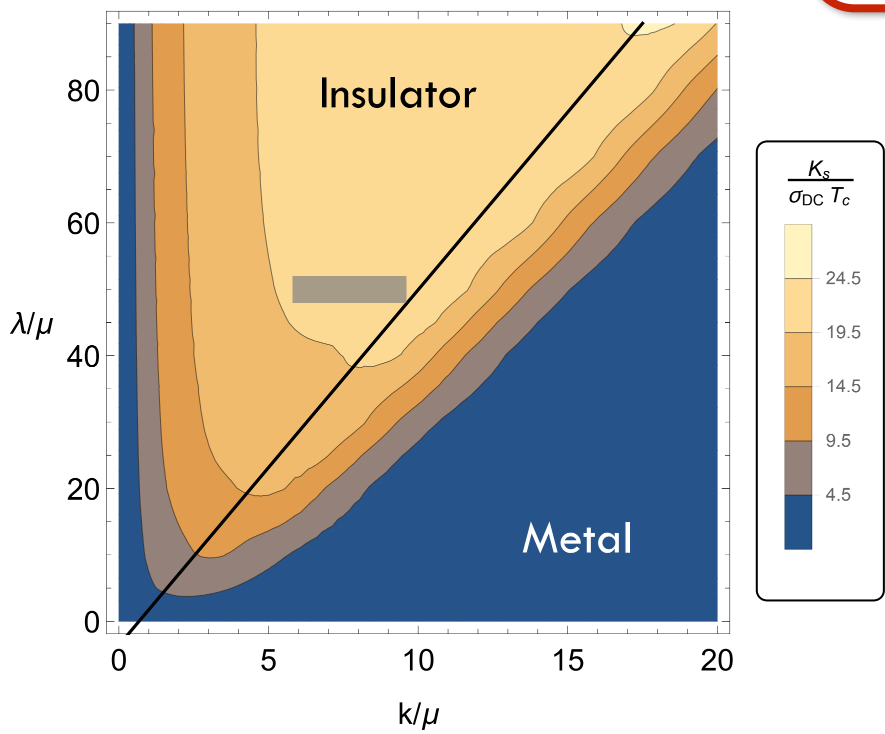
$$B = w(r)\omega_2, \qquad w(\infty) = \lambda,$$
$$\omega_2 = \cos(px)\,\mathrm{d}y - \sin(px)\,\mathrm{d}z$$

Homes' law Uemura's law

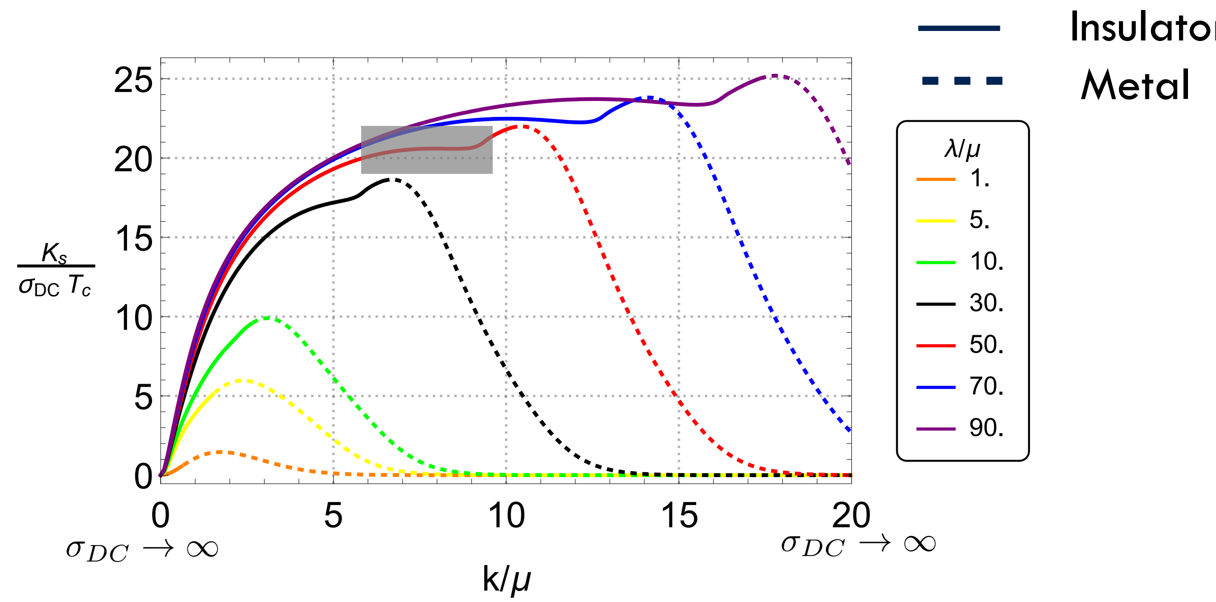
Summary and outlook

Q-lattice model

- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$



At given k, plateau for large λ



Plateau for $\lambda \geq 50$

- Larger parameter space is explored
- "Homes' law" in Q-lattice (and helical lattice) model is due to metal/insulator transition → Further investigation on metal/insulator transition including charge density wave
- No clear relation to phenomenology yet → Other models with linear T resistivity
- Methodology developed:
  systematic, robust and cross-checked
- Superfluid density cross-checked by two methods

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

$$\sigma_{DC} \sim n\tau \sim n/T_c$$

$$\rho_s(T=0) \sim n(T_c)$$

Tanner's law [Tanner et al. :1998 Physica B]

Thank you