Anomalous transport: Theory and Applications - III

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Topological Quantum Matter and Holography, APCTP,, Pohang 2016

Yesterday: stare at these equations until you think they are beautiful ...

$$\vec{J}_A = d_{ABC}\frac{\mu_B}{4\pi^2}\vec{B}_C + \left(d_{ABC}\frac{\mu_B\mu_C}{4\pi^2} + b_A\frac{T^2}{12}\right)\vec{\Omega}$$

$$\vec{J}_\epsilon = \left(d_{ABC}\frac{\mu_B\mu_C}{8\pi^2} + b_A\frac{T^2}{24}\right)\vec{B}_A + \left(d_{ABC}\frac{\mu_A\mu_B\mu_C}{6\pi^2} + b_A\frac{\mu_A T^2}{6}\right)\vec{\Omega}$$

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Today: Rendezvous with the DEVIL !!

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Today: Rendezvous with the DEVIL !!

- Experiment

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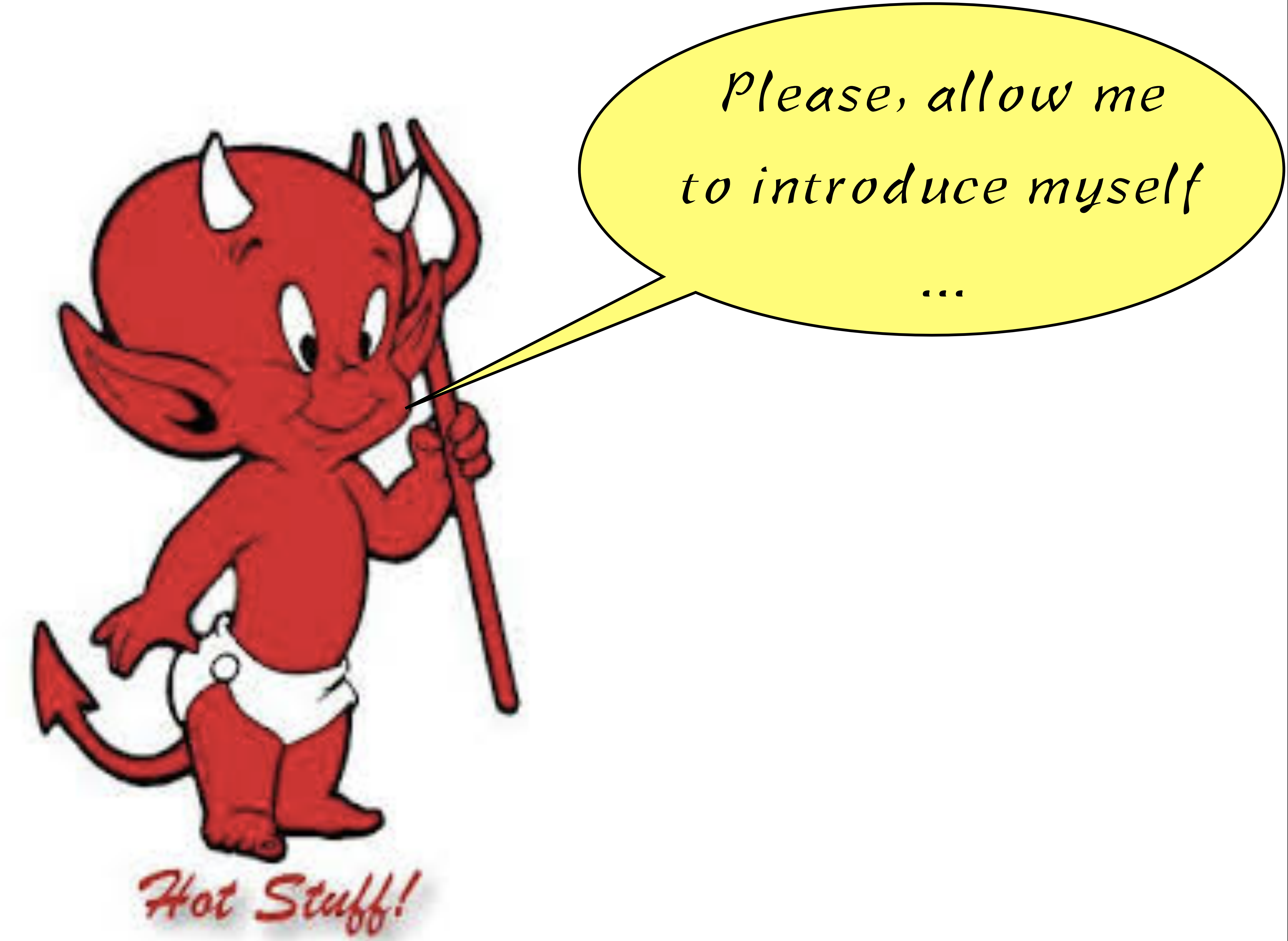
Today: Rendezvous with the DEVIL !!

- Experiment
- Condensed Matter Physics


Please, allow me
to introduce myself
...
Hot Stuff!

Outline:

- Weyl semi-metals
- Physics at the Edge
- A prediction from holography?
- Summary

Weyl semi-metals

Bloch wave functions $\Psi(\vec{x}) = e^{i\vec{k}.\vec{x}} u_k(\vec{x})$

$$u_k(\vec{x}+\vec{a}) = u_k(\vec{x})$$

Physically inequivalent momenta: Brillouin zone

$$\vec{k} \equiv \vec{k} + \vec{K} \qquad \vec{K}.\vec{a} = 2\pi n$$

Berry connection on Brillouin zone

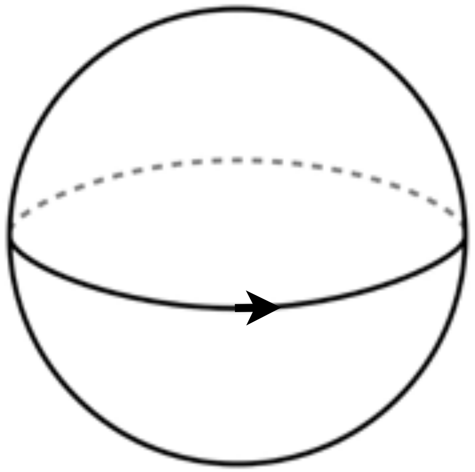
$$\mathcal{A}_i = \langle u_k | \frac{\partial}{\partial k_i} | u_k \rangle$$

$$u_k \to e^{\phi(k)} u_k$$

$$\mathcal{A}_i \to \mathcal{A}_i + \frac{\partial}{\partial k_i}\phi(k)$$

Weyl semi-metals

Berry curvature $\mathcal{F} = d\mathcal{A}$



$$\Delta\phi = \oint \mathcal{A} = \int_{\text{upper}} \mathcal{F} = \int_{\text{lower}} \mathcal{F} + 2\pi n$$

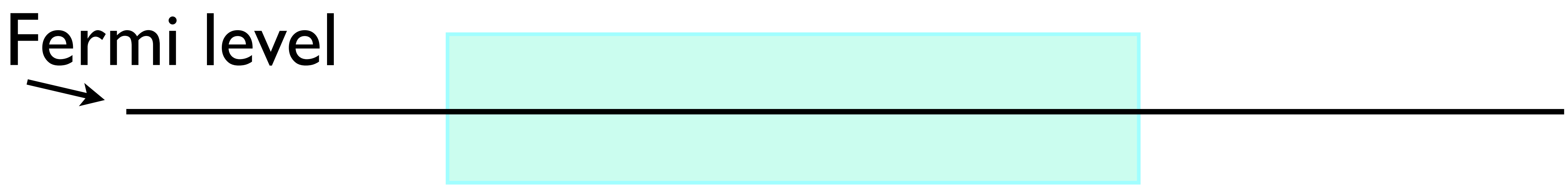
Flux of Berry curvature is quantized $\oint \mathcal{F} = 2\pi n$

Topological Material = non trivial Berry connection

(anti) Chiral fermion = (anti) monopole in Berry connection

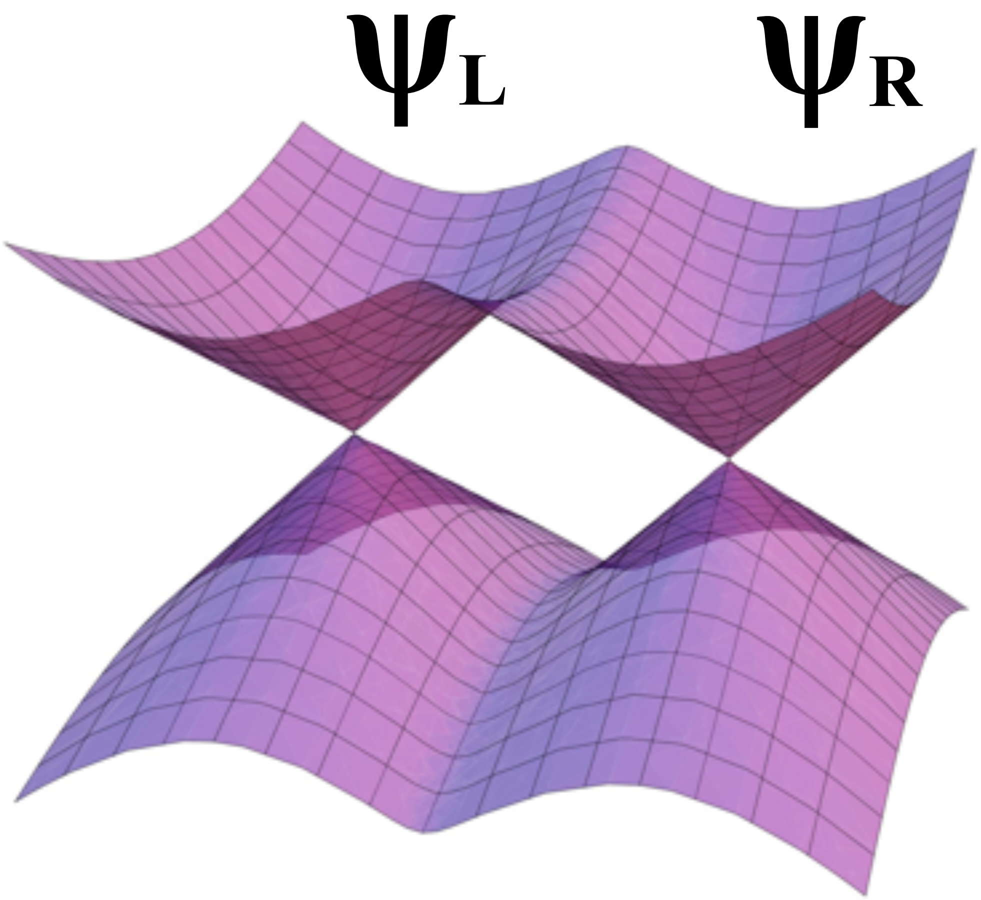
Map to WSMs

Band structure of WSM



Weyl semi-metal

Close up: Linear band touching



$$\gamma^\mu(i\partial_\mu + A^5_\mu\gamma_5)\Psi = 0$$

Nielsen-Ninomiya: left-right come in pairs

Stable: Monopole charge can not vanish
(no mass for chiral fermions)

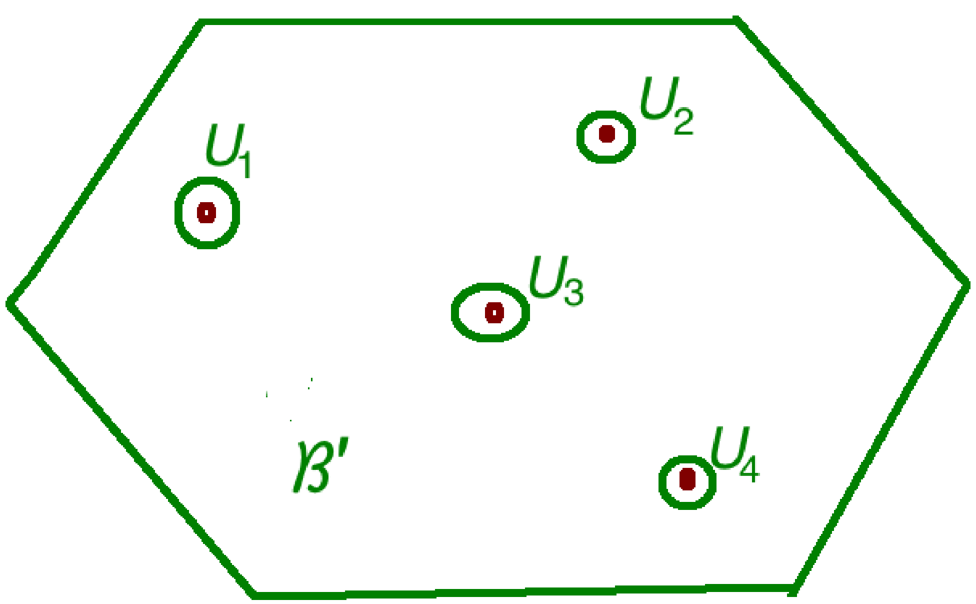
Weyl semi-metal

Topological constraint

Berry connection $$\mathcal{A} = \langle \psi(k) | \frac{\partial}{\partial k_i} | \psi(k) \rangle dk_i$$

$$\mathcal{F}_B = d\mathcal{A}$$

$$d\mathcal{F}_B = 0$$

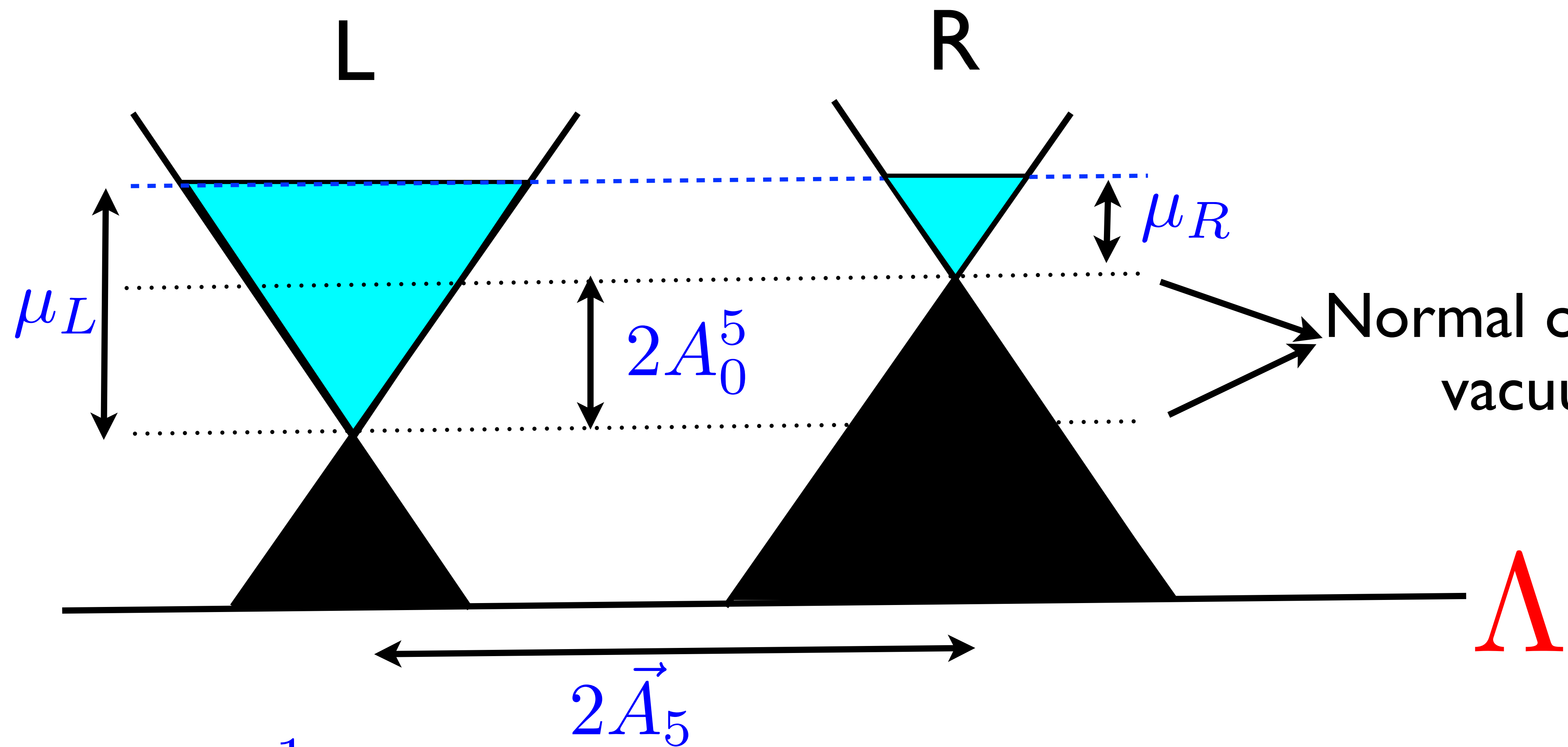


$$\int \frac{d\mathcal{F}_B}{2\pi} = \sum_i \oint_{U_i} \frac{\mathcal{F}_B}{2\pi} = 0$$

[Kiritsis]

BZ has no boundary !

CME in WSMs



$$\mu_5 = \frac{1}{2}(\mu_L - \mu_R) \qquad \mu = \frac{1}{2}(\mu_R + \mu_L)$$

CME: $$\vec{J} = \frac{1}{2\pi^2}\left(\mu_5 - A_0^5\right)\vec{B} = 0$$

CME vanishes in equilibrium in WSM "Bloch theorem"

Negative Magnetoresitivity

CME $\vec{J} = \sigma \vec{E} + \frac{\delta\mu_5}{2\pi^2}\vec{B}$

Anomaly $\partial_t \delta\rho_5 = \frac{1}{2\pi^2}\vec{E}\vec{B}$

Susceptibility $\delta\rho_5 = \chi_5 \delta\mu_5$

$$\vec{J} = \left(\sigma + \frac{iB^2}{\omega\chi_5}\right)\vec{E}$$

$$\Re(\sigma_{\mathrm{tot}}) = \sigma + \frac{B^2}{\chi_5}\delta(\omega)$$

Anomaly induces infinite DC conductivity !!

Negative Magnetoresitivity

CME $$\vec{J} = \sigma \vec{E} + \frac{\delta \mu_5}{2\pi^2} \vec{B}$$

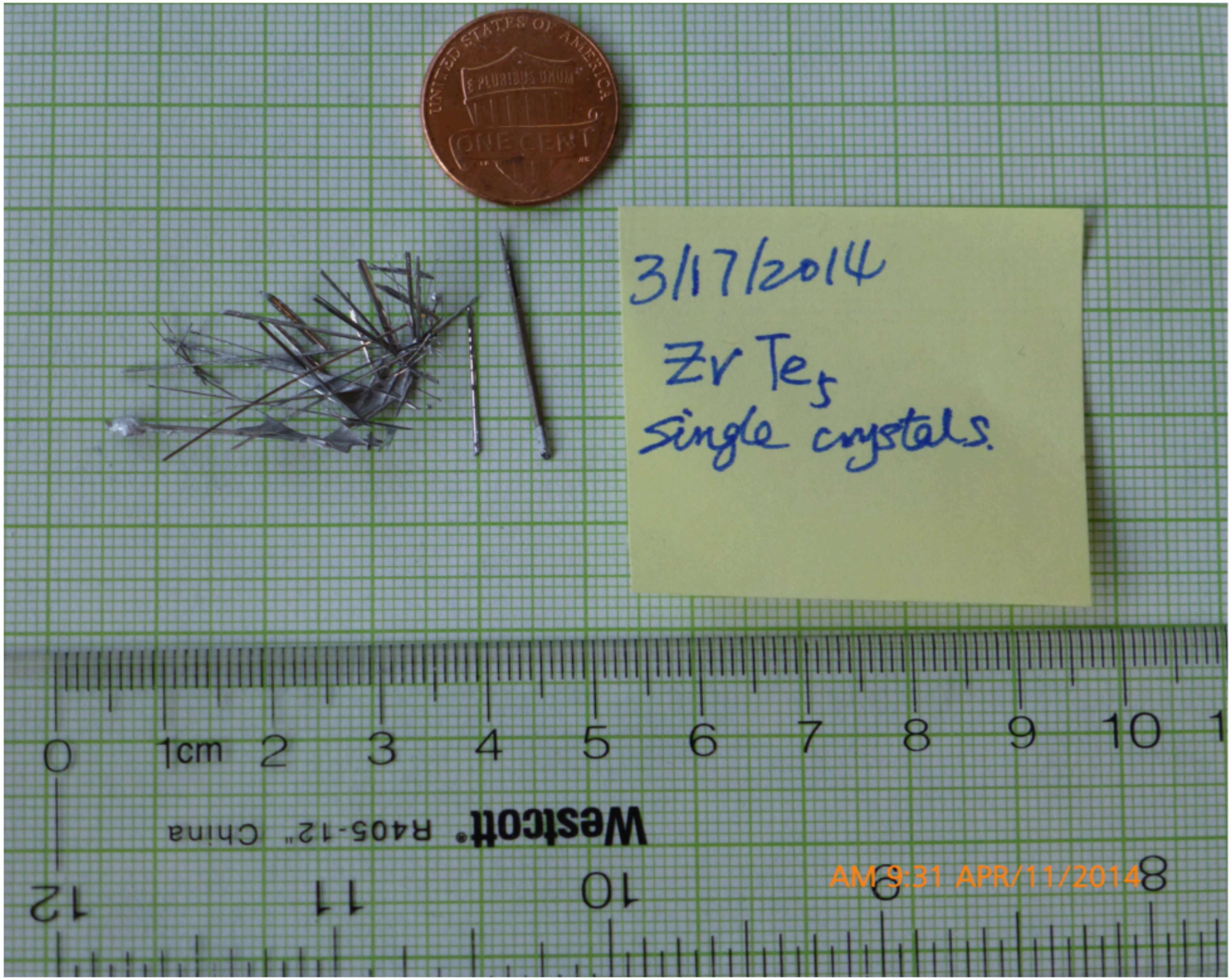
Anomaly $$\partial_t \delta \rho_5 = \frac{1}{2\pi^2} \vec{E} \vec{B} - \tau_c \delta \rho_5$$

Susceptibility $$\delta \rho_5 = \chi_5 \delta \mu_5$$

$$\vec{J} = \left(\sigma + \frac{\tau_c B^2}{\chi_5} \right) \vec{E}$$

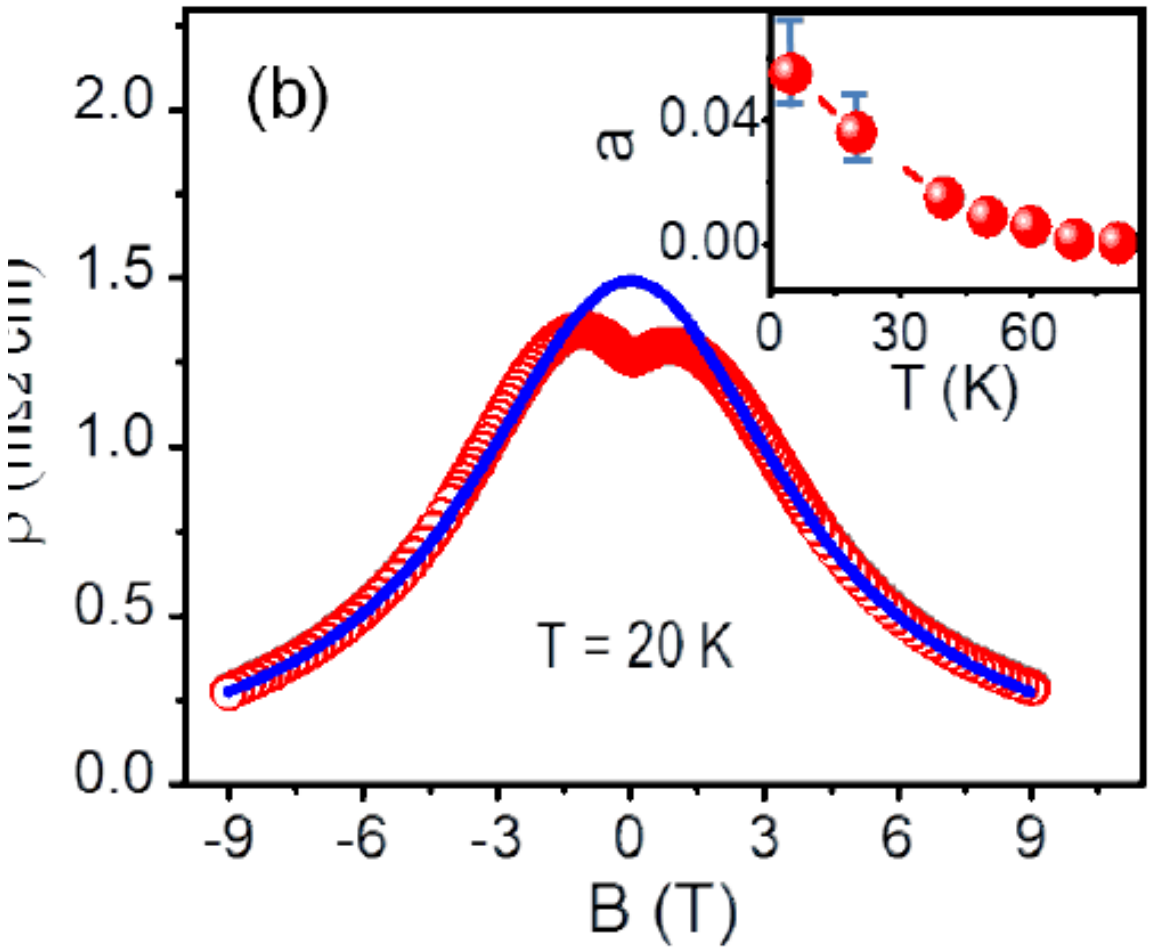
Anomaly induces finite DC conductivity !!

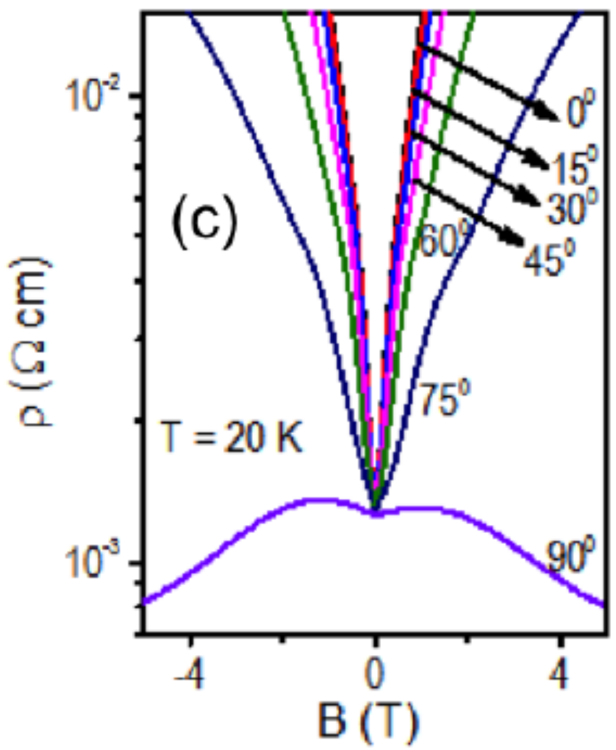
Recent Experiments

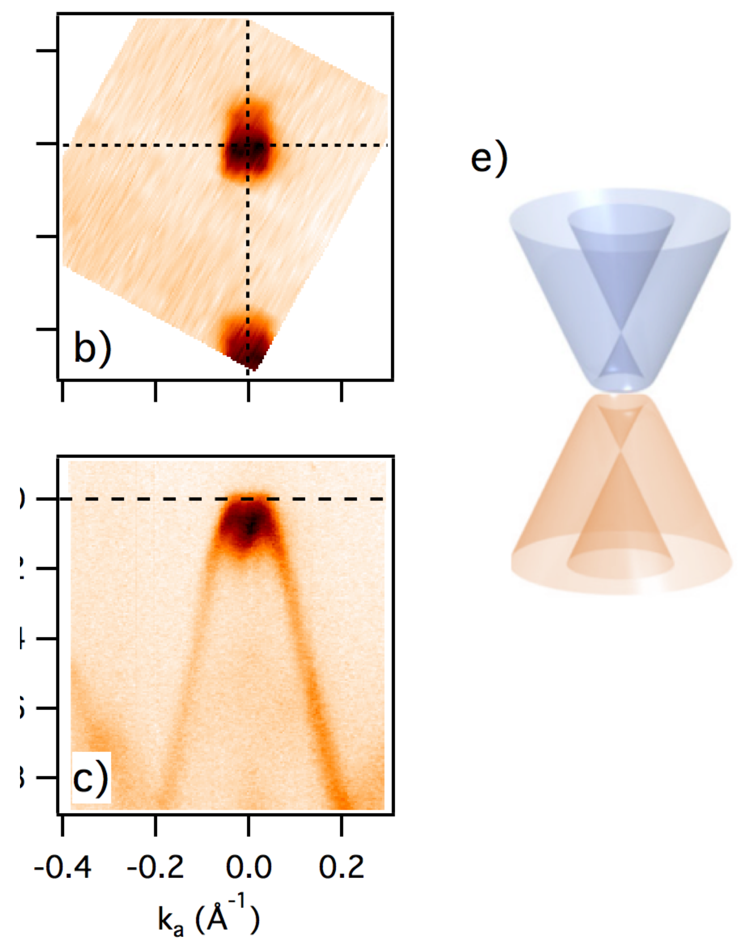


[D. Kharzeev: Lectures at Schladming Winterschool 2015]

Recent Experiments





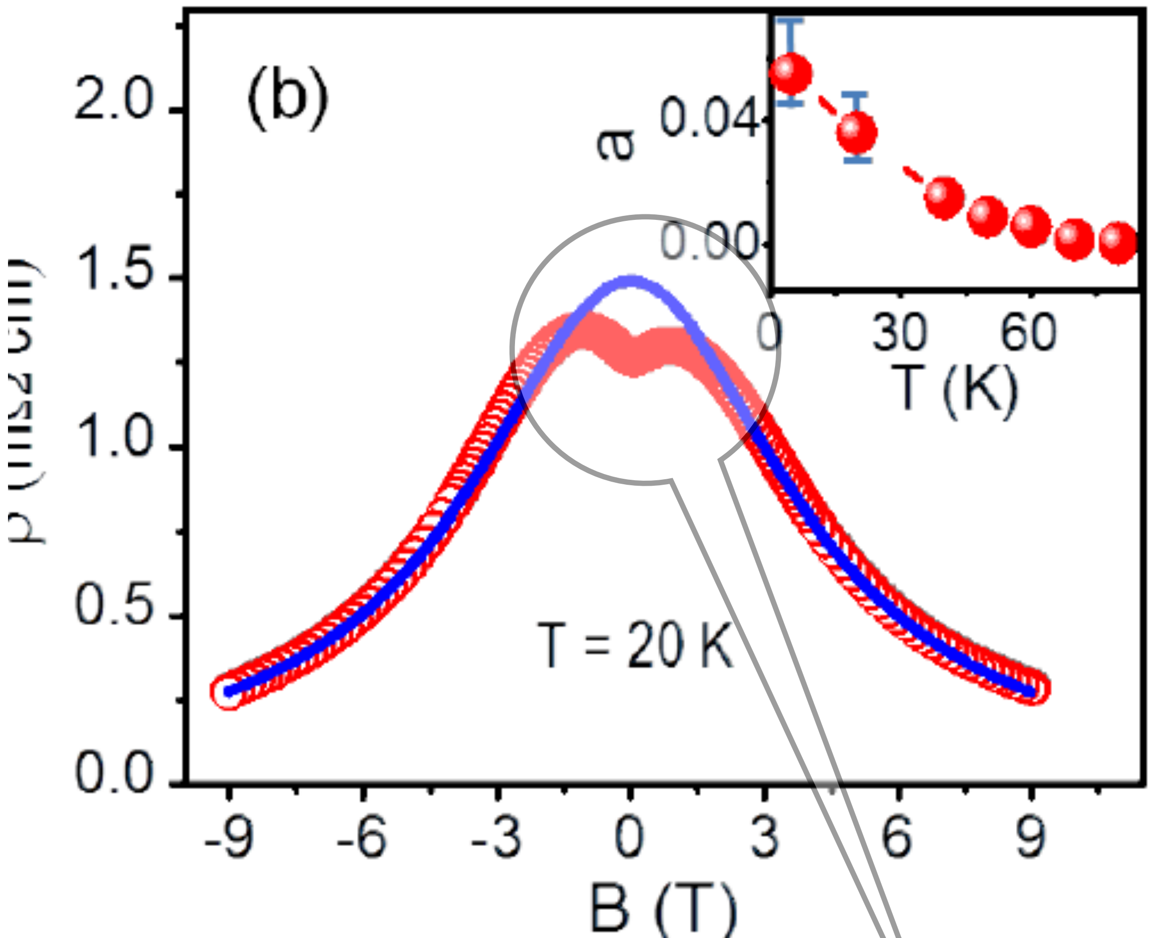


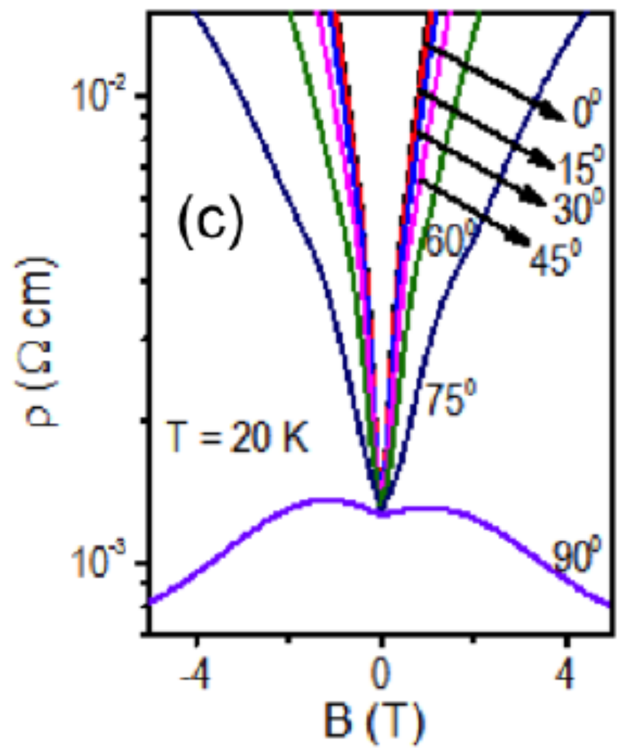
fits to 1/(a+b B^2)

B^2 behaviour!

[Li, Kharzeev at al. NaturePhysics 3648 (2016)]

Recent Experiments





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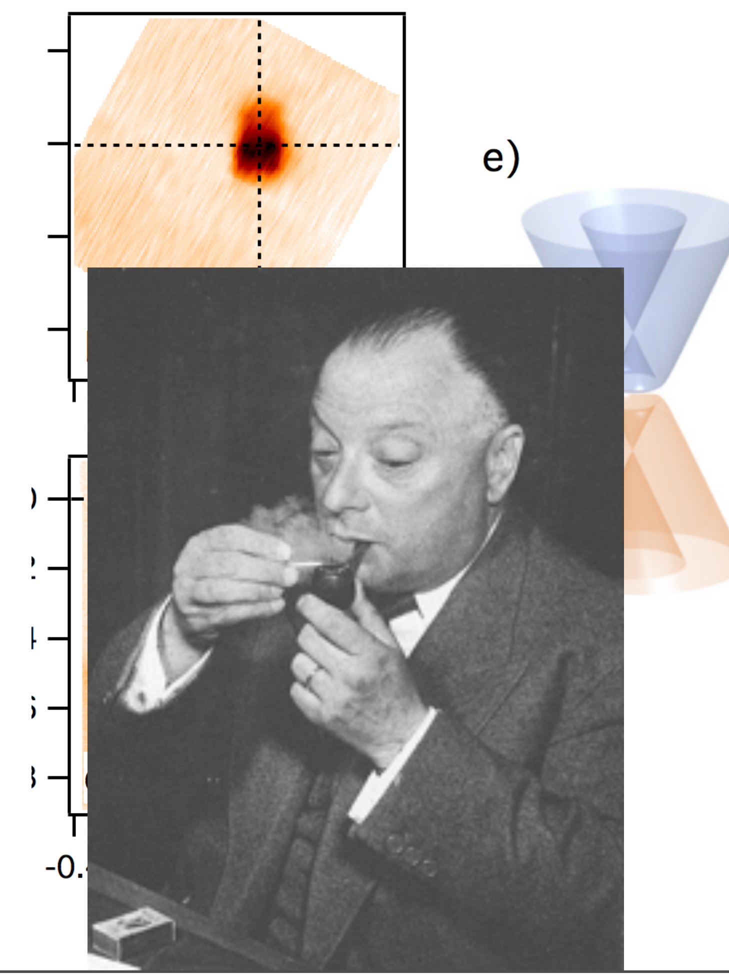
B^2 behaviour!

“Schmutzphysik”
(dirt physics)



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AHE in WSMs

$$J^{\mu}_{cons} = J^{\mu}_{cov} + \frac{1}{4\pi^2}\epsilon^{\mu\nu\rho\lambda}A^5_{\nu}F_{\rho\lambda}$$

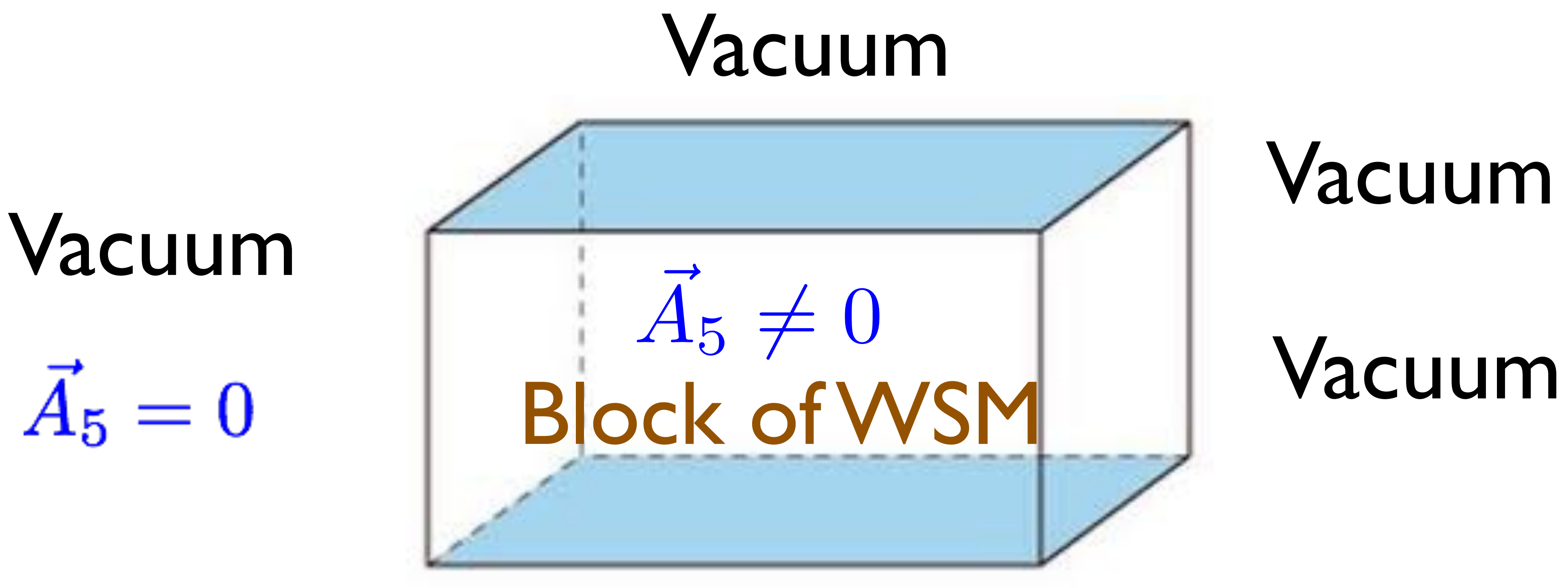
$\vec{A}^5$ Separation of Weyl nodes in Brillouin zone

$$\vec{J} = \frac{1}{2\pi^2}\vec{A}_5 \times \vec{E}$$

No contribution from covariant current

(no analog of chemical potential in momentum)

Edge Physics



- Vacuum: (infinitely) massive Dirac fermions
- Need gradient in axial vector = axial magnetic field

$$\vec{J} = \frac{\mu}{2\pi^2}\vec{B}_5 \qquad \vec{B}_5 = \nabla \times \vec{A}_5$$

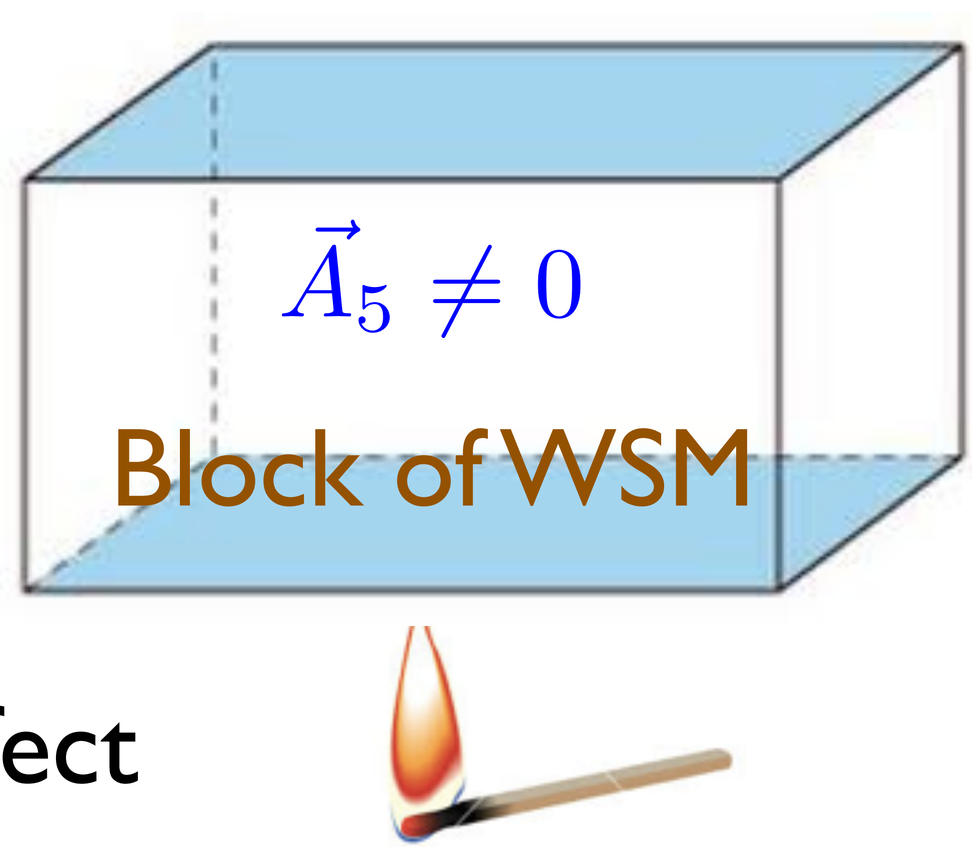
- Edge current
- States: "Fermi arcs" (degeneracy = $|A_5|$)

Edge Physics

Vacuum

Vacuum

$\vec{A}_5 = 0$



Vacuum

Vacuum

- Thermal Hall Effect

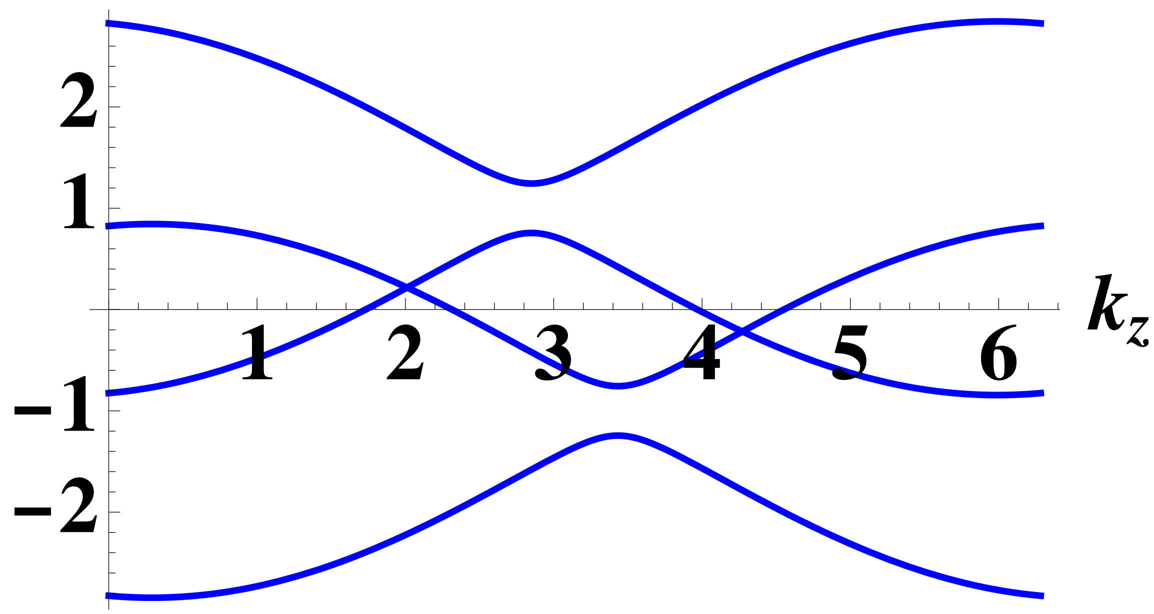
$$\vec{J}_\epsilon = \left(\cdots + \frac{T^2}{12} \right) \vec{B}_5 \qquad \vec{B}_5 = \nabla \times \vec{A}_5$$

- Net energy current at edge
- Effect of *gravitational anomaly!!*

$$J_{\text{thermal}} = \frac{T}{6} |A_5| \Delta T$$

WSM

In crystal left- and right-handed chiralities are connected along the band dispersions



Can we find a QFT and a holographic model with this property?

YES!!

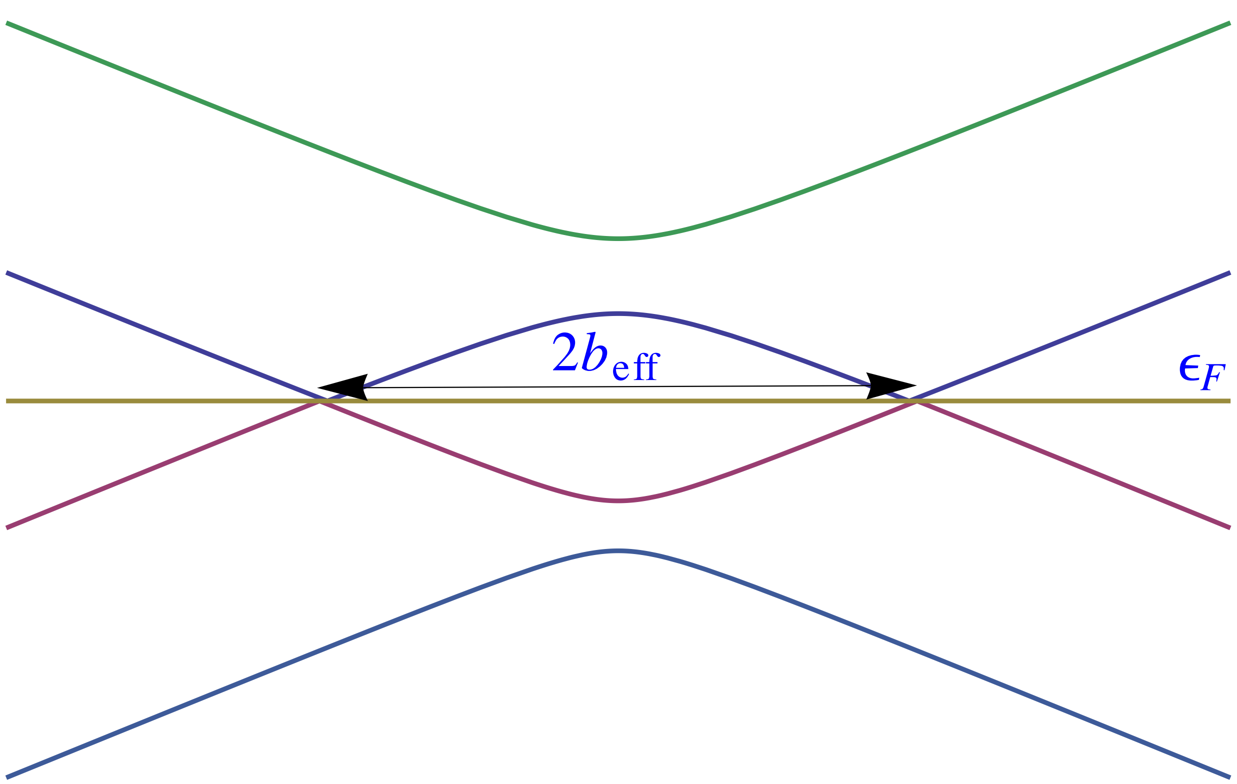
QFT model

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu + M + \gamma_5 \gamma_z b)\psi$$

[Kostolecky et al.] [Jackiw]
[Burkov, Balents] [Grushin]

spectrum:

$M < b:\qquad b_{\rm eff} = \sqrt{b^2 - M^2}$



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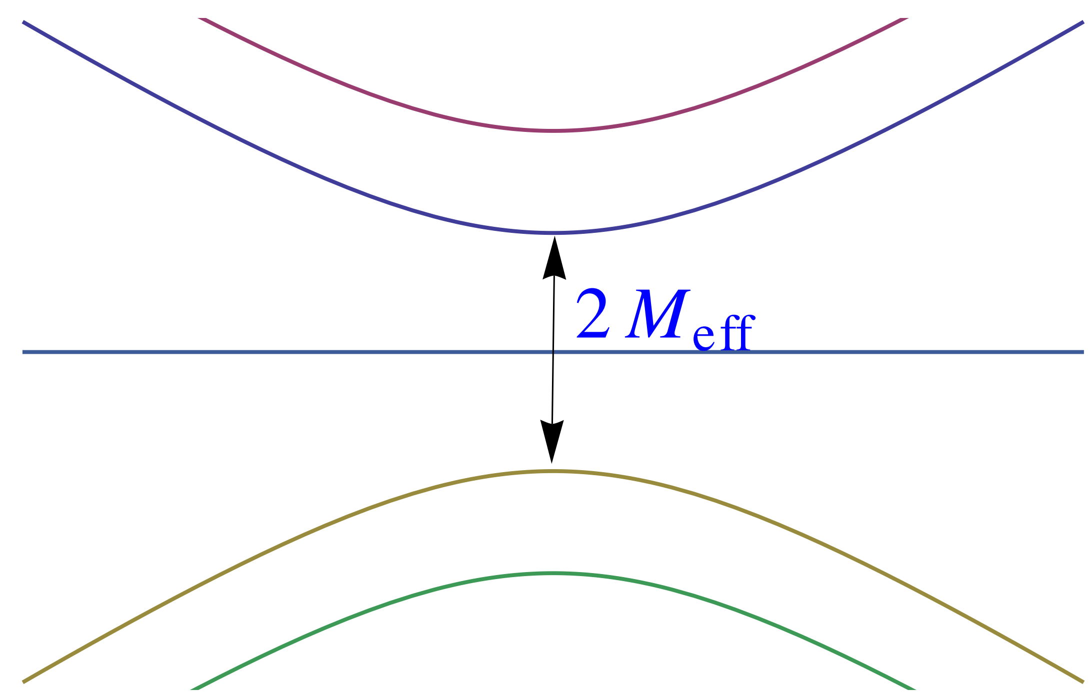


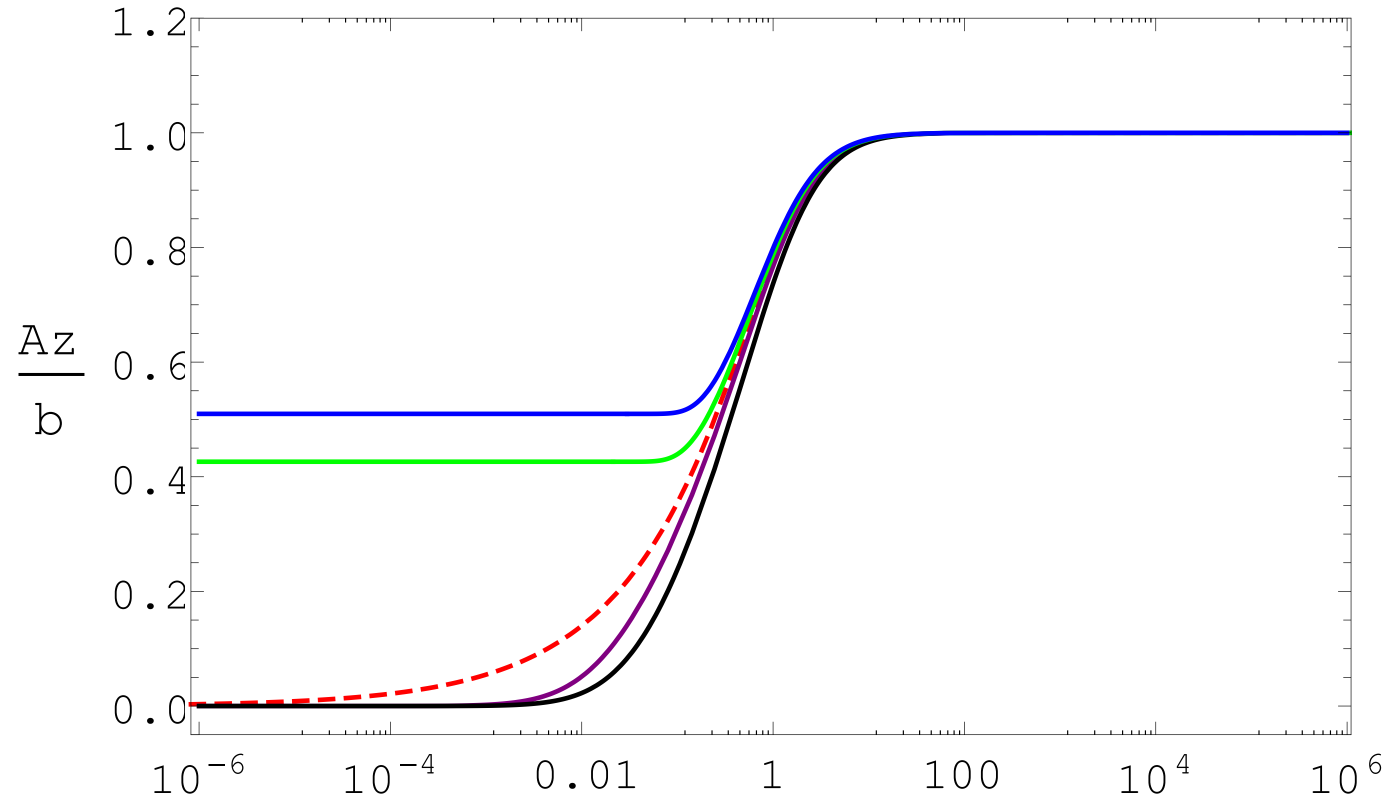
Action of HoloWSM

$$\mathcal{L} = \frac{1}{2\kappa^2}(\mathcal{R} + 12) - \frac{1}{4}\mathcal{F}^2 - \frac{1}{4}F_5^2 + \\ |(\partial_\mu + iqA_\mu^5)\Phi|^2 - V(|\Phi|) + \\ + A_5 \wedge \left(\frac{\alpha}{3}F_5 \wedge F_5 + \alpha\mathcal{F} \wedge \mathcal{F} + \zeta R \wedge R\right)$$

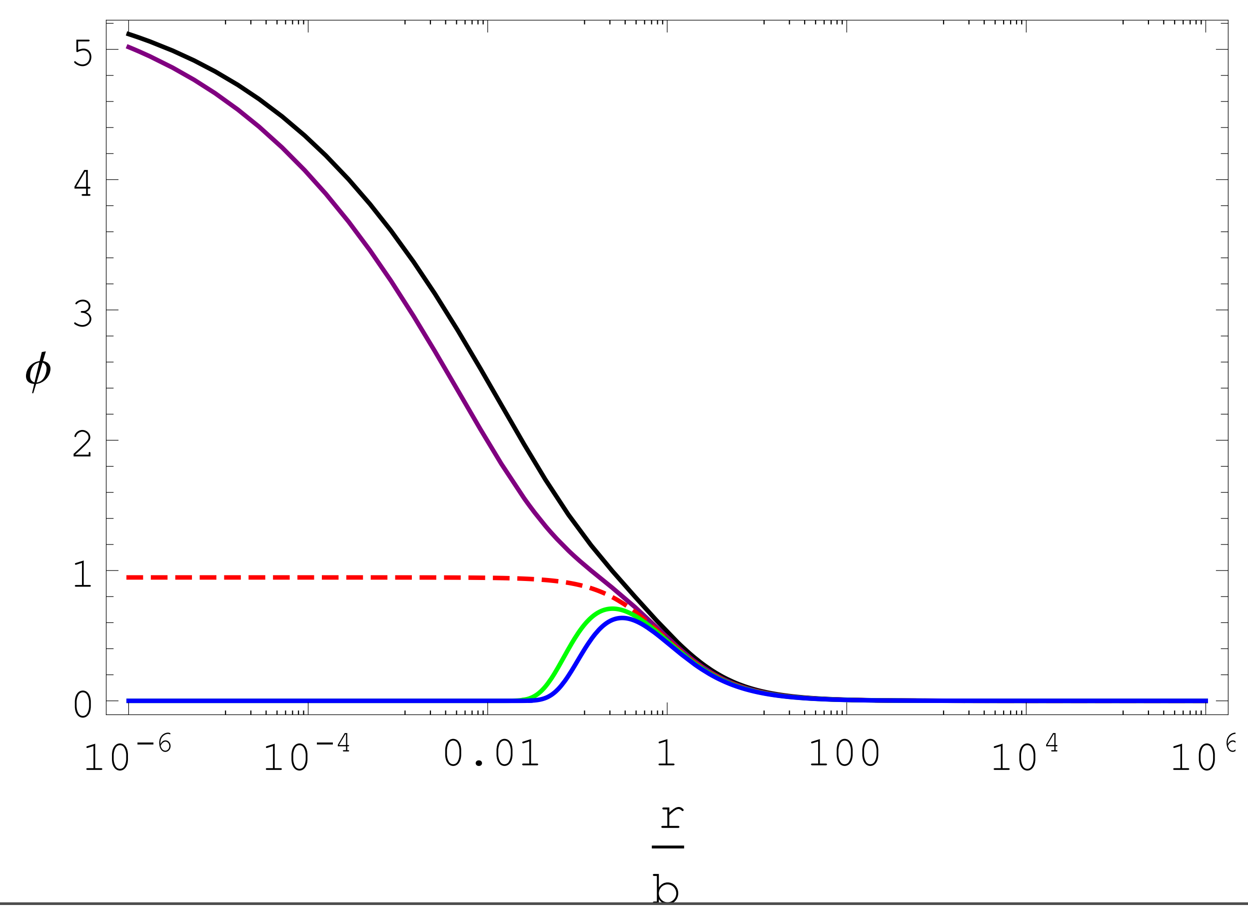
- Cosmological constant = AdS
- Very specific CS term = form of Anomaly
- Scalar potential determines dimension of dual scalar operator (we chose dim=3) i.e. mass deformation

A holographic topological quantum phase transition

Running of axial gauge field:



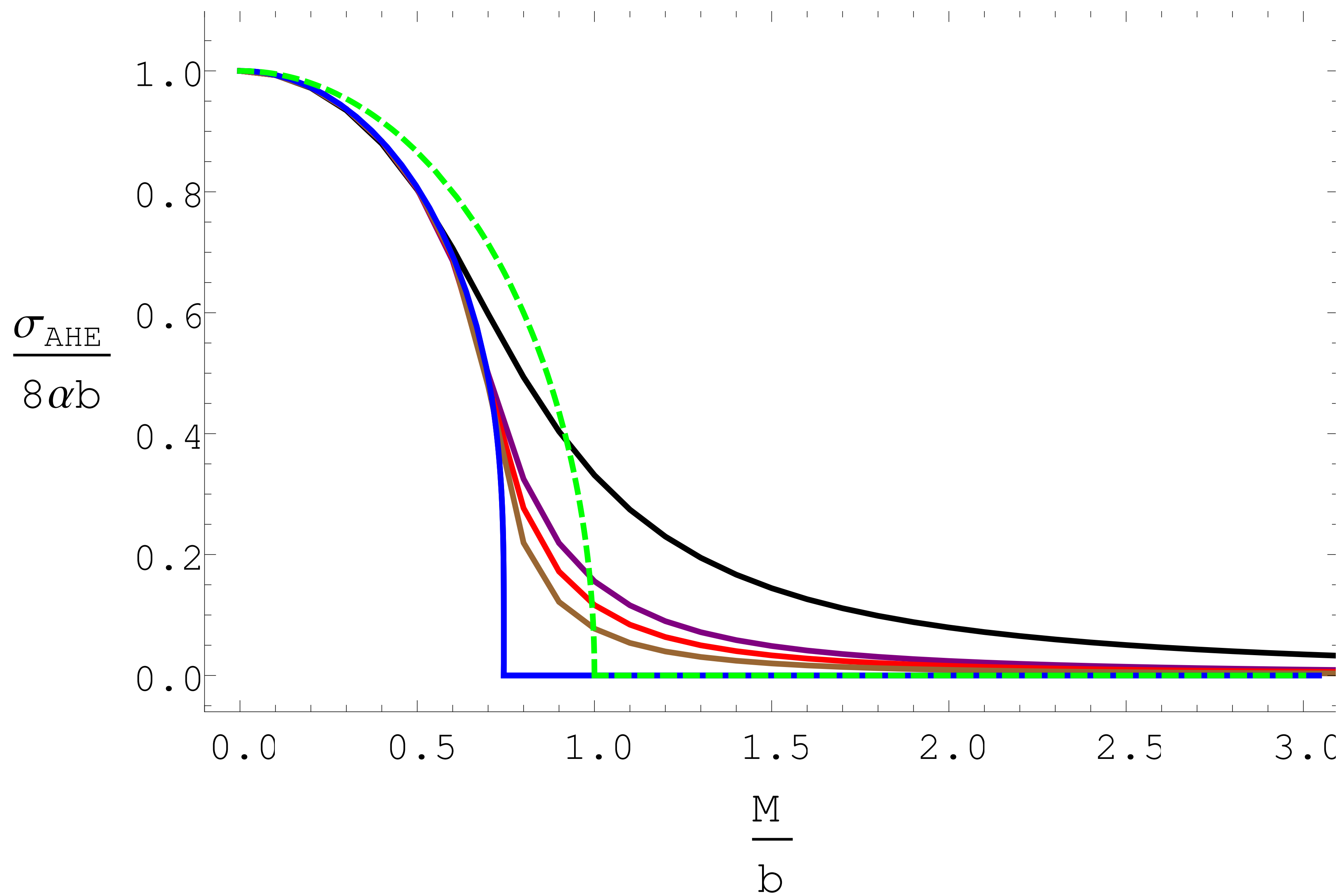
Running of scalar field:



Holographic WSM

Smoking gun of topological state of matter : AHE

$$\sigma_{xy} = 8\alpha A_z^5(0)$$



Odd viscosity

- Hall viscosity in 2D Quantum Hall states [Avron, Seiler, Zograf]
- Time reversal breaking necessary
- 2D : invariant ε tensor
- 3D: need some additional vector
- Hydrodynamics of magnetized plasmas [Landau, Lifshytz Vol. 10]

$$\tau_{xy} = \eta_\perp V_{xy} - \eta_\perp^H (V_{xx} - V_{yy})$$

$$\tau_{xz} = \eta_{||} V_{xz} + \eta_{||}^H V_{yz}$$

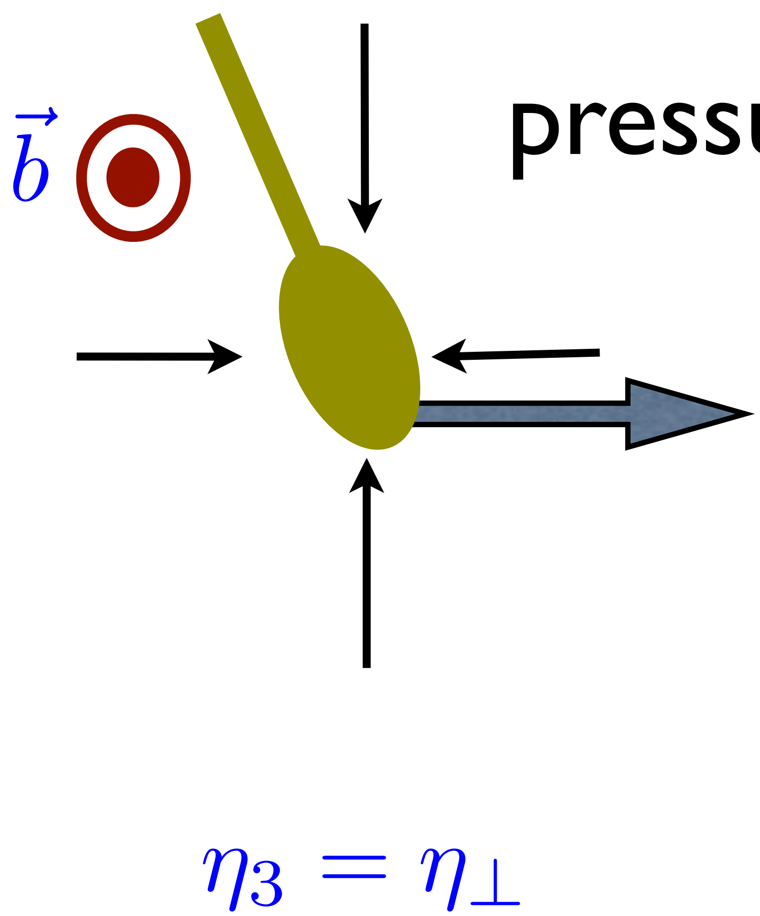
$$\tau_{yz} = \eta_{||} V_{yz} - \eta_{||}^H V_{xz}$$

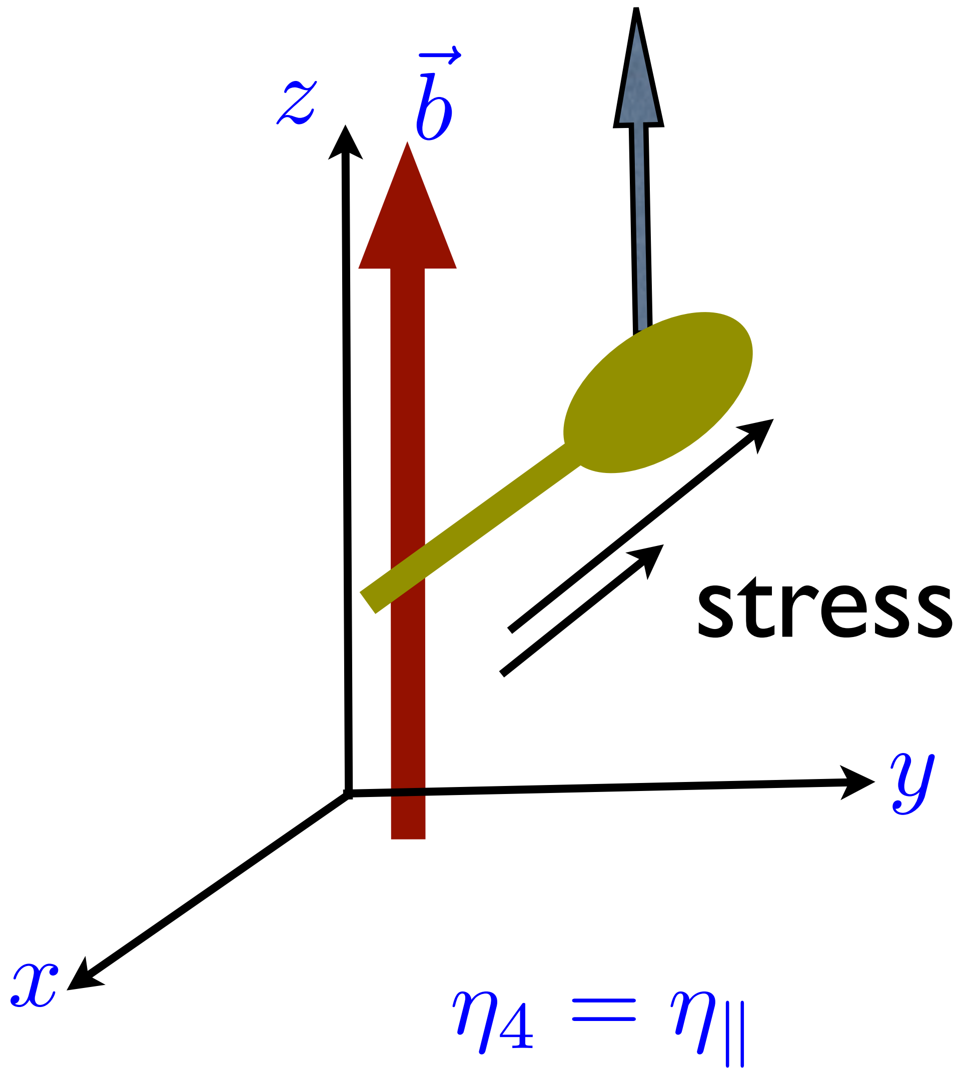
$$V_{ij} = \frac{1}{2}(\partial_i v_j + \partial_j v_i)$$

odd viscosities

- In total: 3 shear, 2 "bulk" and 2 odd viscosities
- $\eta_{odd}(-B) = -\eta_{odd}(B)$

Odd viscosity

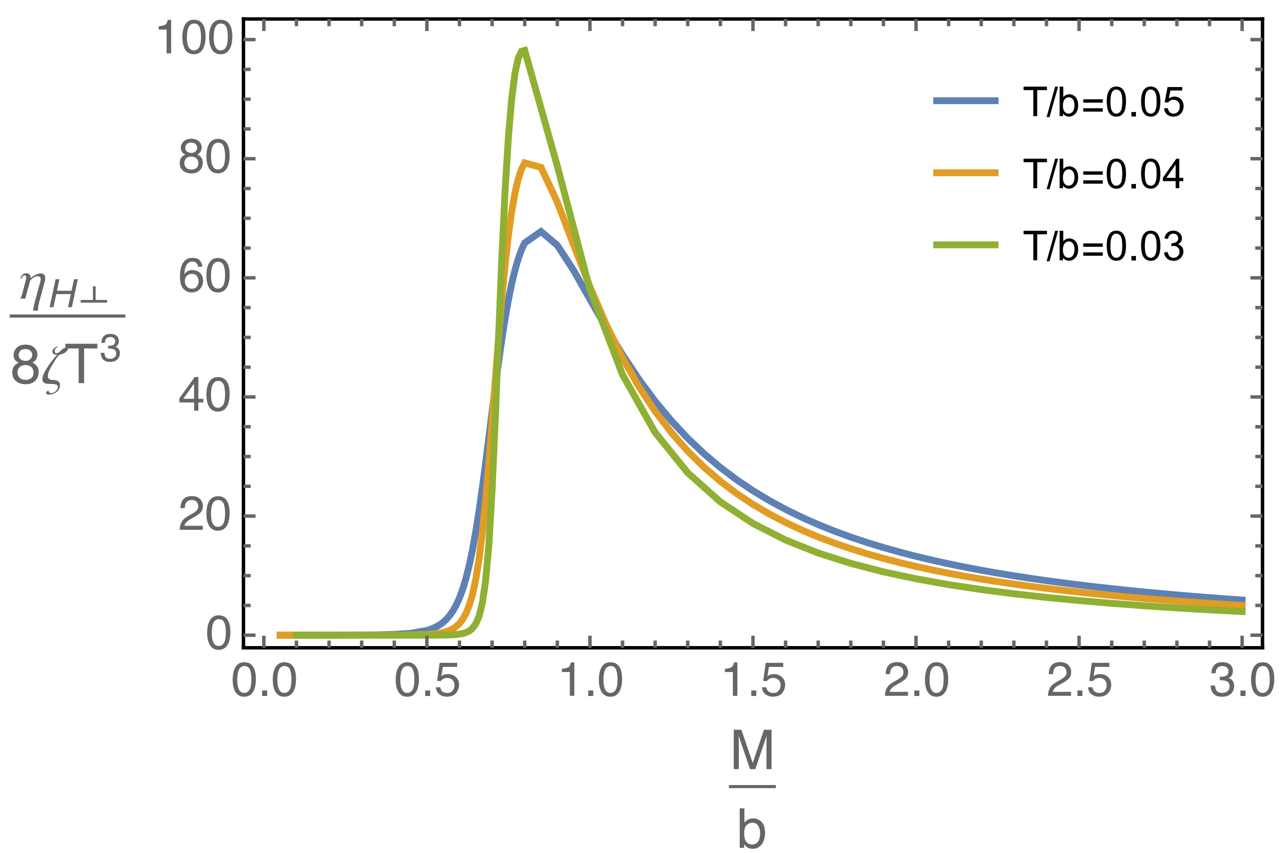




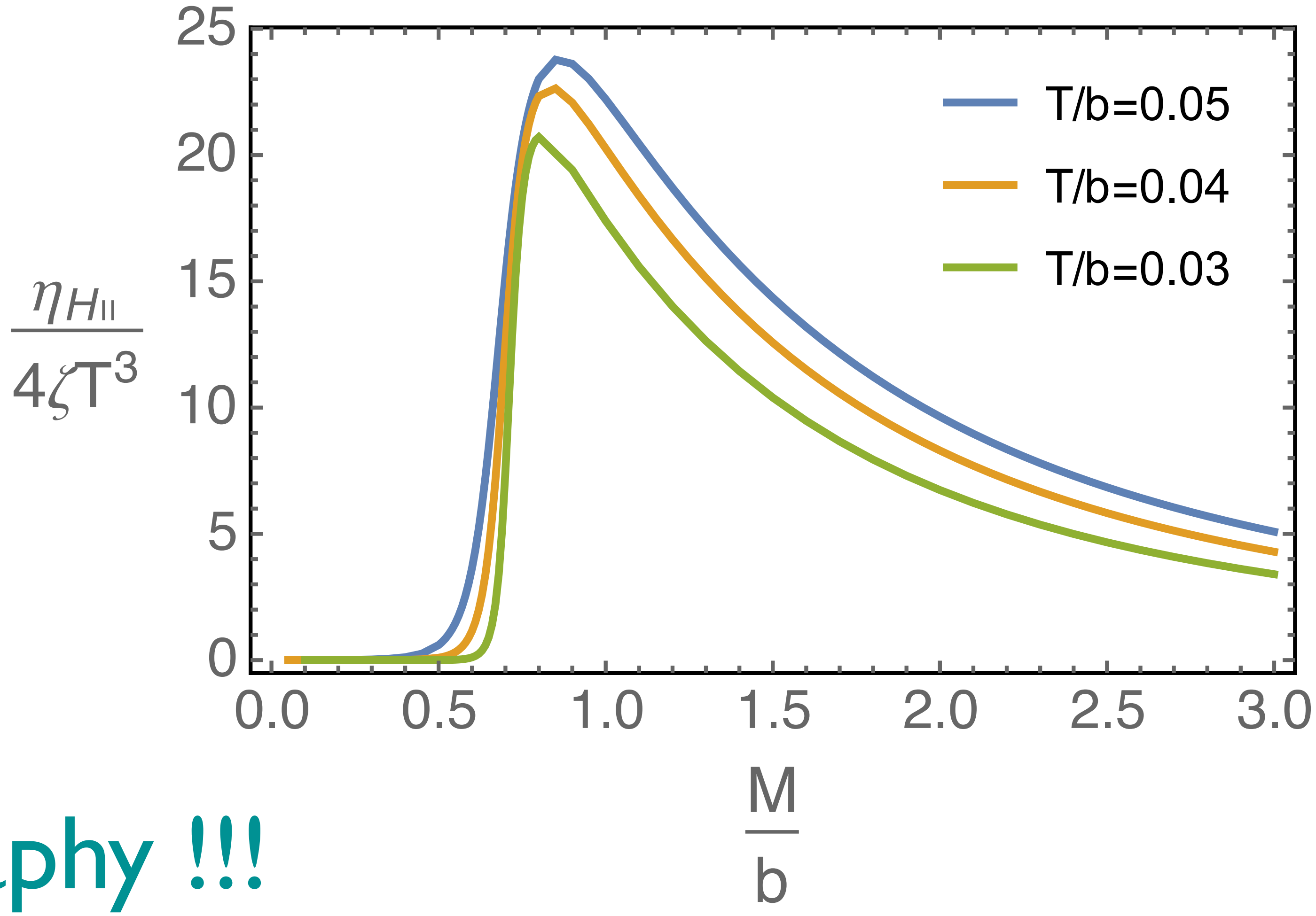
Odd viscosity

- Odd viscosities
- Probe IR region of geometry: Low T

transverse



parallel



- Prediction from Holography !!!
- Again: gravitational Anomaly at first order !

Summary

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