International Workshop for String theory and Cosmology
(17 Aug. 2016, Hanyang Univ., Seoul)

Toward the Renormalizable (Quantum) Gravity

Mu-In Park
Sogang Univ., Korea

We heard about LIGO's detection of gravitational waves.

Selected for a Viewpoint in *Physics*
PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_\odot$ and $29^{+4}_{-4} M_\odot$, and the final black hole mass is $62^{+4}_{-4} M_\odot$, with $3.0^{+0.5}_{-0.5} M_\odot c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

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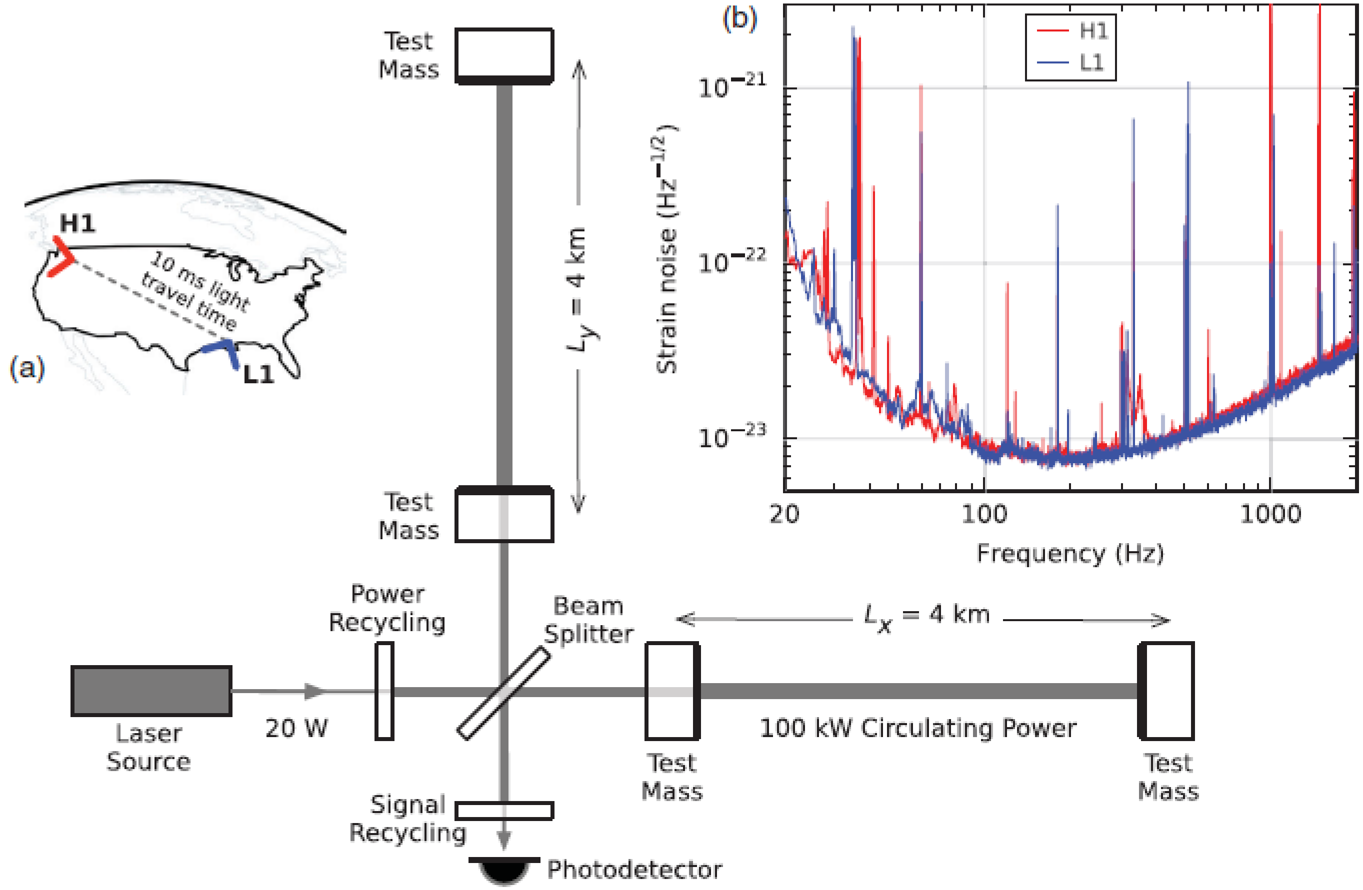
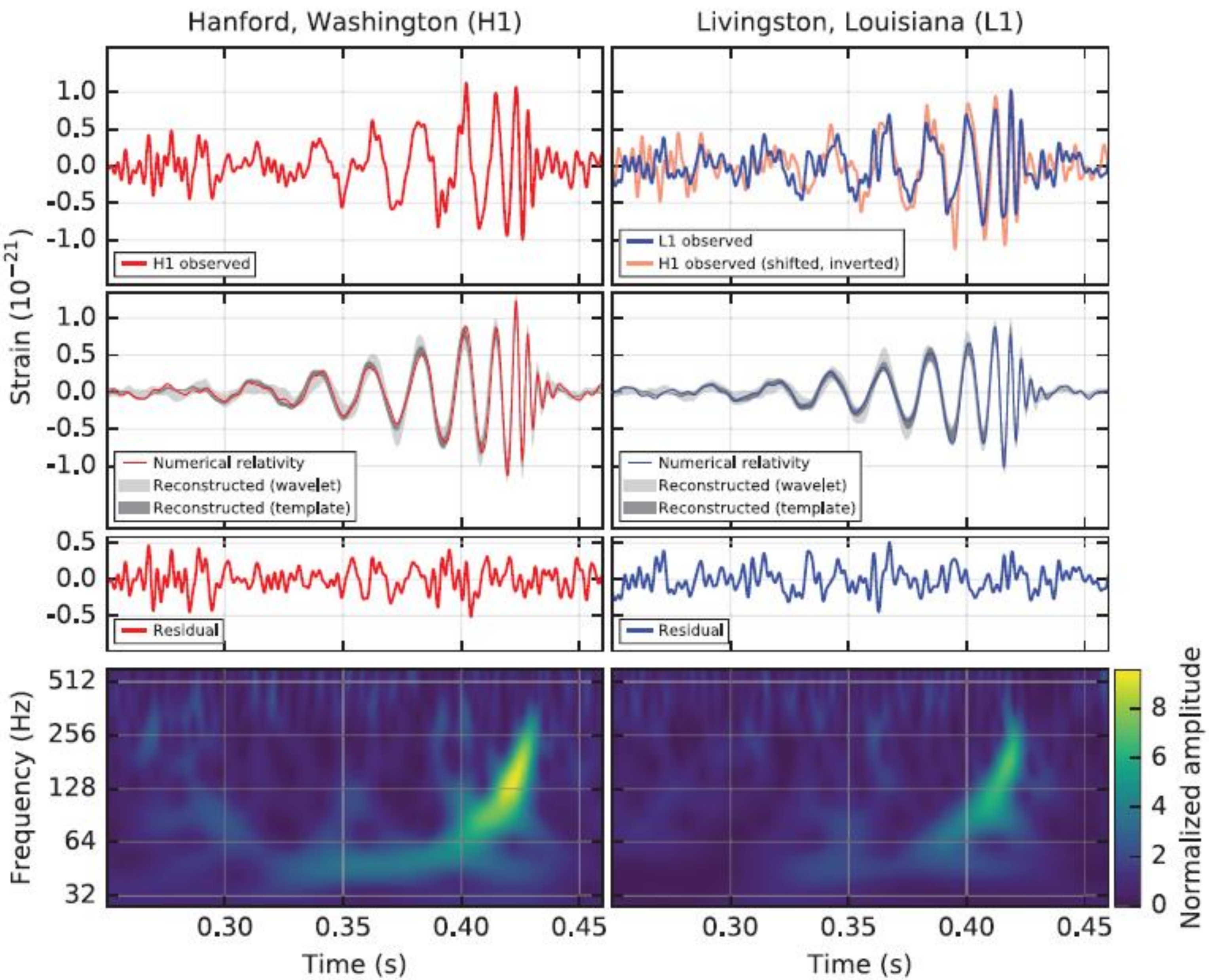
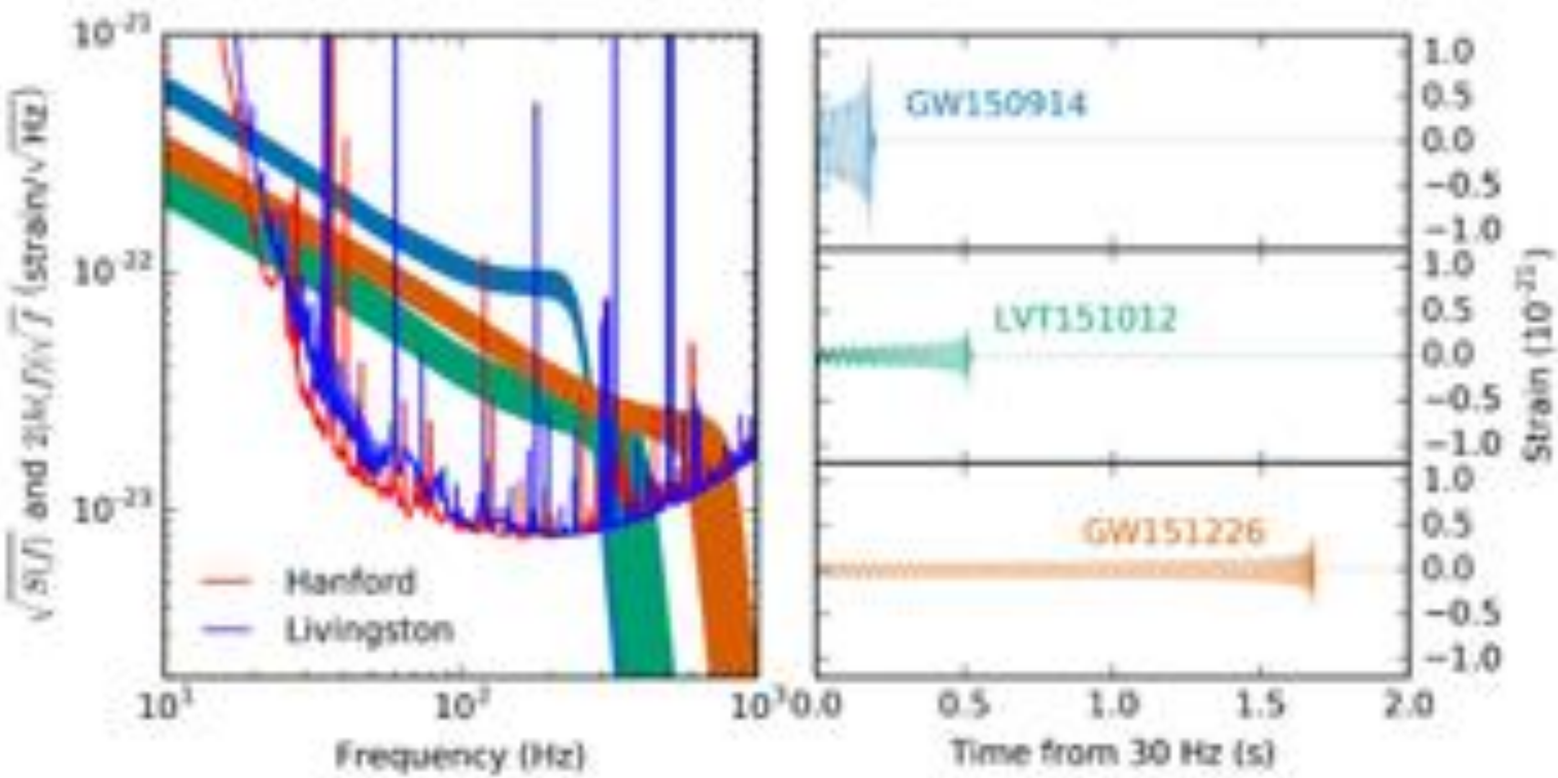


(a)
H1
L1
10 ms light travel time
(b)
H1
L1
Strain noise (Hz$^{-1/2}$)
10^{-21}
10^{-22}
10^{-23}
20
100
1000
Frequency (Hz)
Test Mass
Test Mass
L_y = 4 km
Power Recycling
Beam Splitter
L_x = 4 km
Laser Source
20 W
100 kW Circulating Power
Test Mass
Test Mass
Signal Recycling
Photodetector

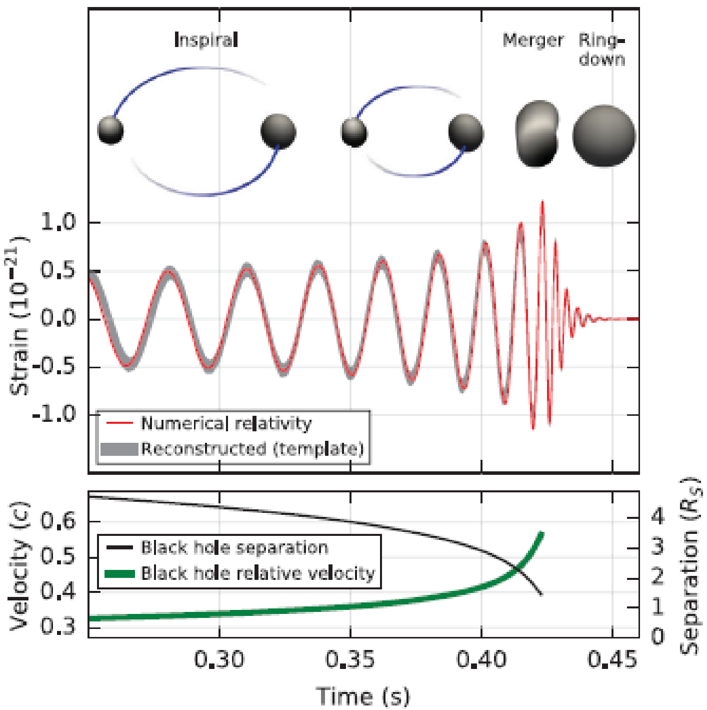


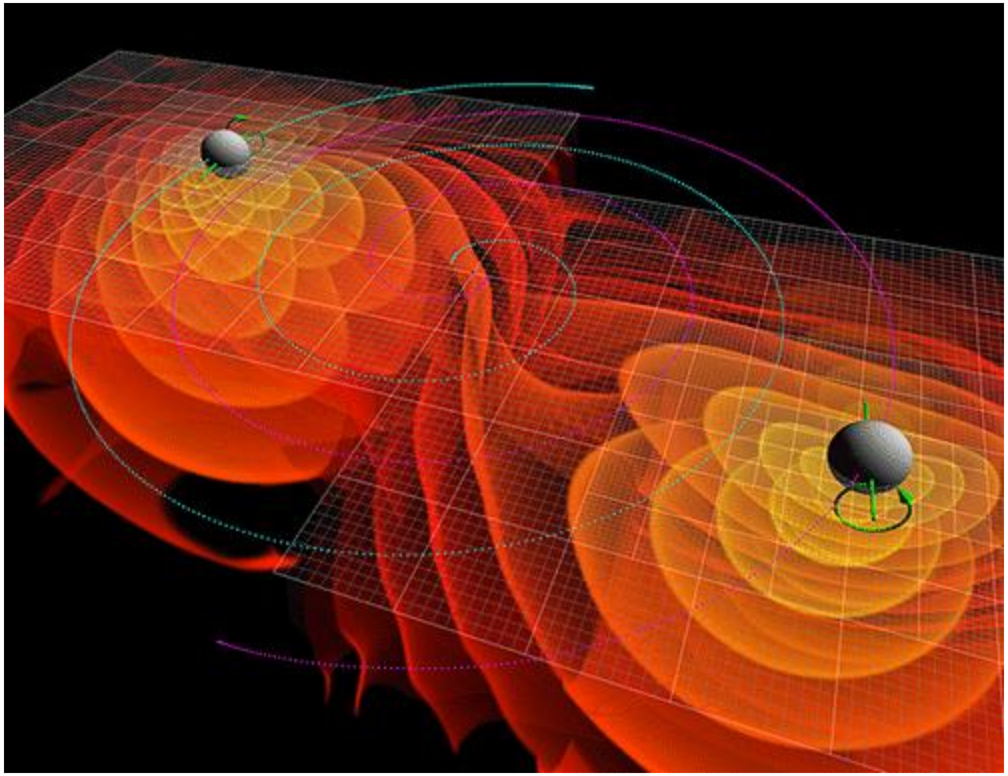
Hanford, Washington (H1)
Livingston, Louisiana (L1)
Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual
Frequency (Hz)
512
256
128
64
32
0.30
0.35
0.40
0.45
Time (s)
Normalized amplitude
8
6
4
2
0

More Detections



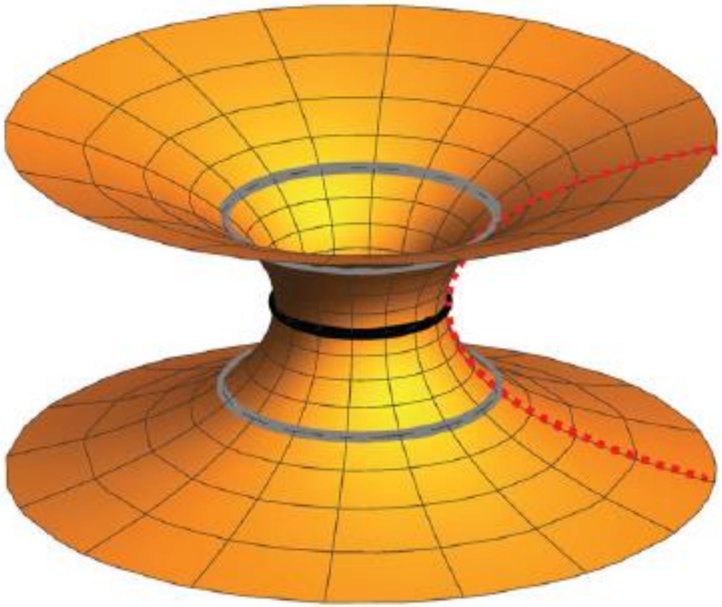
They conclude that these are the results of ``merger” of two spinning black holes.

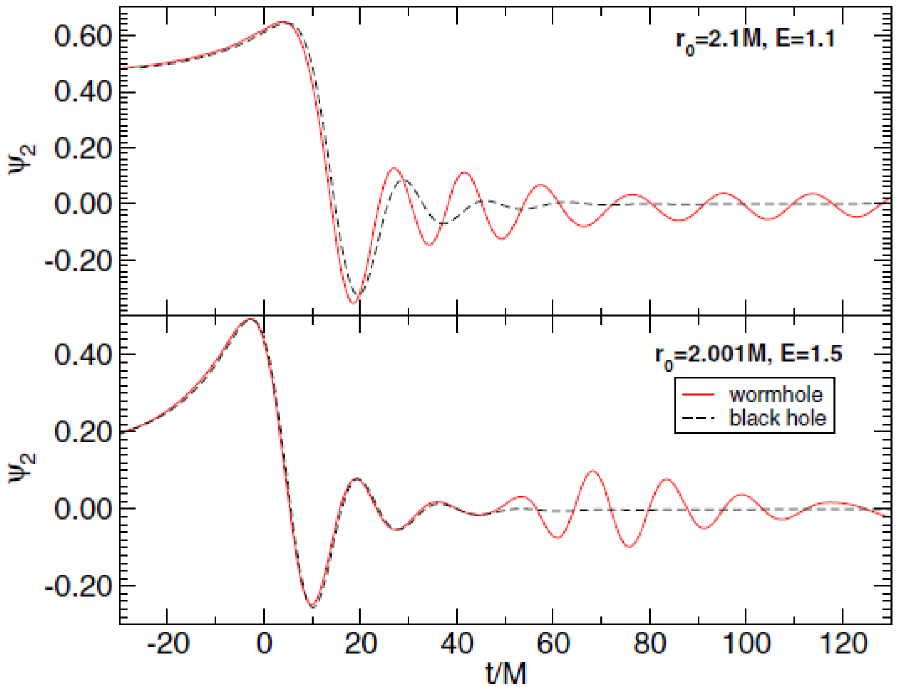




There remains some open issues, though.

- 1. Can we distinguish the black holes with other horizon-less compact objects ? [Cardoso. et al., PRL 116, 171101 (2016)]





- 2. Was there EM radiations from the merging black holes ? [V. Connaughton, et .al. arXiv:1602.03920]

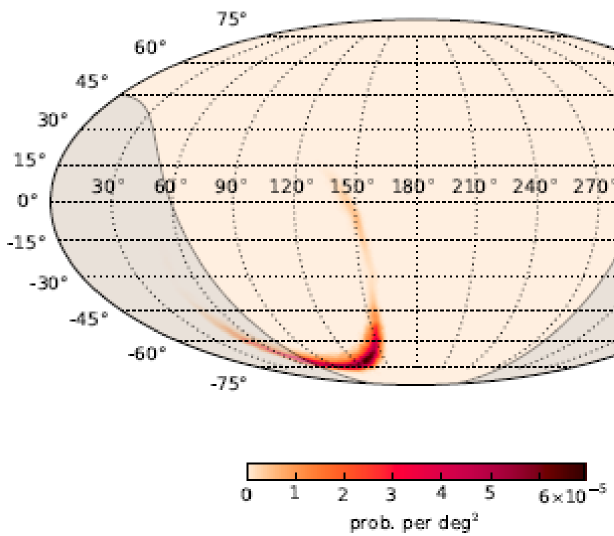


Fig. 1.— Localization map for GW150914, the GW event reported in Abbott et al. (2016). The grey shaded region indicates the region of sky occulted to *Fermi* by the Earth at the time of GW150914. The region not occulted by the Earth contains 75% of the probability of the localization map, with all but 6% of the probability contained in the lower lobe. The entire region was visible to *Fermi* GBM 25 minutes after the GW event was detected.

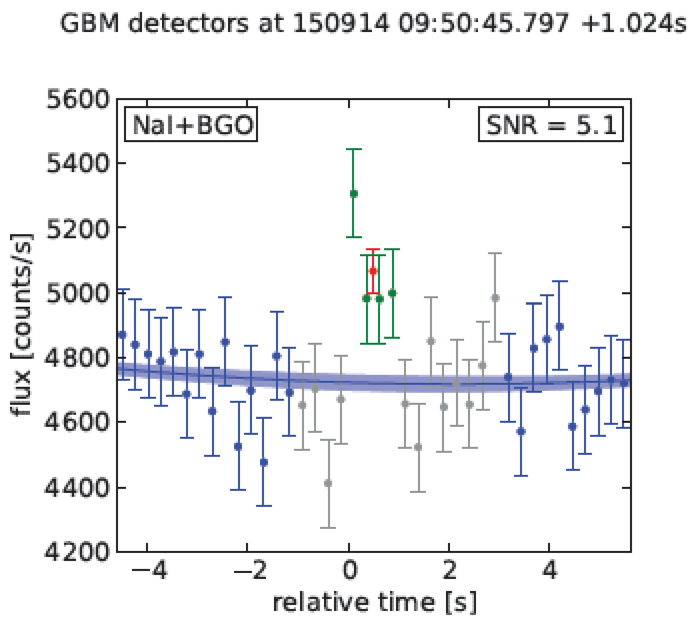


Fig. 2.— Count rates detected as a function of time relative to the start of GW150914-GBM, ~0.4 s after the GW event GW150914, weighted and summed to maximize signal-to-noise for a modeled source. CTIME time bins are 0.256 s wide. The blue data points are used in the background fit.

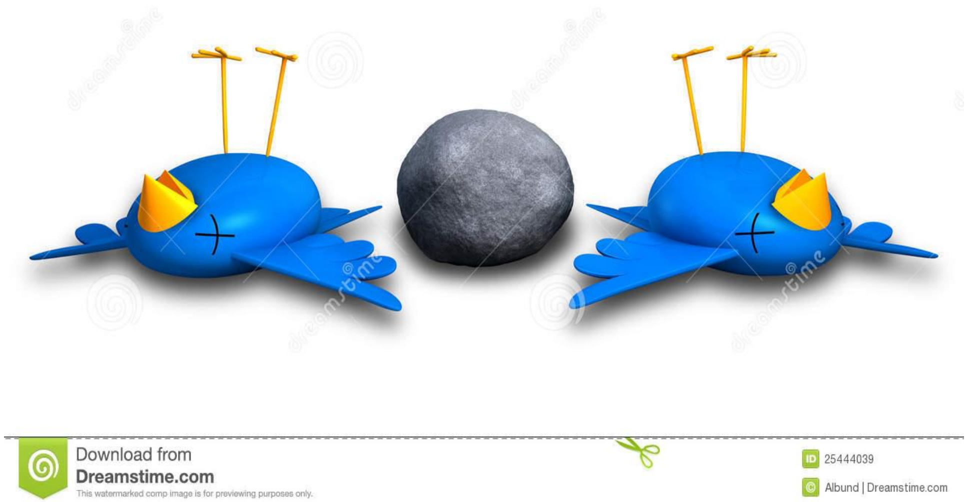
Date updates

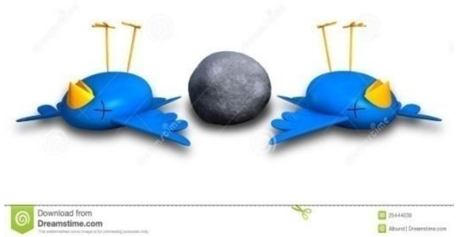
- But, the Fermi data was not confirmed in later analysis by INTEGRAL telescope and AGILE telescope !!??

- Is there similar signal for GW151226 ?

But, everyone will agree that this is the strongest gravity event that we have observed !

- Actually, LIGO got two birds in one stone !



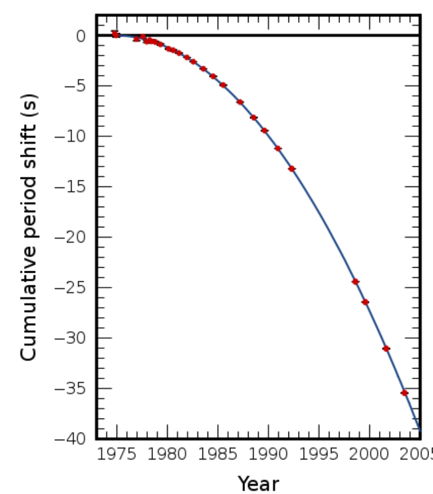


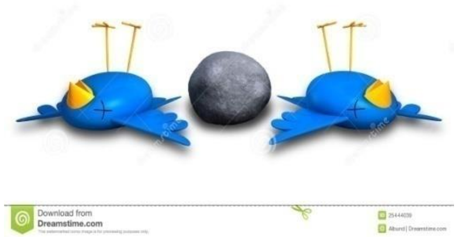
GW150914/151226

- 1. One is, of course, about the first, “direct” detection of gravitational waves.
- Cf. Indirect evidence was found in

Hulse-Taylor’s neutron star binary (1975-2005): Agrees with GW radiation in GR !

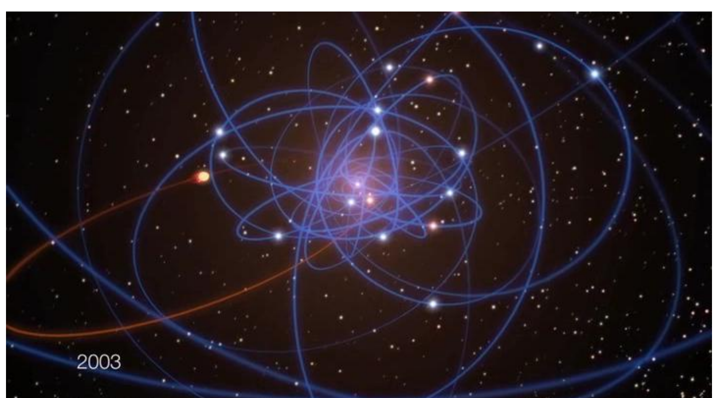
$$-\left\langle \frac{dE}{dt} \right\rangle = \frac{32G^4 m_1^2 m_2^2 (m_1 + m_2)}{5c^5 a^5 (1-e^2)^{7/2}} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right)$$





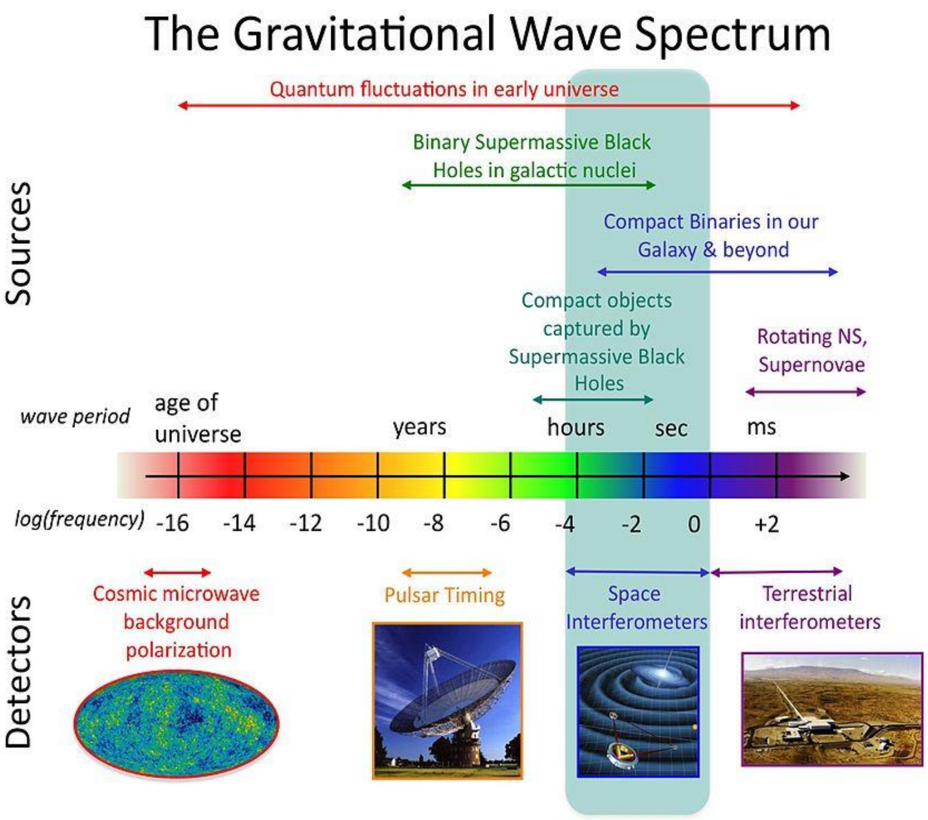
GW150914/151226

- 2. The other is about the first, "direct" detection of (spinning) black holes (if they are).
- Cf. Indirect evidences have been known for many years: supermassive black holes at the galaxy centers.



Messages of LIGO

- 1. We open **GW astronomy** era, beyond **EM Wave astronomy** !



Messages of LIGO

- 2. We open the strong gravity test era of GR, beyond the weak gravity tests in solar systems !!
- 2′. In particular, we open the new era of testing the black hole physics.
- Before LIGO's detection, black hole physics has been just an academic subject.
- Now, it is the time we should consider it more seriously !!

Now, we may ask that "do we understand BH completely from GR ?"

- Maybe NOT ! Why ??

GR predicts the existence of black holes.

- Black holes have the horizons which divides the casually connected two reasons.
- Usually, black hole has singularity inside horizons (Cosmic censorship conjecture).
- The metric outside a collapsed object settles down to Kerr sol. with (positive mass) m and angular momentum a (D=4, asymptotically flat): Uniqueness theorem; No hair theorem.

- When “ordinary” matters (i.e., positive energy density, pressure (weak energy condition), accretes to a black hole, its “horizon area A” increases: Not so strange !?
- When one black hole (BH1) falls into another black hole (BH2), the resulting horizon area (BH3) is greater than sum of BH1 and BH2 (Hawking’s area theorem):
- $A_3 > A_1 + A_2$
- This is quite non-trivial result !

- For example, for collision of two rotating black holes, this gives

$$m_3(m_3 + (m_3{}^2 - a_3{}^2)^{\frac{1}{2}}) > m_1(m_1 + (m_1{}^2 - a_1{}^2)^{\frac{1}{2}}) + m_2(m_2 + (m_2{}^2 - a_2{}^2)^{\frac{1}{2}}).$$

- If we consider a_1=a_2=0 for simplicity, the energy emitted in gravitational (or other forms of radiation if there is),
- m_1+m_2-m_3
- is limited by the efficiency (1-2^(-1/2)) ~30% (maximum for m_1=m_2) [Hawking, PRL 1971].

- With non-vanishing a_1, a_2, generally, the efficiency of radiation is limited by
- (1-2^(-3/2)) ~65%.[Hawking, CMP 1972; I don't know how to prove !]

- These all are based on GR.
- These seems to be consistent with GW150914, within the accuracy.
- But, ...

J. Bekenstein and S. Hawking found that

- Black holes satisfy the thermodynamics laws. For example,

- 1st Law: $$dE = \frac{\kappa}{8\pi}\, dA + \Omega\, dJ + \Phi\, dQ,$$

- 2nd Law: $$\frac{dA}{dt} \geq 0.$$

- This is similar to the usual thermodynamics of ordinary thermal systems with

$$T_{\rm H} = \frac{\kappa}{2\pi}. \qquad S_{\rm BH} = \frac{A}{4}.$$

,

There was no way to understand this result until

- Hawking found that BH radiates with the temperature,

$$T = \frac{\hbar c^3}{8\pi G M k_B} \quad \left(\approx \frac{1.227 \times 10^{23}\ \mathrm{kg}}{M}\ \mathrm{K} = 6.169 \times 10^{-8}\ \mathrm{K} \times \frac{\mathrm{M}_\odot}{M}\right)$$

- by considering “quantum fields” living on the black hole background (non-dynamical, classical): Hawking radiation.
- So, now black hole has entropy !!

$$S_{BH} = \frac{kA}{4\ell_P^2} \quad \ell_P = \sqrt{G\hbar/c^3}$$

- So, black holes are classical (exact) solutions of Einstein equations of GR.
- But, it satisfies a thermodynamics-like law which can not be understood at classical level, without quantum effects.
- This means that “the law is originated from classical solutions, but it is also a precursor of quantum theory of gravity (quantum gravity)!! “
- Actually, we do not know how to compute the black hole entropy ! This may be one strong motivation for studying quantum gravity.

Do we know about the quantum gravity, then ?

- Since Einstein's gravity theory can be considered as a field theory, we need to consider the quantization for gravity fields.
- But from usual experience in other field theories, like QED, Standard model, QCD, we need ``renomalizability" in order that divergences do not appear in physical observables and theory predictions can be compared with experiments.
- So, the better question may be…

Do we know about renormalizable gravity, then ?

- It is known that Einstein gravity can not be the renormalizable (quantum) gravity.
- It is known also that Einstein gravity with higher-curvatures may be the renormalizable but there are ghost problems !!??
- Recently(2009 Jan.), Horava gravity was proposed as a renormalizable quantum gravity without ghost problems.

Plan

1. Horava gravity: Introduction [arXiv: 0901.3775[PRD]].
2. Non-rotating black holes: Summary
3. Rotating black holes in D=2+1 Horava gravity.
4. Future directions and open problems.

1. Horava gravity: Introduction [arXiv: 0901.3775[PRD]].

Background:

- The string theory may be a promising candidate for quantum gravity.
- Yet, it is also a rather "large" theory with a huge landscape of the universe.
- Q: Is there any "smaller" framework for the quantum gravity ?
- For example, Yang-Mills theory is complete in QFT already (renormalizable; UV complete): String theory embedding is not necessary.

- Likewise, is there any (perturbatively) renormalizable quantum gravity theory ?
- This idea is, of course, very old, but this can not be realized in Einstein's gravity or its (relativistic) higher-derivative generalizations: There are ghosts, in addition to massless gravitons, and unitarity violation:

$$\frac{1}{k^2} + \frac{1}{k^2} G_N k^4 \frac{1}{k^2} + \frac{1}{k^2} G_N k^4 \frac{1}{k^2} G_N k^4 \frac{1}{k^2} + \ldots = \frac{1}{k^2 - G_N k^4} = \frac{1}{k^2} - \frac{1}{k^2 - 1/G_N}.$$

Massless gravitons

Ghosts (!)

- But, for anisotropic (mass) dimensions,

$$[\mathbf{x}] = -1, \qquad [t] = -z,$$

the propagator becomes(?)

$$\frac{1}{\omega^2 - \mathbf{k}^2 - G(\mathbf{k}^2)^z}$$

G: Dimensionless coupling

At high mom. k with (z>1), this expands as,

$$\frac{1}{\omega^2 - c^2\mathbf{k}^2 - G(\mathbf{k}^2)^z} = \frac{1}{\omega^2 - G(\mathbf{k}^2)^z} + \frac{1}{\omega^2 - G(\mathbf{k}^2)^z} c^2\mathbf{k}^2 \frac{1}{\omega^2 - G(\mathbf{k}^2)^z} + \cdots.$$

Improved UV divergences but no ghosts, i.e., no unitary problem.

- Whereas at low momentum k,

$$\frac{1}{\omega^2 - \mathbf{k}^2 - G(\mathbf{k}^2)^z} = \frac{1}{\omega^2 - \mathbf{k}^2} + \frac{1}{\omega^2 - \mathbf{k}^2} G(\mathbf{k}^2)^z \frac{1}{\omega - \mathbf{k}^2} + \cdots.$$

Flow to z=1

The Action Construction:

- Einstein-Hilbert action:

$$S_{EH} = \frac{1}{16\pi G_N} \int dx^4 \sqrt{-g^{(4)}} \left(R^{(4)} - 2\Lambda \right)$$

Lorentz invariant !

Lorentz scalars

$$= \frac{1}{16\pi G_N} \int d^4x \, \sqrt{g} \, N \left\{ \left(K_{ij} K^{ij} - K^2 \right) + R - 2\Lambda \right\}$$

$R^{(4)}$

in ADM decomposition

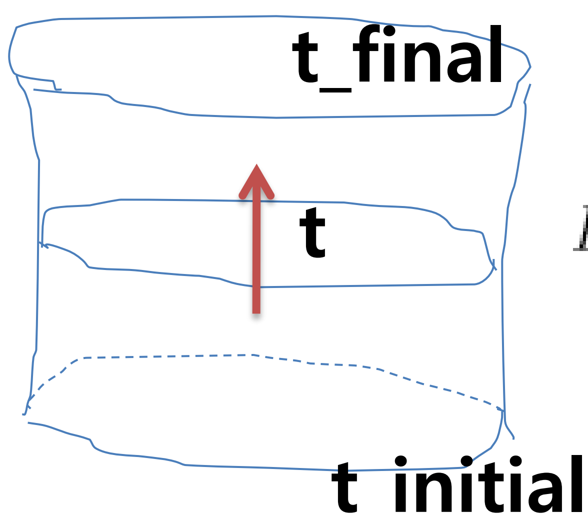
$$ds^2 = -N^2 c^2 dt^2 + g_{ij} \left(dx^i + N^i dt \right) \left(dx^j + N^j dt \right)$$

- **Here, we have used the Gauss-Godacci relation (up to boundary terms)**

$$R^{(4)} = (K_{ij}K^{ij} - K^2) + R$$

Extrinsic curvature of t=constant hypersurface

Intrinsic curvature : 3 curvature



$$K_{ij} = \frac{1}{2N}\left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$$

- In the anisotropic (momentum) dimensions,

$$[\mathbf{x}] = -1, \qquad [t] = -z,$$

we do not need to keep the Lorentz invariant combinations only. (Planck unit)

- For example, we may consider

$$\left(K_{ij}K^{ij} - \lambda K^2\right) + \beta R$$

, in which the Lorentz symmetry is explicitly broken for

$$\lambda \neq 1, \beta \neq 1$$

but there is still Foliation Preserving diffeomorphisms (FPDiff).

- However, in order not to introduce higher-time derivatives to avoid the ``possible” ghost problems, we do not consider “simply” the following terms

$$(K_{ij}K^{ij})^2, K^4 \cdots$$

but only consider

$$R^2,\ R_{ij}R^{ij},\ \nabla_k R_{ij}\nabla^k R^{ij},\ \ \cdots$$

So, the action can be written as

$$S_{\text{Horava}} = \frac{2}{\kappa^2}\int dt\, d^D\mathrm{x}\,\sqrt{g}N\left(K_{ij}K^{ij} - \lambda K^2\right)$$

Kinetic term

$$+ \int dt\, d^D\mathrm{x}\,\sqrt{g}N\, V[g_{ij}]$$

Potential term

Dimension counting

- For an arbitrary spatial dimension D,

$$[g_{ij}] = 0, \qquad [N_i] = z - 1, \qquad [N] = 0.$$

$$[dt\, d^D\mathbf{x}] = -D - z\,,$$

$$[\kappa] = \frac{z - D}{2}.$$

Dimensionless coupling for z=D:
Power counting renormalizable

$$S_V = \int dt\, d^D\mathbf{x}\, \sqrt{g} N\, V[g_{ij}]$$

(Planck unit)

-D-z

D+z

- So, in D=3 (3+1 spacetime), we need the potential V with [V]=6: 6'th-order spatial derivatives with "dimension-less" couplings !
- From

$$[\nabla_k] = [\nabla^k] = 1 \quad [R_{ij}] = [R^{ij}] = 2,$$

we have large numbers of possible terms, which are invariant by themselves, like

$$\nabla_k R_{ij} \nabla^k R^{ij}, \quad \nabla_k R_{ij} \nabla^i R^{jk}, \quad R\Delta R, \quad R^{ij}\Delta R_{ij};$$

$$R^3, \quad R^i_j R^j_k R^k_i, \quad R R_{ij} R^{ij},$$

In D=2 (2+1 dimensions), we need [V]=4: 4'th-order spatial derivatives with "dimension-less" couplings !

- There are too many couplings for explicit computations, though some of them may be constrained by the stability and unitarity. We need some pragmatic way of reducing the number of couplings, in a reliable manner.

- So, Horava adopted "detailed balance" condition from the critical phenomena.

- Horava **required** the potential to be of

$$S_V = \frac{\kappa^2}{8} \int dt\, d^D x \sqrt{g} N\, E^{ij} \mathcal{G}_{ijk\ell} E^{k\ell},$$

by **demanding**

$$\sqrt{g} E^{ij} = \frac{\delta W[g_{k\ell}]}{\delta g_{ij}}$$

D-dimensional Euclidean action

for some action W, and $\mathcal{G}_{ijk\ell}$, the inverse of **De Witt** metric

$$G^{ijk\ell} = \frac{1}{2}\left(g^{ik} g^{j\ell} + g^{i\ell} g^{jk}\right) - \lambda g^{ij} g^{k\ell}$$

Cf. **Kinetic** part is also given by

$$S = \frac{1}{2} \int dt\, d^D x \sqrt{g} \left\{ \frac{1}{\kappa^2 N} \left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right) G^{ijk\ell} \left(\dot{g}_{k\ell} - \nabla_k N_\ell - \nabla_\ell N_k\right)\right.$$

- For D=3, W is 3-dimensional Euclidean action.
- First, we may consider Einstein-Hilbert action,

$$W = \frac{1}{\kappa_W^2} \int d^D x \sqrt{g} (R - 2\Lambda_W).$$

then, this gives 4'th-order spatial derivative potential, with a dimensionful coupling,

$$S_V = \frac{\kappa^2}{8\kappa_W^4} \int dt\, d^D x \sqrt{g} N \left(R^{ij} - \frac{1}{2} R g^{ij} + \Lambda_W g^{ij} \right) \mathcal{G}_{ijk\ell} \left(R^{k\ell} - \frac{1}{2} R g^{k\ell} + \Lambda_W g^{k\ell} \right).$$

- So, this is not enough to get 6'th order !!

- In 3-dim, we also have a peculiar, 3'rd-derivative-order action, called (gravitational) Chern-Simons action.

$$W = \frac{1}{w^2} \int_\Sigma \omega_3(\Gamma).$$

$$\omega_3(\Gamma) = \mathrm{Tr}\left(\Gamma \wedge d\Gamma + \frac{2}{3}\Gamma \wedge \Gamma \wedge \Gamma\right) \equiv \varepsilon^{ijk}\left(\Gamma^m_{i\ell}\partial_j \Gamma^\ell_{km} + \frac{2}{3}\Gamma^n_{i\ell}\Gamma^\ell_{jm}\Gamma^m_{kn}\right) d^3\mathrm{x}$$

- This produces the potential

$$-\frac{\kappa^2}{2w^4} C_{ij} C^{ij}$$

with the Cotton tensor $C^{ij} = \varepsilon^{ik\ell}\nabla_k \left(R^j_\ell - \frac{1}{4} R \delta^j_\ell\right)$

- **Then, in total, he got the 6'th-order action**

$$S = \int dt\, d^3\mathrm{x}\, \sqrt{g}\, N \left\{ \frac{2}{\kappa^2} K_{ij} G^{ijk\ell} K_{k\ell} - \frac{\kappa^2}{2} \left[\frac{1}{w^2} C^{ij} - \frac{\mu}{2} \left(R^{ij} - \frac{1}{2} R g^{ij} + \Lambda_W g^{ij} \right) \right] \times \mathcal{G}_{ijk\ell} \left[\frac{1}{w^2} C^{k\ell} - \frac{\mu}{2} \left(R^{k\ell} - \frac{1}{2} R g^{k\ell} + \Lambda_W g^{k\ell} \right) \right] \right\}.$$

or

$$S = \int dt\, d^3\mathrm{x}\, \sqrt{g}\, N \left\{ \frac{2}{\kappa^2} \left(K_{ij} K^{ij} - \lambda K^2 \right) - \frac{\kappa^2}{2w^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu}{2w^2} \varepsilon^{ijk} R_{i\ell} \nabla_j R^{\ell}_{k} - \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} + \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \left(\frac{1-4\lambda}{4} R^2 + \Lambda_W R - 3\Lambda_W^2 \right) \right\}.$$

from

$$W = \frac{1}{w^2} \int \omega_3(\Gamma) + \mu \int d^3\mathrm{x}\, \sqrt{g} (R - 2\Lambda_W).$$

- **Some improved UV behaviors, without ghosts, are expected, i.e., renormalizability**

➡ **Predictable Quantum Gravity !!(?)**

- **But, it seems that the detailed balance condition is too strong to get general spacetimes with an arbitrary cosmological constant.**
- **For example, there is no Minkowski , i.e., vanishing c.c. vacuum solution ! (Lu, Mei, Pope): There is no Newtonian gravity limit !!**
- **We need to break the detailed balance but without altering UV behaviors: It is called, soft breaking in IR or IR modification.**

- A "soft" breaking of the detailed balance is given by the action :

$$S_g = \int dt d^3x \sqrt{g} N \left[\frac{2}{\kappa^2}\left(K_{ij}K^{ij} - \lambda K^2\right) - \frac{\kappa^2}{2\nu^4} C_{ij}C^{ij} + \frac{\kappa^2 \mu}{2\nu^2}\epsilon^{ijk} R^{(3)}_{i\ell} \nabla_j R^{(3)\ell}{}_k \right.$$
$$\left. - \frac{\kappa^2\mu^2}{8} R^{(3)}_{ij} R^{(3)ij} + \frac{\kappa^2\mu^2}{8(3\lambda-1)}\left(\frac{4\lambda-1}{4}(R^{(3)})^2 - \Lambda_W R^{(3)} + 3\Lambda_W^2\right) + \frac{\kappa^2\mu^2\omega}{8(3\lambda-1)} R^{(3)} \right]$$

IR modification term

- It is found that there does exit the black hole which converges to the usual Schwarzschild solution in Minkowski limit, i.e., $\Lambda_W \to 0$ for $\lambda = 1$ (s.t. Einstein-Hilbert in IR) (KS '09) .

2. Non-rotating black holes

- **The general action with IR-modification is**

$$S = \int dt d^3x \sqrt{g} N \left[\frac{2}{\kappa^2}\left(K_{ij}K^{ij} - \lambda K^2\right) - \frac{\kappa^2}{2\nu^4}C_{ij}C^{ij} + \frac{\kappa^2\mu}{2\nu^2}\epsilon^{ijk}R^{(3)}_{i\ell}\nabla_j R^{(3)\ell}{}_k \right.$$

$$\left. - \frac{\kappa^2\mu^2}{8}R^{(3)}_{ij}R^{(3)ij} + \frac{\kappa^2\mu^2}{8(3\lambda-1)}\left(\frac{4\lambda-1}{4}(R^{(3)})^2 - \Lambda_W R^{(3)} + 3\Lambda_W^2\right) + \frac{\kappa^2\mu^2\omega}{8(3\lambda-1)}R^{(3)}\right]$$

- **From the ansatz (N^i =0)**

$$ds^2 = -N(r)^2c^2dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

- **The equations of motion are**

$$(2\lambda-1)\frac{(f-1)^2}{r^2} - 2\lambda\frac{f-1}{r}f' + \frac{\lambda-1}{2}f'^2 - 2(\omega-\Lambda_W)(1-f-rf') - 3\Lambda_W^2 r^2 = 0 \ ,$$

$$\left(\frac{N}{\sqrt{f}}\right)'\left((\lambda-1)f' - 2\lambda\frac{f-1}{r} + 2(\omega-\Lambda_W)r\right) + (\lambda-1)\frac{N}{\sqrt{f}}\left(f'' - \frac{2(f-1)}{r^2}\right) = 0$$

- **Let's consider $\lambda = 1$, then I obtain**

$$N^2 = f = 1 + (\omega - \Lambda_W)r^2 - \sqrt{r[\omega(\omega - 2\Lambda_W)r^3 + \beta]}$$

- **For $\omega = 0$, this reduces to LMP's solution (with $\beta = -\alpha^2/\Lambda_W$)**

$$f = 1 - \Lambda_W r^2 - \frac{\alpha}{\sqrt{-\Lambda_W}}\sqrt{r}$$

- **For $\Lambda_W = 0$, this reduces to KS's solution (with $\beta = 4\omega M,$) with an arbitrary paremeter ω,**

$$f = 1 + \omega r^2 - \sqrt{r[\omega^2 r^3 + 4\omega M]}$$

- **Black hole solution for $\Lambda_W \to 0$ limit ($\lambda = 1$):**

$$ds^2 = -N(r)^2 c^2 dt^2 + \frac{dr^2}{f(r)} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$$N^2 = f = 1 + \omega r^2 - \sqrt{r[\omega^2 r^3 + 4\omega M]}$$

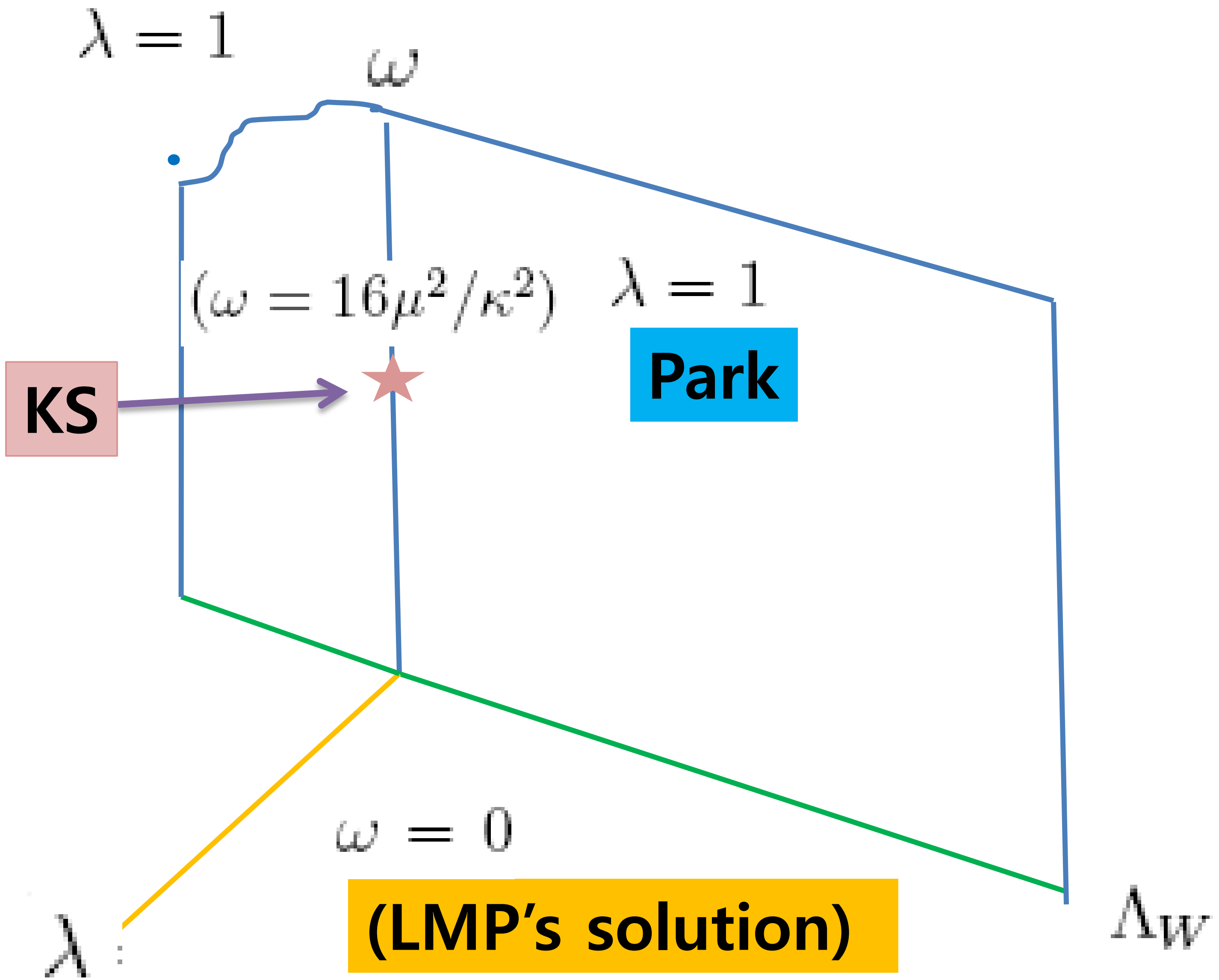
$$\approx 1 - \frac{2M}{r} + \mathcal{O}(r^{-4})$$

~ Schwarzshild Solution

: Independently of ω **!!**

$$(G = c \equiv 1)$$

- So, we obtained the general solution



Remarks

- There more general solutions with arbitrary $\lambda > 1/3$ which reduce to LMP's solution for $\omega = 0$. But, there is no "explicit", analytic solution but only in "implicit" forms. (See Kiritisis-Kofinas)

Other Known Exact Solutions (D=4)

(1) LMP-type:
- Topological (Charged) B.H.(Cai, Cao, Ohta),
- Dyonic (B.H.) Solution (Colgain, Yavartanoo),
- Slowly Rotating B.H. (Ghodsi;Aliev,Senturk)

(2) KS-type:
- Slowly Rotating B.H. (Lee,Kim,Myung)

Unknown Exact Solutions (D=4)

In IR-modified Horava gravity,

- Rotating Solution ?
- Topological (Charged) B.H. ? (Cao, Park)
- Black String ? (Cho, Kang !?)
- …

3. Rotating black holes in D=2+1 Horava gravity

- The D=2+1 renormalizable action is

$$S = \frac{1}{\kappa}\int dt d^2x \sqrt{g} N \left(K_{ij}K_{ij} - \lambda K^2 + \xi R + \alpha R^2 - 2\Lambda \right)$$

- From the ansatz

$$ds^2 = -N^2(r)c^2dt^2 + \frac{1}{f(r)}dr^2 + r^2\left(d\phi + N^\phi(r)dt\right)^2$$

Due to circular symmetry in 2-dim. space

Cf. In higher than D=2+1, this simplicity does not happen.

Cf. Renormalizability in 3D Lorentz invariant actions.

- Topologically massive gravity (TMG): (1) may be renormalizable but no proof beyond one-loop (Deser,Yang('90)) (2) no analogue in 4D.

- New massive gravity (NMG, BHT): ghost (unitarity) problem and renormalization are not compatable (Muneyuki, Ohta (2012))

The reduced action:

$$\mathcal{L} = \frac{2\pi}{\kappa}\frac{N}{\sqrt{f}}\left[\frac{fr^3\left(N^{\phi\prime}\right)^2}{2N^2} - \xi f' + \alpha\frac{f'^2}{r} - 2\Lambda r\right]$$

By varying N, N^{ϕ}, and f, **one can obtain the equations of motions.**

Eqs. of motions.

$$-\frac{fr^3(N^{\phi\prime})^2}{2N^2} - \xi f' + \alpha\frac{f'^2}{r} - 2\Lambda r = 0,$$

Hamiltonian Constraint

$$\left(\frac{\sqrt{f}}{N} r^3 N^{\phi\prime}\right)' = 0,$$

Momentum Constraint

$$\left(\frac{N}{\sqrt{f}}\right)'\left(2\alpha\frac{f'}{r} - \xi\right) + 2\alpha\frac{N}{\sqrt{f}}\left(\frac{f''}{r} - \frac{f'}{r^2}\right) = 0.$$

- These coupled Eqs. can be exactly solved !

Rotating Black Hole Solutions.

$$
\begin{aligned}
f &= -\bar{M} + \frac{br^2}{2}\left[1 - \left\{\sqrt{a + \frac{c}{r^4}} - \sqrt{\frac{c}{r^4}}\, ln\left(\sqrt{\frac{c}{ar^4}} + \sqrt{1 + \frac{c}{ar^4}}\right)\right\}\right], \\
\frac{N}{\sqrt{f}} &\equiv W = 1 - ln\sqrt{1 + \frac{c}{ar^4}}, \\
N^{\phi} &= -\frac{\bar{J}}{2r^2}\left(1 - ln\sqrt{1 + \frac{c}{ar^4}}\right) + \frac{\bar{J}}{2}\left[\sqrt{\frac{a}{c}}\, arctan\left(\sqrt{\frac{c}{ar^4}}\right) - \frac{1}{r^2}\right],
\end{aligned}
$$

- **with**

$$
\begin{aligned}
a &= 1 + \frac{8\alpha\Lambda}{\xi^2}, \\
b &= \frac{\xi}{2\alpha}, \\
c &= \frac{2\alpha \bar{J}^2}{\xi^2}.
\end{aligned}
$$

Two integration constants

Allowed ranges of parameters:

- **For the real-valued metric functions**

  f, N, and N^{ϕ}

  ⇒ $a, c \geq 0$;

- **or** $\frac{8\alpha\Lambda}{\xi^2} \geq -1$ $\alpha \geq 0$,

For large r and small α

$$f = -\frac{\Lambda}{\xi}r^2\left(1-\frac{2\alpha\Lambda}{\xi^2}\right) - \mathcal{M} + \frac{\mathcal{J}^2}{4r^2\xi}\left(1-\frac{4\alpha\Lambda}{\xi^2}\right) - \frac{\alpha\mathcal{J}^4}{24\xi^3}\frac{1}{r^6} + \mathcal{O}(\alpha^2, r^{-10}),$$

$$W = 1 - \frac{\alpha\mathcal{J}^2}{\xi^2}\frac{1}{r^4} + \mathcal{O}(\alpha^2, r^{-8}),$$

$$N^\phi = -\frac{\mathcal{J}}{2r^2} + \frac{\alpha\mathcal{J}^3}{6\xi^2}\frac{1}{r^6} + \mathcal{O}(\alpha^2, r^{-10}).$$

- **In the limit $\alpha \to 0$, the solution reduces to BTZ black hole sol (with $\xi = 1$) :**

$$N^2_{BTZ} = f_{BTZ} = -\frac{\Lambda}{\xi}r^2 - \mathcal{M} + \frac{\mathcal{J}^2}{4r^2\xi}, \; N^\phi_{BTZ} = -\frac{\mathcal{J}}{2r^2}.$$

Curvature Invariants:

$$R = -\frac{f'}{r} = -b\left(1 - \sqrt{a + \frac{c}{r^4}}\right)$$

$$K^{ij}K_{ij} = \frac{r^2}{2W^2}\left(N^{\phi'}\right)^2$$

$$= \frac{\mathcal{J}^2}{2r^4}\frac{\left(ln\sqrt{1+\frac{c}{ar^4}}\right)^2}{\left(1 - ln\sqrt{1+\frac{c}{ar^4}}\right)^2}$$

(Point) singularity at r=0.

Ring singularity at W=0, i.e.,

$$r_{ring} = \left(\frac{c}{a(e^2-1)}\right)^{1/4} \approx \left(0.1565\,\frac{c}{a}\right)^{1/4}$$

Cf. BTZ in Einstein gravity.

- No curvature singularity:

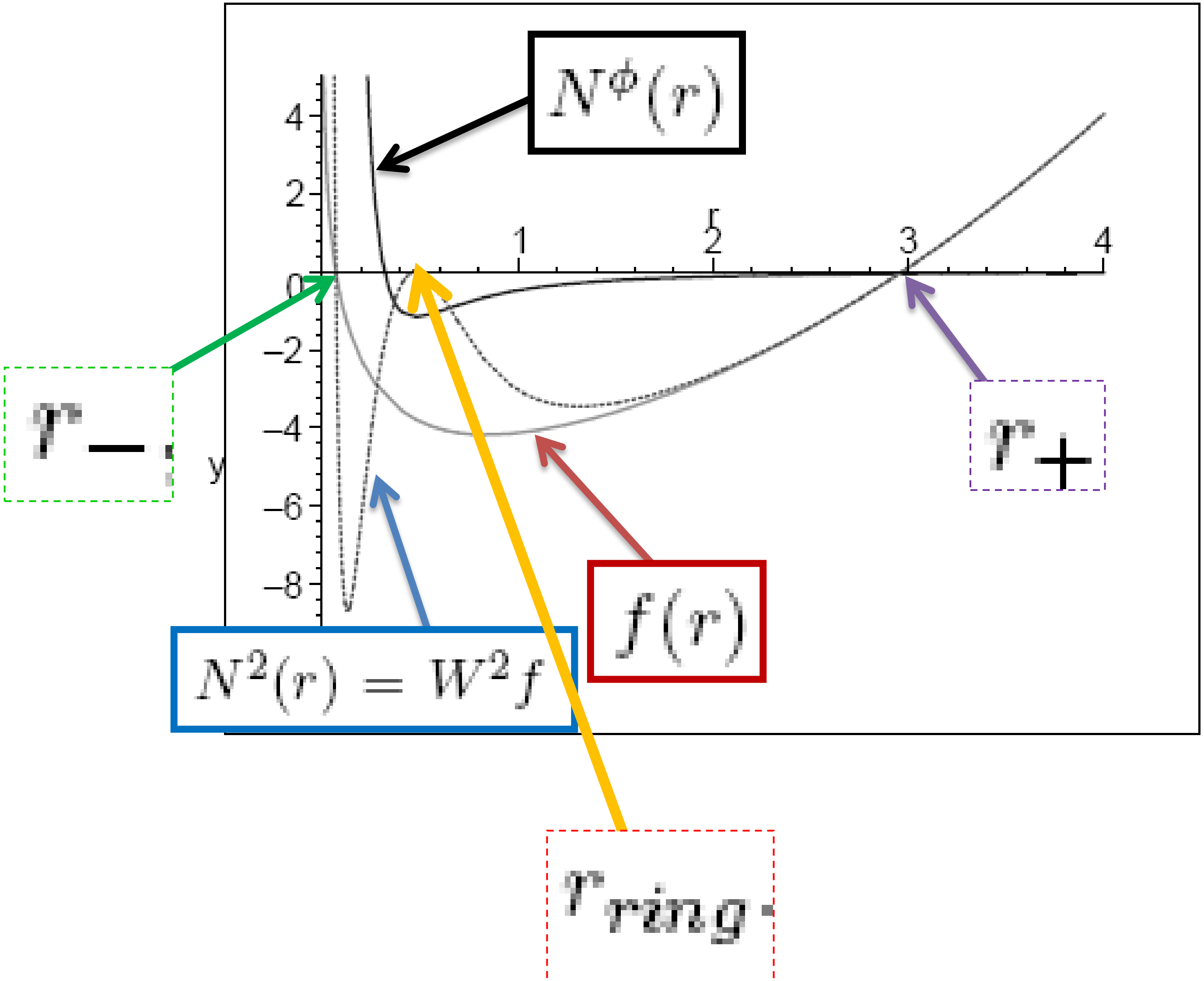
$$R^{(3)} = R + K^{ij}K_{ij} - K^2 \underbrace{- f'/r - f''}_{} = 6\Lambda.$$

Boundary term in Einstein action.

But, important in curvature invariants to cancel the unphysical singularity in R and $K^{ij}K_{ij}$!

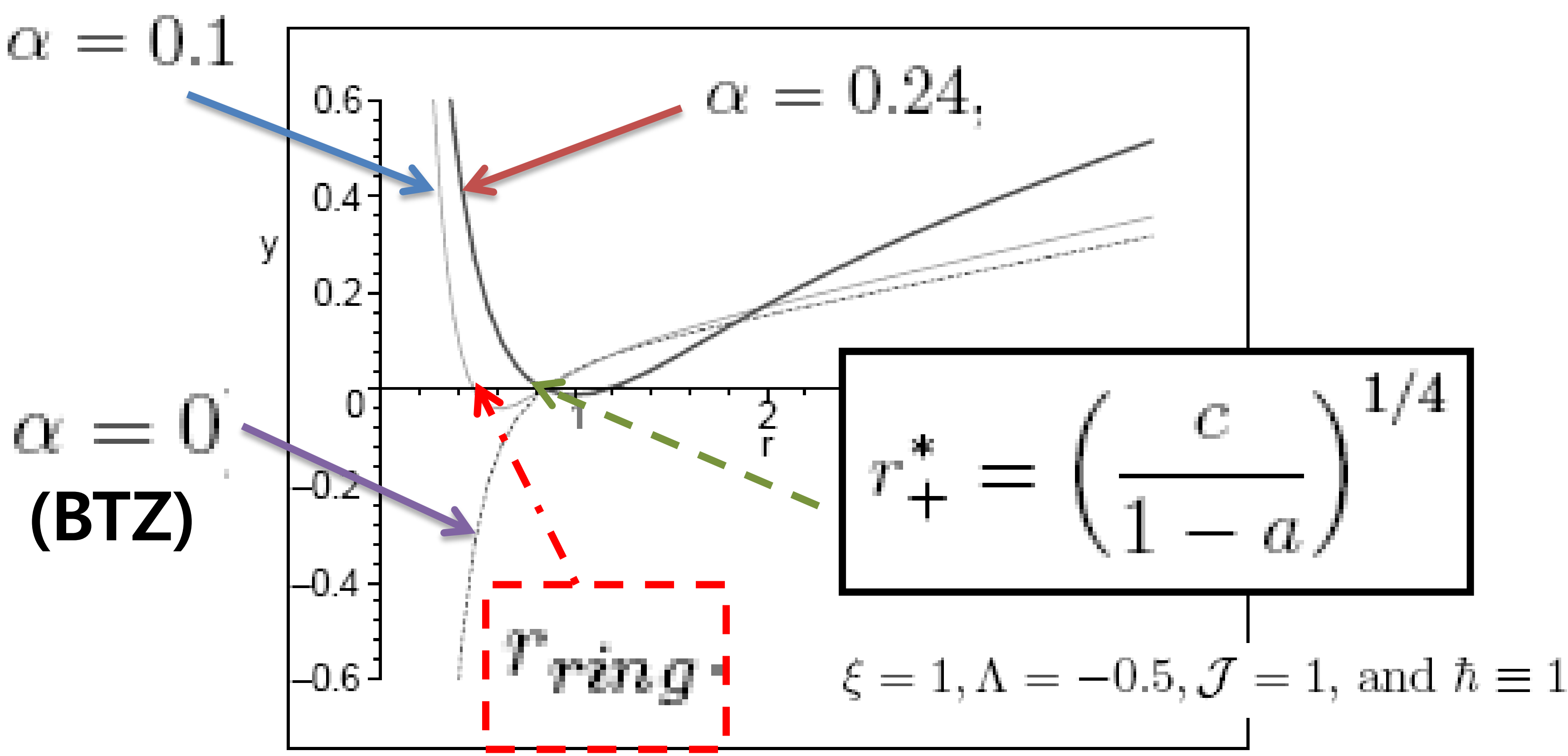
Horizon Structure

$(\xi = 1, \Lambda = -0.5, \mathcal{M} = 5, \mathcal{J} = 1, \alpha = 0.1)$



Hawking Temperature

$$T_+ = \frac{\hbar (W f')|_{r_+}}{4\pi}$$
$$= \frac{\hbar}{4\pi} b r_+ \left(1 - \sqrt{a + \frac{c}{r_+^4}}\right)\left(1 - ln\sqrt{1 + \frac{c}{a r_+^4}}\right)$$



Mass and Angular momentum

- Canonical mass and angular momentum are defined as the time translation and angular rotation generators at the infinite boundary, by the boundary action B

$$S_{total} = S + B,$$
$$B = (t_2 - t_1)(-W(\infty)M + N^{\phi}(\infty)J)$$

- Such that

$$\delta S_{total} = (EOM) + (\delta S)(\infty) + \delta B$$

$$M = \frac{2\pi\xi\sqrt{a}}{\kappa}\bar{M}, \quad J = \frac{2\pi\xi}{\kappa}\bar{J}.$$

Mass bounds (Energy theorem)

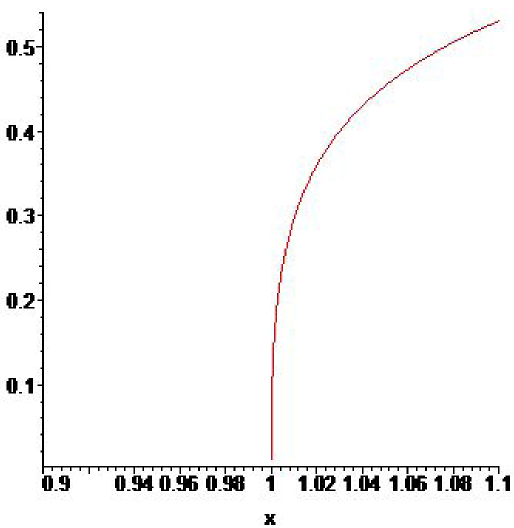
$$M \geq J\sqrt{-\Lambda}\,\chi(x),$$

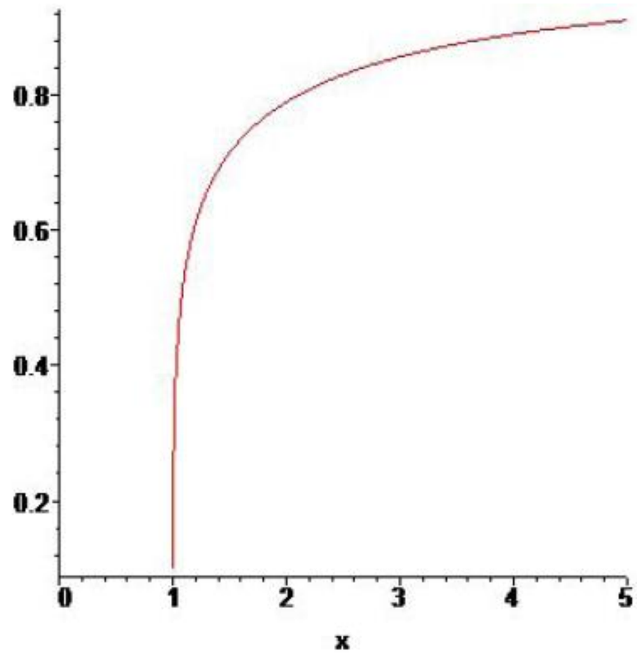
$$\chi(x) = \sqrt{x^2-1}\,ln\left(\frac{1}{\sqrt{x^2-1}}+1\right)$$

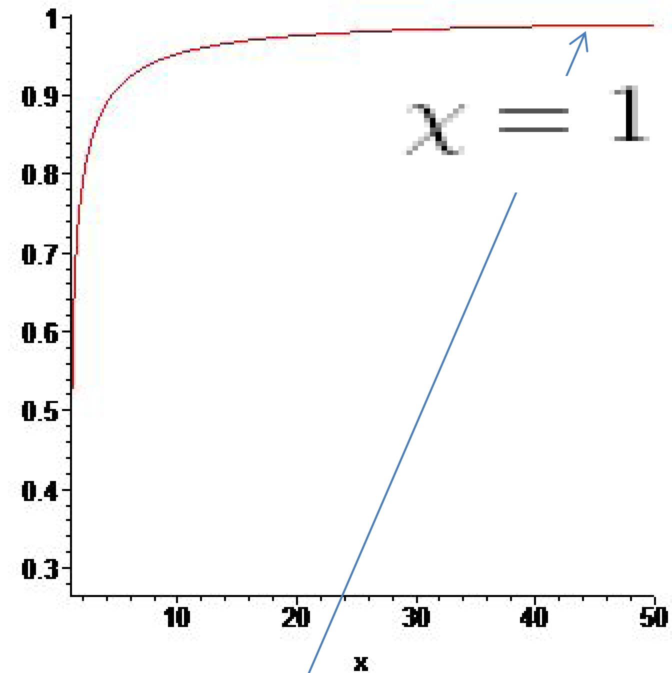
$$x = \xi/(\sqrt{-8\Lambda\alpha})$$

Mass Bounds work still for each x !!

$$\chi(x)$$







$$\chi = 1$$

$$M \geq J\sqrt{-\Lambda}$$

BTZ black hole

The first law of thermodynamics ?

- **Let's consider**

$$dM = AdJ + Bdr_+$$

- **With**

$$A = \frac{\kappa J}{4\pi\xi^2}\sqrt{\frac{a}{c}}\, ln\left(\sqrt{\frac{c}{ar_+^4}} + \sqrt{1 + \frac{c}{ar_+^4}}\right),$$

$$B = \frac{\pi\xi^2}{\kappa\alpha} r_+ \sqrt{a}\left(1 - \sqrt{a + \frac{c}{r_+^4}}\right).$$

The first law of thermodynamics ?

- But, one can **not** get the usual first law (!!)

$$dM = T_+ dS + \Omega_+ dJ$$

- with the usual **Hawking temperature** and the **chemical potential**

$$\begin{aligned} T_+ &= \frac{W(\partial f/\partial r)|_{r_+}}{4\pi} \\ &= \frac{1}{4\pi} br \left(1 - \sqrt{1 + \frac{c}{ar^4}}\right)\left(1 - ln\sqrt{1 + \frac{c}{ar^4}}\right) \end{aligned}$$

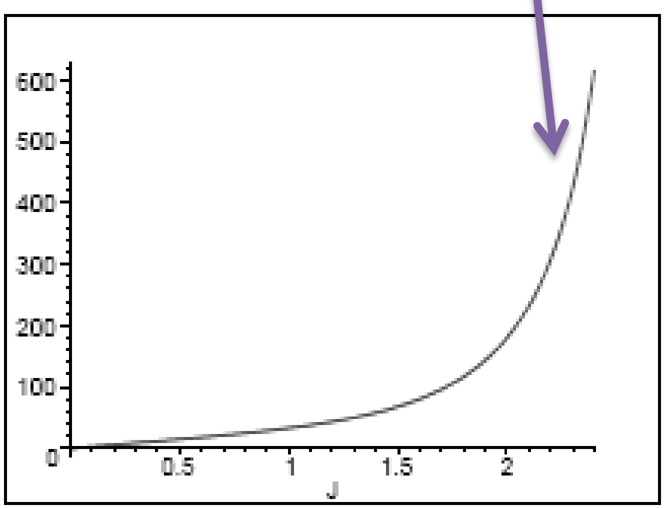
$$\Omega_+ = -N^\phi|_+$$

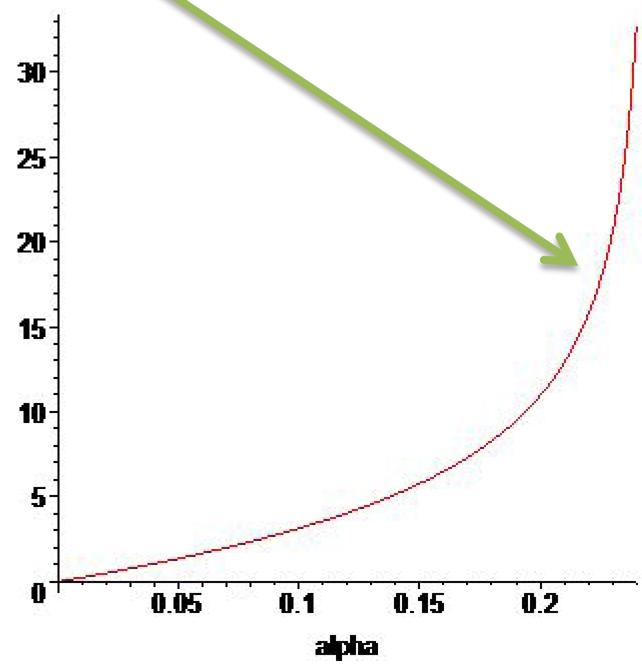
Non-integrability of Entropy.

$$dS \equiv \partial_{r_+} S dr_+ + \partial_J S dJ$$

$$\partial_J \partial_{r_+} S - \partial_{r_+} \partial_J S = \frac{16\pi^2 \bar{J}}{\kappa r_+^4} \alpha + O(\alpha^2).$$

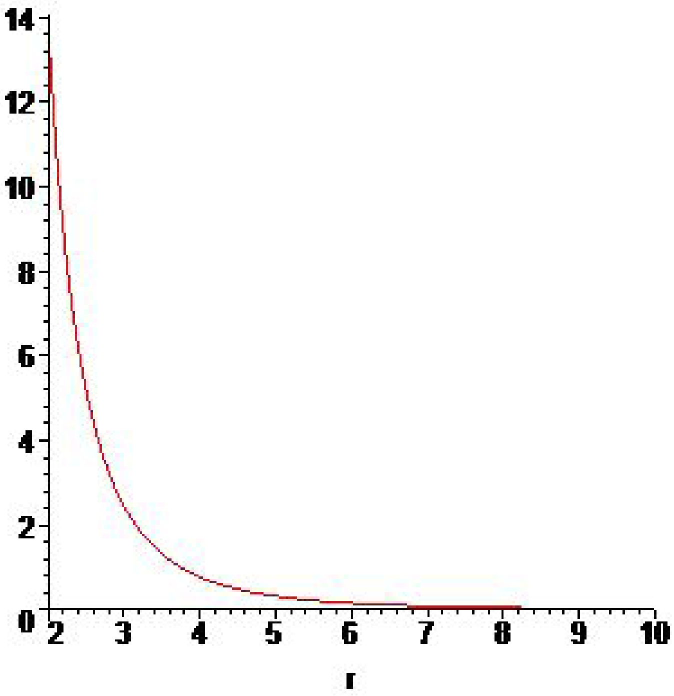
Entropy is not integrable by non-relativistic higher curvature corrections, in company with J.





Non-integrability of Entropy, cont'd

- Or equivalently, as the horizon radius changes



- the entropy is integrable asympotically only.

Summary of Properties

- 1. Ring curvature singularity: Similar to 4D Kerr in Einstein gravity.
- 2. The usual first law does not work: This seems to signal the absence of absolute horizon due to non-relativistic nature of Horava gravity in UV.
- 3. In the limit $\alpha \to 0$, the solution reduces to BTZ black hole:

$$N_{BTZ}^2 = f_{BTZ} = -\frac{\Lambda}{\xi}r^2 - \bar{M} + \frac{\bar{J}^2}{4r^2\xi}, \qquad N_{BTZ}^{\phi} = -\frac{\bar{J}}{2r^2}$$

Summary of Properties, cont'd

- 4. Mass bounds still work for each theory ($x = \xi/(\sqrt{-8\Lambda\alpha})$), but in a modified form.

Discussion

- The Hawking temperature implies the Hawking radiation: So we have "Hawking radiation without black hole entropy" !
- cf. Similar situation in analogue black hole (Visser('98))
- In our case, this seems to be a genuine effect of Lorentz-violating gravity.

4D rotating Horava black hole ?

- Work in progress…

4. Future directions and open problems.

- We need to know about "4D rotating black hole solutions" in order to compute something which can be compared with LIGO or future data.
- On theory side, we need to understand the "full" symmetry of Horava theory and how GR's Diff symmetry are recovered in IR.
- Also, we need to understand the concept of the horizons or universal horizons in our Lorentz violating gravity.

Future directions and open problems.

- We need the rigorous proof of renomalizability: This is in slow progress (3D, ...)
- Cf. Yang-Mills theory, Weinberg-Salam model, ...

Thank you !!