Holographic Dual of the Mass-deformed M2-brane Theory

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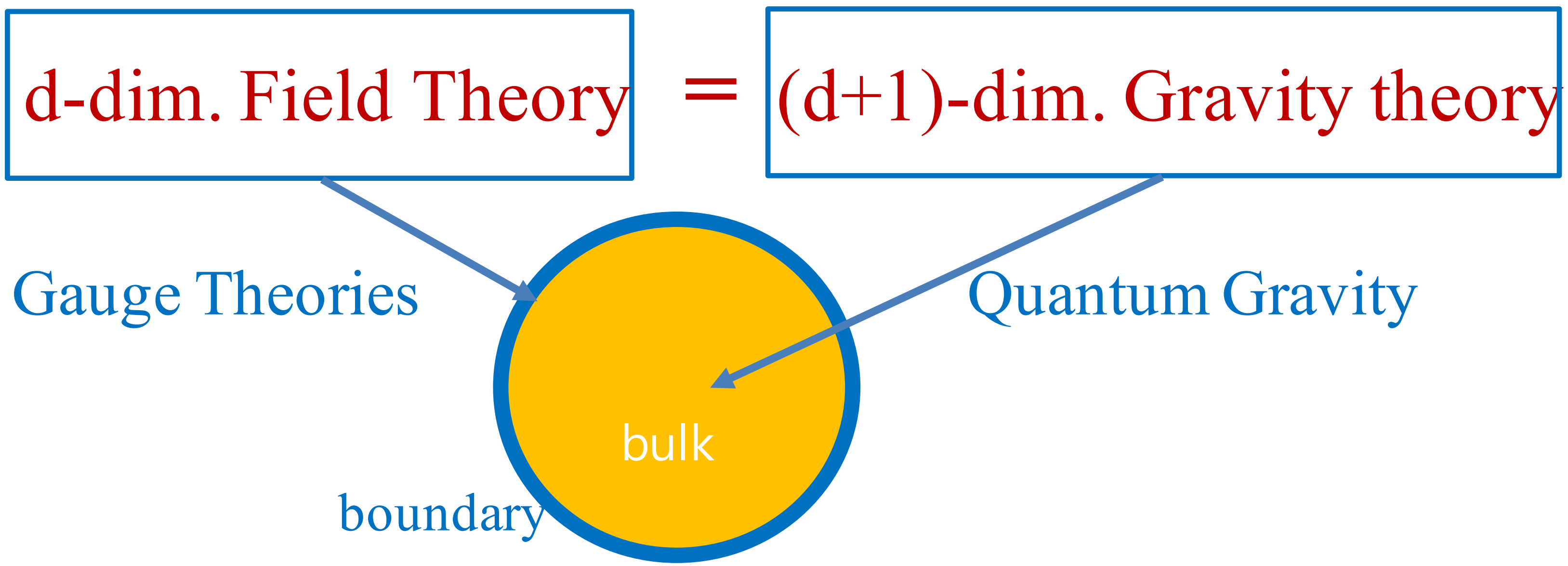
Outline

- Introduction and motivation
- Mass-deformed ABJM theory and LLM geometry
- KK reduction
- Exact holography for finite N
- Summary

Introduction and Motivation

- Gauge/gravity duality



- After the conjecture by Maldacena in 1997, there were many works in this direction, such as string theory, QCD, nulcear physics, condensed matter physics, cosmology, etc.

Introduction and Motivation

- AdS/CFT Conjecture by Maldacena:

We started with a quantum theory and saw that it includes gravity, so it is natural to think that this correspondence goes beyond the supergravity approximation. We are led to the conjecture that *type IIB string theory on $(AdS_5 \times S^5)_N$ plus some appropriate boundary conditions (and possibly also some boundary degrees of freedom) is dual to $\mathcal{N} = 4$, $d = 3 + 1$, $U(N)$ super-Yang–Mills.* The SYM coupling is given by the (complex) IIB string coupling; more precisely

$$\frac{1}{g_{\mathrm{YM}}^2} + i\frac{\theta}{8\pi^2} = \frac{1}{2\pi}\left(\frac{1}{g} + i\frac{\chi}{2\pi}\right)$$

where χ is the value of the RR scalar.

➔No restriction on N, g_{YM}

➔ to support this conjecture he considered the supergravity limit

Introduction and Motivation

- Three versions of conjectures:

1. N=4 SYM in 4-dim for all $N,\ g_{YM}$ is dual to full quantum IIB string theory on $AdS_5 \times S^5$

Strong conjecture!

$$\boxed{g_s = g_{YM}^2, \quad L^4 = 4\pi g_s N \alpha'^2}$$

2. 't Hooft limit of N=4 SYM is dual to classical type IIB string theory on $AdS_5 \times S^5$

$$\boxed{\lambda = g_{YM}^2 N = fixed, \quad N \to \infty}$$ planar limit

➔ 1/N expansion: leading order ➔ planar limit,
1/N correction ➔ non-planar diagram

➔ Perturbation for g_s

➔ String loop expansion

Introduction and Motivation

3. Large 't Hooft coupling limit of N=4 SYM is dual to type IIB supergravity on $AdS_5 \times S^5$

$$\boxed{\lambda = g_{YM}^2 N \gg 1 \qquad N \to \infty}$$

weak conjecture!

$$L^4 = 4\pi g_s N \alpha'^2 = 4\pi \lambda \alpha'^2$$

➔Expansion α'
➔Higher derivative terms are neglected

- Other examples:
  - **N=6 ABAJM theory** is dual to M-theory on $\mathbf{AdS}_4 \times \mathbf{S}^7 / \mathrm{Z}_k$
  - deformations of N=4 SYM and N=6 ABJM and some other cases

Introduction and Motivation

- Duality properties of field theory and gravity theory

Quantum field theory	duality ⟺	Quantum gravity
Strongly coupled gauge theory	Large N ⟺	Weakly curved gravity (classical gravity)

- Very useful but difficult to check the duality!
  For some BPS objects which have no quantum corrections, it was possible to check the duality using supersymmetry and conformal symmetry **in the large N limit.**

Introduction and Motivation

- Duality properties of field theory and gravity theory

Quantum field theory	$\xleftrightarrow{\text{duality}}$	Quantum gravity
Strongly coupled gauge theory	$\xleftrightarrow{\text{Large N}}$	Weakly curved gravity (classical gravity)

- To test Maldacena's conjecture, **large N is necessary.**
- Is Gauge/gravity duality conjecture valid **for finite N?**
- Is there some example to see **the duality for finite N?**

Results: exact duality for finite N

- one example: **mass-deformed ABJM theory** and 11d gravity on **the LLM geometry**

- **Exact correspondence for finite N** between the vevs of the chiral primary operator (CPO) with dimension one in the field theory side and asymptotic coefficients in the LLM geometry

  using holographic renormalization method.

- We checked for **all possible supersymmetric vacua** in field theory side and **all LLM geometries** in gravity side.

  ➔ infinity examples!!

Discrete Higgs vacua of mABJM

- N=6 Aharony-Bergman-Jafferis-Maldacena(ABJM) theory : low energy effective action of N coincident M2-branes on the C^4/Z_k orbifold fixed point

***N* =6 ABJM**

$\xrightarrow[k \ll N^{\frac{1}{5}}]{\text{dual gravity(large N)}}$ **M-theory on** $\mathrm{AdS}_4 \times \mathrm{S}^7/\mathrm{Z}_k$

$\xrightarrow[N^{\frac{1}{5}} \ll k \ll N]{\text{dual gravity(large N)}}$ **IIA theory on** $\mathrm{AdS}_4 \times \mathbb{CP}^3$

Still large N is necessary!!

$\downarrow$ SUSY preserving mass deformation

$\boxed{\textbf{\textit{N} =6 mABJM}}$

[Hosomichi et al 08]
[Gomis et al 08]

➔ Effective action of M2-branes in the presence of the transverse self-dual 4-form flux

Discrete Higgs vacua of mABJM

- N=6 Aharony-Bergman-Jafferis-Maldacena(ABJM) theory : low energy effective action of N coincident M2-branes on the C^4/Z_k orbifold fixed point

***N* =6 ABJM**

dual gravity(large N) →

$k \ll N^{\frac{1}{5}}$

M-theory on $\mathrm{AdS}_4 \times \mathrm{S}^7/\mathrm{Z}_k$

dual gravity(large N) →

$N^{\frac{1}{5}} \ll k \ll N$

IIA theory on $\mathrm{AdS}_4 \times \mathbb{CP}^3$

Still large N is necessary!!

SUSY preserving mass deformation

↓

***N* =6 mABJM**

[Hosomichi et al 08]
[Gomis et al 08]

➔ Mass deformation ➔ breakdown of conformal symmetry

Discrete Higgs vacua of mABJM

- Original ABJM theory ➔ SU(4) global symmetry
- Mass-deformation: SU(4) → SU(2)XSU(2)XU(1)

$$Y^a = (Z^a, W^{\dagger a}), \quad \Psi_a = (\epsilon_{ab}\xi^b, -\epsilon_{ab}\omega^{\dagger b})$$

- Two gauge fields A_μ and $\hat{A}_\mu$
- Matter fields are in the bifundamental representation

Discrete Higgs vacua of mABJM

- Classical supersymmetric vacuum equation

$$S = \int dx^3 (\mathcal{L}_0 + \mathcal{L}_{\rm CS} + \mathcal{L}_{\rm ferm} + \mathcal{L}_{\rm bos})$$

$$\begin{aligned}\mathcal{L}_{\rm bos} = & -\left|\frac{2\pi}{k}(Z^b Z_b^\dagger Z^a - Z^a Z_b^\dagger Z^b + Z^a W_b W^{\dagger b} - W^{\dagger b} W_b Z^a) - \mu Z^a\right|^2 \\ & -\left|\frac{2\pi}{k}(Z_b^\dagger Z^b W_a - W_a Z^b Z_b^\dagger + W_a W^{\dagger b} W_b - W_b W^{\dagger b} W_a) + \mu W_a\right|^2 \\ & -\left|\frac{4\pi}{k}(W^{\dagger b} Z_b^\dagger W^{\dagger a} - W^{\dagger a} Z_b^\dagger W^{\dagger b})\right|^2 - \left|\frac{4\pi}{k}(Z_a^\dagger W^{\dagger b} Z_b^\dagger - Z_b^\dagger W_b^{\dagger b} Z_a^\dagger)\right|^2,\end{aligned}$$

$$Y^A = (Z^a, W^{\dagger a}) \qquad A = 1,2,3,4$$

$$a = 1,2.$$

Discrete Higgs vacua of mABJM

- Classical supersymmetric vacuum equation

$$S = \int dx^3 (\mathcal{L}_0 + \mathcal{L}_{\mathrm{CS}} + \mathcal{L}_{\mathrm{ferm}} + \mathcal{L}_{\mathrm{bos}})$$

$$\begin{aligned}\mathcal{L}_{\mathrm{bos}} = &- \left| \frac{2\pi}{k} \left(\boxed{Z^b Z_b^\dagger Z^a} - \boxed{Z^a Z_b^\dagger Z^b} + \cancel{Z^a W_b W^{\dagger b}} - \cancel{W^{\dagger b} W_b Z^a} \right) - \boxed{\mu Z^a} \right|^2 \\ &- \left| \frac{2\pi}{k} \left(\cancel{Z_b^\dagger Z^b W_a} - \cancel{W_a Z^b Z_b^\dagger} + \boxed{W_a W^{\dagger b} W_b} - \boxed{W_b W^{\dagger b} W_a} \right) + \boxed{\mu W_a} \right|^2 \\ &- \left| \frac{4\pi}{k} \left(\cancel{W^{\dagger b} Z_b^\dagger W^{\dagger a}} - \cancel{W^{\dagger a} Z_b^\dagger W^{\dagger b}} \right) \right|^2 - \left| \frac{4\pi}{k} \left(\cancel{Z_a^\dagger W^{\dagger b} Z_b^\dagger} - \cancel{Z_b^\dagger W^{\dagger b} Z_a^\dagger} \right) \right|^2 ,\end{aligned}$$

$$\boxed{Z^a W_b = 0}$$

Discrete Higgs vacua of mABJM

- Classical supersymmetric vacuum equation

$$\boxed{Z^a W_b = 0}$$

$$S = \int dx^3 (\mathcal{L}_0 + \mathcal{L}_{\mathrm{CS}} + \mathcal{L}_{\mathrm{ferm}} + \mathcal{L}_{\mathrm{bos}})$$

$$Z^a Z_b^\dagger Z^b - Z^b Z_b^\dagger Z^a = -\frac{\mu k}{2\pi} Z^a$$

$$W^{\dagger a} W_b W^{\dagger b} - W^{\dagger b} W_b W^{\dagger a} = \frac{\mu k}{2\pi} W^{\dagger a}$$

$\downarrow$

Discrete vacuum solutions (GRVV matrix)

Discrete Higgs vacua of mABJM

$$Z^a = \sqrt{\frac{\mu k}{2\pi}} \begin{pmatrix} \mathcal{M}_a^{(n_1)} & & & & & \\ & \ddots & & & & \\ & & \mathcal{M}_a^{(n_i)} & & & \\ & & & \mathbf{0}_{(n_{i+1}+1)\times n_{i+1}} & & \\ & & & & \ddots & \\ & & & & & \mathbf{0}_{(n_f+1)\times n_f} \end{pmatrix}$$

$$W^{\dagger a} = \sqrt{\frac{\mu k}{2\pi}} \begin{pmatrix} \mathbf{0}_{n_1\times(n_1+1)} & & & & & \\ & \ddots & & & & \\ & & \mathbf{0}_{n_i\times(n_i+1)} & & & \\ & & & \bar{\mathcal{M}}_a^{(n_{i+1})} & & \\ & & & & \ddots & \\ & & & & & \bar{\mathcal{M}}_a^{(n_f)} \end{pmatrix}$$

$$\mathcal{M}_1^{(n)} = \begin{pmatrix} \sqrt{n} & 0 & & & & \\ & \sqrt{n-1} & 0 & & & \\ & & \ddots & \ddots & & \\ & & & \sqrt{2} & 0 & \\ & & & & 1 & 0 \end{pmatrix}, \qquad \mathcal{M}_2^{(n)} = \begin{pmatrix} 0 & 1 & & & & \\ & 0 & \sqrt{2} & & & \\ & & \ddots & \ddots & & \\ & & & 0 & \sqrt{n-1} & \\ & & & & 0 & \sqrt{n} \end{pmatrix}$$

Discrete Higgs vacua of mABJM

$n_i \times (n_i+1)$ **matrices**

$$Z^a = \sqrt{\frac{\mu k}{2\pi}} \begin{pmatrix} \boxed{\begin{matrix} \mathcal{M}_a^{(n_1)} & & \\ & \ddots & \\ & & \mathcal{M}_a^{(n_i)} \end{matrix}} & & & \\ & \mathbf{0}_{(n_{i+1}+1)\times n_{i+1}} & & \\ & & \ddots & \\ & & & \mathbf{0}_{(n_f+1)\times n_f} \end{pmatrix}$$

$(n_i+1) \times n_i$ **matrices**

$$W^{\dagger a} = \sqrt{\frac{\mu k}{2\pi}} \begin{pmatrix} \mathbf{0}_{n_1\times(n_1+1)} & & & \\ & \ddots & & \\ & & \mathbf{0}_{n_i\times(n_i+1)} & \\ & & & \boxed{\begin{matrix} \bar{\mathcal{M}}_a^{(n_{i+1})} & & \\ & \ddots & \\ & & \bar{\mathcal{M}}_a^{(n_f)} \end{matrix}} \end{pmatrix}$$

Block-diagonal *N* X *N* matrices

Discrete Higgs vacua of mABJM

$n_i \times (n_i+1)$ **matrices**

$$Z^a = \sqrt{\frac{\mu k}{2\pi}} \begin{pmatrix} \mathcal{M}_a^{(n_1)} & & & & & \\ & \ddots & & & & \\ & & \mathcal{M}_a^{(n_i)} & & & \\ & & & \mathbf{0}_{(n_{i+1}+1)\times n_{i+1}} & & \\ & & & & \ddots & \\ & & & & & \mathbf{0}_{(n_f+1)\times n_f} \end{pmatrix}$$

$(n_i+1) \times n_i$ **matrices**

$$W^{\dagger a} = \sqrt{\frac{\mu k}{2\pi}} \begin{pmatrix} \mathbf{0}_{n_1\times(n_1+1)} & & & & & \\ & \ddots & & & & \\ & & \mathbf{0}_{n_i\times(n_i+1)} & & & \\ & & & \bar{\mathcal{M}}_a^{(n_{i+1})} & & \\ & & & & \ddots & \\ & & & & & \bar{\mathcal{M}}_a^{(n_f)} \end{pmatrix}$$

- Constraint: $\sum_{n=0}^{N-1} [nN_n + (n+1)N'_n] = N, \qquad \sum_{n=0}^{N-1} [(n+1)N_n + nN'_n] = N$
- Condition for supersymmetric vacua: $\boxed{0 \le N_n \le k, \qquad 0 \le N'_n \le k}$

[Kim-Kim 10]

Discrete Higgs vacua of mABJM

Vacua are classified by occupation numbers

$$\{N_n, N'_n\}$$

Number of vacua for a given N (k=1)= partition of N

P(N)

For large N, P(N) $\sim e^{\pi\sqrt{\frac{2}{3}}\sqrt{N}}$

LLM geometry in 11-dim. SUGRA

- Half-BPS solutions with SO(2,1)XSO(4)XSO(4) isometry in 11-dimensional supergravity
  [04, Lin-Lunin-Maldacena]

$$ds^2 = -G_{tt}\left(-dt^2 + dw_1^2 + dw_2^2\right) + G_{xx}(dx^2 + dy^2) + G_{\theta\theta} ds^2_{S^3/\mathbb{Z}_k} + G_{\tilde{\theta}\tilde{\theta}} ds^2_{\tilde{S}^3/\mathbb{Z}_k}$$

$$-G_{tt} = \left(\frac{4\mu_0^2 y\sqrt{\frac{1}{4} - z^2}}{f^2}\right)^{2/3},$$

$$G_{xx} = \left(\frac{f\sqrt{\frac{1}{4} - z^2}}{2\mu_0 y^2}\right)^{2/3},$$

$$G_{\theta\theta} = \left(\frac{fy\sqrt{\frac{1}{2} + z}}{2\mu_0\left(\frac{1}{2} - z\right)}\right)^{2/3},$$

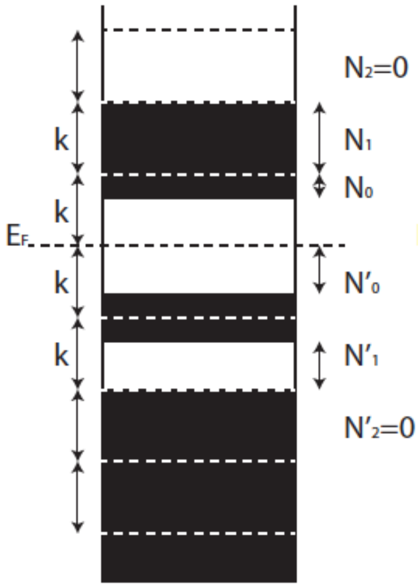
$$G_{\tilde{\theta}\tilde{\theta}} = \left(\frac{fy\sqrt{\frac{1}{2} - z}}{2\mu_0\left(\frac{1}{2} + z\right)}\right)^{2/3}.$$

$$\boxed{f(x,y) = \sqrt{1 - 4z^2 - 4y^2V^2}}$$

LLM geometry in 11-dim. SUGRA

- This solution is completely determined by two functions:

$$z(x,y)=\sum_{i=1}^{2m+1}\frac{(-1)^{i+1}(x-x_i)}{2\sqrt{(x-x_i)^2+y^2}},\qquad V(x,y)=\sum_{i=1}^{2m+1}\frac{(-1)^{i+1}}{2\sqrt{(x-x_i)^2+y^2}}.$$



$y = 0\,\text{case}$

$\boxed{z=\frac{1}{2}}$ or $\boxed{z=-\frac{1}{2}}$

white strip   black strip

Vacuum is identified by the occupation numbers :

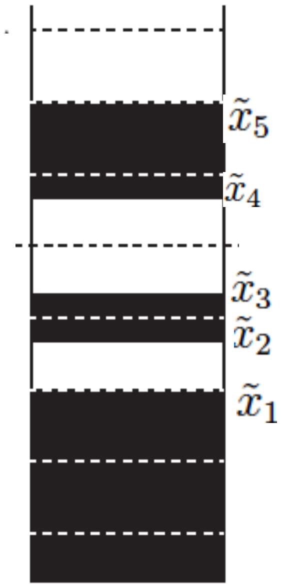
$$\{N_n, N'_n\}$$

[Cheon-Kim-Kim 2011]

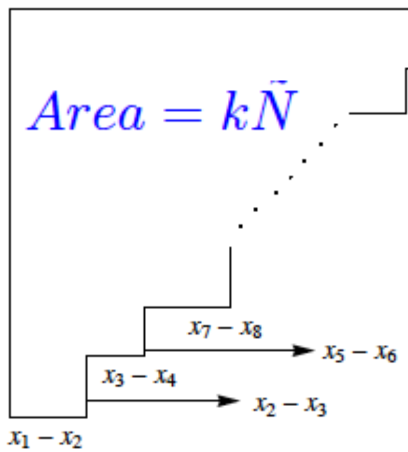
LLM geometry in 11-dim. SUGRA

- This solution is completely determined by two functions:

$$z(x,y) = \sum_{i=1}^{2m+1} \frac{(-1)^{i+1}(x-x_i)}{2\sqrt{(x-x_i)^2+y^2}}, \qquad V(x,y) = \sum_{i=1}^{2m+1} \frac{(-1)^{i+1}}{2\sqrt{(x-x_i)^2+y^2}}$$



droplet



Young diagram

$$C_p = \sum_{i=1}^{2N_B+1} (-1)^{i+1} \left(\frac{\tilde{x}_i}{2\pi l_P^3 \mu_0 \sqrt{k\tilde{N}}} \right)^p$$

$$C_2 - C_1^2 = 2$$

$$\tilde{x}_i \rightarrow C_p$$

new parametrization

LLM geometry in 11-dim. SUGRA

- Known results up to now:
  - LLM geometry can be dual to the massive M2-brane theory
    [Lin-Lunin-Maldacena, 2004]
  - **one-to-one correspondence** between vacua of the mABJM theory and the LLM geometry
    [Cheon-Kim-Kim 2011]
- To show the dual relation of two theories, correspondence between vacua of QFT and BPS solutions in gravity is not enough.
  - **mABJM theory: 3-dim. gauge theory**
  - **LLM geometry: BPS solution in 11-dim. SUGRA**

$$\boxed{\text{11-dim. SUGRA}} \xrightarrow{\text{KK-reduction}} \boxed{\textbf{4-dim. Gravity theory}}$$

Kaluza-Klein Reduction

- Equation of motion for g_{pq} and C_{pqr} :

$$R_{pq} - \frac{1}{2} g_{pq} R = \frac{1}{48} \Big(- \frac{1}{2} g_{pq} F_{rstu} F^{rstu} + 4 F_{pstu} F_q{}^{stu} \Big)$$

$$\partial_p (F^{pqrs}) + \frac{e}{2 \cdot (4!)^2} \epsilon^{qrsu_1 \cdots u_4 v_1 \cdots v_4} F_{u_1 \cdots u_4} F_{v_1 \cdots v_4} = 0 \qquad e = \sqrt{-g}$$

- Fluctuations around $\mathrm{AdS}_4 \times S^7$ (k=1)

$$\boxed{\begin{aligned} g_{pq} &= g^0_{pq} + h_{pq}, \\ C_{pqr} &= C^0_{pqr} + c_{pqr} \quad \Longleftrightarrow \quad F_{pqrs} = F^0_{pqrs} + f_{pqrs} \end{aligned}}$$

- LLM geometry is asymptotically $\mathrm{AdS}_4 \times S^7$ + fluctuations
- From the asymptotic expansion of the LLM geometry, we can read all h_{pq} and f_{pqrs}

Kaluza-Klein Reduction

- Expansion in S^7 spherical harmonics:

$$h_{\mu\nu}(x,y) = h^{I_1}_{\mu\nu}(x)Y^{I_1}(y),$$

$$h_{\mu a}(x,y) = V^{I_7}_{\mu}(x)Y^{I_7}_{a}(y) + S^{I_1}_{\mu}(x)\nabla_a Y^{I_1}(y),$$

$$h_{(ab)}(x,y) = T^{I_{27}}(x)Y^{I_{27}}_{(ab)}(y) + V^{I_7}(x)\nabla_{(a}Y^{I_7}_{b)}(y) + S^{I_1}(x)\nabla_{(a}\nabla_{b)}Y^{I_1}(y)$$

$$h^a{}_a(x,y) = \phi^{I_1}(x)Y^{I_1}(y),$$

$$\boxed{T_{(ab)} = \frac{1}{2}(T_{ab} + T_{ba}) - \frac{1}{7}g_{ab}T^c{}_c}$$

$$f_{\mu\nu\rho\sigma}(x,y) = 4\nabla_{[\mu}\tilde{S}^{I_1}_{\nu\rho\sigma]}(x)Y^{I_1}(y),$$

$$f_{\mu\nu\rho a}(x,y) = 3\nabla_{[\mu}\tilde{V}^{I_7}_{\nu\rho]}(x)Y^{I_7}_{a}(y) + \left[3\nabla_{[\mu}\tilde{S}^{I_1}_{\nu\rho]}(x) - \tilde{S}^{I_1}_{\mu\nu\rho}(x)\right]\nabla_a Y^{I_1}(y)$$

$$f_{\mu\nu ab}(x,y) = 2\nabla_{[\mu}\tilde{T}^{I_{21}}_{\nu]}(x)Y^{I_{21}}_{[ab]}(y) + 2\left[\nabla_{[\mu}\tilde{V}^{I_7}_{\nu]}(x) + \tilde{V}^{I_7}_{\mu\nu}(x)\right]\nabla_{[a}Y^{I_7}_{b]}(y)$$

$$f_{\mu abc}(x,y) = \nabla_{\mu}\tilde{T}^{I_{35}}(x)Y^{I_{35}}_{[abc]}(y) + \left[\nabla_{\mu}\tilde{T}^{I_{21}}(x) - 3\tilde{T}^{I_{21}}_{\mu}(x)\right]\nabla_{[a}Y^{I_{21}}_{bc]}(y)$$

$$f_{abcd}(x,y) = 4\tilde{T}^{I_{35}}(x)\nabla_{[a}Y^{I_{35}}_{bcd]}(y),$$

[H.J. Kim et al 1985, S.Lee et al 1998
Skenderis-Taylor 2006,...]

Kaluza-Klein Reduction

$$\Big[\Big(2\Box + 3\Lambda^{L_1} - \frac{42}{L^2} + \frac{F^2}{48}\Big)h^{L_1} - 2\nabla^\mu\nabla^\nu h^{L_1}_{\mu\nu} + \frac{1}{6}h^{L_1}_{\mu\nu}F^\mu{}_{\rho\sigma\lambda}F^{\nu\rho\sigma\lambda} - 6\Lambda^{L_1}\nabla^\mu S^{L_1}_\mu$$
$$+ 3\Big(\Box + \frac{8}{7}\Lambda^{L_1} + \frac{8}{L^2}\Big)\phi^{L_1} - 24\Lambda^{L_1}\Big(\frac{1}{7}\Lambda^{L_1} + \frac{1}{L^2}\Big)S^{L_1} - \frac{2}{3}\nabla_{[\mu}\tilde{S}^{L_1}_{\nu\rho\sigma]}F^{\mu\nu\rho\sigma}\Big] = 0$$

$$\Big[\Big(-\Box - \frac{6}{L^2} + \frac{F^2}{48}\Big)S^{L_1}_\mu + \nabla_\mu\nabla^\nu S^{L_1}_\nu + \Big(\frac{6}{7}\Lambda^{L_1} + \frac{6}{L^2}\Big)\nabla_\mu S^{L_1} - \frac{6}{7}\nabla_\mu\phi^{L_1}$$
$$+ \nabla^\nu h^{L_1}_{\mu\nu} - \nabla_\mu h^{L_1} + \frac{1}{2}\nabla_{[\rho}\tilde{S}^{L_1}_{\sigma\tau]}F_\mu{}^{\rho\sigma\tau} - \frac{1}{6}\tilde{S}^{L_1}_{\rho\sigma\tau}F_\mu{}^{\rho\sigma\tau}\Big] = 0$$

…………

➔ Equations of motion with gauge invariant fluctuations!!

Scalar field equations

- We concentrate on the scalar field equations.

$$\left(\Box + \Lambda^{I_1} + \frac{12}{L^2}\right)\hat{\phi}^{I_1} - \frac{14}{3L^2}\hat{\psi}^{I_1} = 0$$
$$\left(\Box + \Lambda^{I_1} - \frac{84}{L^2}\right)\hat{\psi}^{I_1} + \frac{216}{7}\left(\Lambda^{I_1} + \frac{7}{L^2}\right)\hat{\phi}^{I_1} = 0$$

$$\hat{\psi}^{I_1} = (18h^{I_1} - \tilde{U}^{I_1}), \qquad \hat{\phi}^{I_1} = (\phi^{I_1} - \Lambda^{I_1} S^{I_1})$$

$$\tilde{U}^{I_1} \equiv L\epsilon^{\mu\nu\rho\sigma}\nabla_\mu \tilde{S}^{I_1}_{\nu\rho\sigma}$$

➔ diagonalized scalar field equations

$$\left(\Box - \frac{(n+6)(n+12)}{L^2}\right)\check{\phi}^{I_1} = 0,$$
$$\left(\Box - \frac{n(n-6)}{L^2}\right)\check{\psi}^{I_1} = 0,$$

Scalar field equations

➔ diagonalized scalar field equations

$$\left(\Box - \frac{(n+6)(n+12)}{L^2}\right)\check{\phi}^{I_1} = 0$$

$$\left(\Box - \frac{n(n-6)}{L^2}\right)\check{\psi}^{I_1} = 0$$

$$\check{\phi}^{I_1} = \frac{(n+7)\left[18(n-1)\hat{\phi}^{I_1} + 7\hat{\psi}^{I_1}\right]}{14(n+3)}, \longrightarrow \Phi^{I_1}$$

$$\check{\psi}^{I_1} = \frac{(n-1)\left[-18(n+7)\hat{\phi}^{I_1} + 7\hat{\psi}^{I_1}\right]}{14(n+3)} \longrightarrow \Psi^{I_1}$$

11-dim gravity fields **4-dim gravity fields**

Field equations and action in 4-dim

$$\left(\Box - \frac{(I_1+6)(I_1+12)}{L^2}\right)\Phi^{I_1}(x) = 0,$$
$$\left(\Box - \frac{I_1(I_1-6)}{L^2}\right)\Psi^{I_1}(x) = 0,$$

EOM in 4-dim gravity

$$S_{sc} = \int d^4x\sqrt{-g^0}\Big[-\frac{1}{2}\partial_\mu\tilde{\Phi}^{I_1}\partial^\mu\tilde{\Phi}^{I_1} - \frac{1}{2}m^2_{\Phi^{I_1}}(\tilde{\Phi}^{I_1})^2 - \frac{1}{2}\partial_\mu\tilde{\Psi}^{I_1}\partial^\mu\tilde{\Psi}^{I_1} - \frac{1}{2}m^2_{\Psi^{I_1}}(\tilde{\Psi}^{I_1})^2\Big],$$

Infinity number of KK-scalar fields
➔ We have to choose one of them.

Vevs of CPO with $\Delta = 1$

- **The mass m^2 of a scalar field on the gravity side is related to a gauge invariant operator with the conformal dimension Δ :**

$$m^2 R^2_{AdS_{d+1}} = \Delta(\Delta - d)$$

$$R^2_{AdS_4} = \frac{L^2}{4}$$ L is the radius of S^7

$$\frac{m^2 L^2}{4} = \Delta(\Delta - 3) \Rightarrow m^2 L^2 = 2\Delta(2\Delta - 6)$$

Vevs of CPO with $\Delta = 1$

$$\boxed{\begin{aligned}\left(\Box - \frac{(I_1+6)(I_1+12)}{L^2}\right)\Phi^{I_1}(x) = 0,\\ \left(\Box - \frac{I_1(I_1-6)}{L^2}\right)\Psi^{I_1}(x) = 0,\end{aligned}}$$

- Scalar field Φ^{I_1} $\Delta = 6, 7, 8, \ldots$

$$(I_1+12)(I_1+6) = 2\Delta(2\Delta-6) \Rightarrow \Delta = \frac{I_1+12}{2}, \quad \{I_1 = 0, 2, 4, 6\cdots\}$$

- Scalar field Ψ^{I_1} $\Delta = 1, 2, 3, \ldots$

$$I_1(I_1-6) = 2\Delta(2\Delta-6) \Rightarrow \Delta = \frac{I_1}{2}, \quad \{I_1 = 2, 4, 6\cdots\}$$

Vevs of CPO with Δ = 1

$$\left(\Box - \frac{(I_1+6)(I_1+12)}{L^2}\right)\Phi^{I_1}(x) = 0,$$
$$\left(\Box - \frac{I_1(I_1-6)}{L^2}\right)\Psi^{I_1}(x) = 0,$$

- **Scalar field** Φ^{I_1} $\Delta = 6, 7, 8, \ldots$

$$(I_1+12)(I_1+6) = 2\Delta(2\Delta-6) \Rightarrow \Delta = \frac{I_1+12}{2}, \quad \{I_1 = 0, 2, 4, 6\cdots\}$$

- **Scalar field** Ψ^{I_1} $\Delta = 1, 2, 3, \ldots$

$$I_1(I_1-6) = 2\Delta(2\Delta-6) \Rightarrow \Delta = \frac{I_1}{2}, \quad \{I_1 = 2, 4, 6\cdots\}$$

Vevs of CPO with Δ = 1

$$\left(\Box - \frac{(I_1+6)(I_1+12)}{L^2}\right)\Phi^{I_1}(x) = 0,$$
$$\left(\Box - \frac{I_1(I_1-6)}{L^2}\right)\Psi^{I_1}(x) = 0,$$

- Holographic renormalization:

$\boxed{\Delta = 1}$ ⇒ $\boxed{\tilde{\Psi} \equiv \tilde{\Psi}^{I_1=2}}$ dual gravity field

$$S_{sc} = \int d^4x\sqrt{-g^0}\left[-\frac{1}{2}\partial_\mu\tilde{\Psi}\partial^\mu\tilde{\Psi} - \frac{1}{2}m^2_{\tilde{\Psi}}(\tilde{\Psi})^2\right]$$

$$\tilde{\Psi} = \phi_{(1)}z + \cdots$$

Asymptotic expansion near the boundary

$$\langle\mathcal{O}^{(\Delta=1)}\rangle = \#\phi_{(1)}$$

Gauge/gravity mapping (GKP-W relation)

Vevs of CPO with $\Delta = 1$

$$\left(\Box - \frac{(I_1+6)(I_1+12)}{L^2}\right)\Phi^{I_1}(x) = 0,$$
$$\left(\Box - \frac{I_1(I_1-6)}{L^2}\right)\Psi^{I_1}(x) = 0,$$

- Asymptotic expansion of the general LLM geometry:

$$\Phi^{I_1} = \frac{(I_1+7)\big(18(I_1-1)\hat{\phi}^{I_1} + 7\hat{\psi}^{I_1}\big)}{14(I_1+3)},$$
$$\Psi^{I_1} = \frac{(I_1-1)\big(-18(I_1+7)\hat{\phi}^{I_1} + 7\hat{\psi}^{I_1}\big)}{14(I_1+3)}$$

$\Downarrow$ Read from general LLM

$$\Phi^{I_1=2} = \mathcal{O}(\mu_0^3)$$
$$\Psi^{I_1=2} = 24(6C_1 + C_1^3 - C_3)\mu_0 z + \mathcal{O}(\mu_0^3)$$

Exact holography for finite N

$$\phi_{(1)} = 24(6C_1 + C_1^3 - C_3)\mu_0$$

$$\langle \mathcal{O}^{(\Delta=1)} \rangle = \#_1 \phi_{(1)} = \#_2 (6C_1 + C_1^3 - C_3)\mu_0$$

GKP-W gauge/gravity mapping

Exact holography for finite N

- Read the normalization factor
  from the field theory calculations (k=1):

$$\boxed{\langle \mathcal{O}^{(\Delta=1)} \rangle_{Y^A=Y_0^A} = -\frac{2}{3\pi} N^{\frac{3}{2}} (6C_1 + C_1^3 - C_3)\mu_0, \quad (N \geq 2)}$$

chiral primary operator

$$\mathcal{O}_\Delta = C^{B_1, \cdots, B_\Delta}_{A_1, \cdots, A_\Delta} \mathrm{Tr}(Y^{A_1} Y^\dagger_{B_1} \cdots Y^{A_\Delta} Y^\dagger_{B_\Delta})$$

$$\mathcal{O}^{(\Delta=1)} = \mathrm{Tr}(Z^a Z^\dagger_a - W^{\dagger a} W_a)$$

reflect the broken symmetry of mABJM
SU(4) → SU(2)XSU(2)XU(1)

$$Y^A = (Z^a, W^{\dagger a})$$

Exact holography for finite N

- Read the normalization factor
  from the field theory calculation:

$$\boxed{\langle \mathcal{O}^{(\Delta=1)} \rangle_{Y^A = Y_0^A} = -\frac{2}{3\pi} N^{\frac{3}{2}} (6C_1 + C_1^3 - C_3)\mu_0, \quad (N \geq 2)}$$

Field theory side **Gravity side**

➔ **Exact relation without 1/N correction for finite N!**

Conclusion

- Origin of this exact duality for finite N? supersymmetry? Other reason?

- $\Delta = 2$ case for general vacua?
- $\Delta = 3$ case for symmetric vacua?

- Correlation functions? And other issue for gauge/gravity duality

- Entanglement entropy?

- Physical phenomena of Chern-Simons theory with massive matter fields in strong coupling limit?