SUPERSYMMETRIC GAUGED DOUBLE FIELD THEORY: SYSTERMATIC DERIVATION BY VIRTUE OF TWIST

Wonyoung Cho

August 19, 2016.

Sogang University

Talk based on

- Series of DFT papers written by Imtak Jeon, Kanghoon Lee and Jeong-Hyuck Park:

  1011.1324, 1102.0419, 1105.6294, 1109.2035, 1112.0069, 1206.3478, 1210.5078, 1304.5946, 1307.8377.

- Supersymmetric gauged Double Field Theory: Systematic derivation by virtue of *Twist*

  with J.J. Fernandez-Melgarejo, Imtak Jeon and Jeong-Hyuck Park, JHEP 08 (2015) 084, arXiv:1505.01301

INTRODUCTION

Introduction

- In Riemannian geometry, the fundamental object is the metric, $g_{\mu\nu}$.
  - Diffeomorphism: $\partial_\mu \longrightarrow \nabla_\mu = \partial_\mu + \Gamma_\mu$
  - $\nabla_\lambda g_{\mu\nu} = 0,\ \Gamma^\lambda_{[\mu\nu]} = 0 \ \longrightarrow \ \Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\rho}(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu})$
  - Curvature: $[\nabla_\mu, \nabla_\nu] \ \longrightarrow \ R_{\kappa\lambda\mu\nu} \ \longrightarrow \ R$

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- In general relativity, the metric is the only geometric object . All other fields are viewed as matter or radiation.

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- Such an idea has been materialize in recent years through the developments of Double Field Theory .

DOUBLE FIELD THEORY

Double Field Theory

- A "generalized metric" and a redefined dilaton,

$$\mathcal{H}_{AB} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}, \qquad e^{-2d} = \sqrt{-g}e^{-2\phi}$$

- O(D, D) metric,

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freely raises or lowers the $(D+D)$-dimensional vector indices, A, B.

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Double Field Theory

- DFT action for NS-NS sector is ,

$$S_{\mathrm{DFT}} = \int dy^{2D} e^{-2d} L_{\mathrm{DFT}} (\mathcal{H}, d)$$

where

$$L_{\mathrm{DFT}} = \mathcal{H}^{AB} \left(4\partial_A\partial_B d - 4\partial_A d\partial_B d + \tfrac{1}{8}\partial_A\mathcal{H}^{CD}\partial_B\mathcal{H}_{CD} - \tfrac{1}{2}\partial_A\mathcal{H}^{CD}\partial_C\mathcal{H}_{BD}\right)$$
$$+4\partial_A\mathcal{H}^{AB}\partial_B d - \partial_A\partial_B\mathcal{H}^{AB}$$

Hull & Zwiebach later with Hohm

- O(D, D) structure is manifest and background independent.
- All spacetime dimension is 'formally doubled', $y^A = (\tilde{x}_\mu, x^\nu)$, $A = 1, 2, \cdots, D + D$.

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Double Field Theory

- A characteristic of DFT is the section condition ,

$$\partial_A \partial^A \sim 0 \, .$$

- Explicitly, the section condition implies

$$\begin{aligned} \partial^A \varphi \partial_A \Phi = 0 \qquad & \text{(strong constraint)} \, , \\ \partial_A \partial^A \Phi = 0 \qquad & \text{(weak constraint)} \, . \end{aligned}$$

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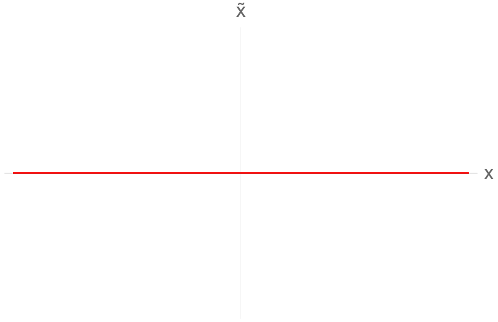

Figure: We choose x-coordinate with $\frac{\partial}{\partial \tilde{x}_\mu} \sim 0$.

- The section condition ensures that DFT lives not on the doubled $(D+D)$-dimensional space but on a D-dimensional null hyperspace, *i.e. section.*

Double Field Theory

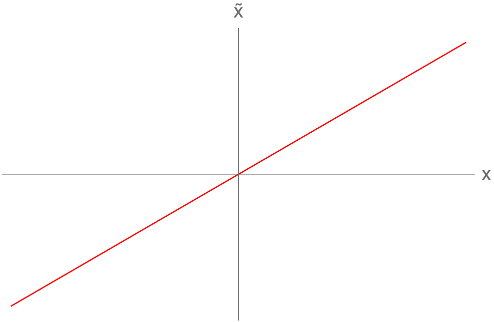

Figure: Other section.

- There is isometry, we can choose any section.

Double Field Theory

- DFT action is (locally) equivalent to the effective action:

$$S_{\mathrm{DFT}} \Rightarrow S_{eff} = \int dx^D \sqrt{-g} e^{-2\phi} \left(R_g + 4 \left(\partial \phi \right)^2 - \frac{1}{12} H^2 \right) .$$

Double Field Theory

Dilaton and a pair of two-index projectors.

- The geometric objects in DFT consist of a dilaton, d, and a pair of symmetric projection operators,

$$P_{AB} = P_{BA}\,, \qquad \bar{P}_{AB} = \bar{P}_{BA}\,, \qquad P_A{}^B P_B{}^C = P_A{}^C\,, \qquad \bar{P}_A{}^B \bar{P}_B{}^C = \bar{P}_A{}^C\,.$$

- Further, the projectors are orthogonal and complementary,

$$P_A{}^B \bar{P}_B{}^C = 0\,, \qquad P_{AB} + \bar{P}_{AB} = \mathcal{J}_{AB}\,.$$

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- The difference of the two projectors, $P_{AB} - \bar{P}_{AB} = \mathcal{H}_{AB}$, corresponds to the “generalized metric”.

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- In supersymmetric double field theories, it appears that the projectors are more fundamental than the “generalized metric”.

Double Field Theory

Dilaton and a pair of two-index projectors.

- The six-index projection operators are

$$\mathcal{P}_{CAB}{}^{DEF} := P_C{}^D P_{[A}{}^{[E} P_{B]}{}^{F]} + \tfrac{2}{D-1} P_{C[A} P_{B]}{}^{[E} P^{F]D}\,, \qquad \mathcal{P}_{ABC}{}^{DEF}\mathcal{P}_{DEF}{}^{GHI} = \mathcal{P}_{ABC}{}^{GHI}\,,$$
$$\bar{\mathcal{P}}_{CAB}{}^{DEF} := \bar{P}_C{}^D \bar{P}_{[A}{}^{[E} \bar{P}_{B]}{}^{F]} + \tfrac{2}{D-1} \bar{P}_{C[A} \bar{P}_{B]}{}^{[E} \bar{P}^{F]D}\,, \qquad \bar{\mathcal{P}}_{ABC}{}^{DEF}\bar{\mathcal{P}}_{DEF}{}^{GHI} = \bar{\mathcal{P}}_{ABC}{}^{GHI}\,.$$

They are symmetric and traceless,

$$\mathcal{P}_{ABCDEF} = \mathcal{P}_{DEFABC}\,, \qquad \mathcal{P}_{ABCDEF} = \mathcal{P}_{A[BC]D[EF]}\,, \qquad P^{AB}\mathcal{P}_{ABCDEF} = 0\,,$$
$$\bar{\mathcal{P}}_{ABCDEF} = \bar{\mathcal{P}}_{DEFABC}\,, \qquad \bar{\mathcal{P}}_{ABCDEF} = \bar{\mathcal{P}}_{A[BC]D[EF]}\,, \qquad \bar{P}^{AB}\bar{\mathcal{P}}_{ABCDEF} = 0\,.$$

Double Field Theory

Integral measure.

- While the projectors are weightless, the dilaton gives rise to the O(D,D) invariant integral measure with weight one, after exponentiation,

$$e^{-2d}\,.$$

- Naturally the cosmological constant term in DFT is given by

$$e^{-2d}\Lambda_{\mathrm{DFT}}$$

which deviates from the conventional one in Riemannian GR, and hence reformulates the cosmological constant problem in a novel manner.

Jeon-Lee-JHP 2011

c.f. Meissner-Veneziano 1991

Integral measure.

- Naturally the cosmological constant term in DFT is given by

$$e^{-2d}\Lambda_{\mathrm{DFT}}\,.$$

- Scherk-Schwarz-type dimensional reductions from $D=10$ half-maximal SDFT can produce $\Lambda_{\mathrm{DFT}}>0$ (as well as $\Lambda_{\mathrm{DFT}}<0$),

  Cho-Fernández-Melgarejo-Jeon-Park 2015

  once the section condition is 'relaxed' for the twisting ansatz.

  Geissbuhler, Grana-Marques, Berman-Lee

Diffeomorphism.

- Diffeomorphism symmetry in $\mathrm{O}(D,D)$ DFT is generated by a generalized Lie derivative Siegel, Courant, Grana

$$\begin{aligned}\hat{\mathcal{L}}_X T_{A_1\cdots A_n} := X^B\partial_B T_{A_1\cdots A_n} \quad &+ \quad \omega_{{}_T}\partial_B X^B T_{A_1\cdots A_n} \\ &+ \quad \textstyle\sum_{i=1}^{n}(\partial_{A_i}X_B - \partial_B X_{A_i})T_{A_1\cdots A_{i-1}}{}^{B}{}_{A_{i+1}\cdots A_n}\,,\end{aligned}$$

where $\omega_{{}_T}$ denotes the weight.

Diffeomorphism.

- In particular, the generalized Lie derivative of the O(D, D) invariant metric is trivial,

$$\hat{\mathcal{L}}_X \mathcal{J}_{AB} = 0\,.$$

- The commutator is closed by C-bracket Hull-Zwiebach

$$\left[\hat{\mathcal{L}}_X, \hat{\mathcal{L}}_Y\right] = \hat{\mathcal{L}}_{[X,Y]_{\mathrm{C}}}\,, \qquad [X, Y]^A_{\mathrm{C}} = X^B \partial_B Y^A - Y^B \partial_B X^A + \tfrac{1}{2} Y^B \partial^A X_B - \tfrac{1}{2} X^B \partial^A Y_B\,.$$

Semi-covariant derivative.

- We define a semi-covariant derivative,

$$\nabla_C T_{A_1 A_2 \cdots A_n} := \partial_C T_{A_1 A_2 \cdots A_n} - \omega_T \Gamma^B{}_{BC} T_{A_1 A_2 \cdots A_n} + \sum_{i=1}^{n} \Gamma_{CA_i}{}^B T_{A_1 \cdots A_{i-1} B A_{i+1} \cdots A_n} .$$

- It is compatible with the $\mathrm{O}(D, D)$ quatities,

$$\nabla_C d = 0, \qquad \nabla_C P_{AB} = 0, \qquad \nabla_C \bar{P}_{AB} = 0,$$
$$\nabla_C \mathcal{J}_{AB} = 0 \quad (\Leftrightarrow \Gamma_{ABC} + \Gamma_{ACB} = 0),$$

- With the torsionless condition,

$$\Gamma_{[ABC]} = 0 \quad (\Leftrightarrow \hat{\mathcal{L}}_X(\partial) = \hat{\mathcal{L}}_X(\nabla)),$$

we may uniquely determine the (torsionelss) connection,

$$\begin{aligned}\Gamma_{CAB} = \quad & 2\left(P\partial_C P\bar{P}\right)_{[AB]} + 2\left(\bar{P}_{[A}{}^{D}\bar{P}_{B]}{}^{E} - P_{[A}{}^{D}P_{B]}{}^{E}\right)\partial_D P_{EC} \\ & -\tfrac{4}{D-1}\left(\bar{P}_{C[A}\bar{P}_{B]}{}^{D} + P_{C[A}P_{B]}{}^{D}\right)\left(\partial_D d + (P\partial^E P\bar{P})_{[ED]}\right)\,,\end{aligned}$$

satisfying

$$\mathcal{P}_{ABC}{}^{DEF}\Gamma_{DEF} = 0\,, \qquad \bar{\mathcal{P}}_{ABC}{}^{DEF}\Gamma_{DEF} = 0\,.$$

Jeon-Lee-Park 2011

- It is the DFT generalization of the Christoffel connection.

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Master semi-covariant derivative.

- We generalize the semi-covariant with the spin connections, Φ_A and $\bar{\Phi}_A$, for the two local Lorentz groups, $\mathrm{Spin}(1, D-1)_L$ and $\mathrm{Spin}(1, D-1)_R$, called *a master 'semi-covariant' derivative*,

$$\mathcal{D}_A = \nabla_A + \Phi_A + \bar{\Phi}_A = \partial_A + \Gamma_A + \Phi_A + \bar{\Phi}_A\,.$$

- It is also compatible with these quantities,

$$\mathcal{D}_A V_{Bp} = \partial_A V_{Bp} + \Gamma_{AB}{}^C V_{Cp} + \Phi_{Ap}{}^q V_{Bq} = 0,$$
$$\mathcal{D}_A \bar{V}_{Bp} = \partial_A \bar{V}_{Bp} + \Gamma_{AB}{}^C \bar{V}_{Cp} + \bar{\Phi}_{Ap}{}^q \bar{V}_{Bq} = 0,$$
$$\mathcal{D}_A d = 0, \qquad \mathcal{D}_A \mathcal{J}_{BC} = 0, \qquad \mathcal{D}_A \eta_{pq} = 0, \qquad \mathcal{D}_A \bar{\eta}_{\bar{p}\bar{q}} = 0,$$
$$\mathcal{D}_A (\gamma^p)^\alpha{}_\beta = 0, \qquad \mathcal{D}_A (\bar{\gamma}^{\bar{p}})^{\bar{\alpha}}{}_{\bar{\beta}} = 0.$$

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Semi-covariant curvature.

- A semi-covariant Riemann curvature is defined by,

$$S_{ABCD} := \frac{1}{2}\left(R_{ABCD} + R_{CDAB} - \Gamma^{E}{}_{AB}\Gamma_{ECD}\right).$$

- Here R_{ABCD} denotes the ordinary "field strength" of a connection,

$$R_{CDAB} = \partial_A \Gamma_{BCD} - \partial_B \Gamma_{ACD} + \Gamma_{AC}{}^{E}\Gamma_{BED} - \Gamma_{BC}{}^{E}\Gamma_{AED}.$$

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- It satisfies, just like the Riemann curvature,

$$S_{ABCD} = \frac{1}{2}\left(S_{[AB][CD]} + S_{[CD][AB]}\right) ,$$

$$S_{A[BCD]} = 0 : \text{Bianchi identity.}$$

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- A semi-covariant curvature of the spin connections is,

$$\begin{aligned}\mathcal{G}_{ABCD} &= S_{ABCD} + \tfrac{1}{2}(\Gamma - \Phi - \bar{\Phi})_{EAB}(\Gamma - \Phi - \bar{\Phi})^{E}{}_{CD} \\ &= S_{ABCD} + \tfrac{1}{2}(V_{A}{}^{p}\partial_{E}V_{Bp} + \bar{V}_{A}{}^{\bar{p}}\partial_{E}\bar{V}_{B\bar{p}})(V_{C}{}^{q}\partial^{E}V_{Dq} + \bar{V}_{C}{}^{\bar{q}}\partial^{E}\bar{V}_{D\bar{q}}),\end{aligned}$$

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- These are Not covariant tensors, but contracting with projection operators, we can obtain covariant quatities.
- The ordinary derivative of a covariant tensor is no longer covariant under diffeomorphisms.

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- A semi-covariant Riemann curvature is defined by,

$$S_{ABCD} := \frac{1}{2}\left(R_{ABCD} + R_{CDAB} - \Gamma^{E}{}_{AB}\Gamma_{ECD}\right) .$$

- With projectors,

$$\left(P^{AB}P^{CD} + \bar{P}^{AB}\bar{P}^{CD}\right) S_{ACBD} \sim 0 ,$$

$$P_I{}^A P_J{}^B \bar{P}_K{}^C \bar{P}_L{}^D S_{ABCD} \sim 0 ,$$

$$P_I{}^A \bar{P}_J{}^B P_K{}^C \bar{P}_L{}^D S_{ABCD} \sim 0 , etc$$

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- Rank two-tensor:

$$P_I{}^A\bar{P}_J{}^B S_{AB}\,, \qquad \textit{where} \qquad S_{AB} := S^C{}_{ACB}\,,$$

- Scalar curvature:

$$\left(P^{AB}P^{CD} - \bar{P}^{AB}\bar{P}^{CD}\right) S_{ACBD}\,.$$

- Upon the section condition,

$$(\delta_X - \hat{\mathcal{L}}_X)\Gamma_{CAB} \sim 2\left[(\mathcal{P}+\bar{\mathcal{P}})_{CAB}{}^{FDE} - \delta_C^F\delta_A^D\delta_B^E\right]\partial_F\partial_{[D}X_{E]}\,,$$

$$(\delta_X - \hat{\mathcal{L}}_X)\nabla_C T_{A_1\cdots A_n} \sim \sum_{i=1}^{n} 2(\mathcal{P}+\bar{\mathcal{P}})_{CA_i}{}^{BDEF}\partial_D\partial_E X_F\, T_{A_1\cdots A_{i-1}BA_{i+1}\cdots A_n}\,.$$

- For the four-index curvatures,

$$\begin{aligned}(\delta_X - \hat{\mathcal{L}}_X)\mathcal{G}_{ABCD} &\sim (\delta_X - \hat{\mathcal{L}}_X)S_{ABCD}\\ &\sim 2\nabla_{[A}\Big((\mathcal{P}+\bar{\mathcal{P}})_{B][CD]}{}^{EFG}\partial_E\partial_F X_G\Big) + \Big[(A,B)\leftrightarrow(C,D)\Big]\,.\end{aligned}$$

Double Field Theory

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Double Field Theory

- The anomalous terms can be easily projected out through appropriate contractions with the two-index projectors.

- This also explains or motivates the naming, '*semi-covariant*': we say a tensor is semi-covariant if its diffeomorphic anomaly, if any, is governed by the six-index projectors.

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Check point

- Understanding the section condition in DFT is subtle and difficult.
- The section condition is sufficient but not the necessary condition for the algebra closure and action invariance.
- "Relaxing" the section condition to some extent has been understood. [Aldazabal, Baron, Nunez, Grana, Marqus, Geissbhler]
- The relaxation of the section condition is allowed when doing Sherk-Schwarz reduction in DFT and it gets gauged DFT.

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- To have systematic understanding the low dimensional gauged SDFT in the semi-covariant formulation
  - We twist the semi-covariant formulation of the ungaged SDFT without an ambiguity.
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Double Field Theory

The fundamental fields of D = 10 Maximal SDFT are precisely,

$$d\,,\qquad V_{Ap}\,,\qquad \bar{V}_{A\bar{p}}\,,\qquad \mathcal{C}^{\alpha}{}_{\bar{\alpha}}\,,\qquad \rho^{\alpha}\,,\quad \rho'^{\bar{\alpha}}\,,\qquad \psi^{\alpha}_{\bar{p}}\,,\qquad \psi'_{p}{}^{\bar{\alpha}}\,.$$

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- The DFT-dilaton is a scalar,

$$e^{-2d}.$$

- The vielbeins satisfy the following four *defining properties* :

$$V_{Ap}V^{A}{}_{q} = \eta_{pq}\,,\quad \bar{V}_{A\bar{p}}\bar{V}^{A}{}_{\bar{q}} = \bar{\eta}_{\bar{p}\bar{q}}\,,\quad V_{Ap}\bar{V}^{A}{}_{\bar{q}} = 0\,,\quad V_{Ap}V_{B}{}^{p} + \bar{V}_{A\bar{p}}\bar{V}_{B}{}^{\bar{p}} = \mathcal{J}_{AB}.$$

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- The vielbeins generate a pair of symmetric, orthogonal and complete two-index projectors ,

$$P_{AB} = P_{BA} = V_A{}^p V_{Bp}\,,\quad \bar{P}_{AB} = \bar{P}_{BA} = \bar{V}_A{}^p \bar{V}_{Bp}\,,$$

satisfying

$$P_A{}^B P_B{}^C = P_A{}^C\,,\quad \bar{P}_A{}^B \bar{P}_B{}^C = \bar{P}_A{}^C\,,\quad P_A{}^B \bar{P}_B{}^C = 0\,,$$
$$\mathrm{tr}(P) = P_A{}^A = D,\quad \mathrm{tr}(\bar{P}) = \bar{P}_A{}^A = \bar{D},$$

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$$P_{AB}=P_{BA}=V_A{}^pV_{Bp}\,,\quad \bar{P}_{AB}=\bar{P}_{BA}=\bar{V}_A{}^p\bar{V}_{Bp}\,,$$

and related to $\mathcal{H}$ and $\mathcal{J}$,

$$P_{AB}-\bar{P}_{AB}=\mathcal{H}_{AB}\,,\qquad P_{AB}+\bar{P}_{AB}=\mathcal{J}_{AB}\,.$$

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- We further define a pair of six-index projection,

$$\mathcal{P}_{ABC}{}^{DEF} = P_A{}^D P_{[B}{}^{[E} P_{C]}{}^{F]} + \tfrac{2}{D-1} P_{A[B} P_{C]}{}^{[E} P^{F]D}\,,\quad \mathcal{P}_{ABC}{}^{DEF}\mathcal{P}_{DEF}{}^{GHI} = \mathcal{P}_{ABC}{}^{GHI}\,,$$
$$\bar{\mathcal{P}}_{ABC}{}^{DEF} = \bar{P}_A{}^D \bar{P}_{[B}{}^{[E} \bar{P}_{C]}{}^{F]} + \tfrac{2}{D-1} \bar{P}_{A[B} \bar{P}_{C]}{}^{[E} \bar{P}^{F]D}\,,\quad \bar{\mathcal{P}}_{ABC}{}^{DEF}\bar{\mathcal{P}}_{DEF}{}^{GHI} = \bar{\mathcal{P}}_{ABC}{}^{GHI}\,,$$

which are symmetric and traceless,

$$\mathcal{P}_{ABCDEF} = \mathcal{P}_{DEFABC} = \mathcal{P}_{A[BC]D[EF]}\,,\quad P^{AB}\mathcal{P}_{ABCDEF} = 0\,,$$
$$\bar{\mathcal{P}}_{ABCDEF} = \bar{\mathcal{P}}_{DEFABC} = \bar{\mathcal{P}}_{A[BC]D[EF]}\,,\quad \bar{P}^{AB}\bar{\mathcal{P}}_{ABCDEF} = 0.$$

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- R-R potential is bi-fundamental spinor representation of $\mathrm{Spin}(1,9)\times\mathrm{Spin}(9,1)$.
- Especially for the torsionless case, the corresponding operators are nilpotent up to the section condition

$$(\mathcal{D}_{\pm})^2\mathcal{C}\sim 0\,.$$

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- Dilatinos and Gravitinos.

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TWISTED DOUBLE FIELD THEORY

Twisted Double Field Theory

- To relax the section condition, we twist the original theory.
  Namely, Sherk-Schwarz reduction.

Twisted Double Field Theory

- For the twisting, we use the two twisting datas:
  a scalar $\lambda(x)$ and $U_A{}^{\dot{A}} \in \mathrm{O}(D,D)$,

$$U\dot{\mathcal{J}}U^t = \mathcal{J}, \qquad \dot{\mathcal{J}}_{\dot{A}\dot{B}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

  using which we set the ansatz for U-twist,

$$T_{A_1 \cdots A_n} = e^{-2\omega\lambda} U_{A_1}{}^{\dot{A}_1} \cdots U_{A_n}{}^{\dot{A}_n} \dot{T}_{\dot{A}_1 \cdots \dot{A}_n}.$$

Twisted Double Field Theory

- The derivatives of the untwisted fields then assume a generic form,

$$\partial_C T_{A_1\cdots A_n} = e^{-2\omega\lambda} U_C{}^{\dot{C}} U_{A_1}{}^{\dot{A}_1} \cdots U_{A_n}{}^{\dot{A}_n} \dot{D}_{\dot{C}} \dot{T}_{\dot{A}_1\cdots\dot{A}_n} \,.$$

- The U-derivative , $\dot{D}_{\dot{C}}$, is defined to act on a twisted field by

$$\dot{D}_{\dot{C}} \dot{T}_{\dot{A}_1\cdots\dot{A}_n} := \dot{\partial}_{\dot{C}} \dot{T}_{\dot{A}_1\cdots\dot{A}_n} - 2\omega\dot{\partial}_{\dot{C}}\lambda\, \dot{T}_{\dot{A}_1\cdots\dot{A}_n} + \sum_{i=1}^{n} \Omega_{\dot{C}\dot{A}_i}{}^{\dot{B}} \dot{T}_{\dot{A}_1\cdots\dot{B}\cdots\dot{A}_n} \,.$$

Twisted Double Field Theory

- Those replacement leads to twisted SDFT Lagrangian,

$$\mathcal{L}^{\mathcal{N}=1}_{D=10}(\mathcal{J}_{AB}, \partial_A, d, V_{Ap}, \bar{V}_{A\bar{p}}, \rho, \psi_{\bar{p}}) = e^{-2\lambda}\dot{\mathcal{L}}^{\text{Half-maximal}}_{\text{Twisted SDFT}}(\dot{\mathcal{J}}_{\dot{A}\dot{B}}, \dot{D}_{\dot{A}}, \dot{d}, \dot{V}_{\dot{A}p}, \dot{\bar{V}}_{\dot{A}\bar{p}}, \rho, \psi_{\bar{p}})\,,$$

$$\mathcal{L}^{\mathcal{N}=2}_{D=10}(\mathcal{J}_{AB}, \partial_A, d, V_{Ap}, \bar{V}_{A\bar{p}}, \mathcal{C}, \rho, \psi_{\bar{p}}, \rho', \psi'_p) = e^{-2\lambda}\dot{\mathcal{L}}^{\text{Maximal}}_{\text{Twisted SDFT}}(\dot{\mathcal{J}}_{\dot{A}\dot{B}}, \dot{D}_{\dot{A}}, \dot{d}, \dot{V}_{\dot{A}p}, \dot{\bar{V}}_{\dot{A}\bar{p}}, \mathcal{C}, \rho, \psi_{\bar{p}}, \rho', \psi'_p)\,.$$

- The twist translates the original section condition as

$$\dot{D}_{\dot{A}}\dot{D}^{\dot{A}} \sim 0\,.$$

- If we impose this, it is nothing but the field redefinition of the untwisted SDFT. We shall look for alternative inequivalent conditions, or the *twistability conditions*.

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- From the closure of the U-twisted generalized Lie derivative,

$$[\dot{\mathcal{L}}_{\dot{X}}, \dot{\mathcal{L}}_{\dot{Y}}] \equiv \dot{\mathcal{L}}_{[\dot{X},\dot{Y}]_{\mathrm{C}}},$$

  we found the twistability conditions.

Twisted Double Field Theory

1. The section condition for all the dotted twisted fields,

$$\dot{\partial}_{\dot{M}}\dot{\partial}^{\dot{M}} \equiv 0\,.$$

2. The orthogonality between the connection and the derivatives,

$$\Omega^{\dot{M}}{}_{\dot{F}\dot{G}}\dot{\partial}_{\dot{M}} \equiv 0\,.$$

3. The Jacobi identity for $f_{\dot{A}\dot{B}\dot{C}} = f_{[\dot{A}\dot{B}\dot{C}]}$,

$$f_{[\dot{A}\dot{B}}{}^{\dot{E}}f_{\dot{C}]\dot{D}\dot{E}} \equiv 0\,.$$

4. The constancy of the structure constant, $f_{\dot{A}\dot{B}\dot{C}}$,

$$\dot{\partial}_{\dot{E}}f_{\dot{A}\dot{B}\dot{C}} \equiv 0\,.$$

5. The triviality of $f_{\dot{A}}$,

$$f_{\dot{A}} = \Omega^{\dot{C}}{}_{\dot{C}\dot{A}} - 2\dot{\partial}_{\dot{A}}\lambda = \partial_C U^C{}_{\dot{A}} - 2\dot{\partial}_{\dot{A}}\lambda \equiv 0\,.$$

Twisted semi-covariant formalism.

- The *U-twisted master semi-covariant derivative* is

$$\dot{\mathcal{D}}_{\dot{A}} = \dot{\nabla}_{\dot{A}} + \dot{\Phi}_{\dot{A}} + \dot{\bar{\Phi}}_{\dot{A}} = \dot{D}_{\dot{A}} + \Gamma_{\dot{A}} + \dot{\Phi}_{\dot{A}} + \dot{\bar{\Phi}}_{\dot{A}}\,,$$

- The twisted torsionless connection reads

$$\begin{aligned}\dot{\Gamma}_{\dot{C}\dot{A}\dot{B}} = \;& 2(\dot{P}\dot{D}_{\dot{C}}\dot{P}\dot{\bar{P}})_{[\dot{A}\dot{B}]} + 2(\dot{\bar{P}}_{[\dot{A}}{}^{\dot{D}}\dot{\bar{P}}_{\dot{B}]}{}^{\dot{E}} - \dot{P}_{[\dot{A}}{}^{\dot{D}}\dot{P}_{\dot{B}]}{}^{\dot{E}})\dot{D}_{\dot{D}}\dot{P}_{\dot{E}\dot{C}} \\ & -\tfrac{4}{D-1}(\dot{\bar{P}}_{\dot{C}[\dot{A}}\dot{\bar{P}}_{\dot{B}]}{}^{\dot{D}} + \dot{P}_{\dot{C}[\dot{A}}\dot{P}_{\dot{B}]}{}^{\dot{D}})\left(\dot{D}_{\dot{D}}\dot{d} + (\dot{P}\dot{D}^{\dot{E}}\dot{P}\dot{\bar{P}})_{[\dot{E}\dot{D}]}\right)\,.\end{aligned}$$

- These are in a completely parallel manner to the untwisted cases.

Twisted Double Field Theory

- Upon all the twistability conditions, we obtain

$$(\delta_{\dot{X}}-\hat{\mathcal{L}}_{\dot{X}})(\dot{\nabla}_{\dot{C}}\dot{T}_{\dot{A}_1\cdots\dot{A}_n})\equiv\sum_{i=1}^{n}(\mathcal{P}+\bar{\mathcal{P}})_{\dot{C}\dot{A}_i}{}^{\dot{B}}\dot{T}_{\dot{A}_1\cdots\dot{A}_{i-1}\dot{B}\dot{A}_{i+1}\cdots\dot{A}_n}\,.$$

- Once again the anomalies are all controlled by the index-six projection operators. Namely, *they are still semi-covariant.*

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U-twisted Double Field Theory

- But, in contrast to the nilpotency of the untwisted differential operators, we get after the twist,

$$(\dot{\mathcal{D}}_{\pm})^2\mathcal{C} \equiv -\frac{1}{24} f_{\dot{A}\dot{B}\dot{C}} f^{\dot{A}\dot{B}\dot{C}}\mathcal{C} \, .$$

- For the consistency , we should impose

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Twisted Supersymmetric Double Field Theory

- Half-maximal supersymmetric gauged double field theory Lagrangian,

$$\dot{\mathcal{L}}^{\text{Half-maximal}}_{\text{Twisted SDFT}} = e^{-2\dot{d}}\left[\tfrac{1}{4}\dot{\mathcal{G}}_{pq}{}^{pq} + i\tfrac{1}{2}\bar{\rho}\gamma^{p}\dot{\mathcal{D}}_{p}\rho - i\bar{\psi}^{\bar{p}}\dot{\mathcal{D}}_{\bar{p}}\rho - i\tfrac{1}{2}\bar{\psi}^{\bar{p}}\gamma^{q}\dot{\mathcal{D}}_{q}\psi_{\bar{p}}\right].$$

- The supersymmetry works, as the induced leading order variation of the Lagrangian vanishes , up to total derivatives and the twistability conditions,

$$\begin{aligned} \delta_{\varepsilon}\dot{\mathcal{L}}^{\text{Half-maximal}}_{\text{Twisted SDFT}} &\equiv -ie^{-2\dot{d}}\bar{\rho}\left[(\gamma^{p}\dot{\mathcal{D}}_{p})^{2} + \dot{\mathcal{D}}_{\bar{p}}\dot{\mathcal{D}}^{\bar{p}} + \tfrac{1}{4}\dot{\mathcal{G}}_{pq}{}^{pq}\right]\varepsilon \\ &\quad + ie^{-2\dot{d}}\bar{\psi}^{\bar{p}}\left[\dot{\mathcal{G}}_{\bar{p}rq}{}^{r}\gamma^{q} + [\dot{\mathcal{D}}_{\bar{p}}, \gamma^{q}\dot{\mathcal{D}}_{q}]\right]\varepsilon \\ &\equiv 0\,. \end{aligned}$$

Twisted Supersymmetric Double Field Theory

- Maximal supersymmetric gauged double field theory Lagrangian,

$$\dot{\mathcal{L}}^{\rm Maximal}_{\rm Twisted\ SDFT} = e^{-2\dot{d}}\Big[\ \tfrac{1}{8}(\dot{\mathcal{G}}_{pq}{}^{pq} - \dot{\mathcal{G}}_{\bar{p}\bar{q}}{}^{\bar{p}\bar{q}}) + \tfrac{1}{2}{\rm Tr}(\dot{\mathcal{F}}\dot{\bar{\mathcal{F}}}) - i\bar{\rho}\dot{\mathcal{F}}\rho' \\
+ i\bar{\psi}_{\bar{p}}\gamma_{q}\dot{\mathcal{F}}\bar{\gamma}^{\bar{p}}\psi'^{q} + i\tfrac{1}{2}\bar{\rho}\gamma^{p}\dot{\mathcal{D}}_{p}\rho - i\bar{\psi}^{\bar{p}}\dot{\mathcal{D}}_{\bar{p}}\rho - i\tfrac{1}{2}\bar{\psi}^{\bar{p}}\gamma^{q}\dot{\mathcal{D}}_{q}\psi_{\bar{p}} \\
- i\tfrac{1}{2}\bar{\rho}'\bar{\gamma}^{\bar{p}}\dot{\mathcal{D}}_{\bar{p}}\rho' + i\bar{\psi}'^{p}\dot{\mathcal{D}}_{p}\rho' + i\tfrac{1}{2}\bar{\psi}'^{p}\bar{\gamma}^{\bar{q}}\dot{\mathcal{D}}_{\bar{q}}\psi'_{p}\ \Big]\,.$$

Twisted Supersymmetric Double Field Theory

- Ignoring total derivatives and up to the twistability conditions, the supersymmetric infinitesimal variation of the Lagrangian is

$$\begin{aligned} &\delta_\varepsilon \dot{\mathcal{L}}^{\text{Maximal}}_{\text{Twisted SDFT}} \\ \equiv\ & i\tfrac{1}{48} e^{-2\dot{d}} \left(\bar{\rho}\varepsilon - \bar{\rho}'\varepsilon' + \bar{\varepsilon}\mathcal{C}\rho' + \bar{\varepsilon}\gamma^p\mathcal{C}\psi'_p + \bar{\rho}\mathcal{C}\varepsilon' + \bar{\psi}_{\bar{p}}\mathcal{C}\bar{\gamma}^{\bar{p}}\varepsilon'\right) \times f_{\dot{A}\dot{B}\dot{C}} f^{\dot{A}\dot{B}\dot{C}} \\ & + i\tfrac{1}{8} e^{-2d} (\bar{\varepsilon}\gamma_p\psi_{\bar{q}} - \bar{\varepsilon}'\bar{\gamma}_{\bar{q}}\psi'_p) \mathrm{Tr}\left(\gamma^p \dot{\mathcal{F}}_- \bar{\gamma}^{\bar{q}} \overline{\dot{\mathcal{F}}_-}\right) . \end{aligned}$$

- Requiring the extra condition which we recall here,

$$f_{\dot{A}\dot{B}\dot{C}} f^{\dot{A}\dot{B}\dot{C}} \equiv 0 \,,$$

the action is supersymmetric invariant modulo the self-duality, up to surface integrals.

Twisted Supersymmetric Double Field Theory

- To compare with the untwisted DFT and to identify the newly added terms after the U-twist up to the twistability conditions,

$$
\begin{aligned}
+\dot{\mathcal{G}}_{pq}{}^{pq} \equiv\ & \tfrac{1}{16}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{\mathcal{H}}_{\dot{C}\dot{D}}\dot{\partial}_{\dot{B}}\dot{\mathcal{H}}^{\dot{C}\dot{D}} + \tfrac{1}{4}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}^{\dot{C}}\dot{\mathcal{H}}_{\dot{A}\dot{D}}\dot{\partial}^{\dot{D}}\dot{\mathcal{H}}_{\dot{B}\dot{C}} - \tfrac{1}{2}\dot{\partial}_{\dot{A}}\dot{\partial}_{\dot{B}}\dot{\mathcal{H}}^{\dot{A}\dot{B}} \\
& -2\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{d}\dot{\partial}_{\dot{B}}\dot{d} + 2\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{\partial}_{\dot{B}}\dot{d} + 2\dot{\partial}_{\dot{A}}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{B}}\dot{d} \\
& +\tfrac{1}{8}f_{\dot{A}\dot{B}\dot{C}}f^{\dot{A}\dot{B}}{}_{\dot{D}}\dot{\mathcal{H}}^{\dot{C}\dot{D}} - \tfrac{1}{24}f_{\dot{A}\dot{B}\dot{C}}f_{\dot{D}\dot{E}\dot{F}}\dot{\mathcal{H}}^{\dot{A}\dot{D}}\dot{\mathcal{H}}^{\dot{B}\dot{E}}\dot{\mathcal{H}}^{\dot{C}\dot{F}} - \tfrac{1}{4}f_{\dot{A}\dot{B}\dot{C}}\dot{\mathcal{H}}^{\dot{B}\dot{D}}\dot{\mathcal{H}}^{\dot{C}\dot{E}}\dot{\partial}_{\dot{D}}\dot{\mathcal{H}}_{\dot{E}}{}^{\dot{A}} \\
& +\tfrac{1}{12}f_{\dot{A}\dot{B}\dot{C}}f^{\dot{A}\dot{B}\dot{C}}\,, \\
-\dot{\mathcal{G}}_{\bar{p}\bar{q}}{}^{\bar{p}\bar{q}} \equiv\ & \tfrac{1}{16}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}_{\dot{A}}\dot{\mathcal{H}}_{\dot{C}\dot{D}}\dot{\partial}_{\dot{B}}\dot{\mathcal{H}}^{\dot{C}\dot{D}} + \tfrac{1}{4}\dot{\mathcal{H}}^{\dot{A}\dot{B}}\dot{\partial}^{\dot{C}}\dot{\mathcal{H}}_{\dot{A}\dot{D}}\dot{\partial}^{\dot{D}}\dot{\mathcal{H}}_{\dot{B}\dot{C}} - \tfrac{1}{2}\dot{\partial}_{\dot{A}}\dot{\partial}_{\dot{B}}\dot{\mathcal{H}}^{\dot{A}\dot{B}} \\
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\end{aligned}
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SUMMARY

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- The semi-covariant formulation also works for the twisted semi-covariant derivative.
- We successfully twisted the semi-covariant formulations of the $\mathcal{N} = 2$ and the $\mathcal{N} = 1$, $D = 10$ SDFT.
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Thank you!