

# Dark Energy and Dark Matter from Quantum Gravity

Hyun Seok Yang  
(양 현 석)

Center for Quantum Spacetime  
Sogang University

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## In this talk

- ☞ I emphasize that noncommutative (NC) spacetime necessarily implies emergent spacetime if spacetime at microscopic scales should be viewed as NC.
- ☞ The emergent gravity from NC  $U(1)$  gauge theory is the large  $N$  duality and the emergent spacetime picture admits a background-independent formulation of quantum gravity.
- ☞ In order to understand NC spacetime correctly, we need to deactivate the thought patterns that we have installed in our brains and taken for granted for so many years.
- ☞ Emergent gravity predicts the existence of the dark composition of our Universe.

# NC spacetime introduces the gauge-gravity duality

1. Recall that quantum mechanics is mechanics on NC phase space whose coordinate generators satisfy the commutation relation

$$[x^i, p_j] = i\hbar\delta_j^i$$

2. The mathematical structure of NC spacetime is essentially the same as the NC phase space in quantum mechanics:

$$[y^\mu, y^\nu] = i\theta^{\mu\nu},$$

where  $p_\mu = B_{\mu\nu} y^\nu$  and  $B_{\mu\nu} \equiv (\theta^{-1})_{\mu\nu}$ .

3. Everything on NC spacetime bears some analogy with quantum mechanics:

NC phase space  $\Rightarrow$  Wave-particle duality

NC spacetime  $\Rightarrow$  Gauge-gravity duality

## NC spacetime necessarily implies emergent spacetime

1. Recall that  $f(x + a) = U(a)^\dagger f(x) U(a)$  where  $U(a) = e^{-ip \cdot a / \hbar}$ , and so every point on NC space is unitarily equivalent.  
There is no space but an algebra  $\mathcal{A}_\theta$  only. Thus the NC *space* is a misnomer.
2. NC space introduces a separable Hilbert space  $\mathcal{H}$  and so  $\mathcal{H}$  has a countable basis.  
Dynamical variables become operators acting on the Hilbert space.
3. NC spacetime implies a paradigm shift: **Geometry  $\rightarrow$  Algebra**  
Hilbert space  $\mathcal{H}$ : **dynamical variables  $\rightarrow N \times N$  matrices**  
where  $N = \dim(\mathcal{H}) \rightarrow \infty$ .
4. Large N duality or gauge/gravity duality such as the AdS/CFT correspondence is an inevitable consequence of the NC spacetime.
5. NC spacetime admits a (dynamical) diffeomorphism symmetry which precisely acts as the novel form of the equivalence principle for electromagnetic force.

# NC Fields As Large N Matrices

Consider a two-dimensional NC space

$$[x, y] = i\theta \quad \Leftrightarrow \quad [a, a^\dagger] = 1 \quad \text{where } a = \frac{x+iy}{\sqrt{2\theta}}.$$

Since  $\mathcal{H} = \{ |n\rangle; n = 0, 1, \dots, \infty \}$  and  $\sum_{n=0}^{\infty} |n\rangle\langle n| = 1_{\mathcal{H}}$ , for  $\phi_1, \phi_2 \in \mathcal{A}_\theta$ ,

$$\phi_1(x, y) = \sum_{n, m=0}^{\infty} |n\rangle\langle n| \phi_1(x, y) |m\rangle\langle m| \equiv M_{nm} |n\rangle\langle m|,$$

$$\phi_2(x, y) = \sum_{n, m=0}^{\infty} |n\rangle\langle n| \phi_2(x, y) |m\rangle\langle m| \equiv N_{nm} |n\rangle\langle m|,$$

$$(\phi_1 \star \phi_2)(x, y) = \sum_{n, l, m=0}^{\infty} |n\rangle\langle n| \phi_1(x, y) |l\rangle\langle l| \phi_2(x, y) |m\rangle\langle m| = M_{nl} N_{lm} |n\rangle\langle m|,$$

NC fields  $\phi_a(x, y)$  in  $\mathcal{A}_\theta =$  adjoint operators acting on a separable Hilbert space  $\mathcal{H} = N \times N$  matrices in  $End(\mathcal{H}) \equiv \mathcal{A}_N$  with  $N = \dim(\mathcal{H}) \rightarrow \infty$ .

Ordering in  $\mathcal{A}_\theta =$  ordering in  $\mathcal{A}_N$  and  $Tr_N = Tr_{\mathcal{H}} = \int \frac{dx dy}{2\pi\theta}$ .

# Large N gauge theory from NC U(1) gauge theory

Consider a  $(d+2n)$ -dimensional NC  $U(1)$  gauge theory on  $\mathbb{R}_C^d \times \mathbb{R}_{NC}^{2n}$  whose coordinates are  $X^M = (x^\mu, y^a)$ ,  $M = 0, 1, \dots, D-1$ ,  $\mu = 0, 1, \dots, d-1$ ,  $a = 1, \dots, 2n$  where

$$[y^a, y^b] = i\theta^{ab}.$$

The  $D=(d+2n)$ -dimensional  $U(1)$  connections are split as

$$D_M(X) = \partial_M - i\hat{A}_M(x, y) = (D_\mu, D_a)(x, y)$$

where  $\partial_a \equiv \text{ad}_{p_a} = -i[p_a, \cdot]$  with  $p_a = B_{ab}y^b$  and

$$D_a(x, y) = -i(p_a + \hat{A}_a(x, y)) \equiv -i\phi_a(x, y).$$

Using the matrix representation  $\mathcal{A}_\theta \rightarrow \mathcal{A}_N$  by

$$\Xi(x, y) \mapsto \Xi(x) \in \mathcal{A}_N,$$

the  $D$ -dimensional NC  $U(1)$  gauge theory is exactly mapped to the  $d$ -dimensional  $U(N)$  Yang-Mills theory

$$\begin{aligned} S &= -\frac{1}{4G_{YM}^2} \int d^D X (\hat{F}_{MN} - B_{MN})^2 \\ &= -\frac{1}{g_{YM}^2} \int d^d x \text{Tr} \left( \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} D_\mu \phi_a D^\mu \phi_a - \frac{1}{4} [\phi_a, \phi_b]^2 \right) \end{aligned}$$

where  $B_{MN} = \begin{pmatrix} 0 & 0 \\ 0 & B_{ab} \end{pmatrix}$ .

$d=0$ : IKKT,  $d=1$ : BFSS,  $d=2$ : DVV,  $\dots$ ,  $d=4$ : AdS/CFT,  $\dots$ .

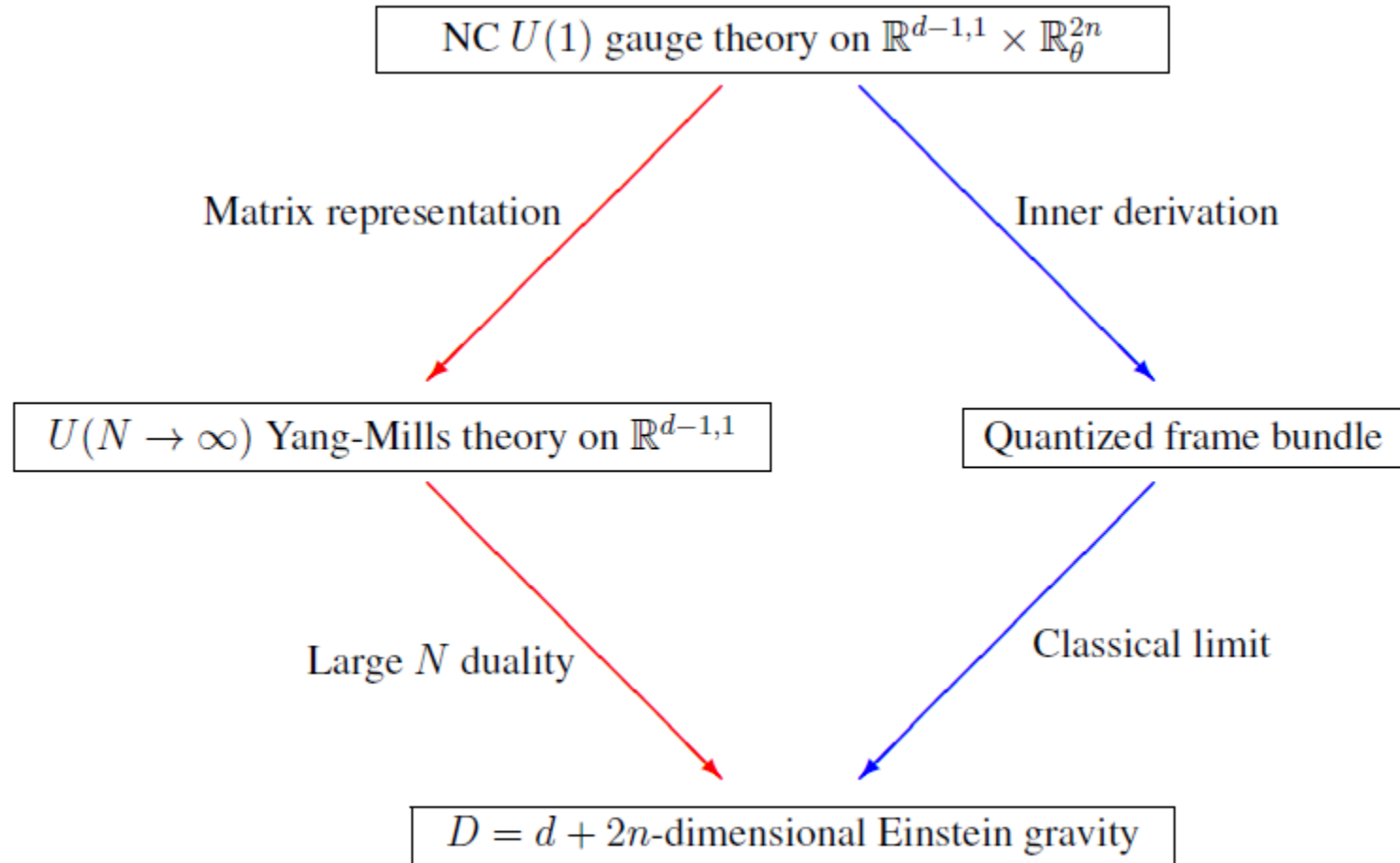


Figure 1: Flowchart for emergent gravity

# NC spacetime as NC Coulomb branch

I will make an important observation that NC spacetime arises as a vacuum solution in the Coulomb branch of a large  $N$  gauge theory to demonstrate the large  $N$  duality.

1. The conventional choice of vacuum in the Coulomb branch of  $U(N)$  Yang-Mills theory is given by

$$[\phi_a, \phi_b]|_{\text{vac}} = 0 \quad \Rightarrow \quad \langle \phi_a \rangle_{\text{vac}} = \text{diag}((\alpha_a)_1, (\alpha_a)_2, \dots, (\alpha_a)_N)$$

In this case the  $U(N)$  gauge symmetry is broken to  $U(1)^N$ .

2. If we consider the  $N \rightarrow \infty$  limit, the large  $N$  limit opens a new phase of the Coulomb branch given by

$$[\phi_a, \phi_b]|_{\text{vac}} = -iB_{ab} \quad \Rightarrow \quad \langle \phi_a \rangle_{\text{vac}} = p_a \equiv B_{ab}y^b$$

where the vacuum moduli  $y^a \in \mathbb{R}_{NC}^{2n}$  satisfy the Moyal-Heisenberg algebra.

3. Suppose that fluctuations around the NC vacuum take the form

$$D_\mu = \partial_\mu - i\hat{A}_\mu(x, y), \quad \phi_a = p_a + \hat{A}_a(x, y)$$

The adjoint scalar fields now obey the deformed algebra given by

$$[\phi_a, \phi_b] = -i(B_{ab} - \hat{F}_{ab})$$



# Large N duality from NC spacetime

Plugging the fluctuations into the  $d$ -dimensional  $U(N \rightarrow \infty)$  Yang-Mills theory, we get the  $D = (d + 2n)$ -dimensional NC  $U(1)$  gauge theory and thus arrive at the reversed version of the equivalence

$$\begin{aligned} S &= -\frac{1}{g_{YM}^2} \int d^d x \operatorname{Tr} \left( \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} D_\mu \phi_a D^\mu \phi_a - \frac{1}{4} [\phi_a, \phi_b]^2 \right) \\ &= -\frac{1}{4G_{YM}^2} \int d^D X (\hat{F}_{MN} - B_{MN})^2 \end{aligned}$$

where  $X^M = (x^\mu, y^a)$  are  $D$ -dimensional coordinates and  $D$ -dimensional connections are defined by

$$D_M(X) = \partial_M - i\hat{A}_M(x, y) = (D_\mu, D_a)(x, y)$$

whose field strength is given by

$$\hat{F}_{MN}(X) = \partial_M \hat{A}_N - \partial_N \hat{A}_M - i[\hat{A}_M, \hat{A}_N]_\star.$$

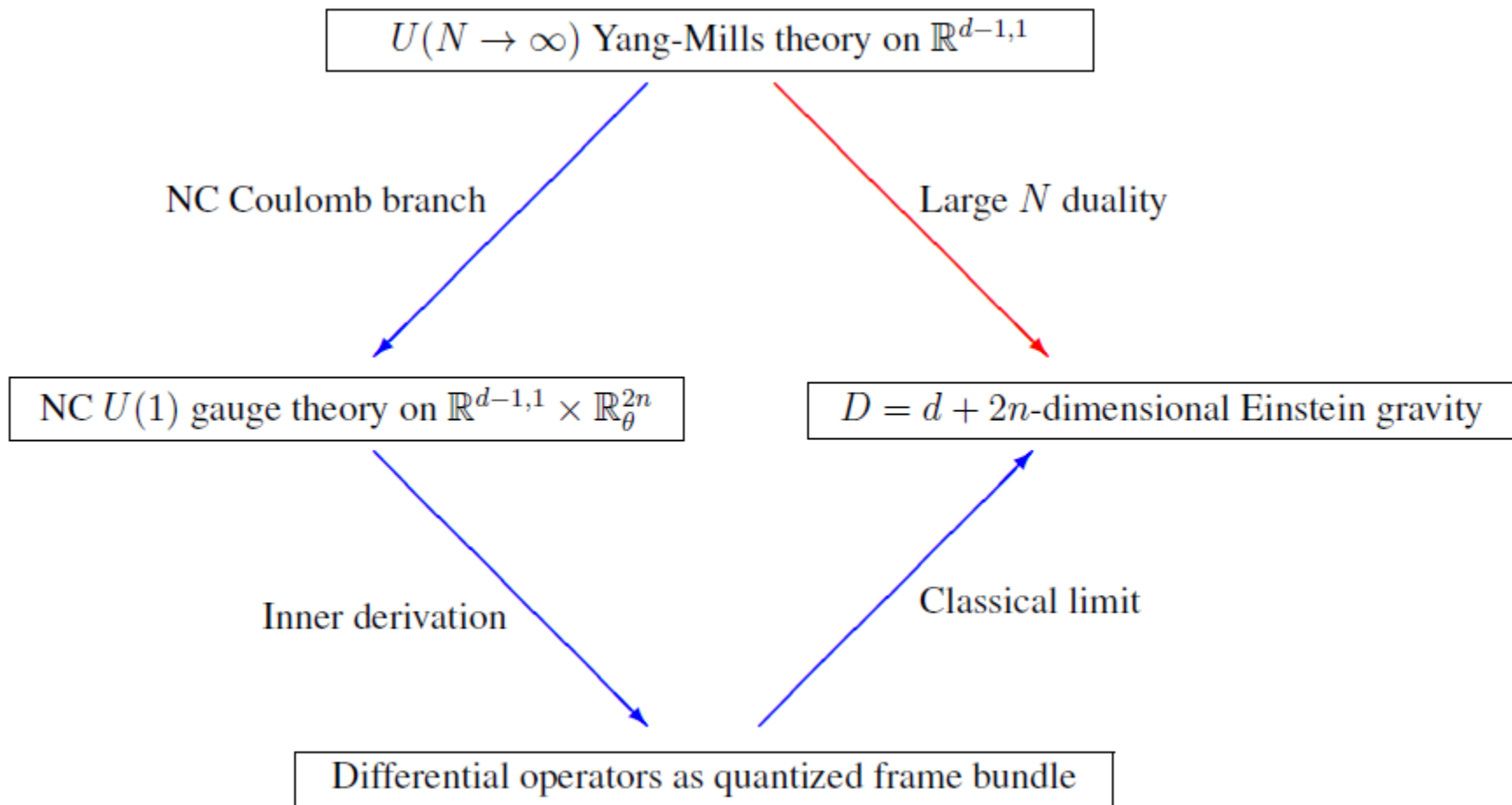


Figure 2: Flowchart for large  $N$  duality

## Inner/Outer automorphism

Therefore there should be some way to map the NC  $U(1)$  gauge theory to the Einstein gravity according to the (conjectural) large  $N$  duality. To be more specific, consider the inverse metric in Einstein gravity given by

$$\left(\frac{\partial}{\partial s}\right)^2 = E_A \otimes E_A = g^{MN}(X)\partial_M \otimes \partial_N$$

where  $E_A = E_A^M(X)\partial_M$  are orthonormal frames on the tangent bundle  $T\mathcal{M}$  of a  $D$ -dimensional spacetime manifold  $\mathcal{M}$ . In order to complete the gauge-gravity duality in Figs. 1 and 2, it is thus necessary to realize the vector fields  $E_A = E_A^M(X)\partial_M \in \Gamma(T\mathcal{M})$  in terms of NC  $U(1)$  gauge fields.

A decisive clue is coming from the fact that the NC  $\star$ -algebra  $\mathcal{A}_\theta$  generated by the Moyal-Heisenberg algebra always admits a nontrivial inner automorphism  $\mathfrak{S}$ . The infinitesimal generators of  $\mathfrak{S}$  form an inner derivation defined by the adjoint operation

$$\mathcal{A}_\theta^d \rightarrow \mathfrak{D}^d : f \mapsto \text{ad}_f = -i[f, \cdot]$$

for any  $f \in \mathcal{A}_\theta^d \equiv \mathcal{A}_\theta(C^\infty(\mathbb{R}^{d-1,1})) \cong \mathcal{A}_\theta \times C^\infty(\mathbb{R}^{d-1,1})$

Definitely the derivation  $\mathfrak{D}^d$  is a Lie algebra homomorphism, i.e.,

$$\text{ad}_{[f,g]} = i[\text{ad}_f, \text{ad}_g]$$

## Vielbeins from inner derivation

Consider the derivation algebra generated by the dynamical variables defined by

$$\widehat{V}_A = \{i \text{ ad}_{D_A} = [D_A, \cdot] | D_A(x, y) = (D_\mu, D_a)(x, y)\} \in \mathfrak{D}^d$$

where  $D_A(x, y) = -i\phi_A(x, y)$ . In a large-distance limit, i.e.  $|\theta| \rightarrow 0$ , one can expand the NC vector fields using the explicit form of the Moyal  $\star$ -product. The result takes the form

$$\widehat{V}_A = V_A^M(x, y) \frac{\partial}{\partial Y^M} + \sum_{p=2}^{\infty} V_A^{a_1 \dots a_p}(x, y) \frac{\partial}{\partial y^{a_1}} \dots \frac{\partial}{\partial y^{a_p}} \in \mathfrak{D}^d,$$

where  $V_A^\mu = \delta_A^\mu$ . Thus the Taylor expansion of NC vector fields generates an infinite tower of the so-called polyvector fields. Note that the leading term gives rise to the ordinary vector fields that will be identified with a frame basis associated with the tangent bundle  $T\mathcal{M}$  of an emergent spacetime manifold  $\mathcal{M}$ .

Let us truncate the above polyvector fields to ordinary vector fields given by

$$\mathfrak{X}(\mathcal{M}) = \left\{ V_A = V_A^M(x, y) \frac{\partial}{\partial X^M} | A, M = 0, 1, \dots, D-1 \right\}$$

where  $X^M = (x^\mu, y^a)$  are local coordinates on a  $D$ -dimensional emergent *Lorentzian* manifold  $\mathcal{M}$ .

# Emergent gravity from NC spacetime

The orthonormal vielbeins on  $T\mathcal{M}$  are then defined by the relation

$$V_A = \lambda E_A \in \Gamma(T\mathcal{M})$$

or on  $T^*\mathcal{M}$

$$v^A = \lambda^{-1} e^A \in \Gamma(T^*\mathcal{M}).$$

The conformal factor  $\lambda \in C^\infty(\mathcal{M})$  is determined by the volume preserving condition

$$\mathcal{L}_{V_A} \nu_t = (\nabla \cdot V_A + (d - 2n) V_A \ln \lambda) \nu_t = 0, \quad \forall A = 0, 1, \dots, D - 1,$$

where

$$\nu_t \equiv d^d x \wedge \nu = \lambda^2 d^d x \wedge v^1 \wedge \dots \wedge v^{2n}$$

is a  $D$ -dimensional volume form on  $\mathcal{M}$ .

In the end, the Lorentzian metric on a  $D$ -dimensional spacetime manifold is given by

$$\begin{aligned} ds^2 &= \mathcal{G}_{MN}(X) dX^M \otimes dX^N = e^A \otimes e^A \\ &= \lambda^2 v^A \otimes v^A = \lambda^2 (\eta_{\mu\nu} dx^\mu dx^\nu + v_b^a v_c^a (dy^b - \mathbf{A}^b)(dy^c - \mathbf{A}^c)) \end{aligned}$$

where  $\mathbf{A}^b := A_\mu^b(x, y) dx^\mu$ . Therefore the NC field theory representation of the  $d$ -dimensional large  $N$  gauge theory in the NC Coulomb branch provides a powerful machinery to identify gravitational variables dual to large  $N$  matrices.

# NC Electromagnetism from Matrix Model

Let us start with a zero-dimensional matrix model with a bunch of  $N \times N$  Hermitian matrices,  $\{\phi_a \in \mathcal{A}_N | a = 1, \dots, 2n\}$ , whose action is given by

$$S = -\frac{1}{4} \sum_{a,b=1}^{2n} \text{Tr} [\phi_a, \phi_b]^2.$$

We require that the matrix algebra  $\mathcal{A}_N$  is associative, which leads to the Jacobi identity

$$[\phi_a, [\phi_b, \phi_c]] + [\phi_b, [\phi_c, \phi_a]] + [\phi_c, [\phi_a, \phi_b]] = 0.$$

We also assume the action principle, from which we yield the equation of motion

$$\sum_{b=1}^{2n} [\phi_b, [\phi_a, \phi_b]] = 0.$$

First suppose that the vacuum configuration of  $\mathcal{A}_N$  is given by

$$\langle \phi_a \rangle_{\text{vac}} = p_a \in \mathcal{A}_N,$$

An obvious solution in the limit  $N \rightarrow \infty$  is given by the Moyal-Heisenberg algebra

$$[p_a, p_b] = -iB_{ab},$$

where  $(B_{ab}) = -L_P^{-2}(\mathbf{1}_n \otimes i\sigma^2)$  is a  $2n \times 2n$  constant symplectic matrix.

A general solution is generated by considering all possible deformations of the Moyal-Heisenberg algebra. They take the form

$$\phi_a = p_a + \hat{A}_a \in \mathcal{A}_N,$$

obeying the deformed algebra

$$[\phi_a, \phi_b] = -i(B_{ab} - \hat{F}_{ab}),$$

where  $\hat{F}_{ab} = \partial_a \hat{A}_b - \partial_b \hat{A}_a - i[\hat{A}_a, \hat{A}_b] \in \mathcal{A}_N$  with the definition  $\partial_a \equiv \text{ad}_{p_a} = -i[p_a, \cdot]$ .

For the general matrix  $\phi_a \in \mathcal{A}_N$  to be a solution the set of matrices  $\hat{F}_{ab} \in \mathcal{A}_N$ , must obey the following equations

$$\hat{D}_a \hat{F}_{bc} + \hat{D}_b \hat{F}_{ca} + \hat{D}_c \hat{F}_{ab} = 0,$$

$$\sum_{b=1}^{2n} \hat{D}_b \hat{F}_{ab} = 0,$$

where

$$\hat{D}_a \hat{F}_{bc} \equiv \text{ad}_{\phi_a} \hat{F}_{bc} = -i[\phi_a, \hat{F}_{bc}] = -[\phi_a, [\phi_b, \phi_c]].$$

Let us apply the Lie algebra homomorphism to yield

$$X_{D_A F_{BC}} = [V_A, [V_B, V_C]] \in \Gamma(TM)$$

It is then straightforward to get the following correspondence

$$\begin{aligned} D^B F_{AB} = 0 & \quad \Leftrightarrow \quad [V^B, [V_A, V_B]] = 0, \\ D_A F_{BC} + \text{cyclic} = 0 & \quad \Leftrightarrow \quad [V_A, [V_B, V_C]] + \text{cyclic} = 0. \end{aligned}$$



Note that the torsion  $T$  and curvature  $R$  are multi-linear differential operators

$$\begin{aligned} T(X, Y) &= \nabla_X Y - \nabla_Y X - [X, Y], \\ R(X, Y)Z &= [\nabla_X, \nabla_Y]Z - \nabla_{[X, Y]}Z, \end{aligned}$$

where  $X, Y$  and  $Z$  are vector fields on  $M$ . It is easy to derive the relation

$$\begin{aligned} T(V_A, V_B) &= \lambda^2 T(E_A, E_B), \\ R(V_A, V_B)V_C &= \lambda^3 R(E_A, E_B)E_C. \end{aligned}$$

After imposing the torsion free condition  $T(E_A, E_B) = 0$ , it is easy to derive the identity below

$$R(E_A, E_B)E_C + \text{cyclic} = \lambda^{-3} \left( [V_A, [V_B, V_C]] + \text{cyclic} \right).$$

Therefore we see that the Bianchi identity for NC  $U(1)$  gauge fields is equivalent to the first Bianchi identity for the Riemann curvature tensors, i.e.,

$$D_A F_{BC} + \text{cyclic} = 0 \quad \Leftrightarrow \quad R(E_A, E_B)E_C + \text{cyclic} = 0.$$

The mission for the equations of motion is more involved.

So let us focus on four dimensions, i.e.,  $n = 2$ .

Basically we are expecting the Einstein equations

$$D^B F_{AB} = 0 \quad \Leftrightarrow \quad R_{AB} = 8\pi G \left( T_{AB} - \frac{1}{2} \delta_{AB} T \right).$$

After a straightforward but tedious calculation, we get a remarkably simple but cryptic result

$$R_{AB} = -\frac{1}{\lambda^2} \left[ g_D^{(+a)} g_D^{(-b)} \left( \eta_{AC}^a \bar{\eta}_{BC}^b + \eta_{BC}^a \bar{\eta}_{AC}^b \right) - g_C^{(+a)} g_D^{(-b)} \left( \eta_{AC}^a \bar{\eta}_{BD}^b + \eta_{BC}^a \bar{\eta}_{AD}^b \right) \right].$$

To get the above result, we have introduced the structure equation of vector fields  $V_A \in \Gamma(TM)$

$$[V_A, V_B] = -g_{AB}{}^C V_C$$

and the canonical decomposition

$$g_{ABC} = g_C^{(+a)} \eta_{AB}^a + g_C^{(-\dot{a})} \bar{\eta}_{AB}^{\dot{a}}.$$

First it is convenient to decompose the energy-momentum tensor into two parts

$$8\pi GT_{AB}^{(M)} = -\frac{1}{\lambda^2} \left( g_{ACD}g_{BCD} - \frac{1}{4}\delta_{AB}g_{CDE}g_{CDE} \right),$$

$$8\pi GT_{AB}^{(L)} = \frac{1}{2\lambda^2} \left( \rho_A\rho_B - \Psi_A\Psi_B - \frac{1}{2}\delta_{AB}(\rho_C^2 - \Psi_C^2) \right),$$

where  $\rho_A \equiv g_{BAB}$  and  $\Psi_A \equiv -\frac{1}{2}\epsilon^{ABCD}g_{BCD}$ .

A close inspection reveals that the first one is the Maxwell energy-momentum tensor given by

$$T_{ab}^{(em)} = \frac{\hbar^2 c^2}{g_{YM}^2} \left( F_{ac}F_{bc} - \frac{1}{4}\delta_{ab}F_{cd}F_{cd} \right),$$

but the second one seems to be very mysterious.

In order to descry closer aspects of the second energy-momentum tensor, let us consider the following decomposition

$$\rho_a\rho_b = \frac{1}{4}\delta_{ab}\rho_c^2 + \left( \rho_a\rho_b - \frac{1}{4}\delta_{ab}\rho_c^2 \right),$$

$$\Psi_a\Psi_b = \frac{1}{4}\delta_{ab}\Psi_c^2 + \left( \Psi_a\Psi_b - \frac{1}{4}\delta_{ab}\Psi_c^2 \right).$$

In a long wavelength limit, the quadruple modes can be ignored and then it behaves like a cosmological constant

$$T_{ab}^{(L)} = -\frac{c^4 R}{32\pi G} \delta_{ab}$$

where  $R = \frac{1}{2\lambda^2} (\rho_a \rho_b + \Psi_a \Psi_b) \delta^{ab}$ . So it may be related to dark energy/dark matter.

In order to get a corresponding result in (3+1)-dimensional Lorentzian spacetime, let us take the analytic continuation defined by  $x^4 = ix^0$

Under this Wick rotation,

$$\delta_{AB} \rightarrow \eta_{AB}, \quad \Psi_A \rightarrow i\Psi_A$$

the so-called Poisson-Liouville energy-momentum (PLEM) tensor is given by

$$T_{\mu\nu}^{(L)} = \frac{1}{16\pi G_4 \lambda^2} \left( \rho_\mu \rho_\nu + \Psi_\mu \Psi_\nu - \frac{1}{2} g_{\mu\nu} (\rho_\lambda^2 + \Psi_\lambda^2) \right),$$

Note that  $\rho_\mu$  and  $\Psi_\mu$  are four vectors and random fluctuations in nature.

So they are classified into two classes:

$(\rho_\mu, \Psi_\mu)$ : spacelike vectors, i.e.,  $\rho_\mu \rho_\nu g^{\mu\nu} > 0$ , etc.

$(\rho_\mu, \Psi_\mu)$ : timelike vectors, i.e.,  $\rho_\mu \rho_\nu g^{\mu\nu} < 0$ , etc.

Given a timelike unit vector field  $u^\mu$ , *i.e.*,  $u^\mu u_\mu = -1$ , the Raychaudhuri equation in four dimensions is given by

$$\dot{\Theta} - \dot{u}^\mu_{;\mu} + \Sigma_{\mu\nu} \Sigma^{\mu\nu} - \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{3} \Theta^2 = -R_{\mu\nu} u^\mu u^\nu$$

where

$$-R_{\mu\nu} u^\mu u^\nu = -\frac{1}{2\lambda^2} u^\mu u^\nu (\rho_\mu \rho_\nu + \Psi_\mu \Psi_\nu)$$

Suppose that all the terms except the expansion evolution,  $\dot{\Theta}$ , in the Raychaudhuri equation vanish or become negligible.

In this case the Raychaudhuri equation reduces to

$$\dot{\Theta} = -\frac{1}{2\lambda^2} u^\mu u^\nu (\rho_\mu \rho_\nu + \Psi_\mu \Psi_\nu).$$

Note that  $\dot{\Theta} \approx \frac{R}{4}$  and  $R < 0$  when  $\rho_\mu$  and  $\Psi_\mu$  are timelike while  $R > 0$  when  $\rho_\mu$  and  $\Psi_\mu$  are spacelike.

$$\dot{\Theta} \approx \frac{R}{4} < 0 \text{ for timelike fluctuations}$$

$$\dot{\Theta} \approx \frac{R}{4} > 0 \text{ for spacelike fluctuations}$$

By a simple dimensional argument, it is natural to assess that  $R \sim \frac{1}{L_H^2}$   
Then the PLEM tensor is given by

$$T_{\mu\nu} \approx -\frac{1}{L_P^2 L_H^2} \approx (10^{-3} eV)^4$$

if we identify  $L_H$  with the size of cosmic horizon of our observable universe,

$$L_H \sim 1.3 \times 10^{26} \text{ m},$$

This extended (nonlocal) energy is in good agreement with the observed value of current dark energy.

I quote the footnote in one of my papers.

<sup>7</sup> In the Lorentzian signature, the sign of the Ricci scalar  $R$  depends on whether fluctuations are spacelike ( $R > 0$ ) or timelike ( $R < 0$ ) [12, 5]. In consequence the spacelike perturbations act as a repulsive force whereas the timelike ones act as an attractive force. When considering the fact that the fluctuations in (79) are random in nature and we are living in (3+1) (macroscopic) dimensions, the ratio of the repulsive and attractive components will end in  $\frac{3}{4} : \frac{1}{4} = 75 : 25$ . Is it outrageous to conceive that this ratio curiously coincides with the dark composition of our universe ?