

Topological Defects and Gravitational Waves from Aligned Axions

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1512.05295, 1603.02090, 1606.05552

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Strong CP problem

QCD Lagrangian: $\mathcal{L}_{\text{QCD}} = -\frac{1}{4}G_{\mu\nu}^a G^{\mu\nu a} + \frac{g_s^2}{32\pi^2} \theta G_{\mu\nu}^a \tilde{G}^{\mu\nu a}$

CP violating phase

neutron electric dipole moment

$$d_n \simeq 4.5 \times 10^{-15} \theta e \text{ cm}$$

CP violation

experimental constraint: $|d_n| < 2.9 \times 10^{-26} e \text{ cm}$

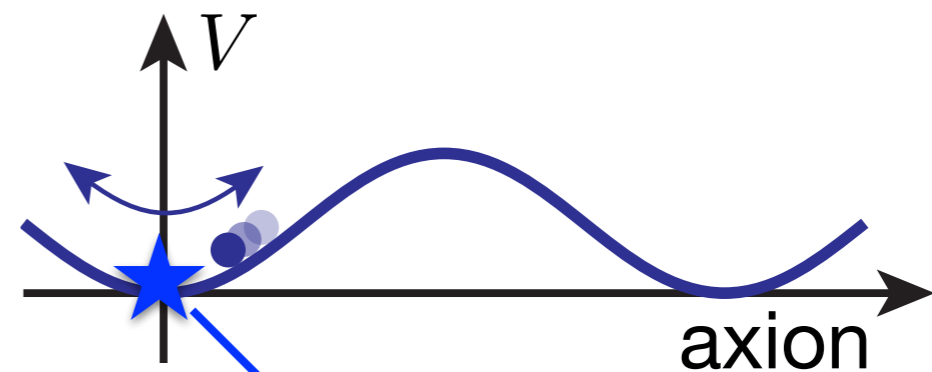
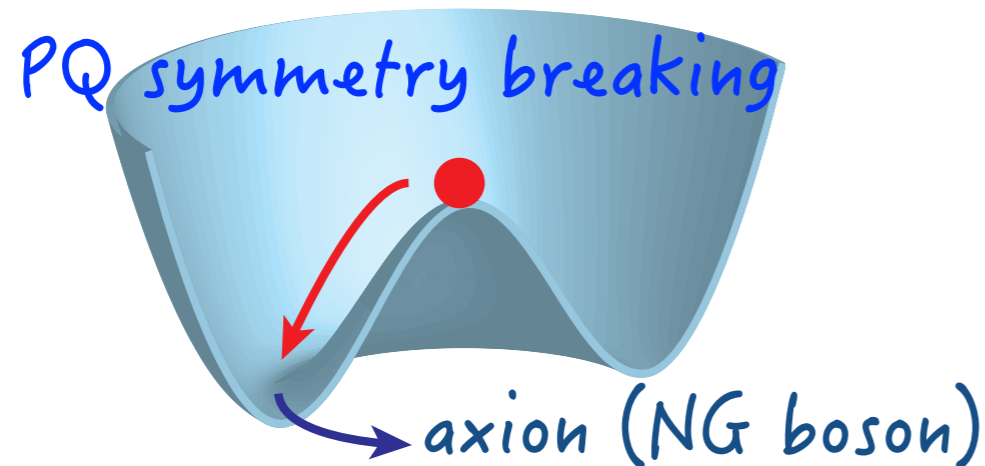
→ $|\theta| < 0.7 \times 10^{-11}$ “strong CP problem”

Why is it extremely small? Fine tuning is necessary!

Solution : Peccei-Quinn mechanism

Peccei, Quinn (1977)

Complex scalar field (PQ field): $\Phi = |\Phi| e^{ia/f_a}$
 +
 U(1) symmetry (PQ symmetry) ↙
axion



$$\mathcal{L} \ni \frac{g_s^2}{32\pi^2} \left(\theta + \frac{a}{F_a} \right) G_{\mu\nu}^a \tilde{G}^{\mu\nu a}$$

↙ dynamical field

$$\theta + \frac{a}{F_a} = 0$$

CP conserving minimum

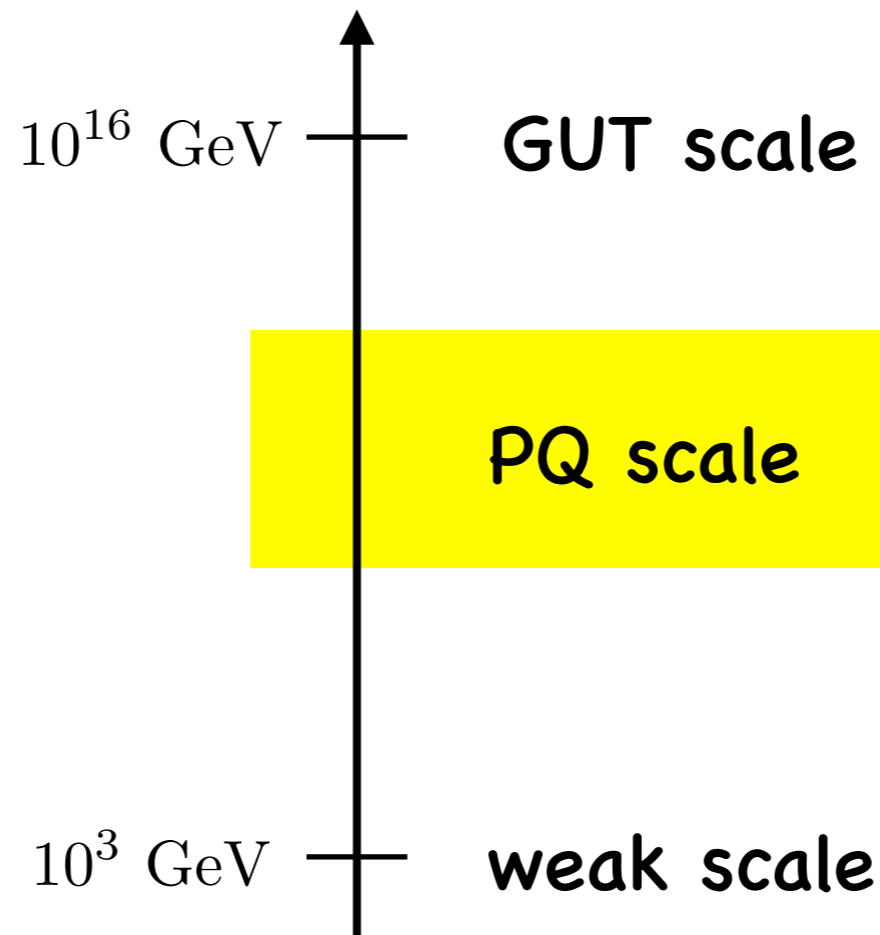
- Strong CP phase is dynamically cancelled!
- New particle "axion" is predicted! → contribute to CDM

Axion puzzle -1-

$$\text{Axion window : } 10^9 \text{ GeV} \lesssim F_a \lesssim 10^{12} \text{ GeV}$$

↑
SN1987A

↑
DM abundance

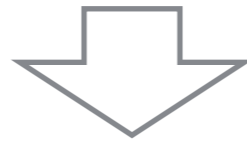


Q1. PQ scale must be in some intermediate scale. Why??

Axion puzzle -2-

Global symmetries must be explicitly broken

by, e.g., Planck suppressed operators : $\mathcal{L} \ni \frac{\Phi^5}{M_P}, \frac{\Phi^6}{M_P^2}, \dots$

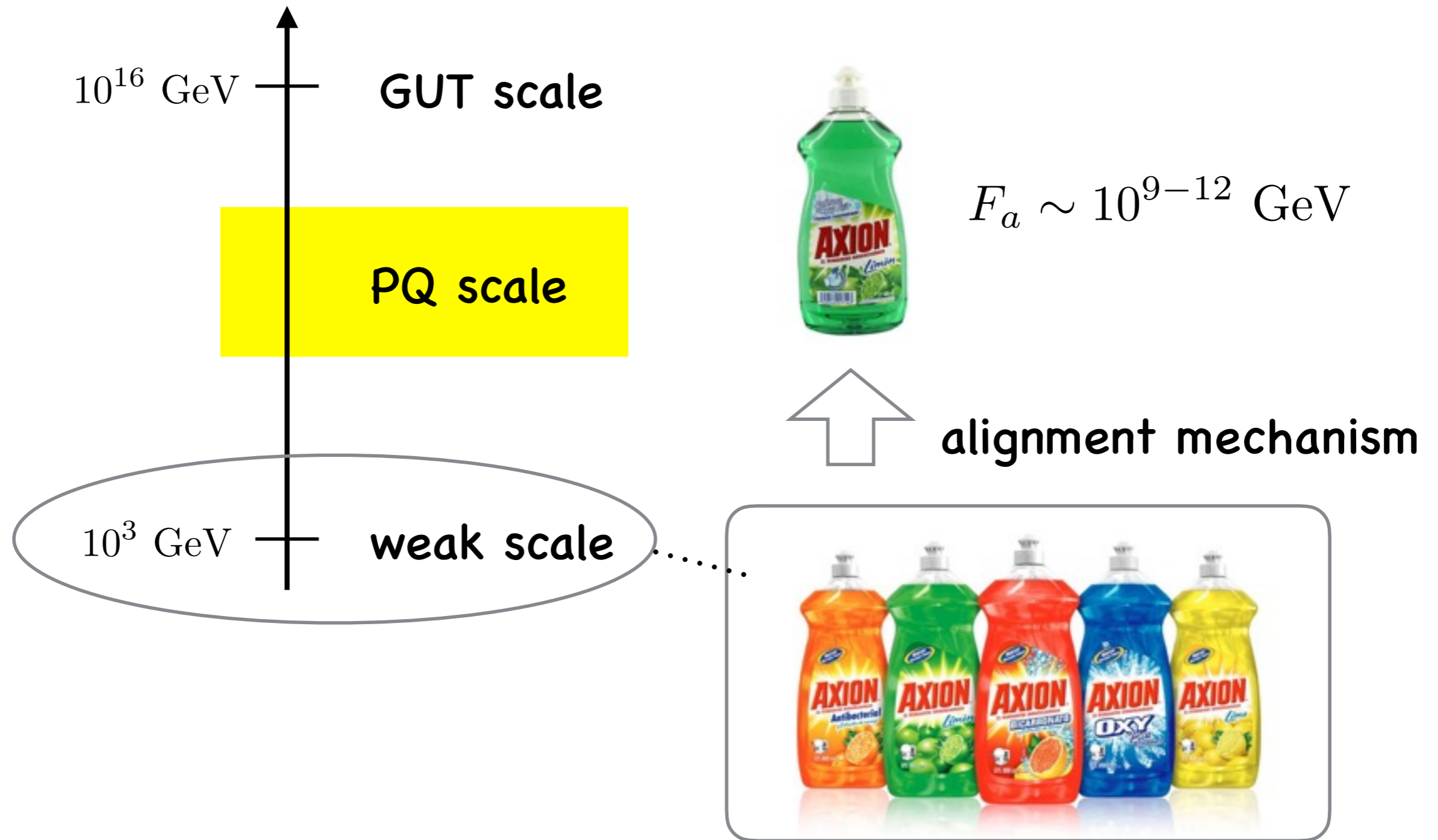


contribute to CP phase

!! PQ mechanism would be broken down

Q2. High quality of PQ symmetry is required. How come?

Our strategy



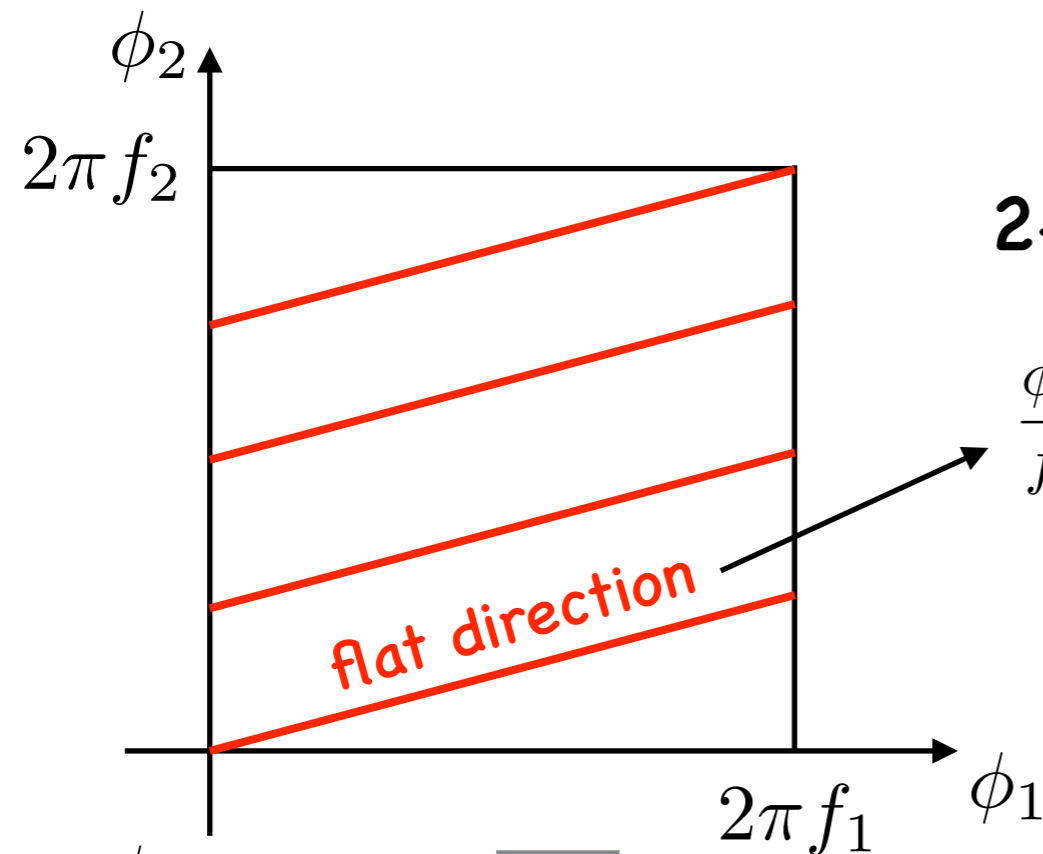
multiple axions with $\langle \Phi_i \rangle = f_i \sim \text{TeV}, \quad i = 1, 2, 3, \dots$

Aligned QCD axion model

T. Higaki, K.S. Jeong, NK, F. Takahashi, 1512.05295; 1603.02090

Alignment mechanism – basic picture

Kim, Niles, Peloso, hep-ph/0419138



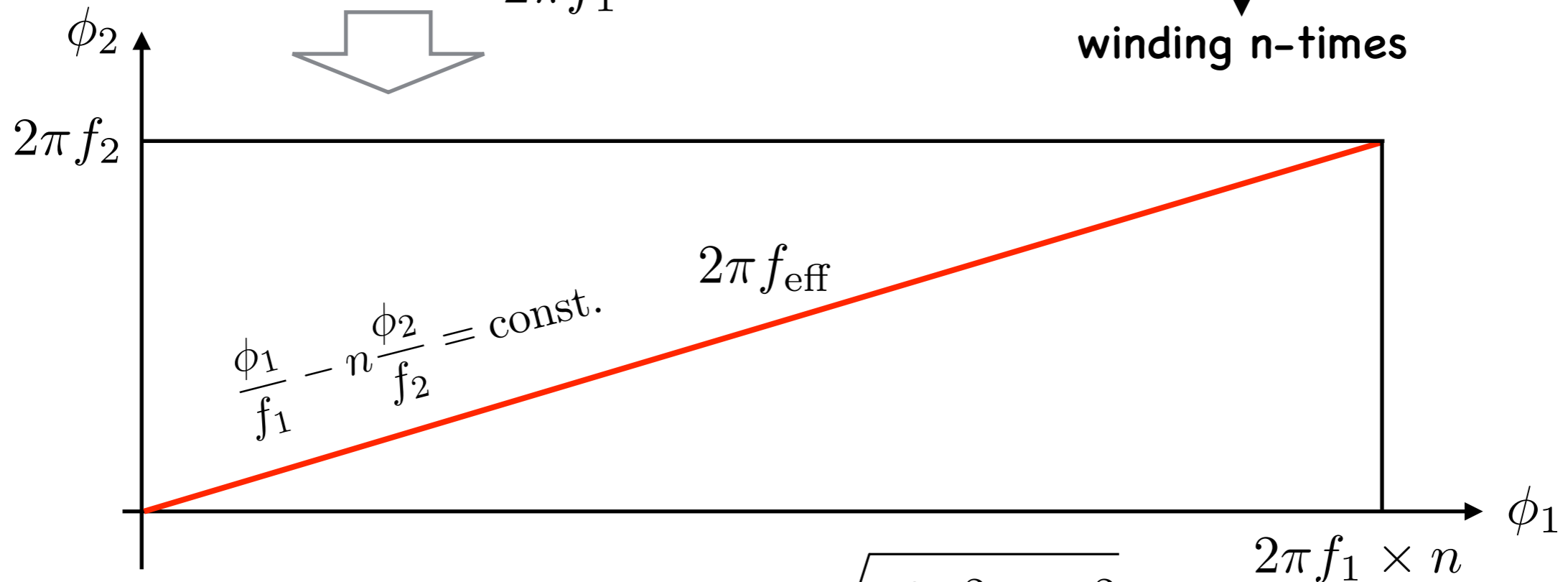
2-axion case $V = \Lambda^4 \left[1 - \cos \left(\frac{\phi_1}{f_1} - \frac{n\phi_2}{f_2} \right) \right]$

$$\frac{\phi_1}{f_1} - n \frac{\phi_2}{f_2} = \text{const.}$$

period of flat direction

$$\phi_1 \rightarrow \phi_1 + 2\pi f_1 \times n, \quad \phi_2 \rightarrow \phi_2 + 2\pi f_2$$

winding n-times



effective decay constant : $f_{\text{eff}} = \sqrt{n^2 f_1^2 + f_2^2}$



Clockwork axion model

K. Choi, H. Kim, S. Yun, 1404.6209

K. Choi, S.H. Im, 1511.00132

D.E. Kaplan, R. Rattazzi, 1511.01827

N-axions: $\phi_i = \phi_i + 2\pi f_i$ ($i = 1, 2, \dots, N$)

potential :
$$V = - \sum_{i=1}^{N-1} \Lambda_i^4 \cos \left(\frac{\phi_i}{f_i} + n_i \frac{\phi_{i+1}}{f_{i+1}} \right)$$

flat direction :

$$\phi_{\text{flat}} = \sum_{i=1}^N (-1)^{i-1} \left(\prod_{j=i}^N n_j \right) \frac{f_i \phi_i}{f_{\text{eff}}} \quad \text{with} \quad f_{\text{eff}} = \sqrt{\sum_{i=1}^N \left(\prod_{j=i}^N n_j^2 \right) f_i^2}$$

$(n_N \equiv 1)$



Aligned "QCD" axion model

T. Higaki, K.S. Jeong, NK, F. Takahashi, 1512.05295

$$\mathcal{L} = \sum_{i=1}^{N-1} \Lambda_i^4 \cos \left(\frac{\phi_i}{f_i} + n_i \frac{\phi_{i+1}}{f_{i+1}} \right) + \frac{g_3^2}{32\pi^2} \frac{k_s \phi_N}{f_N} G^{\mu\nu} \tilde{G}_{\mu\nu} + \frac{g_1^2}{32\pi^2} \frac{k \phi_N}{f_N} B^{\mu\nu} \tilde{B}_{\mu\nu}$$

flat direction = QCD axion

$$a = \sum_{i=1}^N (-1)^{i-1} \left(\prod_{j=i}^N n_j \right) \frac{f_i \phi_i}{f_a} \quad \text{with} \quad f_a = \sqrt{\sum_{i=1}^N \left(\prod_{j=i}^N n_j^2 \right) f_i^2}$$

$$f_i \sim f_N \sim f \quad \text{and} \quad |n_i| \sim n > 0 \quad (i = 1, 2, \dots, N-1)$$

(TeV scale)

$$\longrightarrow f_a \sim e^{N \ln n} f \sim 10^{9-12} \text{ GeV}$$

$f \sim \text{TeV}$, $f_a \sim 10^{10} \text{ GeV}$, $n = 3 \longrightarrow$ we need more than 10 axions

Possible UV completion

N-complex scalar fields with global $U(1)^N$ symmetry

$U(1)^N$ symmetry breaking \Downarrow $V = \sum_{i=1}^N \left(-m_i^2 |\Phi_i|^2 + \frac{\lambda_i}{4} |\Phi_i|^4 \right)$

$$\Phi_i = \left(\rho_i + \frac{f_i}{\sqrt{2}} \right) e^{i\phi_i/f_i} \quad \text{with} \quad f_i = \sqrt{2} \langle |\Phi_i| \rangle$$

$$\Delta\mathcal{L} = \sum_{i=1}^{N-1} \left(\Phi_i \bar{\psi}_i \psi_i + \sum_{a=1}^{n_i} \Phi_{i+1} \bar{\Psi}_{ai} \Psi_{ai} \right) + \text{h.c.}$$

[K. Choi, H. Kim, S. Yun, 1404.6209](#)

$$\Delta V = \sum_{i=1}^{N-1} \epsilon_i \Phi_i \Phi_{i+1}^3 + \text{h.c.} \quad \longrightarrow \quad n_i = 3 \quad \text{and} \quad \Lambda_i = \left(\frac{\epsilon_i}{2} f_i f_{i+1}^3 \right)^{1/4}$$

[D.E. Kaplan, R. Rattazzi, 1511.01827](#)

Quality of the PQ symmetry

T. Higaki, K.S. Jeong, NK, F. Takahashi, 1603.02090

Quality of the PQ symmetry

Carpenter, Dine, Festuccia 0906.1273

potential for QCD axion : QCD instanton + quantum gravity

$$V_{\text{QCD}} = -m_{\text{QCD}}^2 F_a^2 \cos\left(\frac{a}{F_a}\right) - m_{\text{PQ}}^2 \mu^2 \cos\left(\frac{a}{\mu} - \alpha\right)$$

QCD instantonexplicit PQ breaking
from quantum gravity

axion mass : $m_a^2 \simeq m_{\text{QCD}}^2 + m_{\text{PQ}}^2 \cos \alpha$

QCD instanton — $m_{\text{QCD}} \simeq 6 \times 10^{-4} \text{ eV} \left(\frac{\Lambda_{\text{QCD}}}{400 \text{ MeV}}\right)^2 \left(\frac{F_a}{10^{10} \text{ GeV}}\right)^{-1}$

Strong CP phase is modified by the second term

$$\frac{\langle a \rangle}{F_a} \equiv \bar{\theta} \simeq \frac{m_{\text{PQ}}^2 \sin \alpha}{m_{\text{QCD}}^2 + m_{\text{PQ}}^2 \cos \alpha} \frac{\mu}{F_a}$$

1. Planck-suppressed dimension-5 operator (conventional case)

$$\Delta V_5 = \frac{\kappa_5}{5} \frac{\Phi_1^5}{M_P} + \text{h.c.}$$

no alignment mechanism \Downarrow $\langle \Phi_1 \rangle \sim F_a \sim \mu$

$$m_{PQ} \sim 10^6 \text{ GeV} \left(\frac{F_a}{10^{10} \text{ GeV}} \right)^{3/2} \gg m_{\text{QCD}} \quad \& \quad \bar{\theta} \sim \tan \alpha$$

$$\text{QCD instanton} - m_{\text{QCD}} \simeq 6 \times 10^{-4} \text{ eV} \left(\frac{\Lambda_{\text{QCD}}}{400 \text{ MeV}} \right)^2 \left(\frac{F_a}{10^{10} \text{ GeV}} \right)^{-1}$$

too large strong CP phase unless $\tan(\alpha)$ is extremely small...

n.b. $\bar{\theta} \lesssim 10^{-10}$ by NEDM constraint

1. Planck-suppressed dimension-5 operator

$$\Delta V_5 = \frac{\kappa_5}{5} \frac{\Phi_1^5}{M_P} + \text{h.c.}$$

aligned QCD axion



$$m_{\cancel{PQ}}^2 = \frac{5|\kappa_5|}{2\sqrt{2}} \frac{f_1^3}{M_P}, \quad \mu = \frac{f_1}{5} \quad \text{and} \quad \alpha = \arg(\kappa_5)$$

$$\longrightarrow m_{\cancel{PQ}} \simeq 0.03 \text{ MeV} \sqrt{|\kappa_5|} \left(\frac{f_1}{10^3 \text{ GeV}} \right)^{3/2}$$

$$\bar{\theta} \approx 2 \times 10^{-10} \left(\frac{\alpha}{0.1} \right) \left(\frac{F_a/f_1}{10^8} \right)^{-1} \quad \text{for} \quad m_{\cancel{PQ}} \gg m_{\text{QCD}}$$

Experimental bound can be satisfied if alignment mechanism enhances effective decay constant $\sim 10^8$

2. Planck-suppressed dimension-6 operator

$$\Delta V_6 = \frac{\kappa_6}{6} \frac{\Phi_1^6}{M_P^2} + \text{h.c.}$$

$$Z_2 : \Phi_i \rightarrow -\Phi_i$$

aligned QCD axion



$$m_{\cancel{PQ}}^2 = \frac{3|\kappa_6|}{2} \frac{f_1^4}{M_P^2}, \quad \mu = \frac{f_1}{6} \quad \text{and} \quad \alpha = \arg(\kappa_6)$$

$$\longrightarrow m_{\cancel{PQ}} \simeq 0.5 \times 10^{-3} \text{ eV} \sqrt{|\kappa_6|} \left(\frac{f_1}{10^3 \text{ GeV}} \right)^2$$

$$\bar{\theta} \simeq 1.7 \times 10^{-10} \frac{m_{\cancel{PQ}}^2 \cos \alpha}{m_{\text{QCD}}^2 + m_{\cancel{PQ}}^2 \cos \alpha} \left(\frac{\tan \alpha}{0.1} \right) \left(\frac{F_a/f_1}{10^8} \right)^{-1}$$

Experimental bound can be satisfied if alignment mechanism enhances effective decay constant $\sim 10^8$

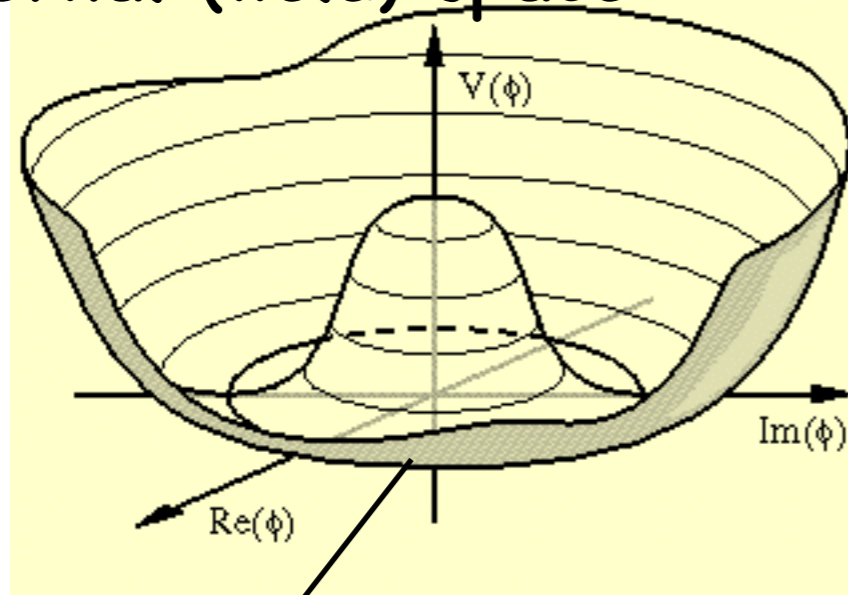
Topological defects in aligned axion models

T. Higaki, K.S. Jeong, NK, T. Sekiguchi F. Takahashi, 1606.05552

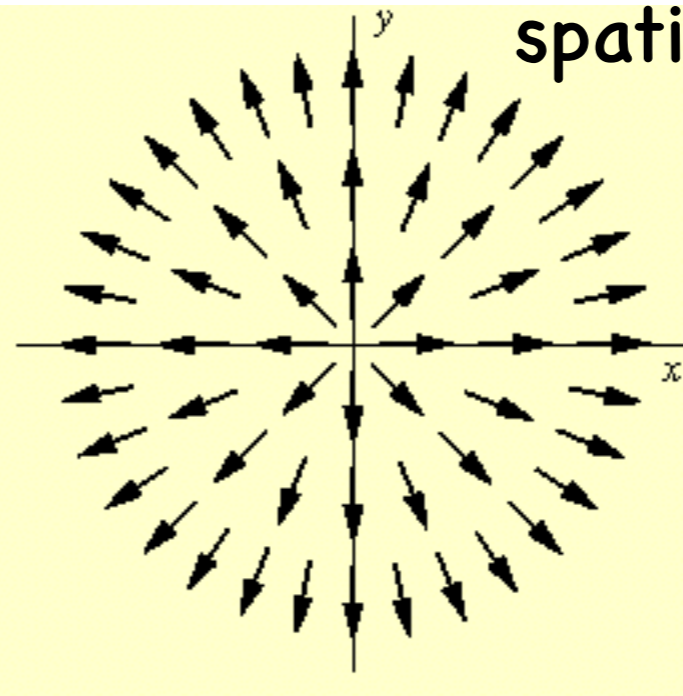
Topological defect 1 : Cosmic string

spontaneous breaking of $U(1)$ symmetry \rightarrow cosmic string

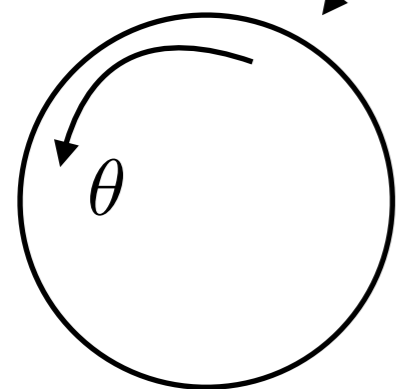
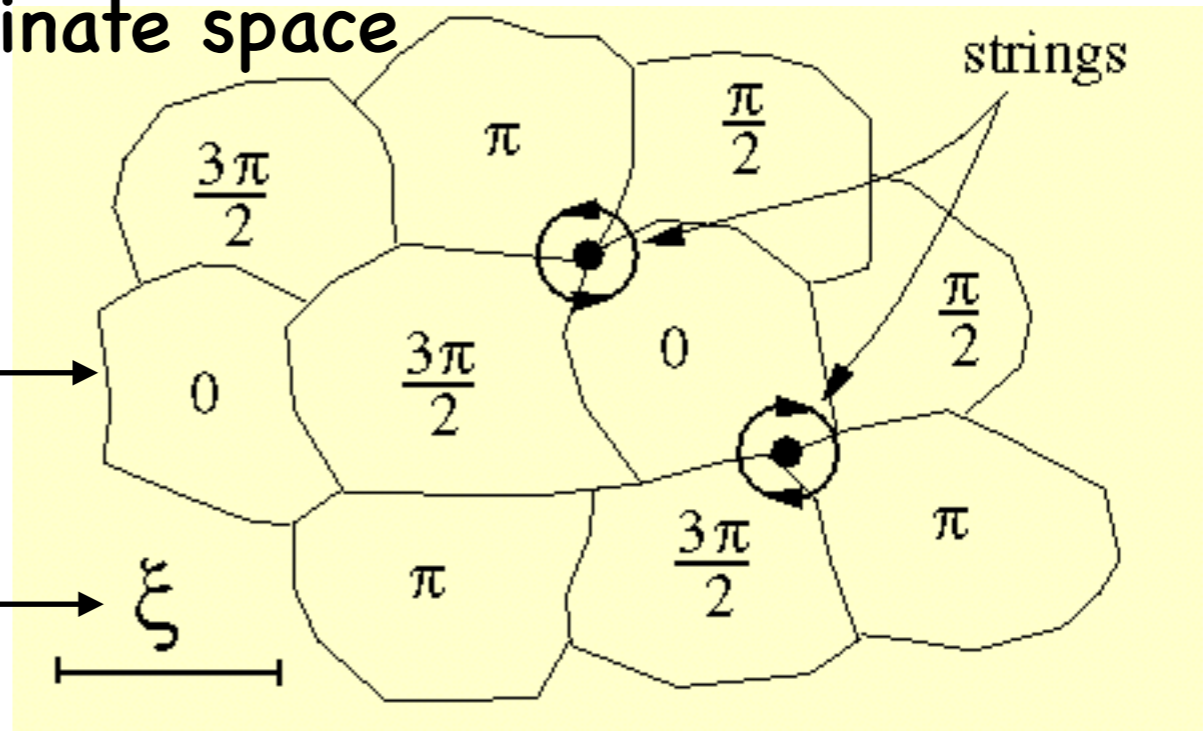
internal (field) space



spatial coordinate



coordinate space



"vacuum circle"

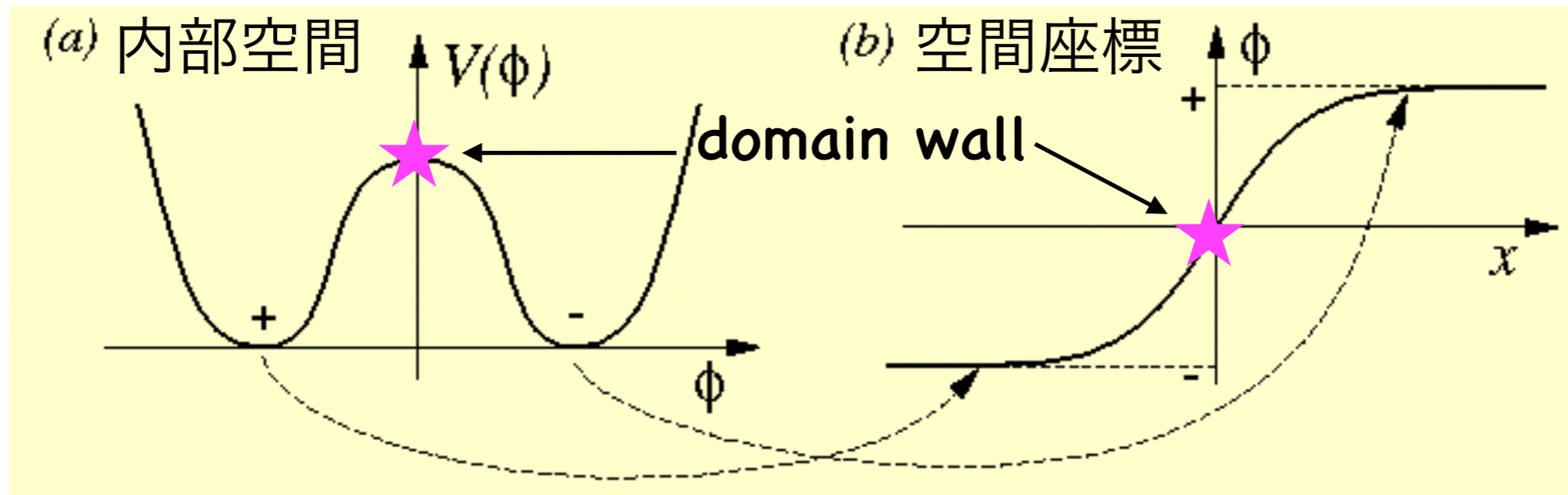
domain wall

horizon scale

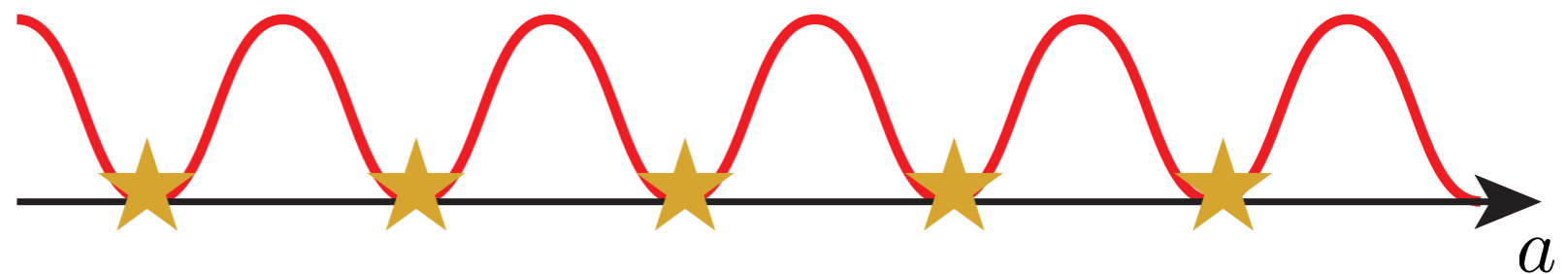
ξ

Topological defect 2 : Domain wall

discrete minima (vacuum) \rightarrow domain wall formation



Axion potential



Topological defects in conventional axion scenario

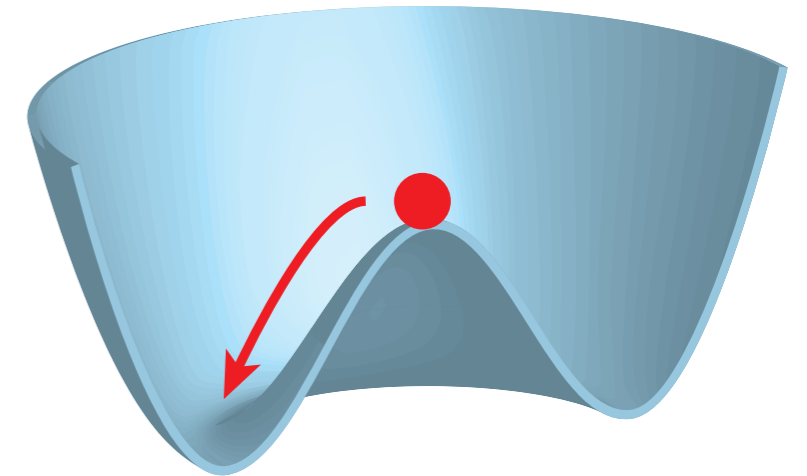
PQ symmetry breaking after inflation



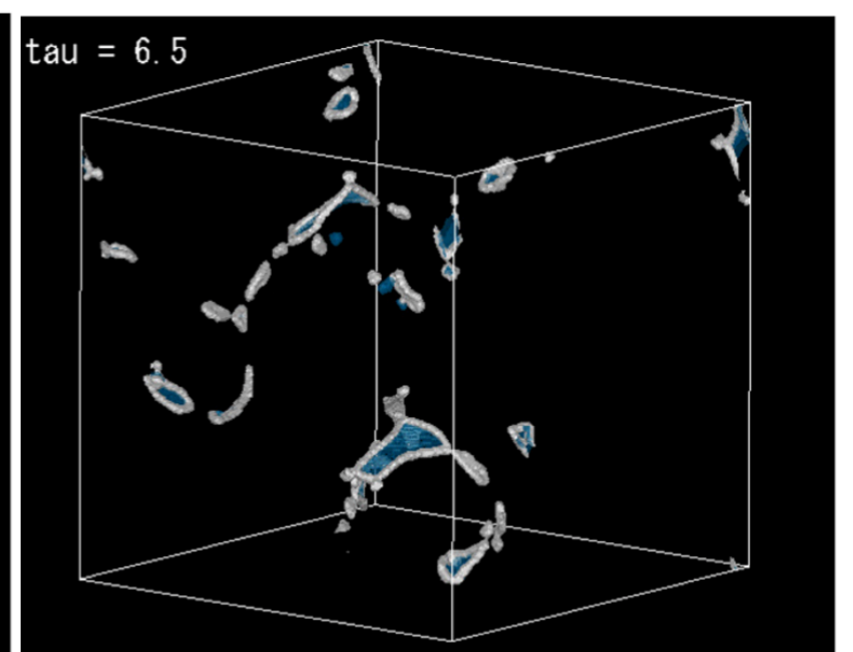
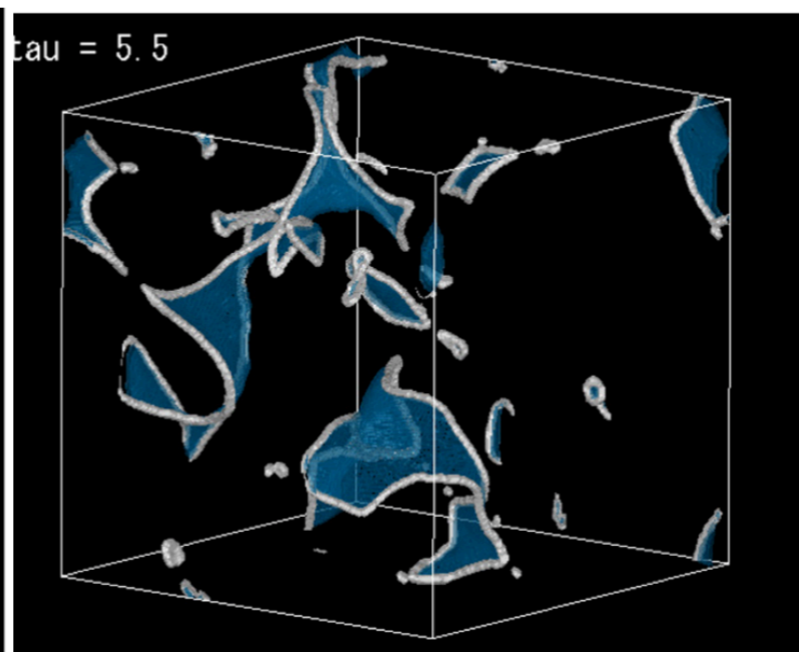
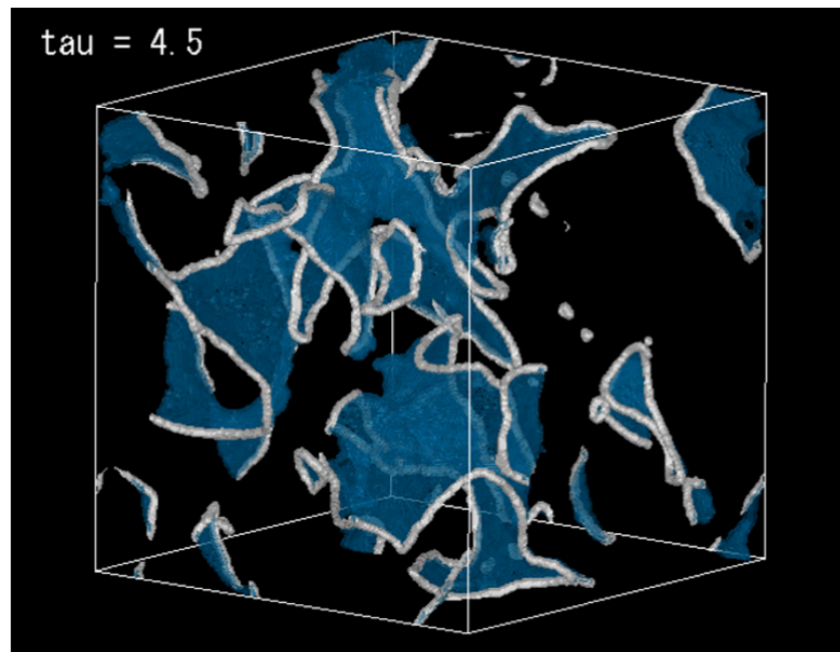
Cosmic string



Domain walls with cosmic strings



$N_{DW}=1 \longrightarrow$ string-wall network decays into axions



Hiramatsu, Kawasaki, Saikawa, Sekiguchi 1202.5851

Axion abundance (conventional scenario)

coherent oscillation : $\Omega_{a,\text{osc}} h^2 = 0.18 \theta_i^2 \left(\frac{F_a}{10^{12} \text{ GeV}} \right)^{1.19}$

with $\langle \theta_i^2 \rangle = \frac{c_{\text{anh}} \pi^2}{3}$

cosmic string : $\Omega_{a,\text{str}} h^2 = 2.0 \xi \left(\frac{F_a}{10^{12} \text{ GeV}} \right)^{1.19} \left(\frac{\Lambda}{400 \text{ MeV}} \right)$

domain wall : $\Omega_{a,\text{dw}} h^2 = (5.8 \pm 2.8) \left(\frac{F_a}{10^{12} \text{ GeV}} \right)^{1.19} \left(\frac{\Lambda}{400 \text{ MeV}} \right)$

total : $\Omega_{a,\text{tot}} h^2 = (8.4 \pm 3.0) \left(\frac{F_a}{10^{12} \text{ GeV}} \right)^{1.19} \left(\frac{\Lambda}{400 \text{ MeV}} \right)$

Defect formation in aligned axion model

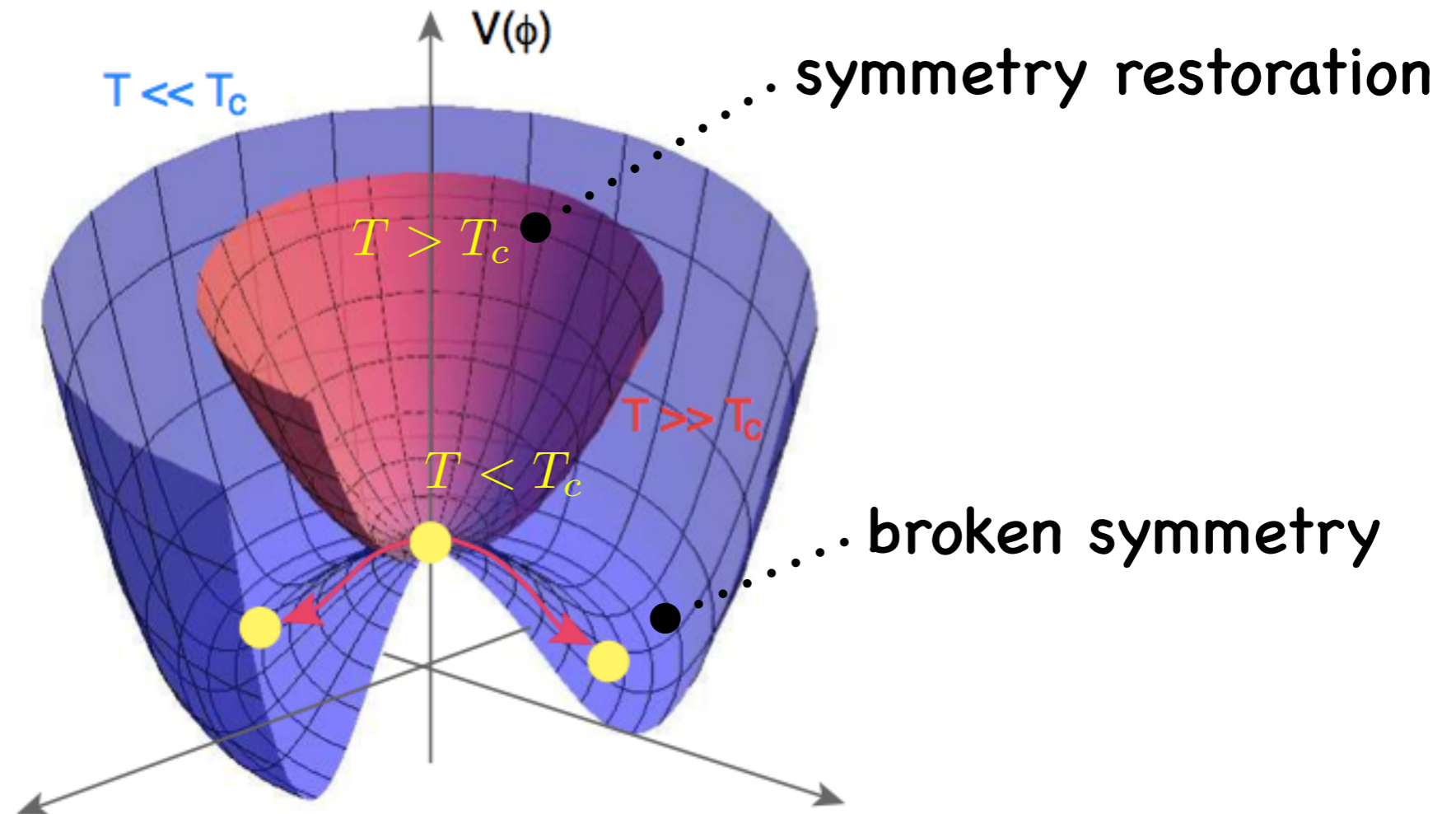


Fig. in Kawasaki, Nakayama 1301.1123

$T_c \sim f \sim$ weak scale for n-axions



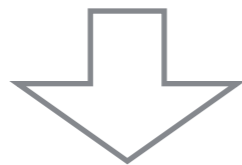
PQ symmetries $U(1)^N$ can easily be restored in a hot universe
(no isocurvature perturbations)

Defect formation in aligned axion model

Re-breaking of $U(1)^N$



N cosmic strings are formed
each string has a tension f^2



~~shift symmetry~~

$N-1$ domain walls are formed
(bounded by cosmic strings)

“string-wall network”

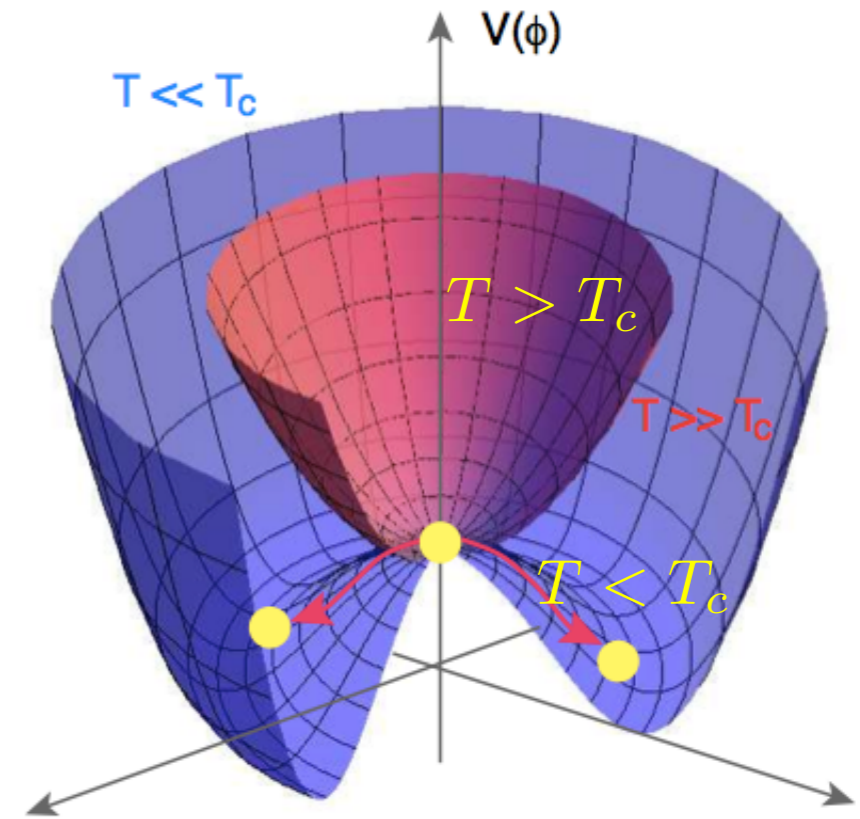


Fig. in Kawasaki, Nakayama 1301.1123

$$V = \sum_{i=1}^N \left(-m_i^2 |\Phi_i|^2 + \lambda_i |\Phi_i|^4 \right)$$

$$\Delta V = \sum_{i=1}^{N-1} \epsilon_i \Phi_i \Phi_{i+1}^3 + \text{h.c.}$$

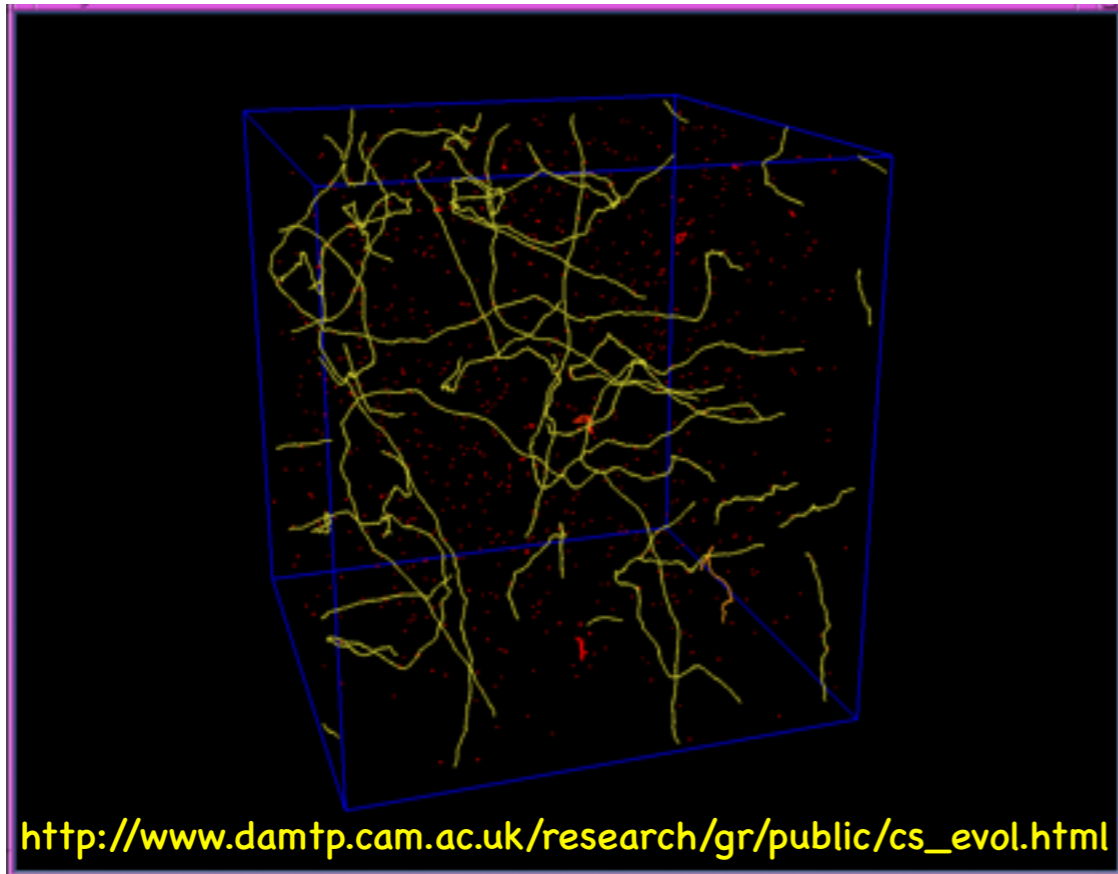
Cosmic strings of N-axions

Cosmic string solution : $\Phi_i = \frac{f_i}{\sqrt{2}} e^{i\phi_i/f_i} = \frac{f_i}{\sqrt{2}} e^{iw_i\theta}$

w_i : winding number for each axion string

θ : angular coordinate in real space

tension : $\mu_i \sim \mu_{\text{core}} + \int_{\delta}^R \left| \frac{1}{r} \frac{\partial \Phi_i}{\partial \theta_i} \right|^2 2\pi r dr \approx \pi w_i^2 f_i^2 \ln \left(\frac{R}{\delta} \right)$

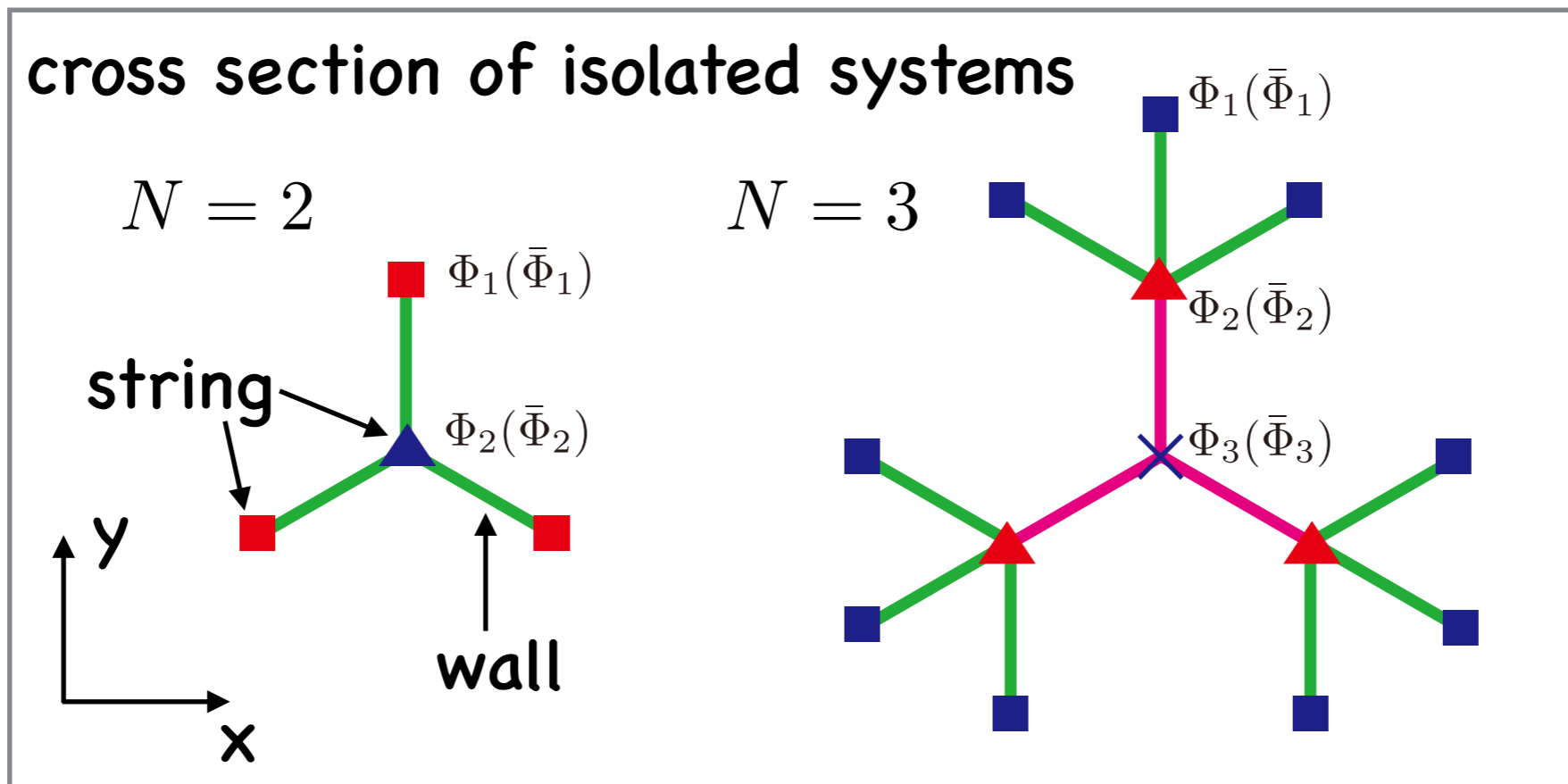


n.b. $f_i \ll F_a$

Each string tension is much smaller than that of the QCD axion string

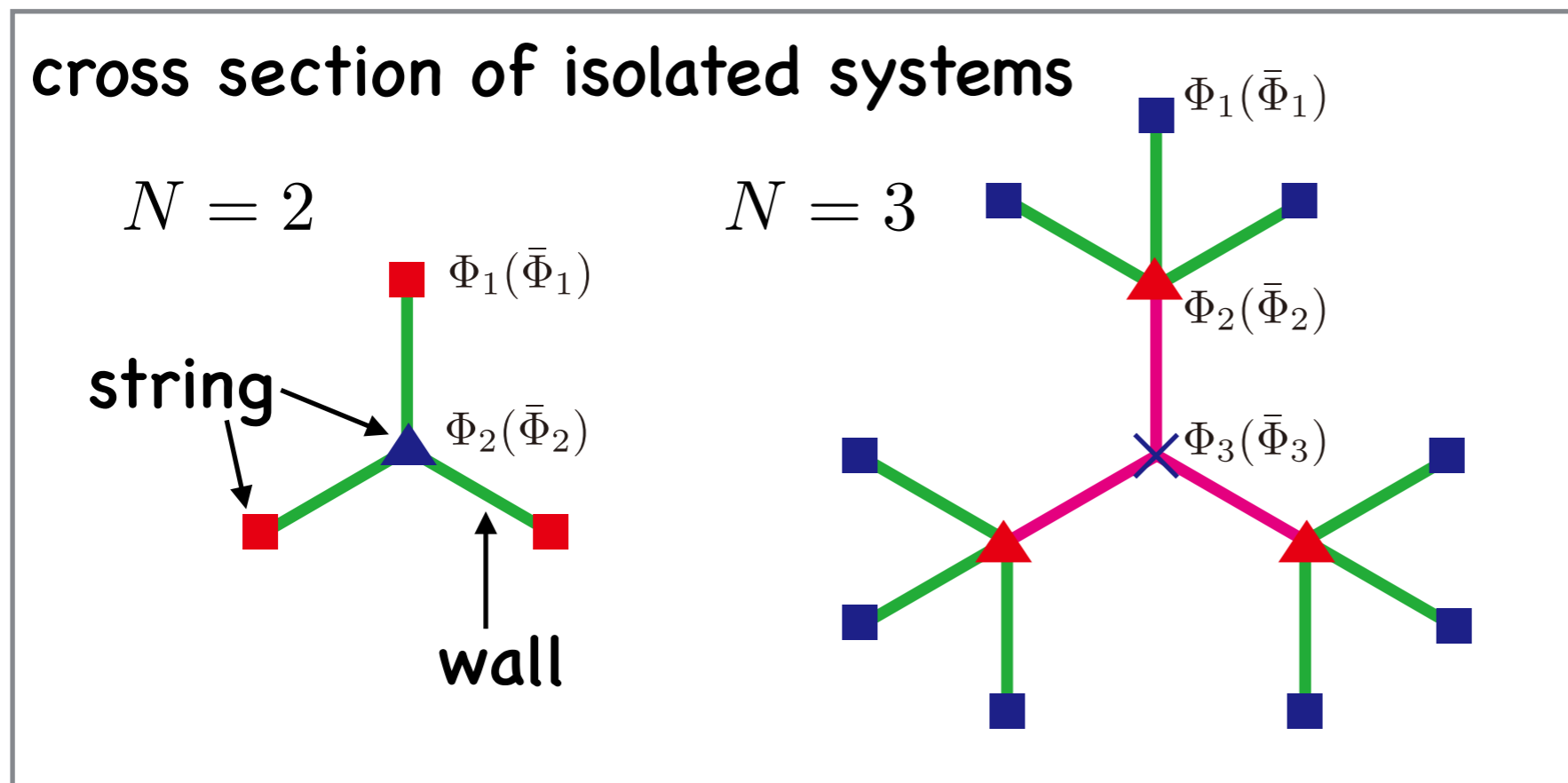
Cosmic string "Bundle"

$$\Delta V = \sum_{i=1}^{N-1} \epsilon_i \Phi_i \Phi_{i+1}^3 + \text{h.c.}$$



Aligned structure in the field (internal) space
appears in the real space!

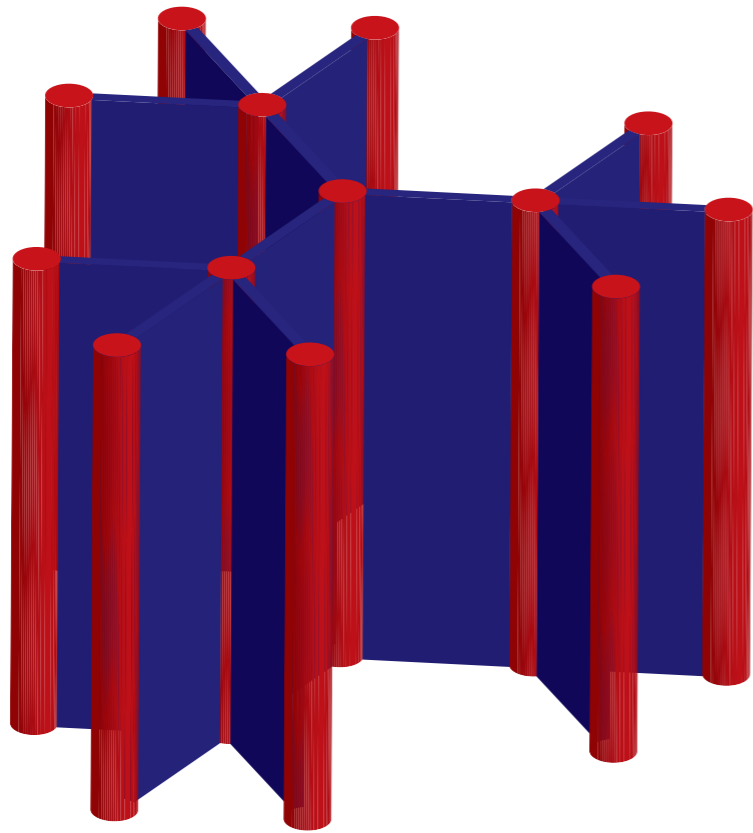
Cosmic string "Bundle"



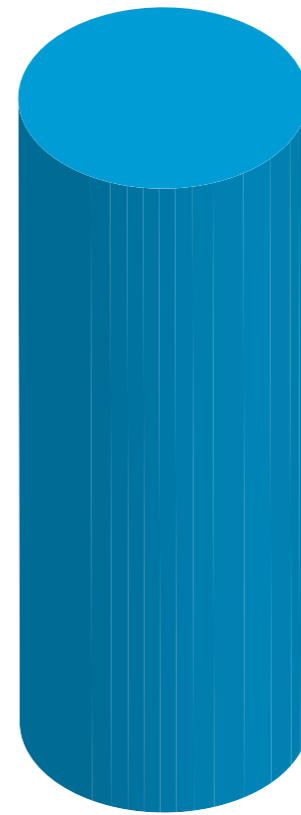
$$F_a = \sqrt{\sum_{i=1}^N f_i^2 \left(\prod_{j=i}^N n_j^2 \right)}$$

"effective" tension : $\mu_{\text{eff}} \simeq \pi(3^2 f_1^2 + f_2^2) \ln \left(\frac{R}{\delta} \right) = \pi F_a^2 \ln \left(\frac{R}{\delta} \right) \quad (\mathbf{N=2})$

$$\mu_{\text{eff}} \simeq \pi(3^{2(N-1)} f_1^2 + \dots + 3^2 f_{N-1}^2 + f_N^2) \ln \left(\frac{R}{\delta} \right) = \pi F_a^2 \ln \left(\frac{R}{\delta} \right)$$



string bundle

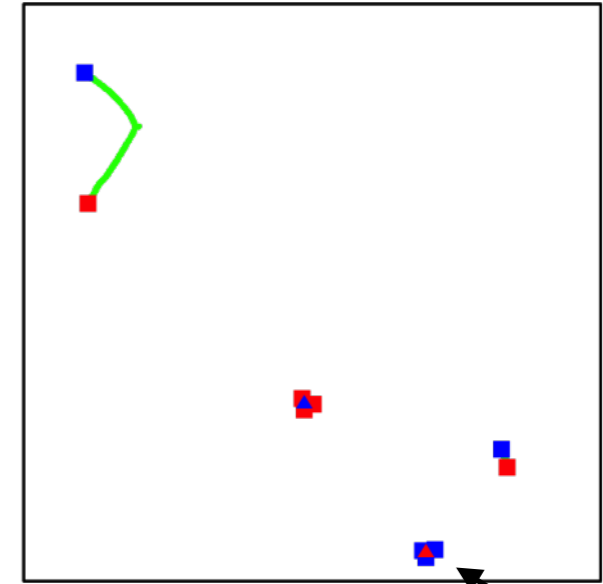
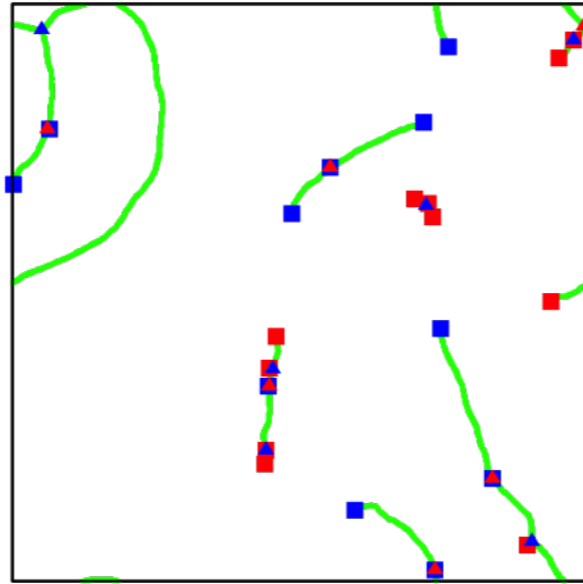
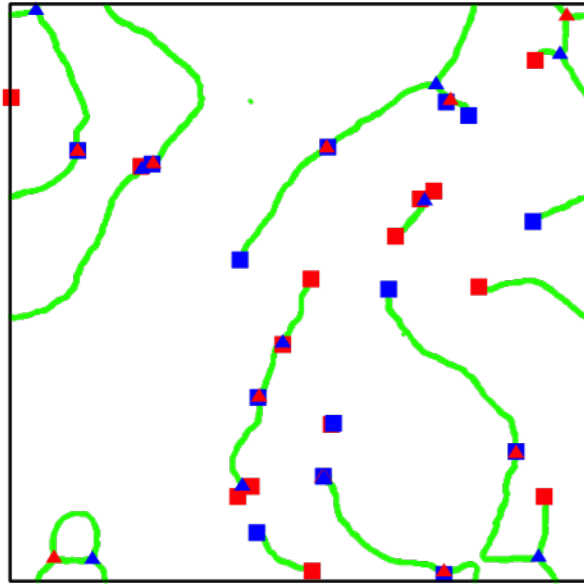


QCD axion string

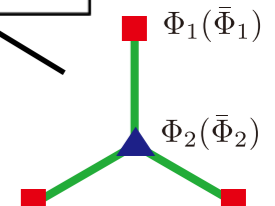
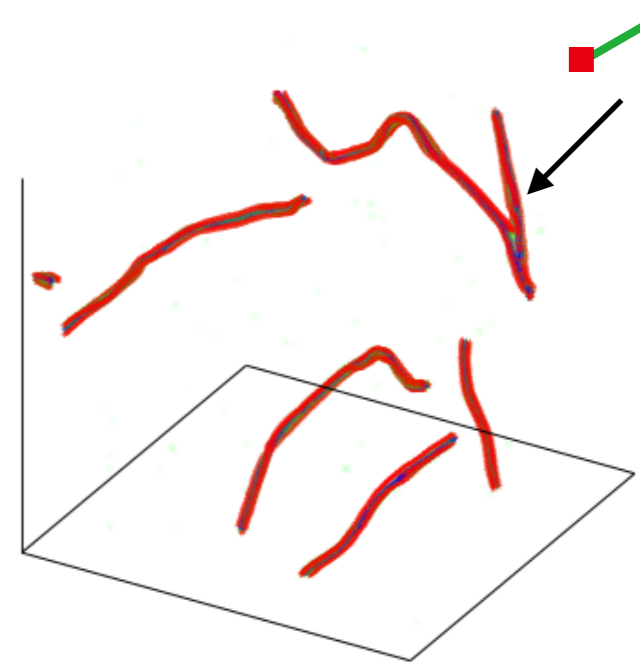
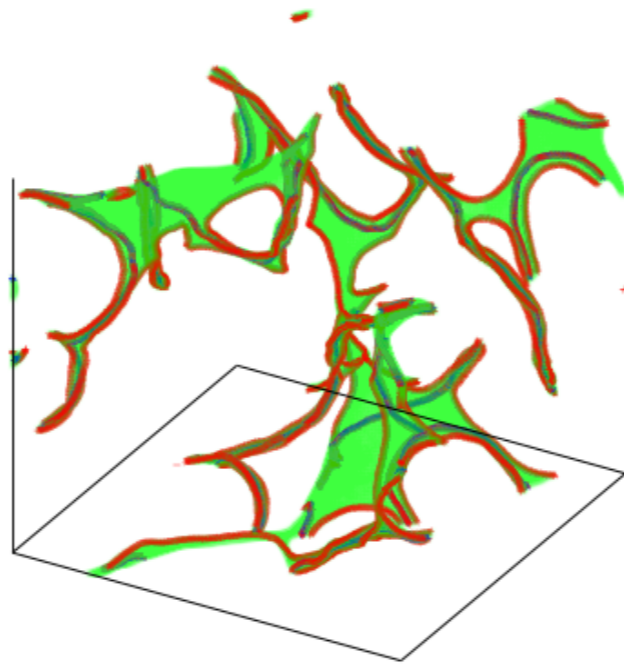
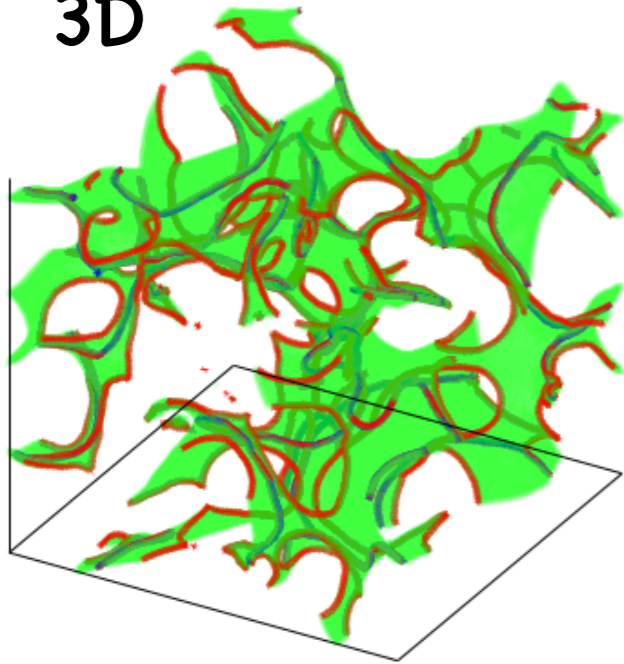
They are equivalent to the QCD axion string!!

Lattice simulation for N=2

2D



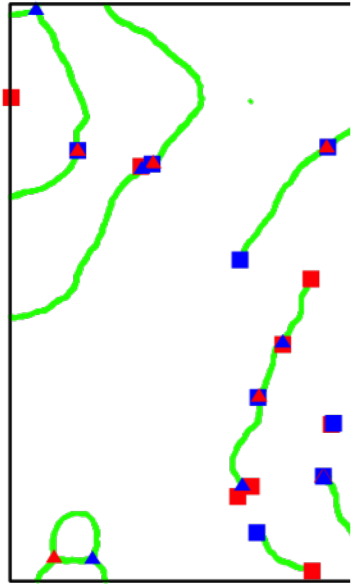
3D



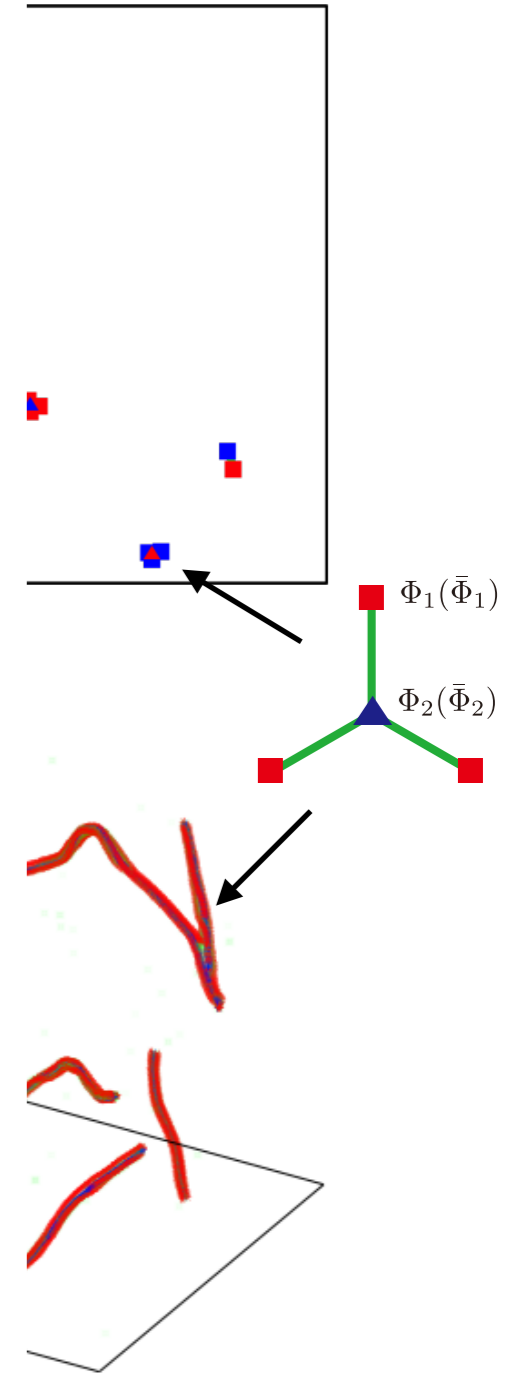
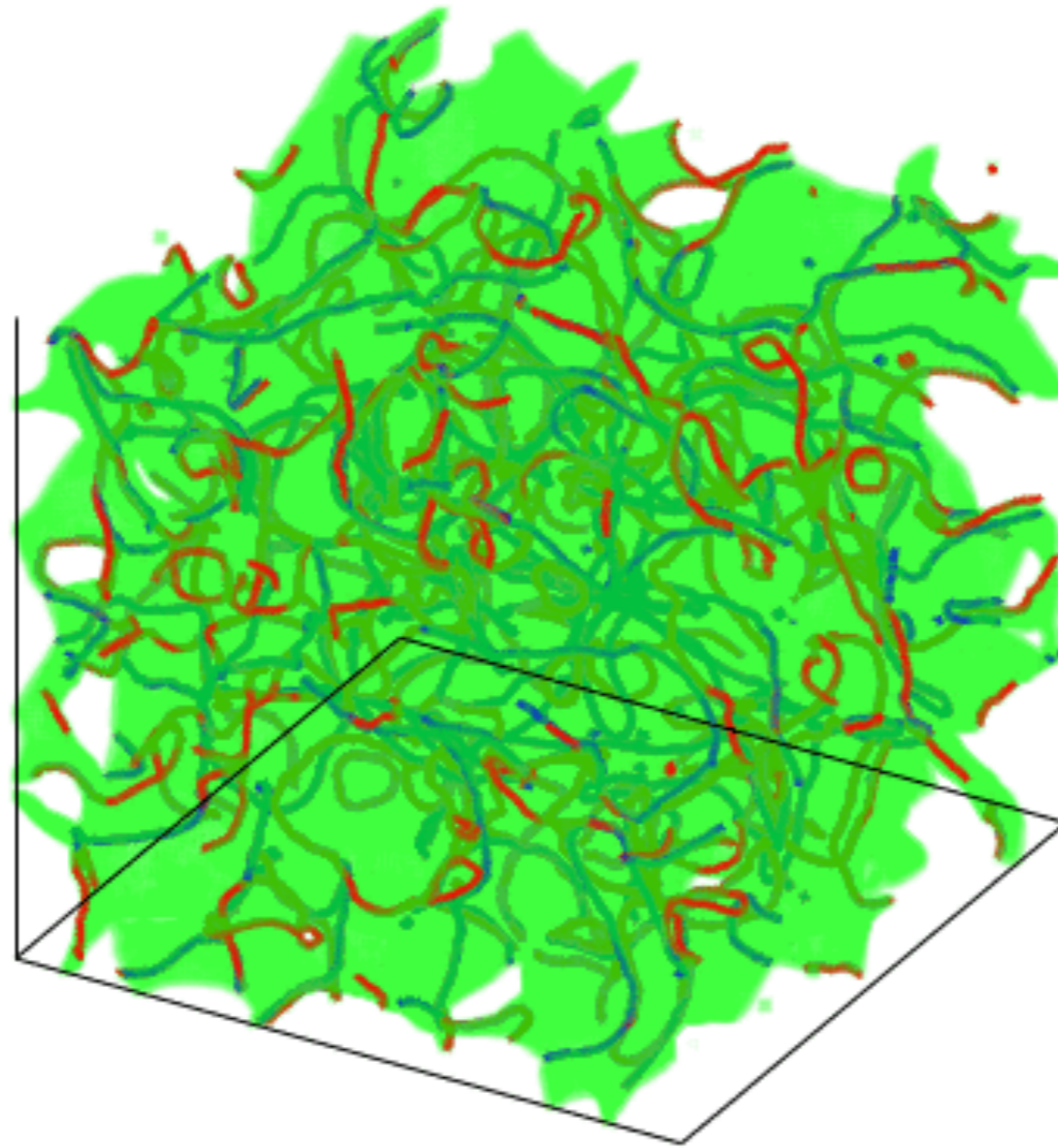
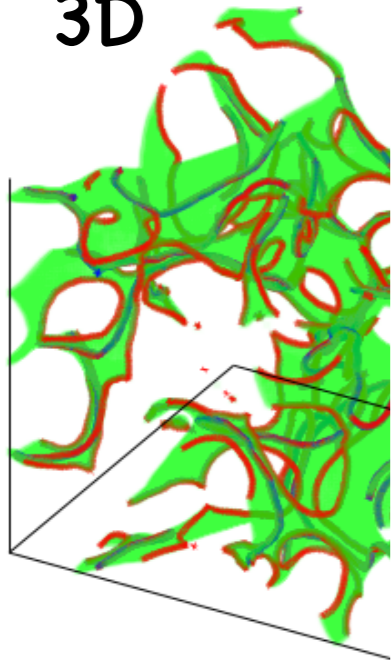
Domain walls disappear and string bundles are formed!!

Lattice simulation for N=2

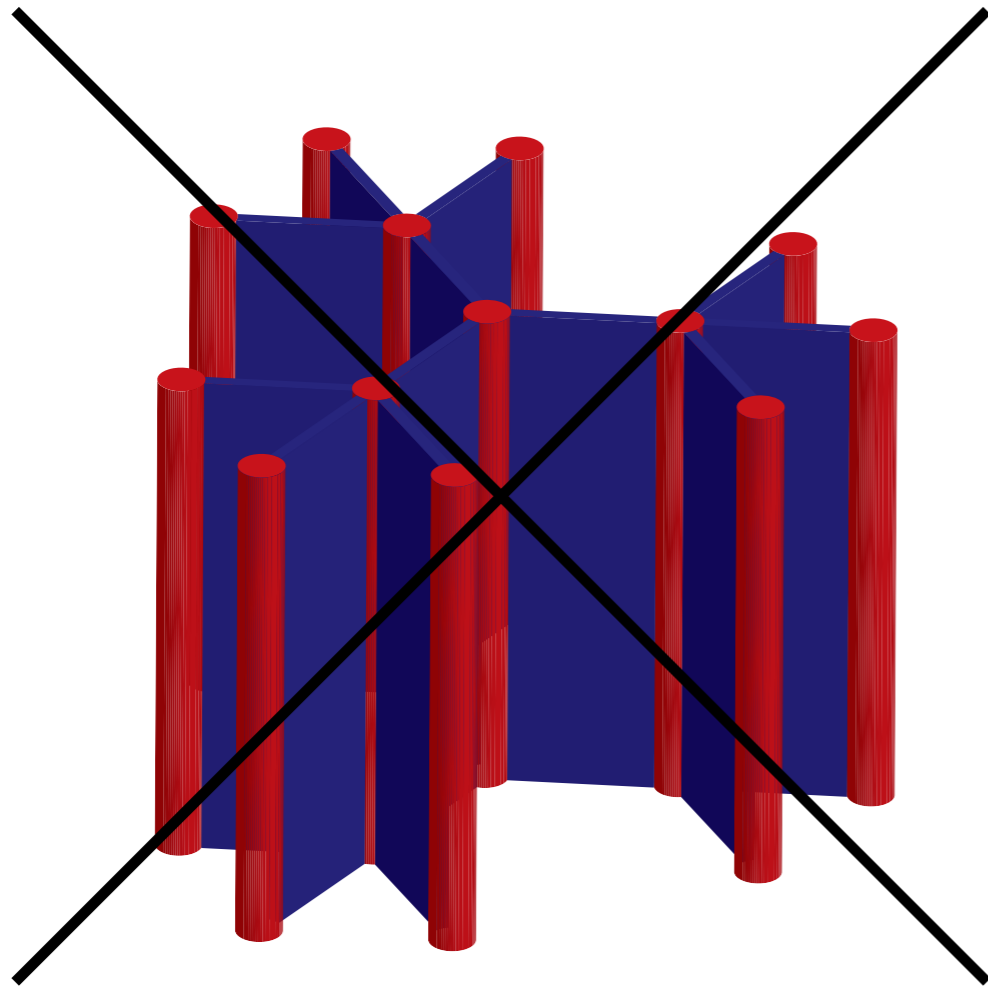
2D



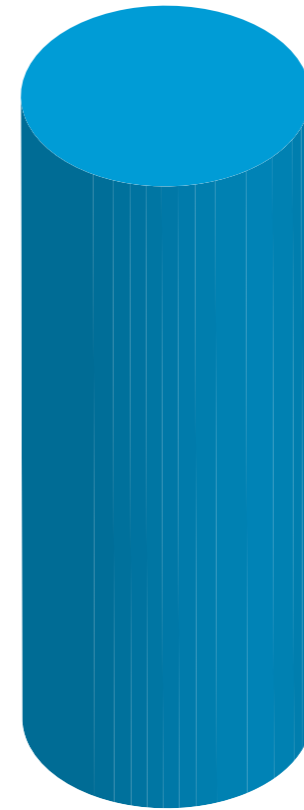
3D



Domain walls disappear and string bundles are formed!!



string bundle

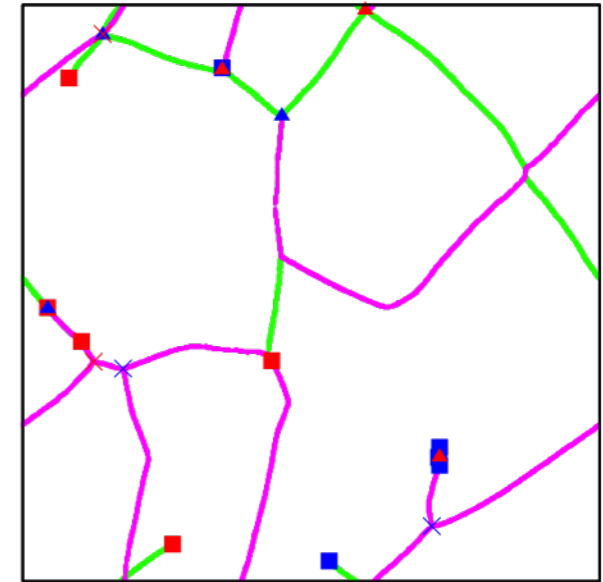
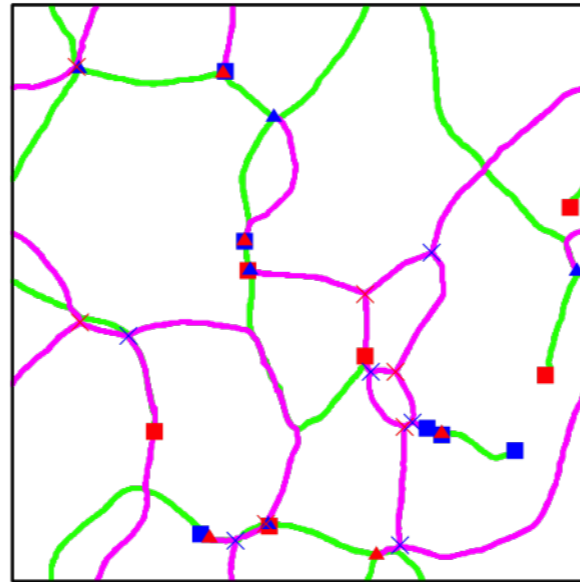
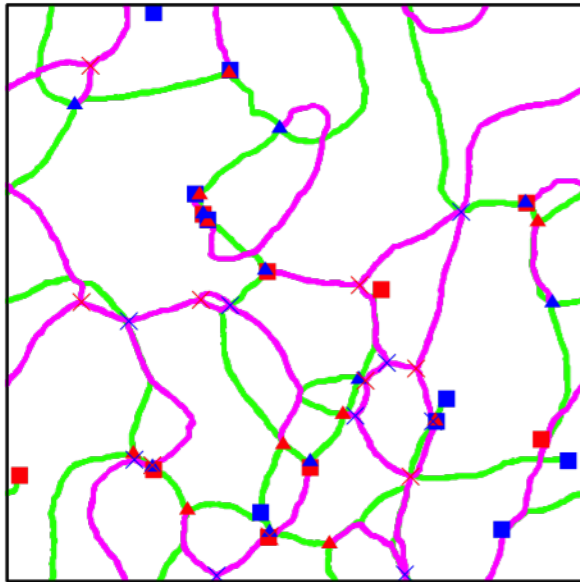


QCD axion string

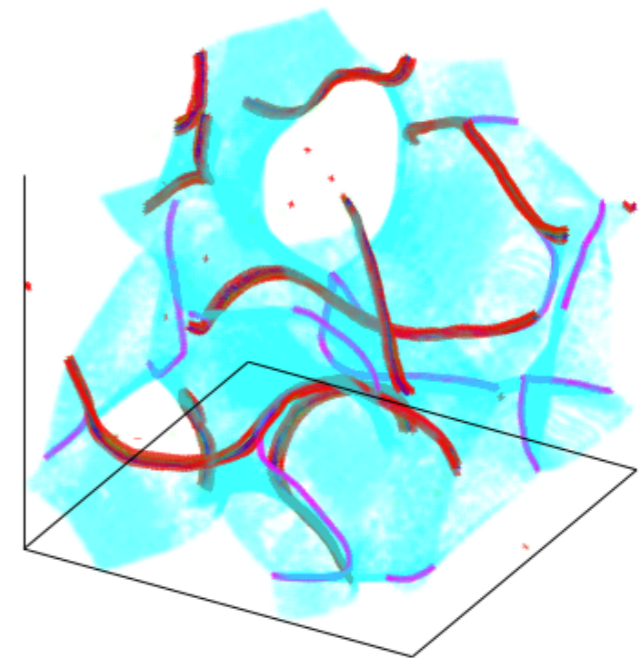
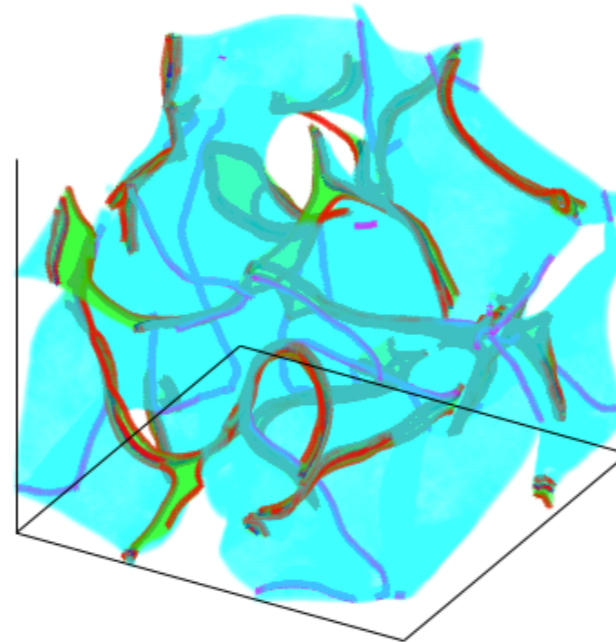
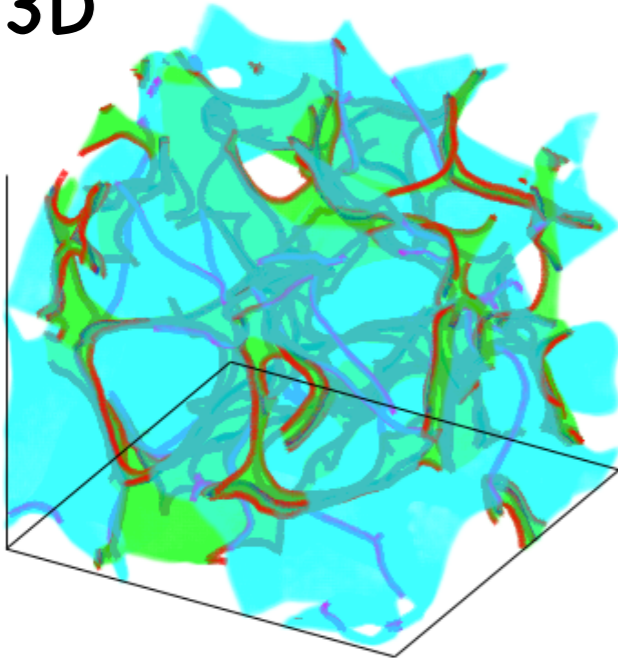
However, formation probability for such structure
is exponentially suppressed
(Φ_N string requires $3^N \Phi_1$ strings
but # of strings decreases by pair annihilation)

Lattice simulation for $N=3$

2D



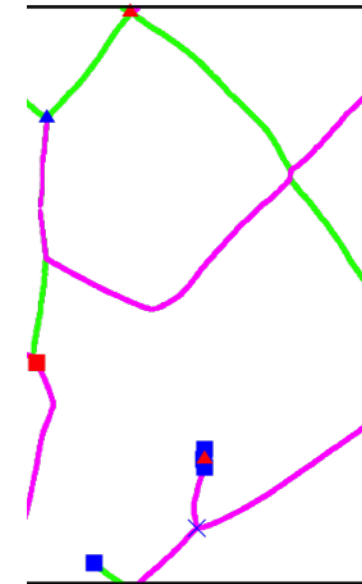
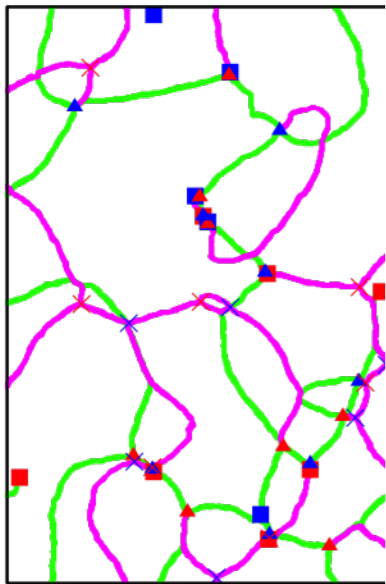
3D



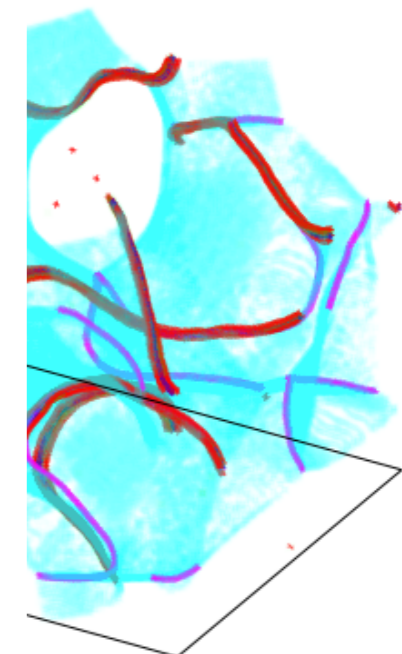
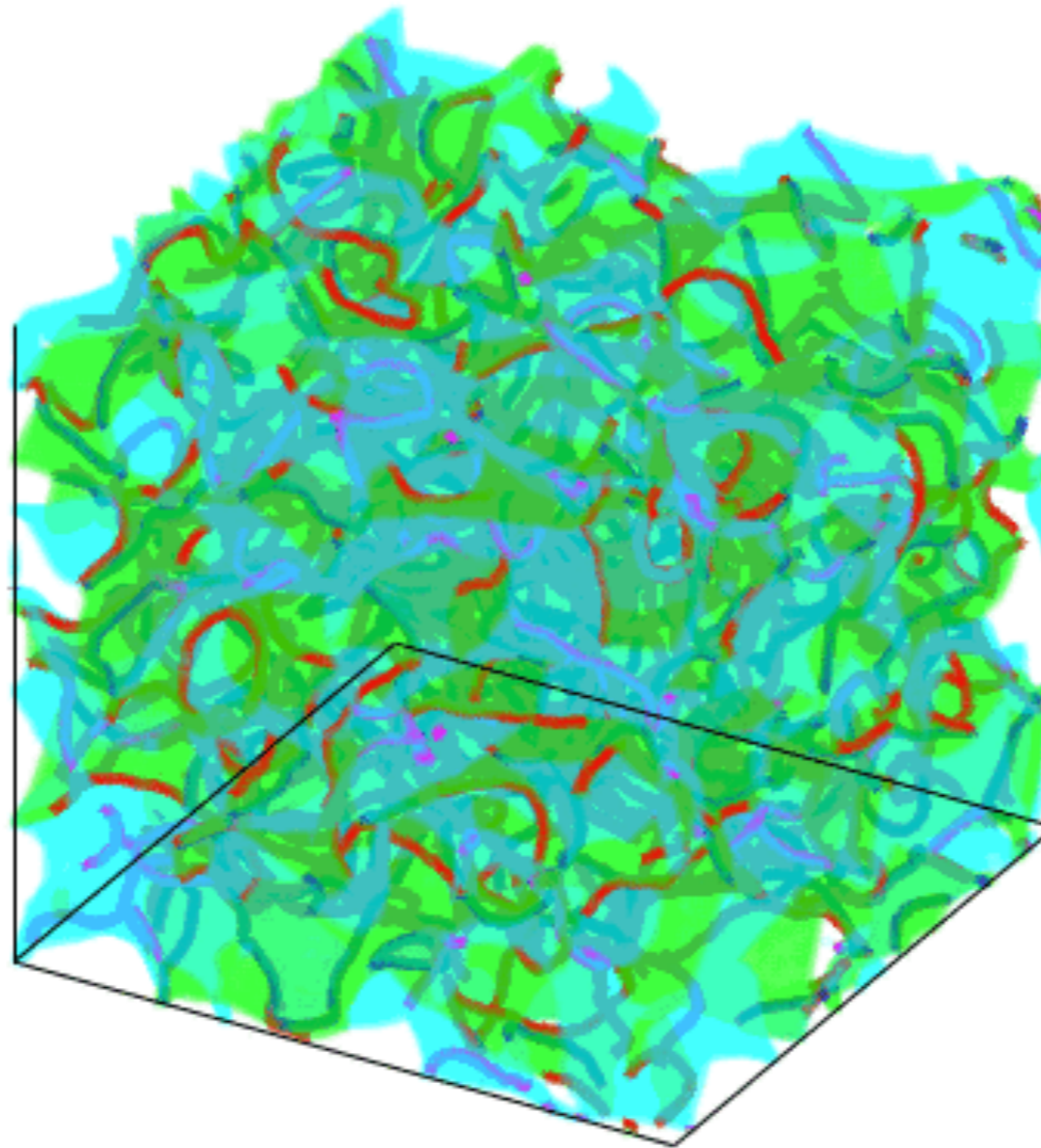
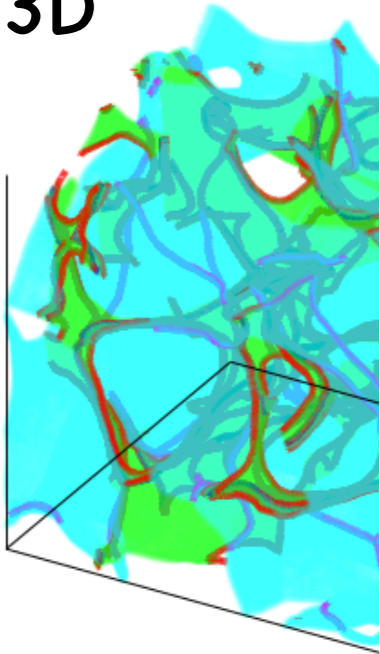
String-wall network is long-lived
and they evolve under the scaling law

Lattice simulation for $N=3$

2D

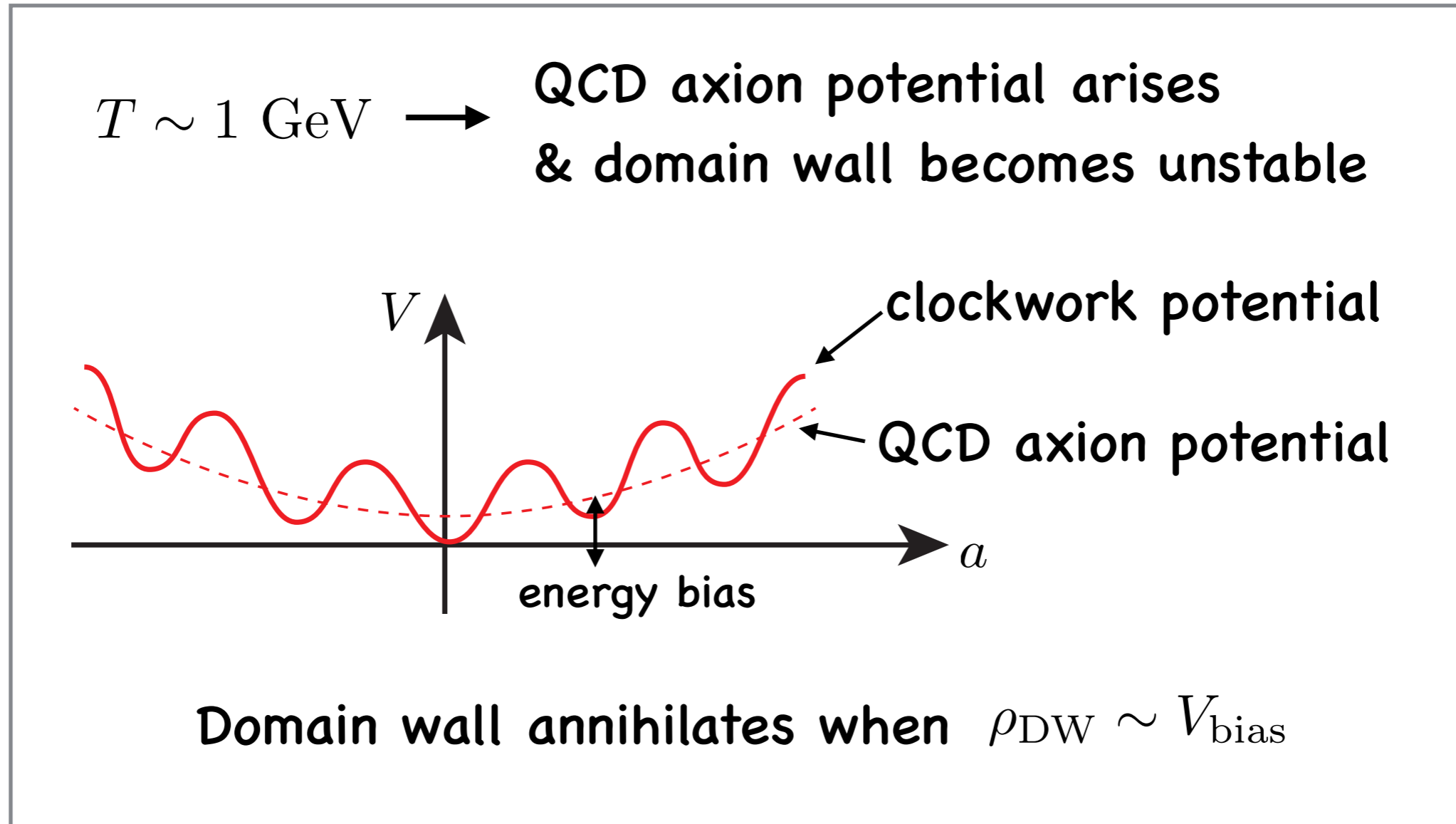


3D



String-wall network is long-lived
and they evolve under the scaling law

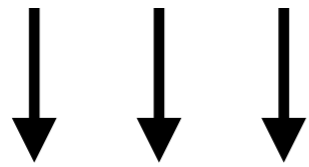
Domain wall annihilation



To avoid domain wall domination
before the annihilation :

$$f \lesssim 400 \text{ TeV } \epsilon^{-1/6} \left(\frac{\Lambda_{\text{QCD}}}{400 \text{ MeV}} \right)^{4/3}$$

Domain wall collapse

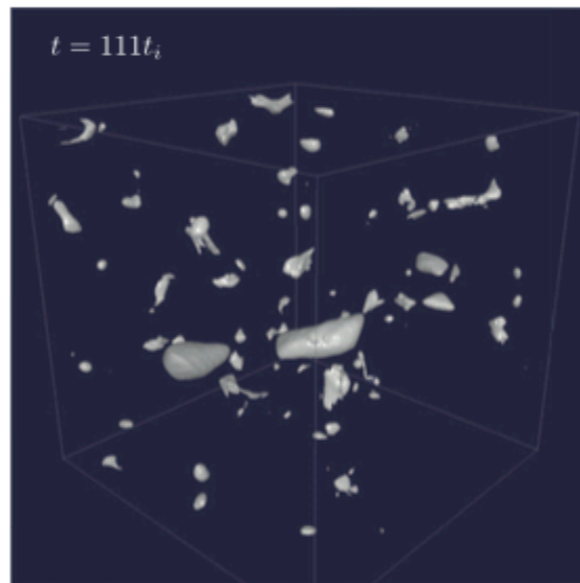
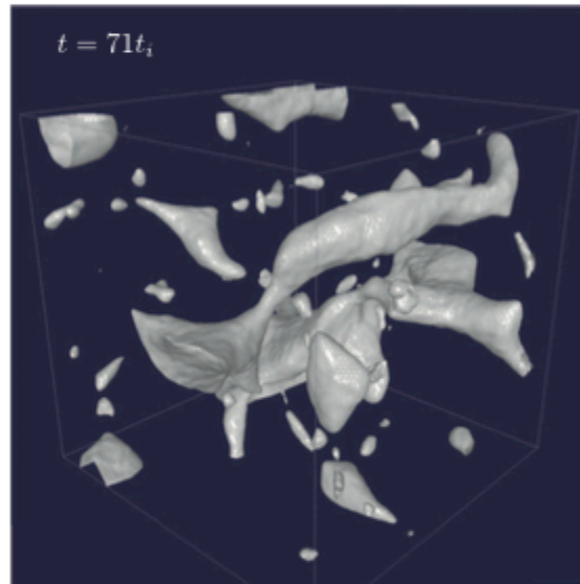
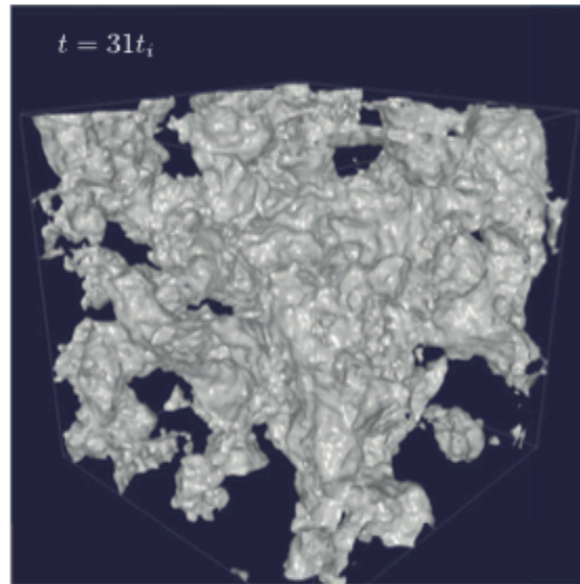


Gravitational waves

Long-lived DW



A large amount of GW



gravitational waves



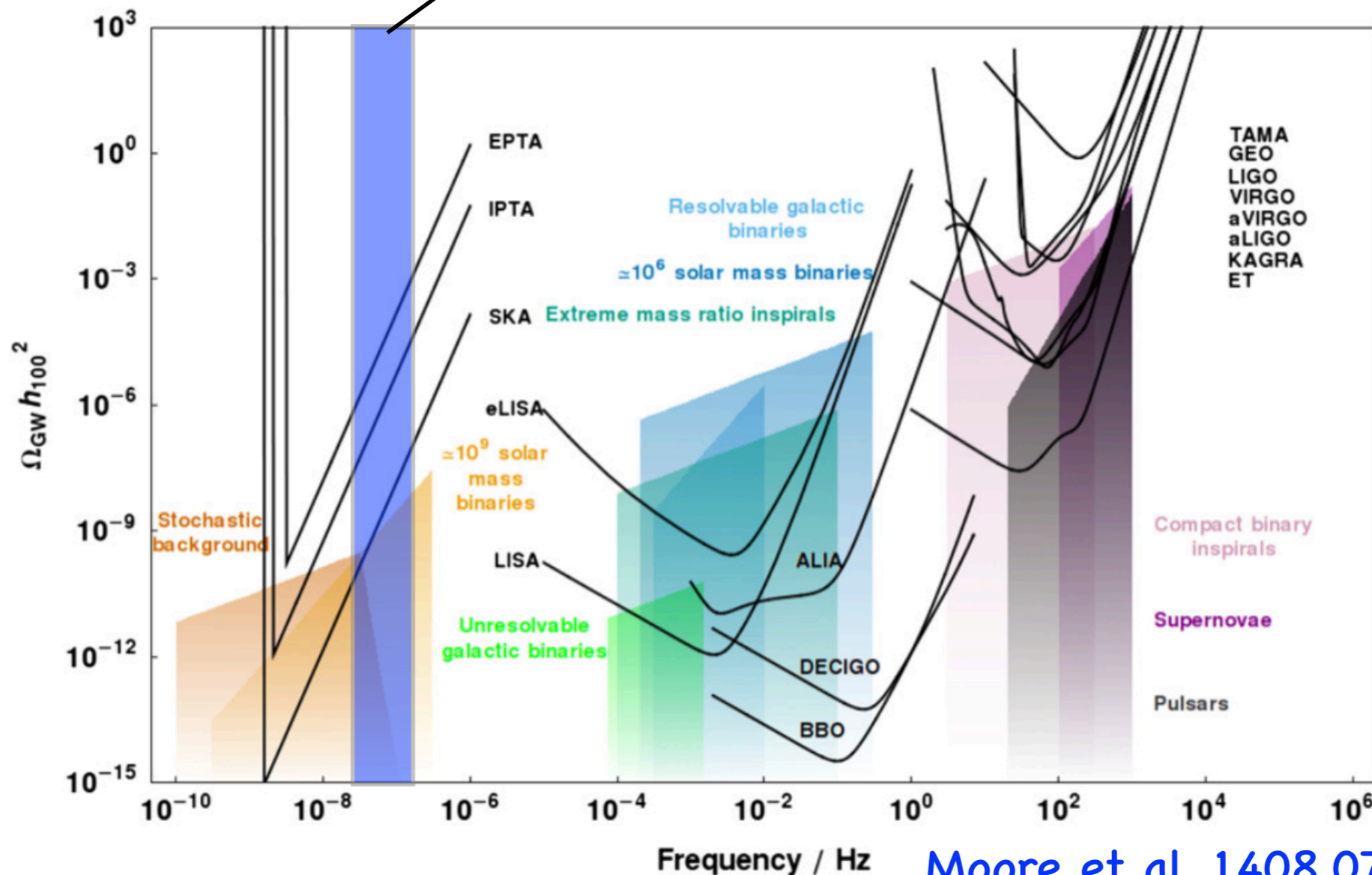
Hiramatsu, Kawasaki, Saikawa (2010)

Gravitational waves from decaying domain walls

Peak frequency of GW – Hubble parameter at the annihilation

$$\nu_{\text{peak}} \simeq 1.6 \times 10^{-7} \text{ Hz} \left(\frac{g_{*\text{ann}}}{80} \right)^{1/6} \left(\frac{T_{\text{ann}}}{1 \text{ GeV}} \right)$$

pulsar timing obs. 



Moore et al. 1408.0740

Constraints from pulsar timing observations

Intensity of GW by DW decay

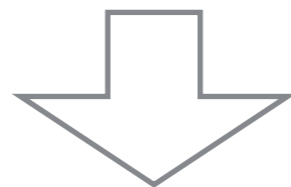
$$\Omega_{\text{gw}}(\nu_{\text{peak}})h^2 \simeq 2 \times 10^{-11} \epsilon \left(\frac{g_{*\text{ann}}}{80} \right)^{-4/3} \left(\frac{T_{\text{ann}}}{1 \text{ GeV}} \right)^{-4} \left(\frac{f}{100 \text{ TeV}} \right)^6$$

frequency dependence : $\Omega_{\text{gw}}(\nu) \propto \nu^3$ for $\nu < \nu_{\text{peak}}$

Hiramatsu, Kawasaki, Saikawa, 1309.5001

current constraint : $\Omega_{\text{gw}}h^2 < 2.3 \times 10^{-10}$ at $\nu_{1\text{yr}} \simeq 3 \times 10^{-8}$ Hz

P. D. Lasky et al. 1511.05994



$$f \lesssim 200 \text{ TeV} \times \epsilon^{-1/6} \left(\frac{\Lambda_{\text{QCD}}}{400 \text{ MeV}} \right)^{28/33}$$

100 TeV (future observation by SKA)

Summary

- Intermediate scale for axion decay constant can be realized by alignment mechanism – aligned QCD axion model

N-axions w/ weak scale decay constant \rightarrow QCD axion

- High quality of PQ scale can be naturally realized
PQ mechanism works well w/o fine tuning / introducing extra Z_N
- Topological defects can easily formed in aligned axion model and it puts upper bound on the decay constant
 $\longrightarrow f \lesssim O(100) \text{ TeV}$

Aligned axion

Kim, Niles, Peloso, hep-ph/0419138; K. Choi, H. Kim, S. Yun, 1404.6209, ...

$$V(\phi_1, \phi_2) = \Lambda_1^4 \left[1 - \cos \left(\frac{n_1 \phi_1}{f_1} + \frac{n_2 \phi_2}{f_2} \right) \right] \\ + \Lambda_2^4 \left[1 - \cos \left(\frac{m_1 \phi_1}{f_1} + \frac{m_2 \phi_2}{f_2} \right) \right]$$

$$V_{\text{eff}}(\phi_{\text{flat}}) = \Lambda_2^4 \left[1 - \cos \left(\frac{\phi_{\text{flat}}}{f_{\text{eff}}} \right) \right]$$

with

$$\phi_{\text{flat}} \propto \frac{n_2 \phi_1}{f_2} - \frac{n_1 \phi_2}{f_1}, \quad f_{\text{eff}} = \frac{\sqrt{n_1^2 f_2^2 + n_2^2 f_1^2}}{|n_1 m_2 - n_2 m_1|}$$

$f_1, f_2 < M_P$ but $f_{\text{eff}} > M_P$

: requirement for inflaton/relaxion excursion