Reconstruction of the Scalar-field Potential for the Cosmological Model with a Gauss-Bonnet term

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Outline

- Review: Slow-roll inflation and/with Gauss-Bonnet term
- Reconstruction of the inflaton potential
- Feature of the reconstructed potential
- Summary and Discussion

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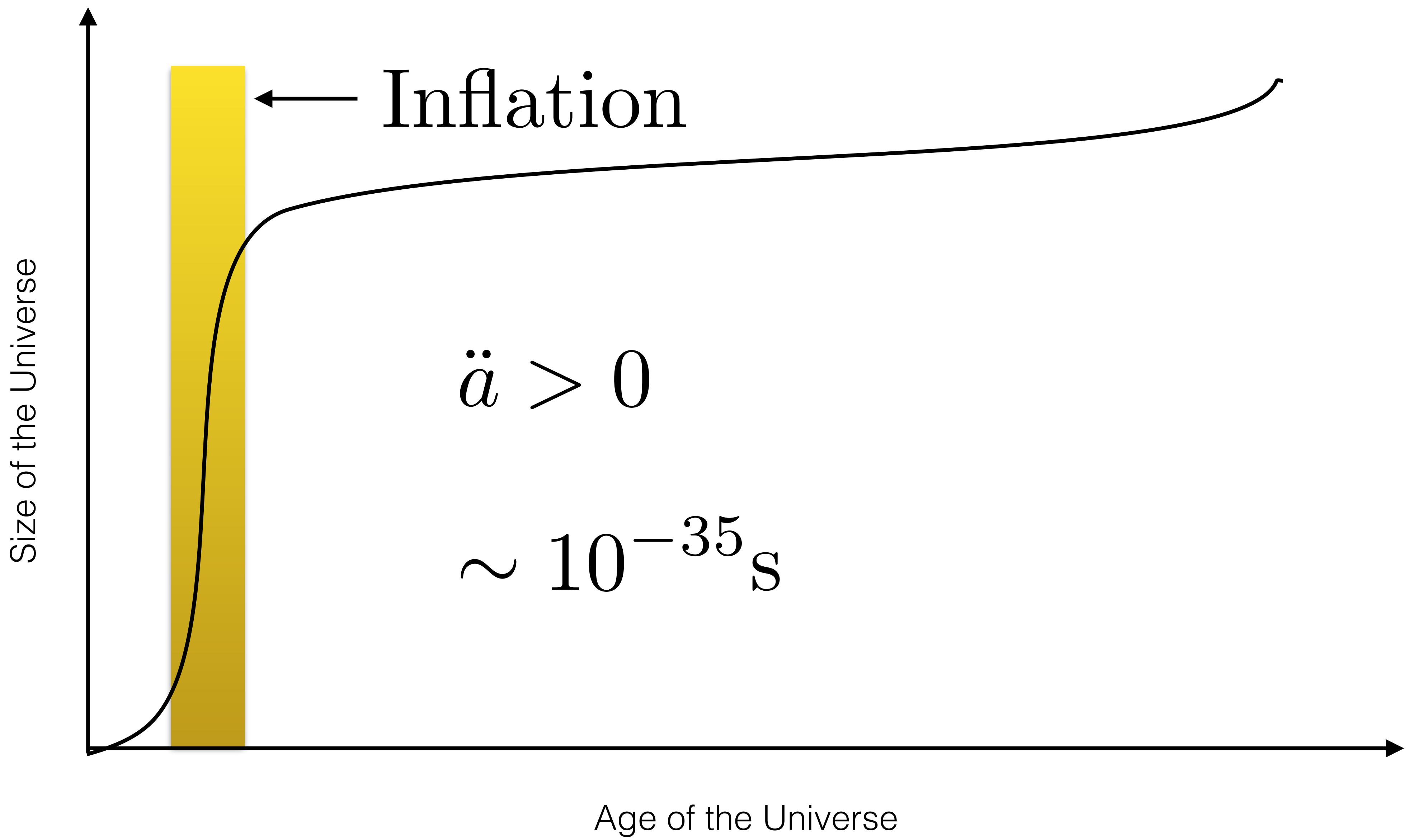
Our main interest!!!

Review: What is inflation?

Inflation is the idea that the very early universe went through a period of accelerated exponential expansion during first fraction of seconds after the Big Bang.



Inflation provides solutions to the problems of the cosmology such as the horizon and flatness.

- Shrinking comoving Hubble radius: $\frac{d}{dt}(aH)^{-1} < 0\,,$
- Slowly varying Hubble parameter: $0 > \frac{d}{dt}(aH)^{-1} = -\frac{\dot{a}H + a\dot{H}}{(aH)^2} = -\frac{1}{a}(1-\epsilon) \quad \Rightarrow \quad \epsilon \equiv -\frac{\dot{H}}{H^2} < 1\,.$
- Acceleration of scale factor: $-\frac{\dot{H}}{H^2} < 1 \quad \Rightarrow \quad \ddot{a} > 0$
- Negative pressure: $\dot{H} + H^2 = -\frac{\kappa^2}{6}\,(\rho + 3p) \quad \Rightarrow (\rho + 3p) < 0 \quad \Leftrightarrow \quad \omega \equiv \frac{p}{\rho} < -\frac{1}{3}\,.$

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Condition for inflation

The simplest model of inflation is based upon a single scalar field, minimally coupled to a gravity, known as *inflaton* field.

- Action: $$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R+\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)\right],$$
- The FRW universe: $$ds^2=-dt^2+a^2\left(\frac{dr^2}{1-Kr^2}+r^2d\Omega^2\right),$$
- Equations of motion with *K*=0: $$\ddot{\phi}+3H\dot{\phi}+V_\phi=0\,,$$

$$H^2 = \frac{\kappa^2}{3}\left[\frac{1}{2}\dot{\phi}^2+V(\phi)\right],$$

$$\dot{H} = -\frac{\kappa^2}{2}\dot{\phi}^2\,.$$

- Equations of state parameter: $$\omega_\phi\equiv\frac{p_\phi}{\rho_\phi}=\frac{\frac{1}{2}\dot{\phi}+V(\phi)}{\frac{1}{2}\dot{\phi}-V(\phi)}\,.$$

Slow-roll Condition

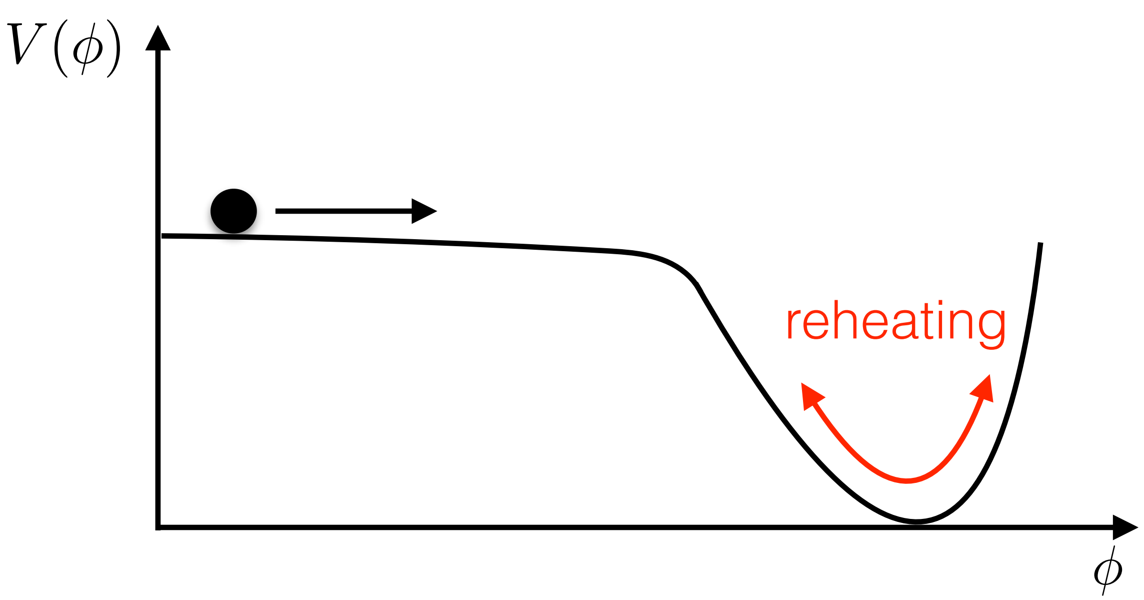
- Slow-roll parameter: $\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\kappa^2}{2}\frac{\dot{\phi}^2}{H^2}\,, \qquad \delta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}}\,.$
- Slow-roll approximation: $\dot{\phi}^2 \ll V\,, \quad |\ddot{\phi}| \ll |3H\dot{\phi}|,\ |V_\phi|.$
- Equations of motion: $3H\dot{\phi} \simeq -V_{,\phi}\,, \quad H^2 \simeq \frac{\kappa^2}{3}V(\phi)\,.$

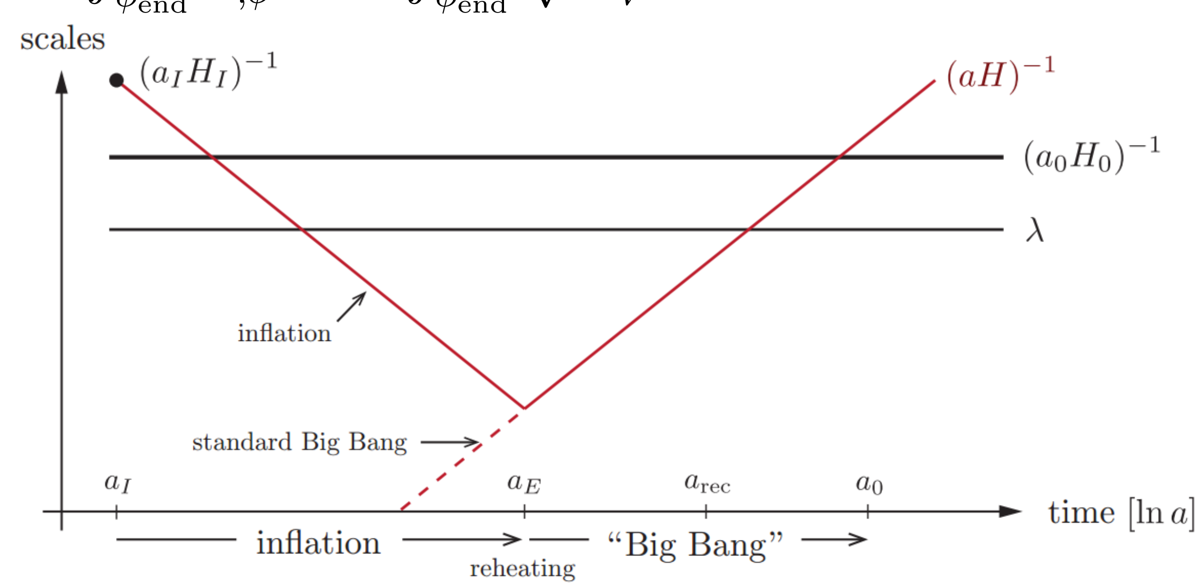
$$\Rightarrow \quad \epsilon_V \equiv \frac{1}{2\kappa^2}\left(\frac{V_{,\phi}}{V}\right)^2 \ll 1\,, \qquad \eta_V \equiv \frac{1}{\kappa^2}\frac{V_{,\phi\phi}}{V} \ll 1\,,$$

- Number of e-folds:

$$N(\phi) \equiv \ln\frac{a_{\rm end}}{a} = \int_t^{t_{\rm end}} H\,dt = \int_\phi^{\phi_{\rm end}} \frac{H}{\dot{\phi}}d\phi \approx \int_{\phi_{\rm end}}^{\phi} \frac{V}{V_{,\phi}}d\phi \approx \int_{\phi_{\rm end}}^{\phi} \frac{\kappa\, d\phi}{\sqrt{2\epsilon_V}}\,.$$

$$\Rightarrow \quad \ln\left(\frac{a_E}{a_I}\right) \gtrsim 64\,,$$





Perturbation Theory

- Let us start with: $S = \int dx^4 \sqrt{-g}\left[\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$, $\quad \delta\phi = 0$, $\quad g_{ij} = a^2[(1+\mathcal{R})\delta_{ij} + h_{ij}]$, $\quad \partial_i h_{ij} = h^i_i = 0$.

$$S^{(2)} = \frac{1}{2}\int d^4x a^3 \frac{\dot\phi^2}{H^2}\left[\dot{\mathcal{R}}^2 - \frac{1}{a^2}(\partial_i\mathcal{R})^2\right].$$
$$v \equiv z\mathcal{R}, \quad z^2 \equiv a^2\dot\phi^2/H^2 = 2a^2\epsilon,$$

$$S^{(2)} = \frac{1}{2}\int d\tau d^3x\left[v'^2 + (\partial_i v)^2 + \frac{z''}{z}v^2\right],$$
$$v(\tau,\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} v_{\mathbf{k}}(\tau) e^{i\mathbf{k}\cdot\mathbf{x}},$$

$$v_k'' + \left(k^2 - \frac{z''}{z}\right)v_k = 0.$$

$$\hat v_{\mathbf{k}} = v_k(\tau)\hat a^-_{\mathbf{k}} + v^*_{-k}(\tau)\hat a^+_{\mathbf{-k}},$$
$$[\hat a^-_{\mathbf{k}}, \hat a^+_{\mathbf{-k'}}] = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}'),$$
$$\frac{i}{\hbar}(v_k^* v_k' - v_k^{*\prime} v_k) = 1.$$
$$v_k = \sqrt{\frac{\hbar}{2k^3}}\left(1 - \frac{i}{k\tau}\right)e^{-ik\tau}.$$

$$\langle \hat v_{\mathbf{k}}(\tau)\,\hat v_{\mathbf{k}'}(\tau)\rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')|v_k(\tau)|^2$$
$$= (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\frac{H^2}{2k^3}(1+k^2\tau^2).$$

On small scales, $k\tau \gg 1$

$$\langle \hat v_{\mathbf{k}}(\tau)\,\hat v_{\mathbf{k}'}(\tau)\rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\frac{H^2}{2k^3}.$$

$$\langle \mathcal{R}_{\mathbf{k}}(t)\,\mathcal{R}_{\mathbf{k}'}(t)\rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')P_S(k)$$
$$= (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\frac{H_*^2}{2k^3}\frac{H_*^2}{\dot\phi_*^2}.$$

$$\langle\mathcal{R}\mathcal{R}\rangle = \int_0^\infty \mathcal{P}_S(k)d\ln k,$$
$$\mathcal{P}_S(k) \equiv \frac{k^2}{2\pi^2}P_S(k) = \frac{H_*^4}{(2\pi)^2\dot\phi_*^2}$$

$$S^{(2)} = \frac{1}{8}\int d^4x a^3\left[(\dot h_{ij})^2 - \frac{1}{a^2}(\partial_l h_{ij})^2\right].$$

$$h_{ij} = \int \frac{d^3k}{(2\pi)^3}\sum_{s=+/\times}\epsilon^s_{ij}(k)h^s_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}},$$
$$h^s_{\mathbf{k}} \equiv h^{+,\times}_{\mathbf{k}}, \;\; \epsilon_{ii} = 0 = k^i\epsilon_{ij} \text{ and } \epsilon^s_{ij}(k)\epsilon^{s'}_{ij}(k) = 2\delta_{ss'}.$$

$$\langle h_{\mathbf{k}}h_{\mathbf{k}'}\rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\frac{1}{2k^3}\frac{H_*^2}{M_p^2}.$$
$$\mathcal{P}_T(k) = \frac{2}{\pi^2}\frac{H_*^2}{M_p^2}.$$

$$n_S - 1 \equiv \frac{d\ln\mathcal{P}_S}{d\ln k} = 2\eta_V - 6\epsilon_V|_{k=aH}$$

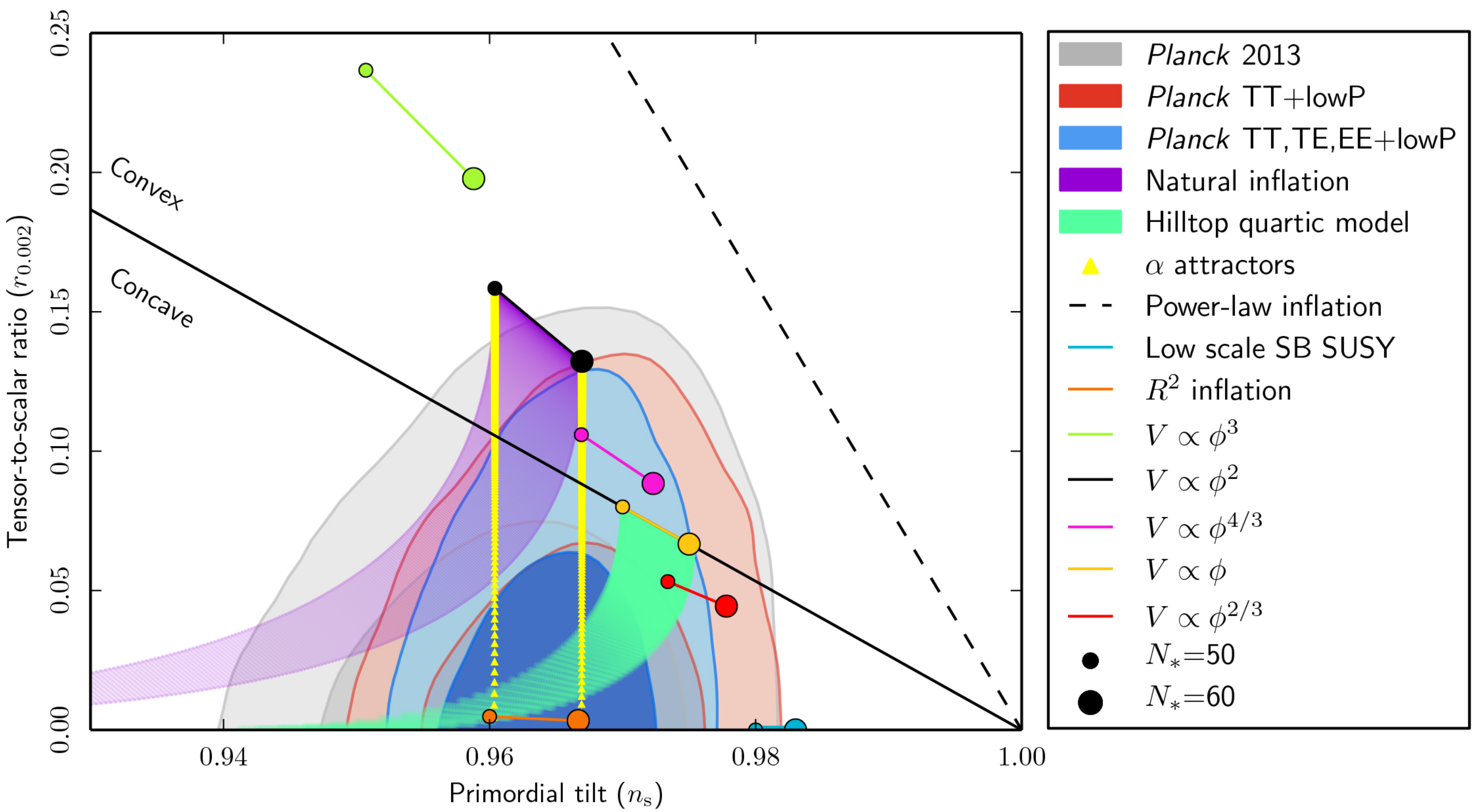
$$n_T \equiv \frac{d\ln\mathcal{P}_T}{d\ln k} = -2\epsilon_V|_{k=aH}.$$

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_S} = 16\epsilon_V,$$
$$r = -8n_T.$$

$$\alpha_T \equiv \left.\frac{dn_T}{d\ln k}\right|_{k=aH}.$$

$$\alpha_S \equiv \left.\frac{dn_s}{d\ln k}\right|_{k=aH}.$$

Inflation Models



$$
\begin{aligned}
n_S &= 0.9655 \pm 0.0062\,, \\
r_{0.002} &< 0.11\,, \\
\alpha_S &= -0.0126^{+0.0098}_{-0.0087}\,.
\end{aligned}
$$

Fig. 12. Marginalized joint 68 % and 95 % CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets, compared to the theoretical predictions of selected inflationary models.

Review: What is inflation with Gauss-Bonnet term?

Inflation with GB term

The action is: $S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)-\frac{1}{2}\xi(\phi)R^2_{\rm GB}\right],$

$\hbar=c=8\pi G=1$

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The background EoMs in the FRW universe with spacetime metric $ds^2=-dt^2+a^2\left(\frac{dr^2}{1-Kr^2}+r^2d\Omega^2\right)$, are:

$$H^2=\frac{\kappa^2}{3}\left[\frac{1}{2}\dot\phi^2+V-\frac{3K}{\kappa^2a^2}+12\dot\xi H\left(H^2+\frac{K}{a^2}\right)\right],$$

$$\dot H=-\frac{\kappa^2}{2}\left[\dot\phi^2-\frac{2K}{\kappa^2a^2}-4\ddot\xi\left(H^2+\frac{K}{a^2}\right)-4\dot\xi H\left(2\dot H-H^2-\frac{3K}{a^2}\right)\right],$$

$$\ddot\phi+3H\dot\phi+V_\phi+12\xi_\phi\left(H^2+\frac{K}{a^2}\right)\left(\dot H+H^2\right)=0,$$

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$$\ddot\phi+3H\dot\phi+V_\phi+12\xi_\phi\left(H^2+\frac{K}{a^2}\right)\left(\dot H+H^2\right)=0,$$

the slow-roll condition

$$\dot\phi^2/2\ll V\,,\quad \ddot\phi\ll 3H\dot\phi\,,\quad 4\dot\xi H\ll 1\,,\quad \text{and}\quad \ddot\xi\ll\dot\xi H\,.$$

$$H^2\simeq\frac{\kappa^2}{3}V\,,$$
$$\dot H\simeq-\frac{\kappa^2}{2}(\dot\phi^2+4\dot\xi H^3)\,,$$
$$3H\dot\phi+V_\phi+12\xi_\phi H^4\simeq 0\,.$$

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The background EoMs in the FRW universe with spacetime metric $ds^2 = -dt^2 + a^2 \left(\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right),$ are:

$$H^2 = \frac{\kappa^2}{3} \left[\frac{1}{2} \dot\phi^2 + V - \frac{3K}{\kappa^2 a^2} + 12 \dot\xi H \left(H^2 + \frac{K}{a^2} \right) \right],$$
$$\dot H = -\frac{\kappa^2}{2} \left[\dot\phi^2 - \frac{2K}{\kappa^2 a^2} - 4\ddot\xi \left(H^2 + \frac{K}{a^2} \right) - 4\dot\xi H \left(2\dot H - H^2 - \frac{3K}{a^2} \right) \right],$$
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$$\epsilon \equiv -\frac{\dot H}{H^2}\,, \quad \eta \equiv \frac{\ddot H}{H\dot H}\,, \quad \delta_1 \equiv 4\kappa^2 \dot\xi H\,, \quad \delta_2 \equiv \frac{\ddot\xi}{\dot\xi H}\,.$$

$$N = \int_t^{t_e} H dt \simeq \int_{\phi_e}^{\phi} \frac{\kappa^2}{Q} d\phi,$$
$$Q \equiv \frac{V_\phi}{V} + \frac{4}{3} \kappa^4 \xi_\phi V\,.$$

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$$N=\int_t^{t_e}Hdt\simeq\int_{\phi_e}^{\phi}\frac{\kappa^2}{Q}d\phi,$$
$$Q\equiv\frac{V_\phi}{V}+\frac{4}{3}\kappa^4\xi_\phi V\,.$$

$$n_s-1\approx -2\epsilon-\frac{2\epsilon(2\epsilon+\eta)-\delta_1(\delta_2-\epsilon)}{2\epsilon-\delta_1}\,,$$
$$r\approx 8(2\epsilon-\delta_1)\,,$$
$$n_t\approx -2\epsilon\,.$$

The main findings!!!

What did previous works find?

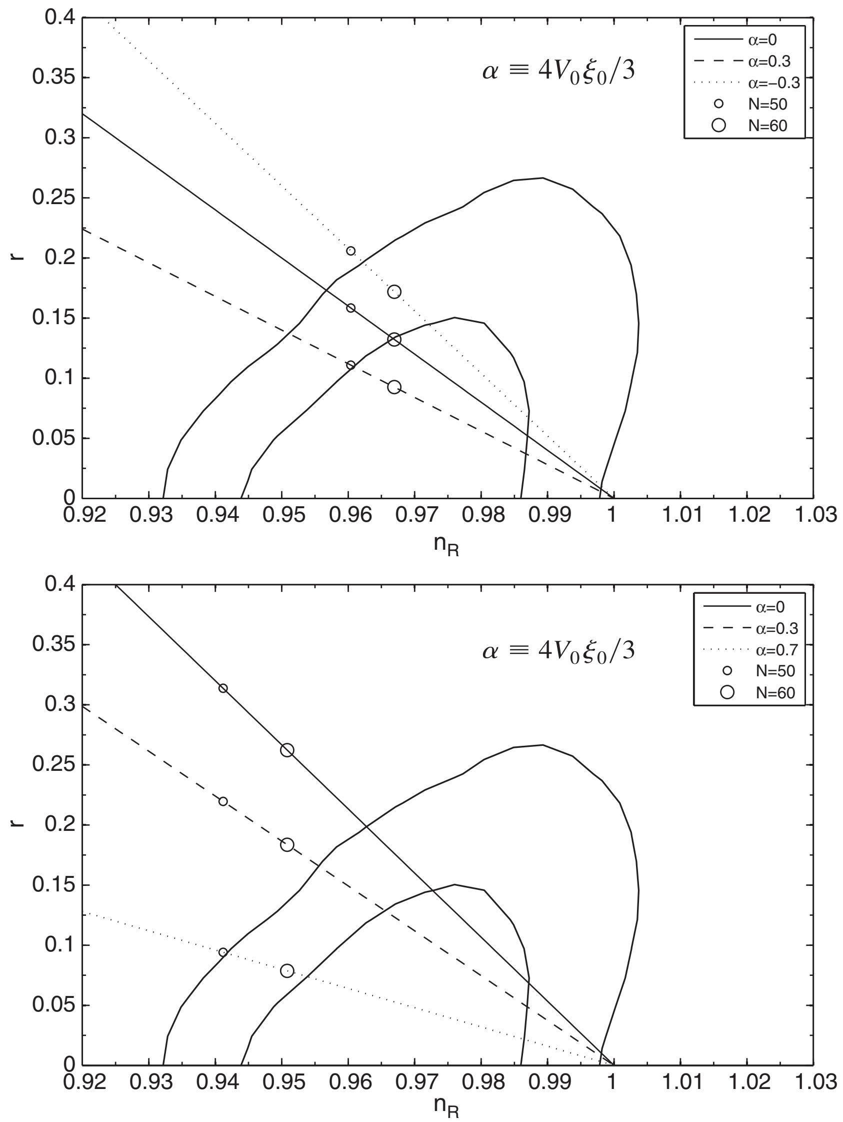

FIG. 2. Tensor-to-scalar ratio r versus the spectral index $n_{\mathcal{R}}$ for the inflation model (49) with $n = 2$ (top panel) and $n = 4$ (bottom panel). The contours show the 68% and 95% confidence level derived from WMAP7 + BAO + H_0 without the consistency relation.

$$V(\phi) = V_0\phi^n, \qquad \xi(\phi) = \xi_0\phi^{-n}.$$

$$n_{\mathcal{R}} - 1 = -\frac{2(n+2)}{4N+n},$$
$$r = \frac{16n(1-\alpha)}{4N+n}.$$

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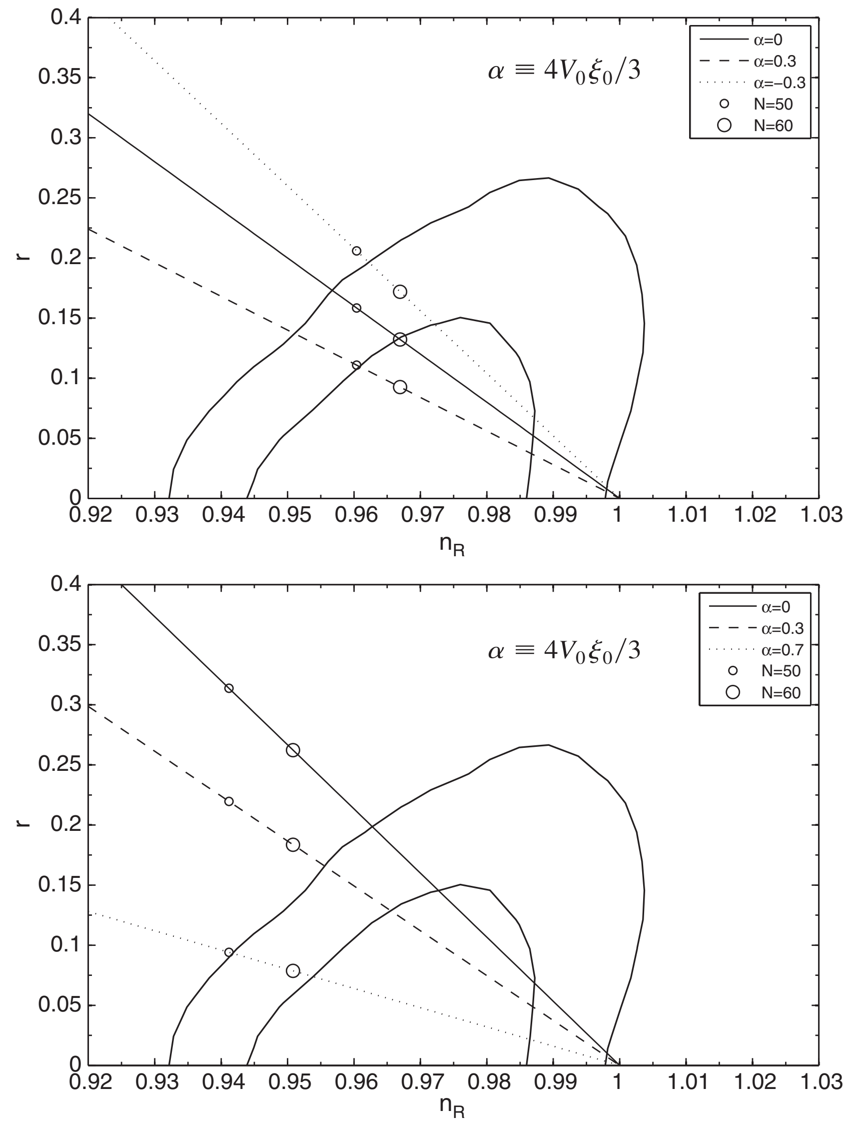

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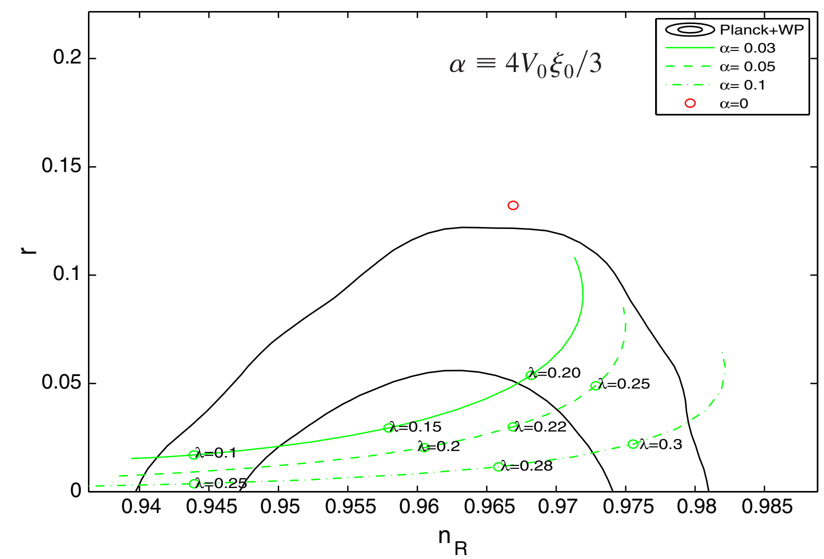

FIG. 2 (color online). Predicted $n_{\mathcal{R}}$ versus r in the model (25) with $n = 2$ for different values of λ and α. Here we choose $N = 60$. The contours show the 68% and 95% CL from the Planck + WP data.

$$V(\phi) = V_0\phi^n, \qquad \xi(\phi) = \xi_0 e^{-\lambda\phi}.$$

$$n_{\mathcal{R}} - 1 = \frac{-n(n+2) + \alpha\lambda e^{-\lambda\phi}\phi^{n+1}(2\lambda\phi - n)}{\phi^2},$$

$$r = \frac{8(n - \alpha\lambda e^{-\lambda\phi}\phi^{n+1})^2}{\phi^2},$$

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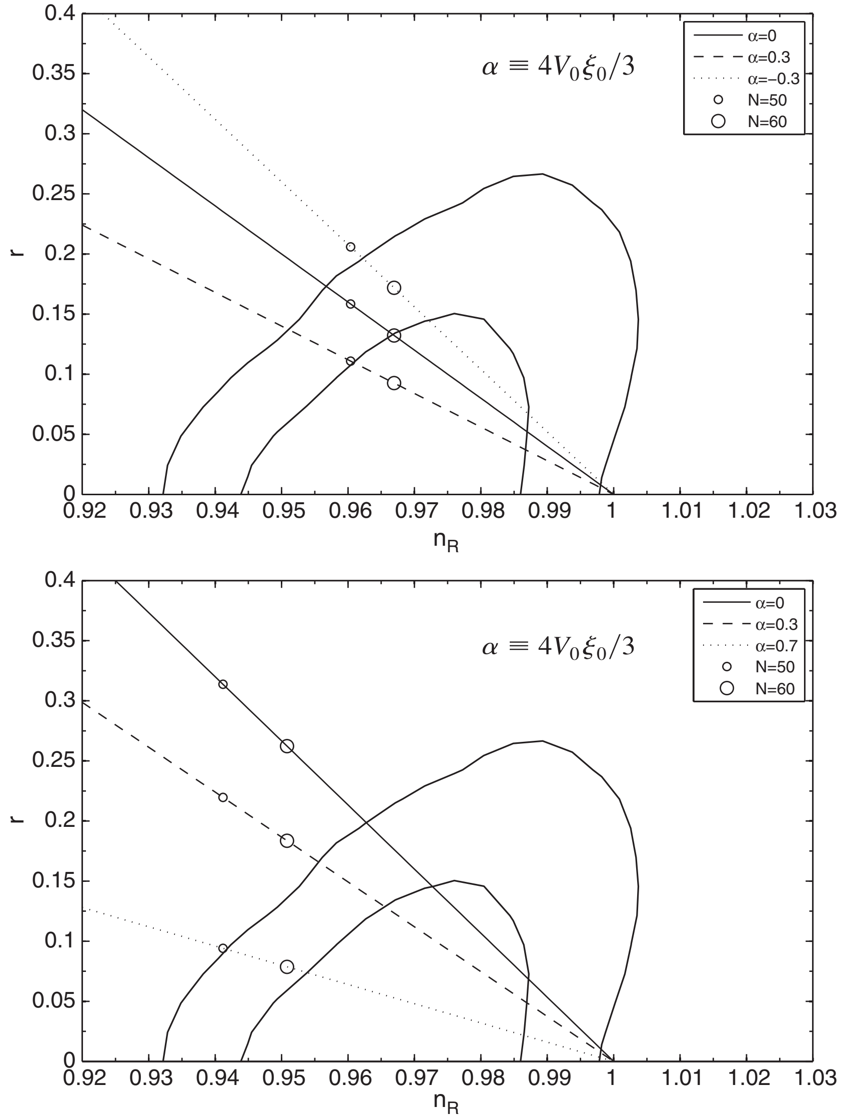

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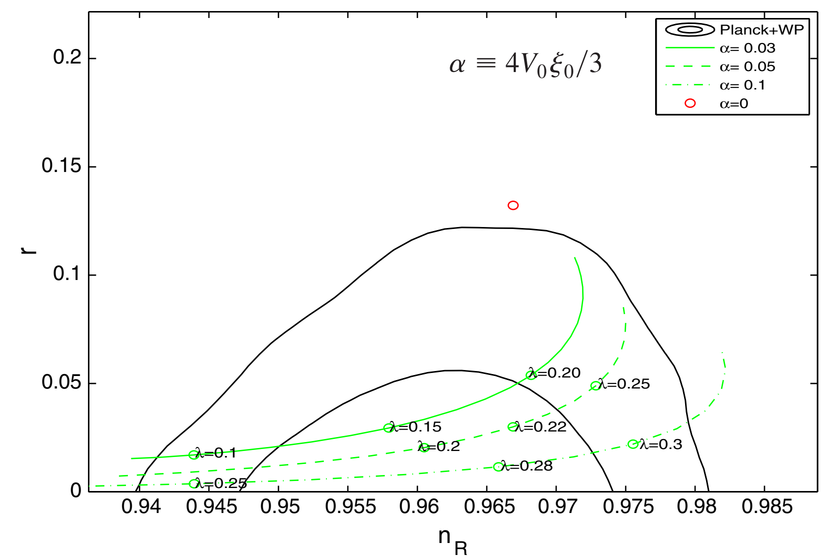

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$$n_{\mathcal{R}} - 1 = \frac{-n(n+2) + \alpha\lambda e^{-\lambda\phi}\phi^{n+1}(2\lambda\phi - n)}{\phi^2},$$
$$r = \frac{8(n - \alpha\lambda e^{-\lambda\phi}\phi^{n+1})^2}{\phi^2},$$

PHYSICAL REVIEW D **90,** 063527 (2014)
KOH *et al.*

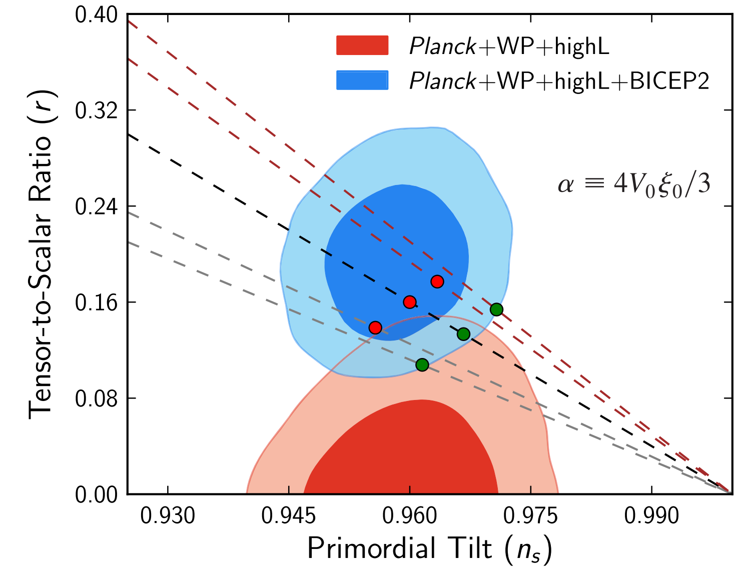

FIG. 5 (color online). Marginalized joint 68% and 95% CL regions for (n_s, r), using observational data sets with and without a running spectral index, compared to the theoretical prediction of the model (73) with $n = 2$. The black dashed line is for the case where model parameter $\alpha = 0$ while gray and brown are for the case where $\alpha = -2 \times 10^{-6}$ and $\alpha = 1.5 \times 10^{-6}$, respectively. The pairs of red and green dots represent the number of e-folds, $N = 50$ and $N = 60$, respectively.

$$V(\phi) = V_0\phi^n, \qquad \xi(\phi) = \xi_0\phi^n.$$

$$n_s - 1 \simeq -\frac{n+2}{2N} + \frac{n(3n+2)(2nN)^n\alpha}{2(1+n)N\kappa^{2n}},$$
$$r \simeq \frac{4n}{N} + \frac{4n(2n+1)(2nN)^n\alpha}{(1+n)N\kappa^{2n}},$$

The Problem

$V(\phi)$ and $\xi(\phi)$

↓

$$
\begin{aligned}
n_s - 1 &\approx -2\epsilon - \frac{2\epsilon(2\epsilon+\eta) - \delta_1(\delta_2 - \epsilon)}{2\epsilon - \delta_1}\,, \\
r &\approx 8(2\epsilon - \delta_1)\,, \\
n_t &\approx -2\epsilon\,.
\end{aligned}
$$

usual way

The Problem in Reverse

$V(\phi)$ and $\xi(\phi)$

⇩

$$
\begin{aligned}
n_s - 1 &\approx -2\epsilon - \frac{2\epsilon(2\epsilon+\eta) - \delta_1(\delta_2 - \epsilon)}{2\epsilon - \delta_1}\,, \\
r &\approx 8(2\epsilon - \delta_1)\,, \\
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\end{aligned}
$$

usual way

$$
\begin{aligned}
n_s - 1 &\approx -2\epsilon - \frac{2\epsilon(2\epsilon+\eta) - \delta_1(\delta_2 - \epsilon)}{2\epsilon - \delta_1}\,, \\
r &\approx 8(2\epsilon - \delta_1)\,, \\
n_t &\approx -2\epsilon\,.
\end{aligned}
$$

⇩

$V(\phi)$ and $\xi(\phi)$

is what we are interested in!!!

Reverse Problem

$$
\begin{aligned}
n_s - 1 &\approx -2\epsilon - \frac{2\epsilon(2\epsilon+\eta) - \delta_1(\delta_2 - \epsilon)}{2\epsilon - \delta_1}\,, \\
r &\approx 8(2\epsilon - \delta_1)\,, \\
n_t &\approx -2\epsilon\,.
\end{aligned}
$$

$$
\epsilon \equiv -\frac{\dot{H}}{H^2}\,, \quad \eta \equiv \frac{\ddot{H}}{H\dot{H}}\,, \quad \delta_1 \equiv 4\kappa^2\dot{\xi}H\,, \quad \delta_2 \equiv \frac{\ddot{\xi}}{\dot{\xi}H}\,.
$$

Reverse Problem

$$\begin{aligned} n_s - 1 &\approx -2\epsilon - \frac{2\epsilon(2\epsilon+\eta) - \delta_1(\delta_2-\epsilon)}{2\epsilon-\delta_1}, \\ r &\approx 8(2\epsilon-\delta_1), \\ n_t &\approx -2\epsilon. \end{aligned}$$

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \eta \equiv \frac{\ddot{H}}{H\dot{H}}, \quad \delta_1 \equiv 4\kappa^2\dot{\xi}H, \quad \delta_2 \equiv \frac{\ddot{\xi}}{\dot{\xi}H}.$$

$$\begin{aligned} H^2 &\simeq \frac{\kappa^2}{3}V, \\ \dot{H} &\simeq -\frac{\kappa^2}{2}(\dot{\phi}^2 + 4\dot{\xi}H^3), \\ 3H\dot{\phi} + V_\phi &+ 12\xi_\phi H^4 \simeq 0. \end{aligned}$$

$$\begin{aligned} \epsilon &= \frac{1}{2\kappa^2}\frac{V_\phi}{V}Q, \\ \eta &= -\frac{1}{\kappa^2}\left(\frac{V_{\phi\phi}}{V_\phi}Q + Q_\phi\right), \\ \delta_1 &= -\frac{4\kappa^2}{3}\xi_\phi V Q, \\ \delta_2 &= -\frac{1}{\kappa^2}\left(\frac{\xi_{\phi\phi}}{\xi_\phi}Q + \frac{1}{2}\frac{V_\phi}{V}Q + Q_\phi\right), \end{aligned}$$

$$Q \equiv \frac{V_\phi}{V} + \frac{4}{3}\kappa^4\xi_\phi V.$$

Reverse Problem

$$
\begin{aligned}
n_s - 1 &\approx -2\epsilon - \frac{2\epsilon(2\epsilon+\eta) - \delta_1(\delta_2 - \epsilon)}{2\epsilon - \delta_1}\,,\\
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$$

$$
\begin{aligned}
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\delta_1 &= -\frac{4\kappa^2}{3}\xi_\phi V Q,\\
\delta_2 &= -\frac{1}{\kappa^2}\left(\frac{\xi_{\phi\phi}}{\xi_\phi}Q + \frac{1}{2}\frac{V_\phi}{V}Q + Q_\phi\right)\,,
\end{aligned}
$$

$$
Q \equiv \frac{V_\phi}{V} + \frac{4}{3}\kappa^4\xi_\phi V\,.
$$

$$
N = \int_t^{t_e} H\,dt \simeq \int_{\phi_e}^{\phi}\frac{\kappa^2}{Q}d\phi,
$$

$$
\begin{aligned}
\epsilon &= \frac{1}{2}\frac{V_N}{V}\,,\\
\eta &= -\frac{V_{NN}}{V_N} = -2\epsilon - \frac{d\ln\epsilon}{dN}\,,\\
\delta_1 &= -\frac{4}{3}\kappa^4\xi_N V\,,\\
\delta_2 &= -\frac{\xi_{NN}}{\xi_N} - \frac{1}{2}\frac{V_N}{V} = \epsilon - \frac{d\ln\delta_1}{dN}\,.
\end{aligned}
$$

Reverse Problem

$$\begin{aligned} n_s - 1 &\approx -2\epsilon - \frac{2\epsilon(2\epsilon+\eta) - \delta_1(\delta_2 - \epsilon)}{2\epsilon - \delta_1}\,, \\ r &\approx 8(2\epsilon - \delta_1)\,, \\ n_t &\approx -2\epsilon\,. \end{aligned}$$

$$\epsilon \equiv -\frac{\dot H}{H^2}\,, \quad \eta \equiv \frac{\ddot H}{H\dot H}\,, \quad \delta_1 \equiv 4\kappa^2 \dot\xi H\,, \quad \delta_2 \equiv \frac{\ddot\xi}{\dot\xi H}\,.$$

$$\begin{aligned} \epsilon &= \frac{1}{2\kappa^2}\frac{V_\phi}{V}Q\,, \\ \eta &= -\frac{1}{\kappa^2}\left(\frac{V_{\phi\phi}}{V_\phi}Q + Q_\phi\right)\,, \\ \delta_1 &= -\frac{4\kappa^2}{3}\xi_\phi V Q\,, \\ \delta_2 &= -\frac{1}{\kappa^2}\left(\frac{\xi_{\phi\phi}}{\xi_\phi}Q + \frac{1}{2}\frac{V_\phi}{V}Q + Q_\phi\right)\,, \end{aligned}$$

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$$\begin{aligned} \epsilon &= \frac{1}{2}\frac{V_N}{V}\,, \\ \eta &= -\frac{V_{NN}}{V_N} = -2\epsilon - \frac{d\ln\epsilon}{dN}\,, \\ \delta_1 &= -\frac{4}{3}\kappa^4 \xi_N V\,, \\ \delta_2 &= -\frac{\xi_{NN}}{\xi_N} - \frac{1}{2}\frac{V_N}{V} = \epsilon - \frac{d\ln\delta_1}{dN}\,. \end{aligned}$$

$$\begin{aligned} n_s(N) - 1 &= \left[\ln\left(\frac{V_N}{V^2} + \frac{4}{3}\kappa^4\xi_N\right)\right]_{,N}\,, \\ r(N) &= 8\left(\frac{V_N}{V} + \frac{4}{3}\kappa^4 \xi_N V\right) = 8Q^{(N)}\,, \\ n_t(N) &= -\frac{V_N}{V}\,. \end{aligned}$$

Reverse Problem

$$
\begin{aligned}
n_s - 1 &\approx -2\epsilon - \frac{2\epsilon(2\epsilon+\eta) - \delta_1(\delta_2 - \epsilon)}{2\epsilon - \delta_1}\,, \\
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$$

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Q \equiv \frac{V_\phi}{V} + \frac{4}{3}\kappa^4\xi_\phi V\,.
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\epsilon &= \frac{1}{2}\frac{V_N}{V}\,, \\
\eta &= -\frac{V_{NN}}{V_N} = -2\epsilon - \frac{d\ln\epsilon}{dN}\,, \\
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\end{aligned}
$$

If these are known!!!

$$
\begin{aligned}
n_s(N) - 1 &= \left[\ln\left(\frac{V_N}{V^2} + \frac{4}{3}\kappa^4\xi_N\right)\right]_{,N}\,, \\
r(N) &= 8\left(\frac{V_N}{V} + \frac{4}{3}\kappa^4\xi_N V\right) = 8Q^{(N)}\,, \\
n_t(N) &= -\frac{V_N}{V}\,.
\end{aligned}
$$

$$
V(N) = \frac{1}{8c_1} r(N) e^{-\int [n_s(N)-1]dN}\,.
$$

$$
\xi(N) = \frac{3}{4\kappa^4}\left[\frac{1}{V(N)} + \int \frac{r(N)}{8V(N)}dN + c_2\right]\,.
$$

Reverse Problem

$$\begin{aligned} n_s - 1 &\approx -2\epsilon - \frac{2\epsilon(2\epsilon+\eta) - \delta_1(\delta_2-\epsilon)}{2\epsilon-\delta_1}\,, \\ r &\approx 8(2\epsilon-\delta_1)\,, \\ n_t &\approx -2\epsilon\,. \end{aligned}$$

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$$Q \equiv \frac{V_\phi}{V} + \frac{4}{3}\kappa^4\xi_\phi V\,.$$

$$N = \int_t^{t_e} H dt \simeq \int_{\phi_e}^{\phi}\frac{\kappa^2}{Q}d\phi,$$

$$V(\phi) \quad \text{and} \quad \xi(\phi)$$

$$\begin{aligned} \epsilon &= \frac{1}{2}\frac{V_N}{V}\,, \\ \eta &= -\frac{V_{NN}}{V_N} = -2\epsilon - \frac{d\ln\epsilon}{dN}\,, \\ \delta_1 &= -\frac{4}{3}\kappa^4\xi_N V\,, \\ \delta_2 &= -\frac{\xi_{NN}}{\xi_N} - \frac{1}{2}\frac{V_N}{V} = \epsilon - \frac{d\ln\delta_1}{dN}\,. \end{aligned}$$

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$$V(N) = \frac{1}{8c_1}r(N)e^{-\int[n_s(N)-1]dN}\,.$$

$$\xi(N) = \frac{3}{4\kappa^4}\left[\frac{1}{V(N)} + \int\frac{r(N)}{8V(N)}dN + c_2\right]\,.$$

Let’s check with some examples…

General Ansatz

$$\begin{aligned} n_s - 1 &= -\frac{\beta}{N+\alpha}, \\ r &= \frac{q}{N^p + \gamma N + \alpha}, \end{aligned}$$

$\gamma = 1$

we set $\beta = p = 2$ and $q = 8$

Takeshi Chiba, PTEP 2015 (2015) no. 7, 073E02

$$\begin{aligned} n_s - 1 &= -\frac{2}{N+\alpha}, \\ r &= \frac{8}{N^2 + N + \alpha}. \end{aligned}$$

$$\begin{aligned} V(\phi) &= c_1 \tanh^2\left(\frac{1}{2}\kappa(\phi - C)\right), \\ \xi(\phi) &= \frac{3}{4\kappa^4}(c_1 - c_2). \end{aligned}$$

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$\gamma = 0$

$$n_s(N) - 1 = -\frac{\beta}{N+\alpha}\,, \qquad r(N) = \frac{q}{N^p + \alpha}\,.$$

$p = 1$ **case**

$p = 2$ **case**

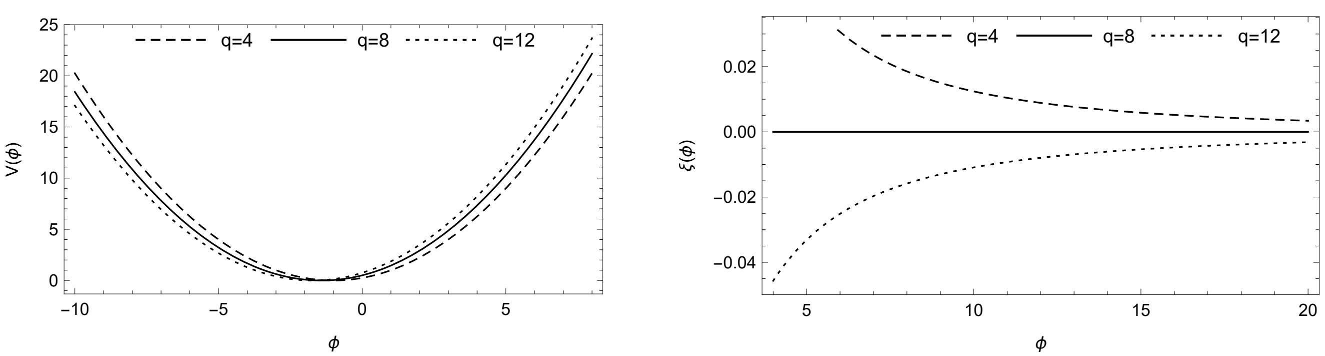
$V(\phi)$ and $\xi(\phi)$

$\gamma = 0$ Model

$$
\begin{aligned}
n_s(N) - 1 &= -\frac{\beta}{N+\alpha}\,, \\
r(N) &= \frac{q}{N^p+\alpha}\,.
\end{aligned}
$$

$p = 1$ case

$$
\begin{aligned}
V(\phi) &= \frac{q}{8c_1}\left(\alpha + \frac{2}{q}\kappa^2\phi^2 + \sqrt{\frac{8\alpha}{q}}\kappa\phi\right)^{\beta-1}, \\
\xi(\phi) &= \frac{3}{4\kappa^4}\left[\frac{q+8(1-\beta)}{q(1-\beta)}\left(\alpha + \frac{2}{q}\kappa^2\phi^2 + \sqrt{\frac{8\alpha}{q}}\kappa\phi\right)^{1-\beta} c_1 + c_2\right].
\end{aligned}
$$



(a) (b)

Figure 1: Numerical plot of Eq. (50) and Eq. (51) with $c_1 = 1$, $c_2 = 0$, $\kappa^2 = 1$ and $n = 2$.

For $\alpha \to 0$ limit

$$
\begin{aligned}
V(\phi) &= \frac{q}{8c_1}\left(\frac{2}{q}\kappa^2\phi^2\right)^{\beta-1}, \\
\xi(\phi) &= \frac{3}{4\kappa^4}\left[\frac{q+8(1-\beta)}{q(1-\beta)}\left(\frac{2}{q}\kappa^2\phi^2\right)^{1-\beta} c_1 + c_2\right],
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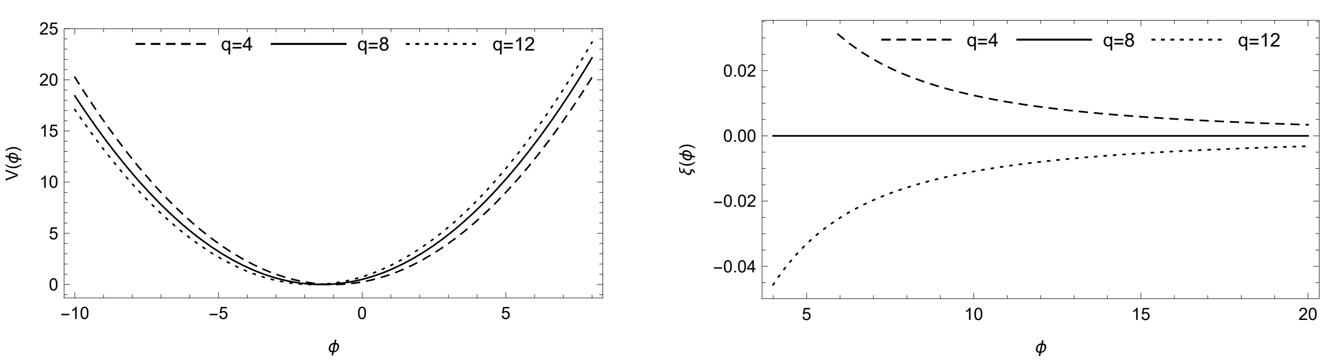
$$



(a) (b)

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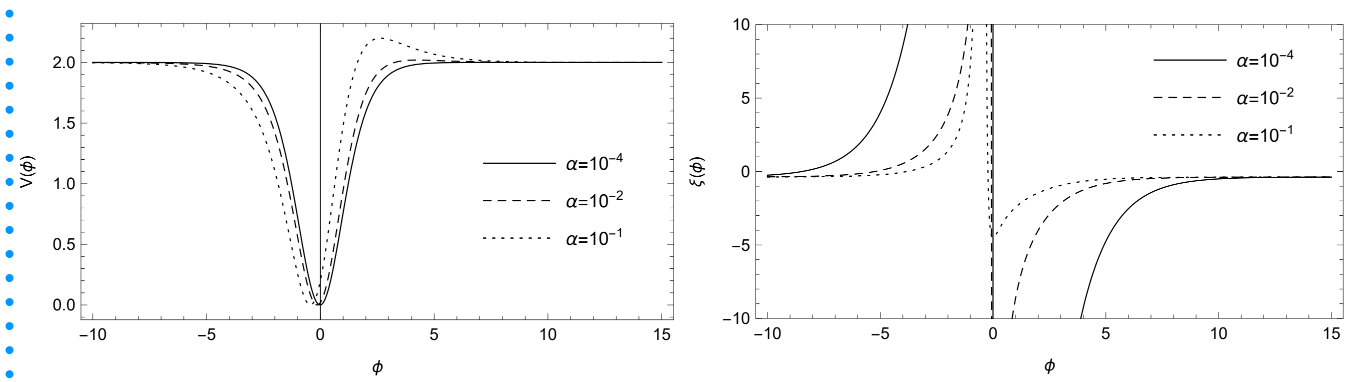
\begin{aligned}
V(\phi) &= \frac{q}{8c_1}\left(\frac{2}{q}\kappa^2\phi^2\right)^{\beta-1},\\
\xi(\phi) &= \frac{3}{4\kappa^4}\left[\frac{q+8(1-\beta)}{q(1-\beta)}\left(\frac{2}{q}\kappa^2\phi^2\right)^{1-\beta} c_1 + c_2\right],
\end{aligned}
$$

$p = 2$ case

$$
\begin{aligned}
V(\phi) &= \frac{q}{8c_1\alpha}\mathrm{sech}^2\left(\sqrt{\frac{8}{q}}\kappa\phi\right)\left[\alpha + \sqrt{\alpha}\sinh\left(\sqrt{\frac{8}{q}}\kappa\phi\right)\right]^{\beta},\\
\xi(\phi) &= \frac{3}{4\kappa^4}\left[\frac{q\left(\alpha + \sqrt{\alpha}\sinh\left(\sqrt{\frac{8}{q}}\kappa\phi\right)\right) + 8(1-\beta)\alpha\cosh^2\left(\sqrt{\frac{8}{q}}\kappa\phi\right)}{q(1-\beta)\left(\alpha + \sqrt{\alpha}\sinh\left(\sqrt{\frac{8}{q}}\kappa\phi\right)\right)^{\beta}} c_1 + c_2\right],
\end{aligned}
$$



(a) (b)

Figure 2: Numerical plots of Eqs. (56)–(57) with $c_1 = 1$, $c_2 = 0$, $\kappa^2 = 1$, $\beta = 2$ and q=16. The bump shown in Fig. 2(a) increases as α increases and vice versa.

For $\alpha \to 0$ limit

$$
\begin{aligned}
V(\phi) &\sim \tanh^2\left(\sqrt{\frac{8}{q}}\kappa\phi\right),\\
\xi(\phi) &\sim -\frac{3c_1}{4\sqrt{\alpha}\kappa^4}\mathrm{csch}\left(\sqrt{\frac{8\kappa^2}{q}}\phi\right).
\end{aligned}
$$

Potential & Coupling Function

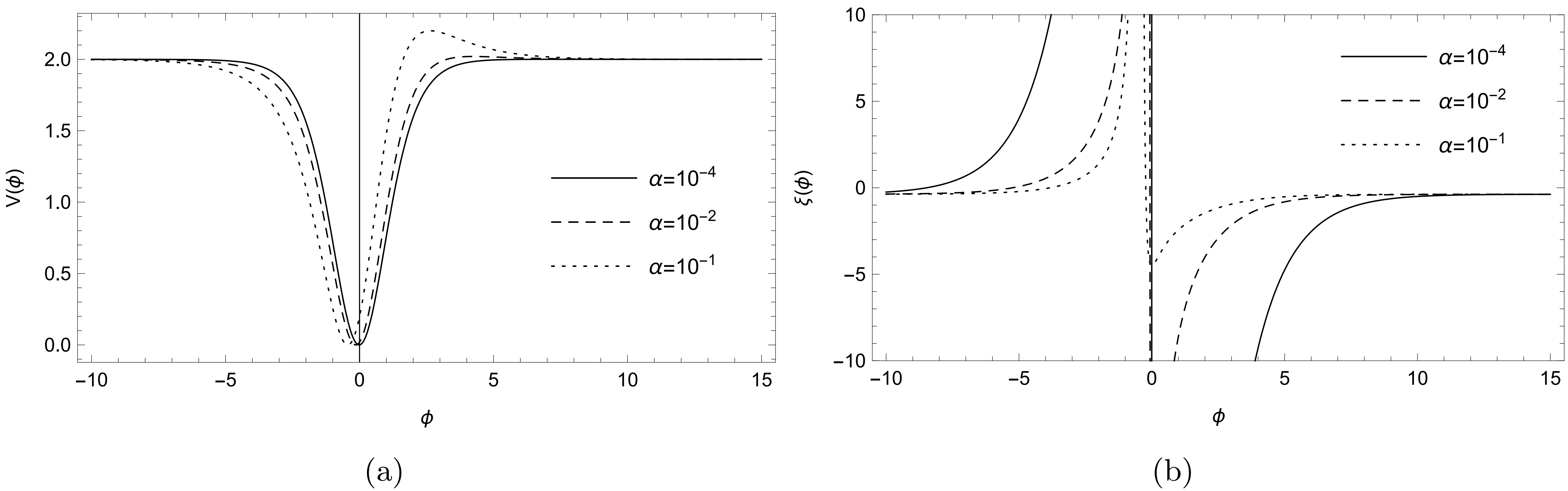


Figure 2: Numerical plots of Eqs. (56)–(57) with $c_1 = 1$, $c_2 = 0$, $\kappa^2 = 1$, $\beta = 2$ and q=16. The bump shown in Fig. 2(a) increases as α increases and vice versa.

What is interesting?

Blue-spectrum for $p = 2$

For the conventional models of inflation, $(\dot{H} < 0)$, and it is implied that $\epsilon > 0$. Hence $n_t < 0$.

Blue-spectrum for $p = 2$

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But such case is violated in our model:

Blue-spectrum for $p = 2$

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But such case is violated in our model:

The spectrum of the tensor mode can be blue-tilted $n_t > 0$, if: JCAP11 (2010) 024

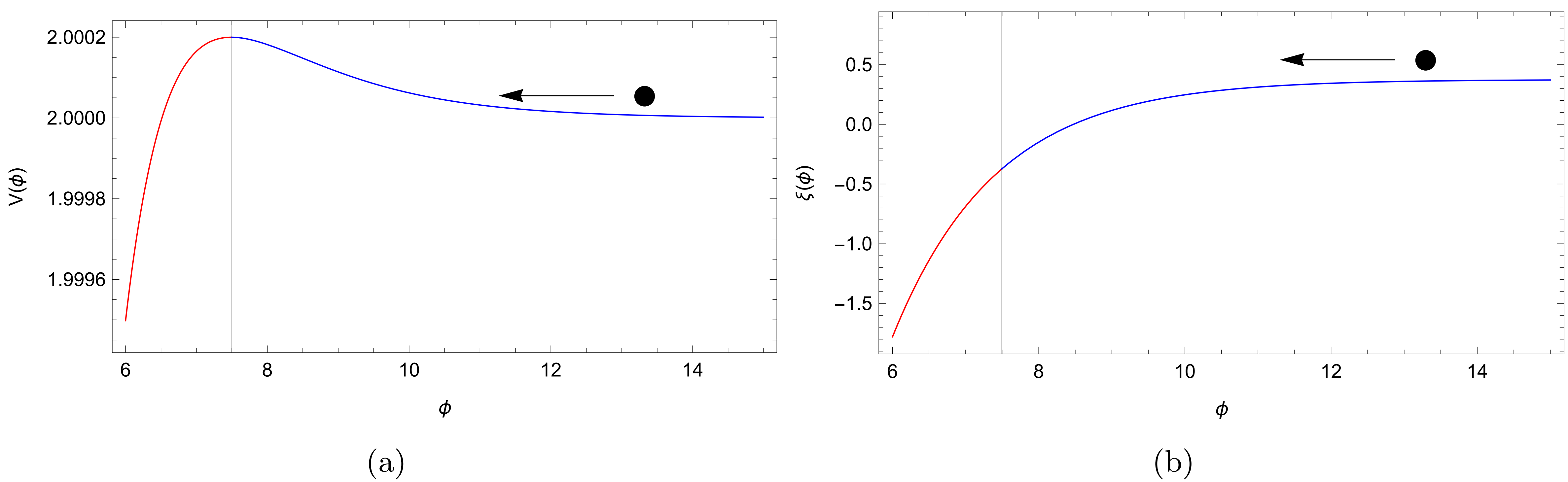
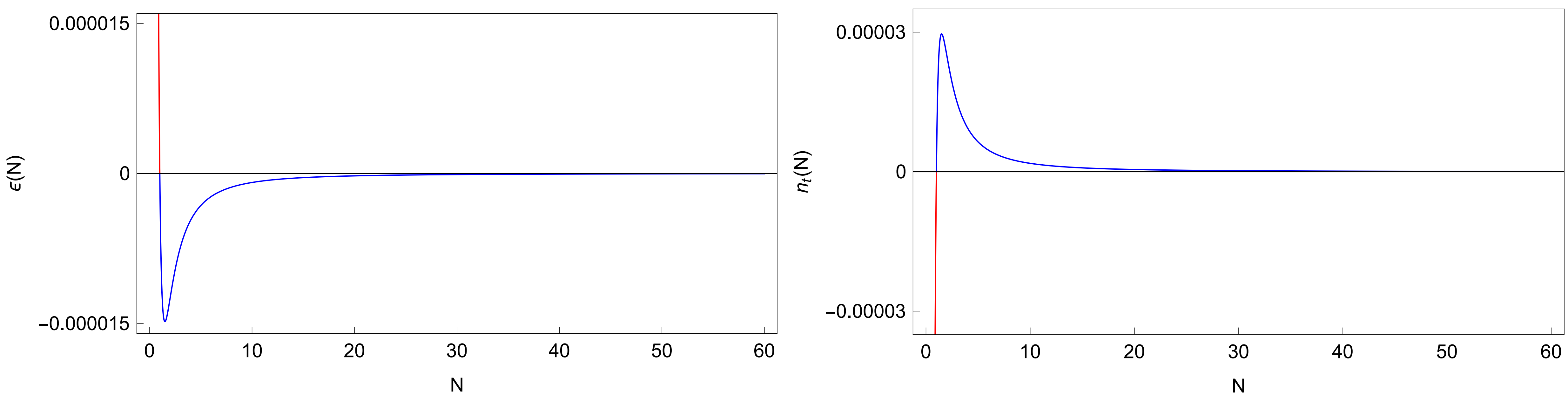


Figure 4: Marginalized part of the potential (left) and the Gauss-Bonnet coupling (right) shown in Figs. 2(a) and 2(b) where we set $\alpha = 10^{-4}$. Vertical line corresponds to the field value, ϕ_*, at which the potential takes its maximum value. At early stage, the effective potential ξ makes ϕ climb up the potential slope. At late stage, ϕ rolls down as usual.

Blue-spectrum for $p = 2$

On the other hand, to achieve the blue-tilted spectrum for the tensor fluctuations, $n_t > 0$, in our model, $\epsilon < 0$ must be satisfied from $n_t \approx -2\epsilon$ such that $\dot{H} > 0$ is necessary from $\epsilon \equiv -\dot{H}/H^2$

Condition for the coupling function: $\xi_{,\phi} > -\frac{3}{4\kappa^4}\frac{V_{,\phi}}{V^2}\,,$ → $\cosh\left(\sqrt{\frac{8\kappa^2}{q}}\phi\right)\left(\sqrt{\alpha} + \sinh\left(\sqrt{\frac{8\kappa^2}{q}}\phi\right)\right)^2 > 0\,,$



(a) (b)

Figure 5: $\epsilon(N)$ and $n_t(N)$ plot where we use Eqs. (56)–(57) with $\kappa^2 = 1$, $c_1 = 1$, $c_2 = 0$, $\alpha = 10^{-4}$, $\beta = 2$ and $q = 16$. At $N = 1$, both ϵ and n_t is zero, $\epsilon = 0 = n_t$.

Blue-spectrum for $p = 2$

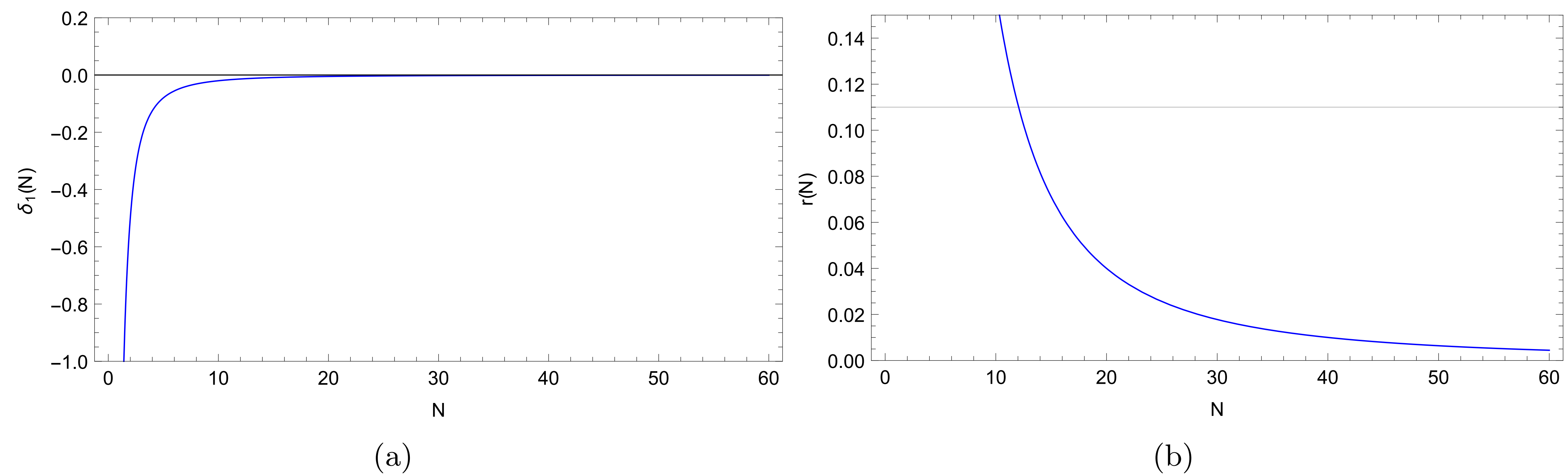


Figure 6: $\delta_1(N)$ and $r(N)$ plot where we use Eq. (56) and Eq. (57) with $c_1 = 1$, $c_2 = 0$, $\kappa^2 = 1$, $\alpha = 10^{-4}$, $\beta = 2$ and $q = 16$. Horizontal line in Fig. 6(b) represents the current upper limit of the tensor-to-scalar ratio.

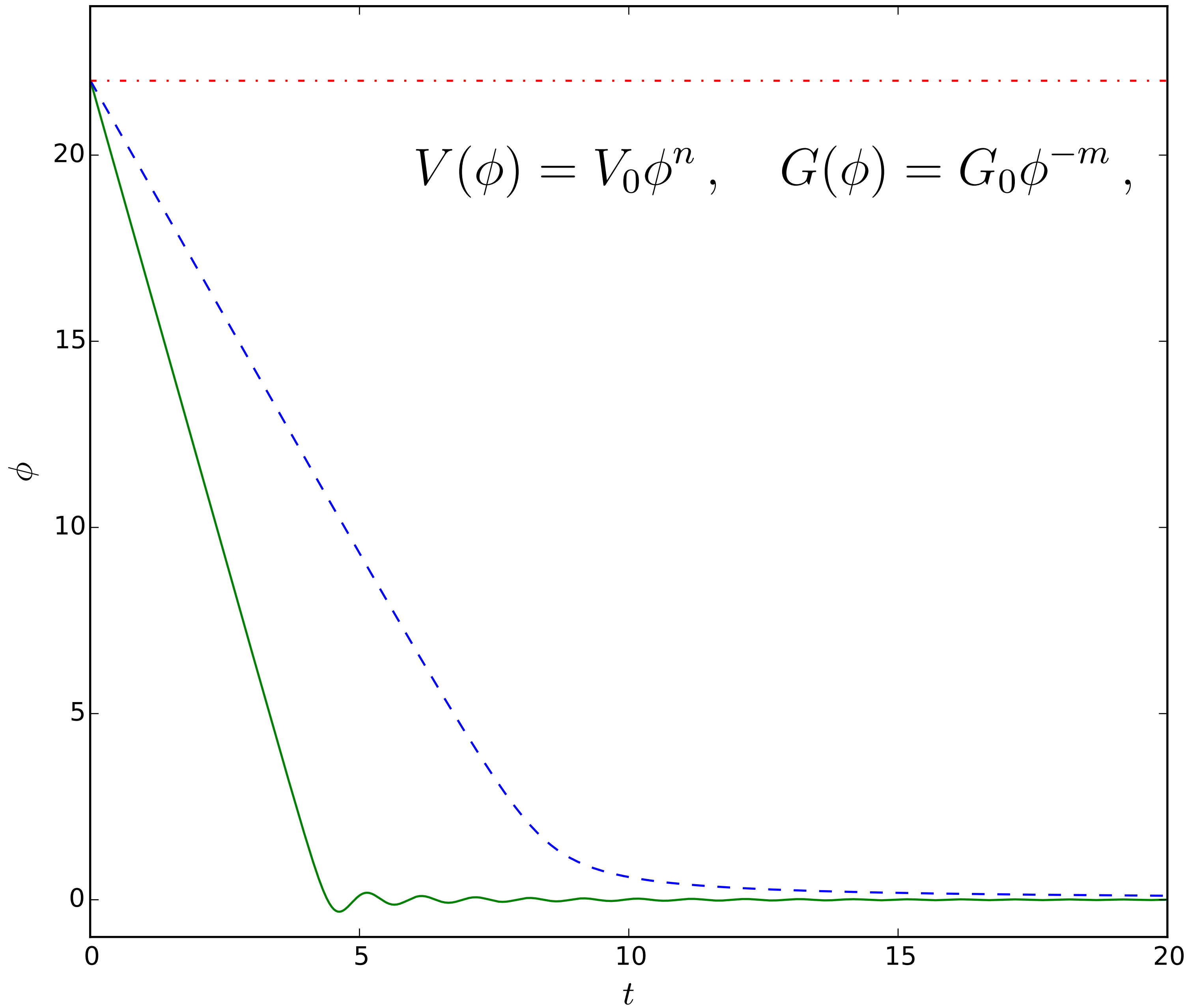
Findings!!!

- Potential is reconstructed: $n_S(N), \quad r(N).$
- The spectrum of the tensor mode can be blue-tilted

What could be possible extension?

Reheating?!

Carsten, et al PRD 94, 023506 (2016)

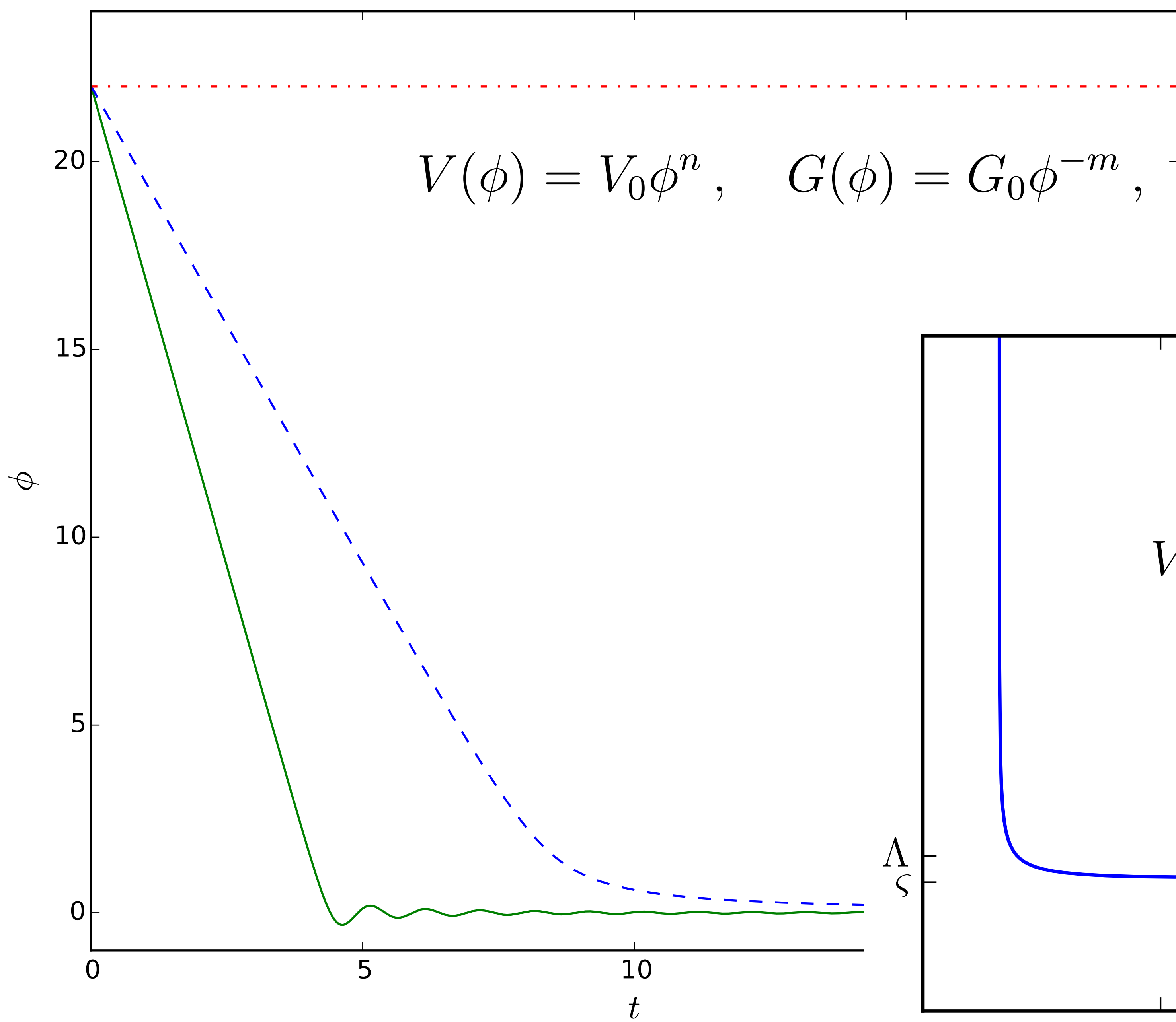


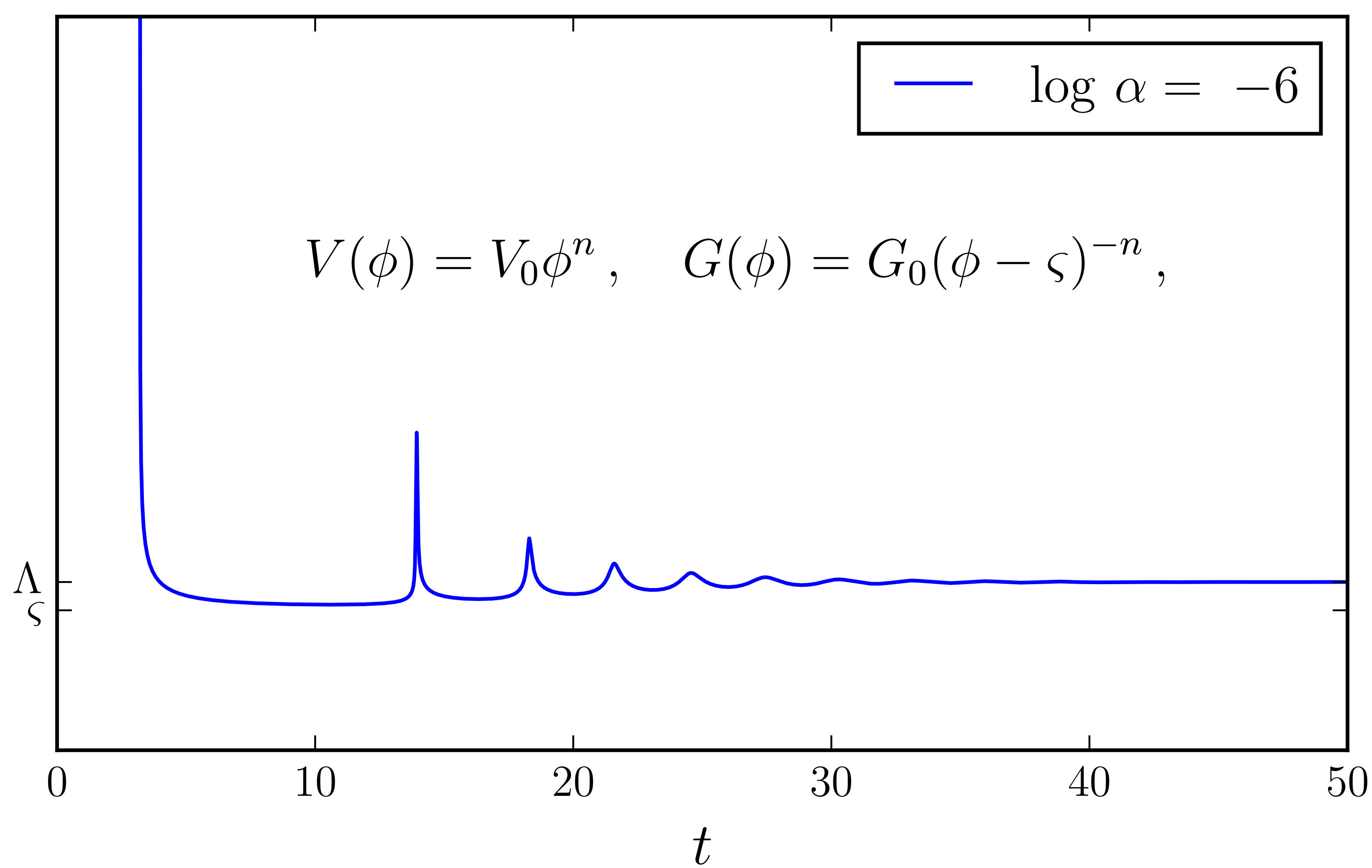
Reheating?!

Carsten, et al PRD 94, 023506 (2016)

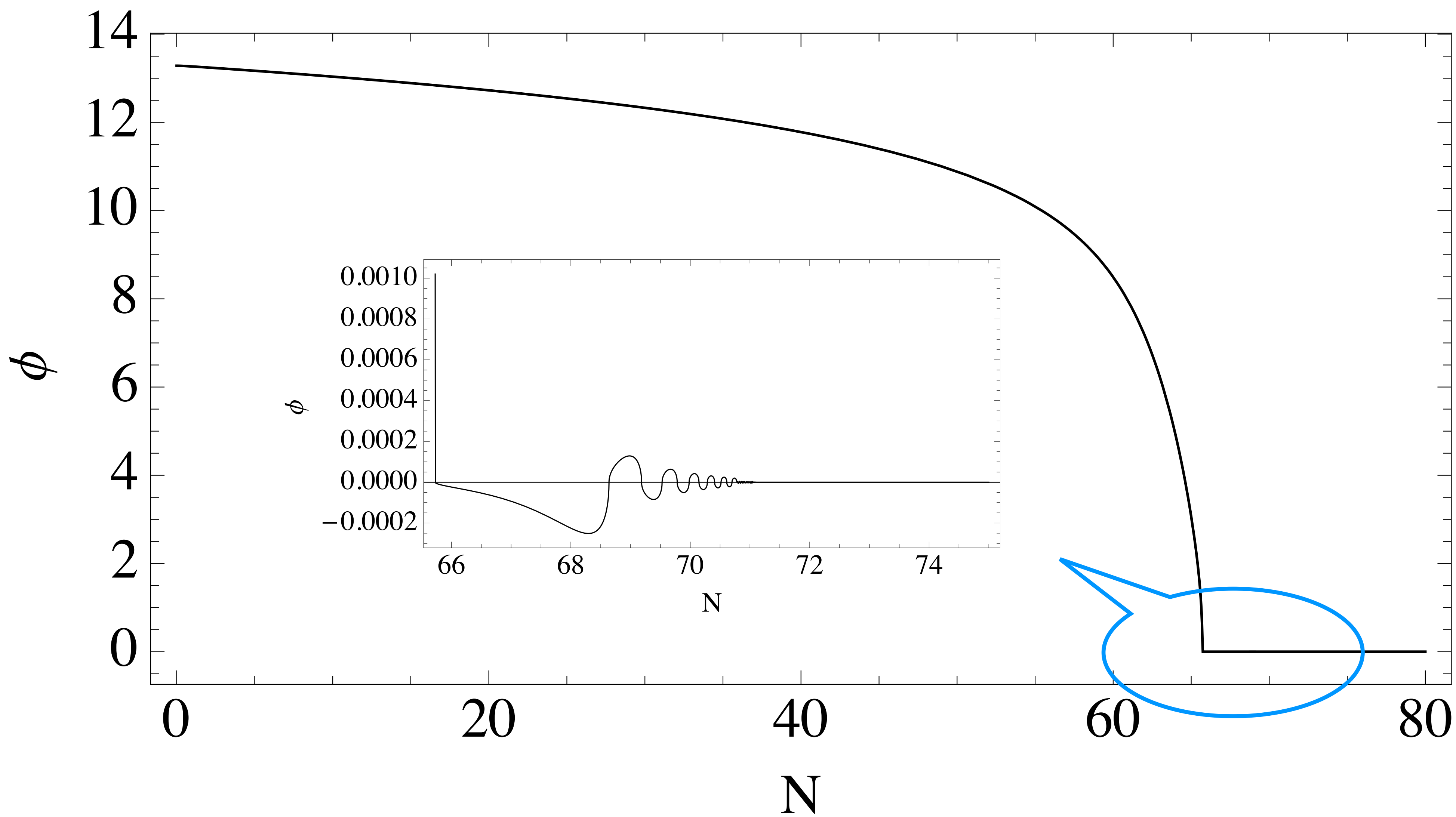
$$V(\phi) = V_0(\phi + \varsigma)^n\,, \quad G(\phi) = G_0\phi^{-n}\,.$$

$$\phi \to \phi - \varsigma,$$



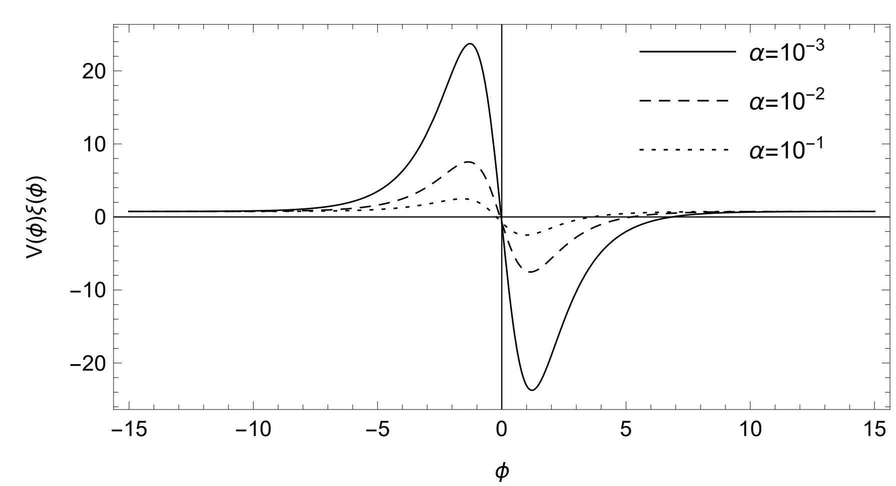


Reheating?!



$$V(\phi) = \frac{q}{8c_1\alpha}\text{sech}^2\left(\sqrt{\frac{8}{q}}\kappa\phi\right)\left[\alpha+\sqrt{\alpha}\sinh\left(\sqrt{\frac{8}{q}}\kappa\phi\right)\right]^{\beta},$$

$$\xi(\phi) = \frac{3}{4\kappa^4}\left[\frac{q\left(\alpha+\sqrt{\alpha}\sinh\left(\sqrt{\frac{8}{q}}\kappa\phi\right)\right)+8(1-\beta)\alpha\cosh^2\left(\sqrt{\frac{8}{q}}\kappa\phi\right)}{q(1-\beta)\left(\alpha+\sqrt{\alpha}\sinh\left(\sqrt{\frac{8}{q}}\kappa\phi\right)\right)^{\beta}}c_1+c_2\right],$$



Thank you for your attentions!!!