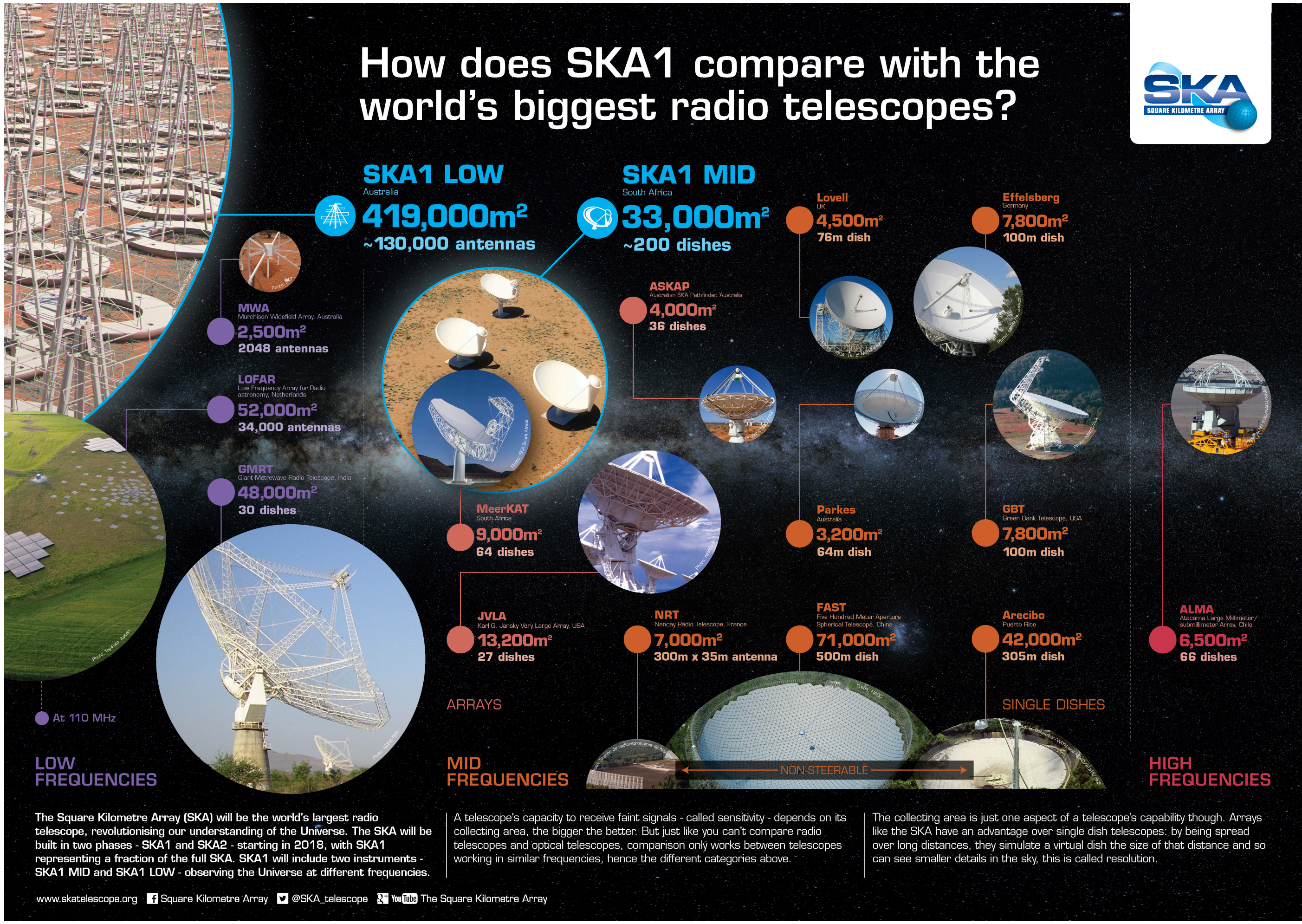
STCOS 2016, Hanyang University 2016.8.18

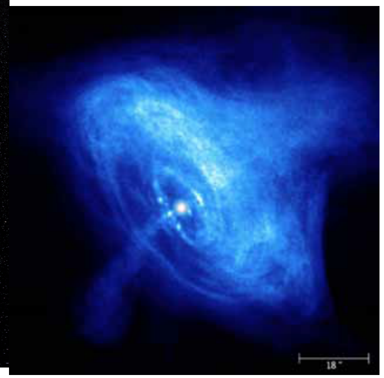
Probing Ultralight Axion Dark Matter

Jiro Soda
Kobe University

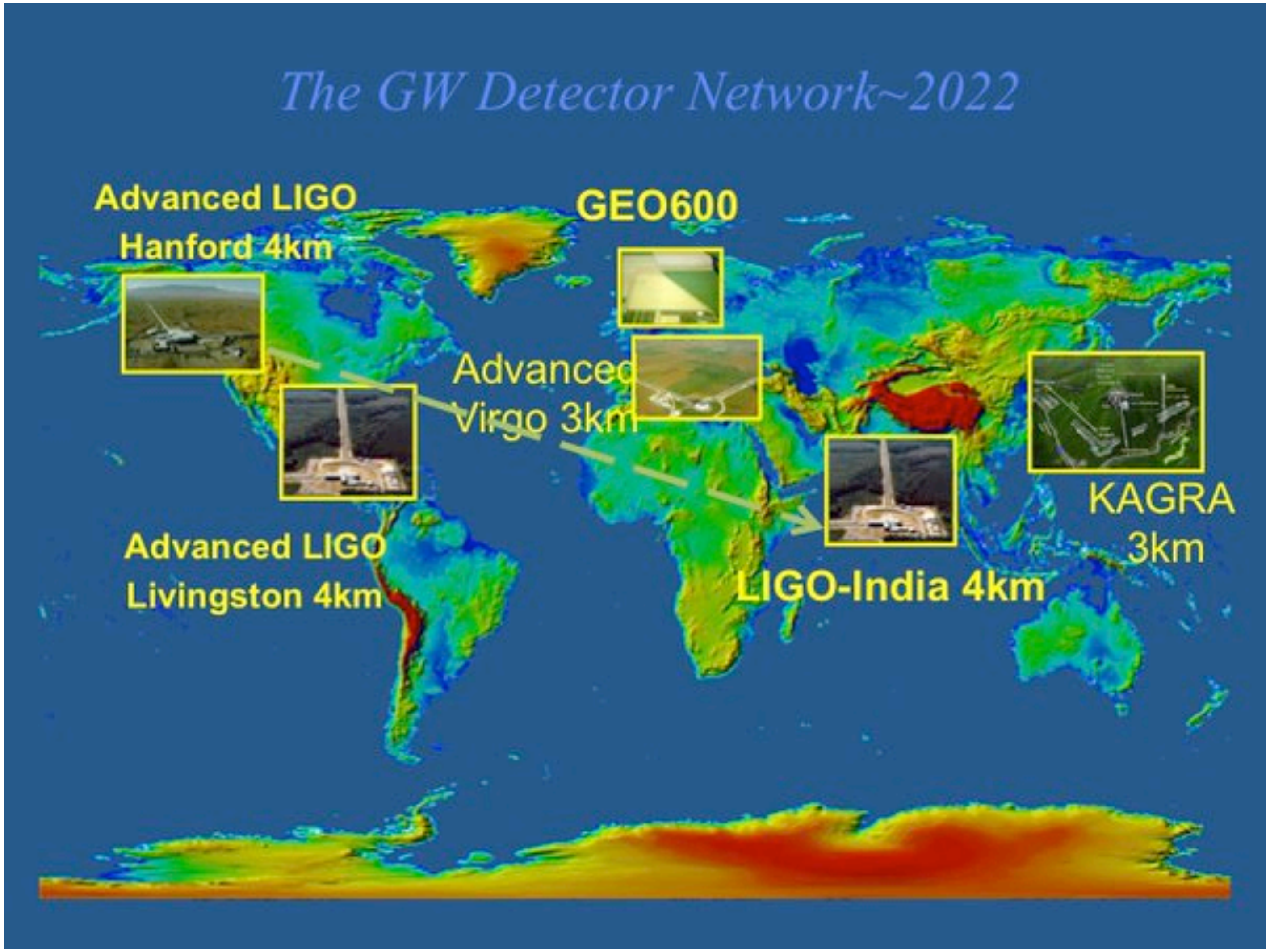
A. Aoki, J.S., Phys.Rev.D93 (2016) 083503
A. Aoki, J.S., arXiv:1608.xxxxx

Pulsar timing array experiments

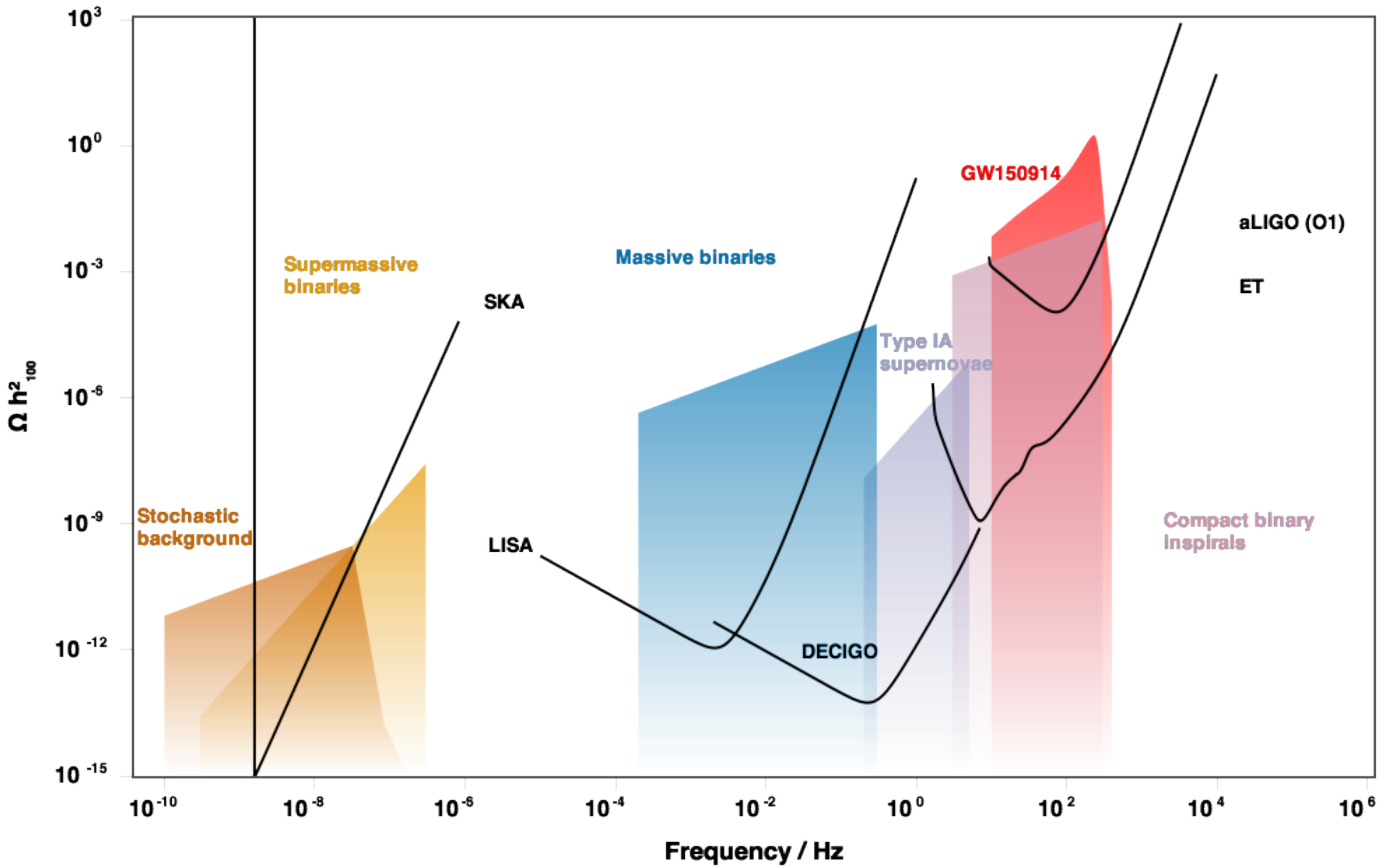




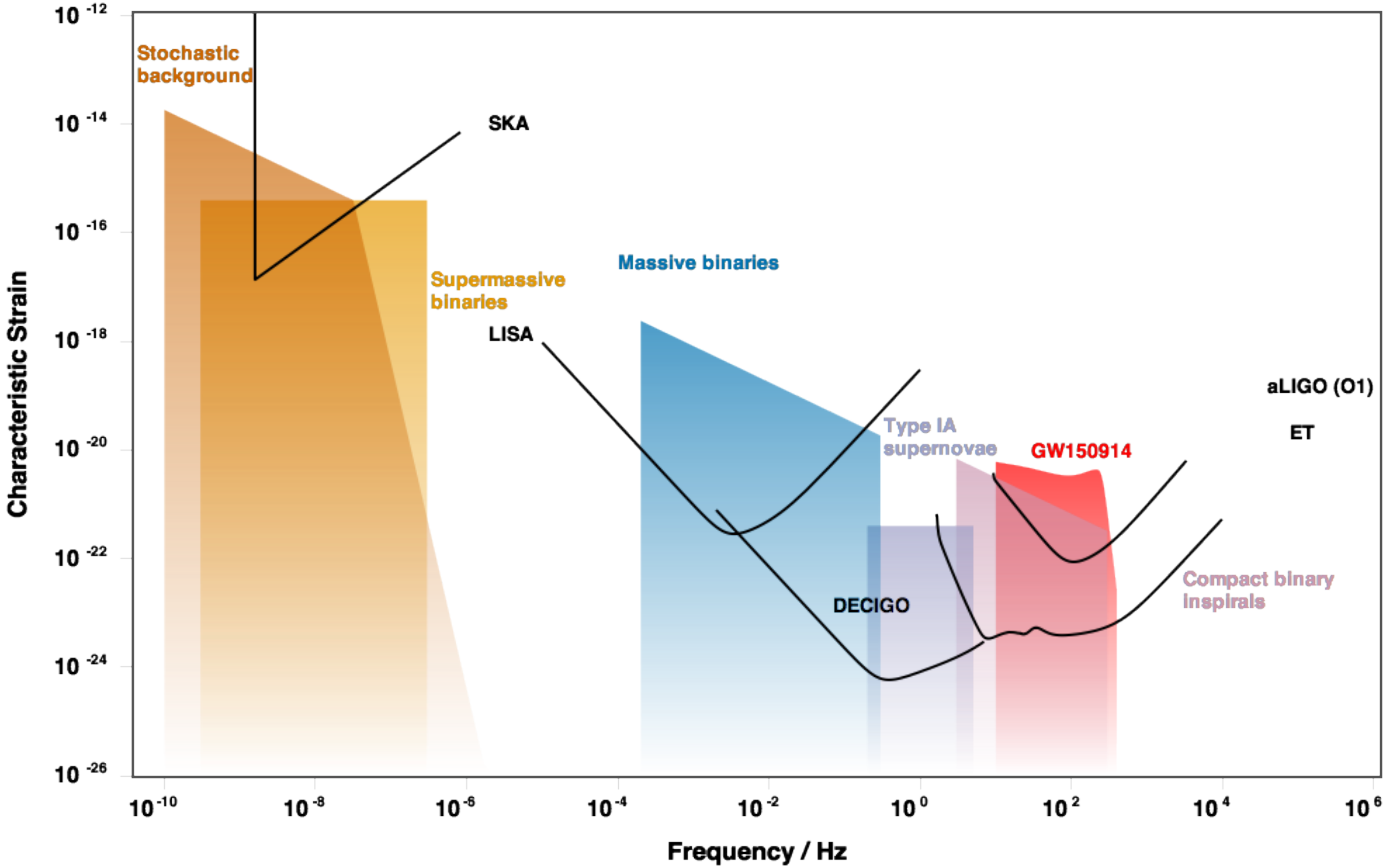
Laser interferometer experiments



Sources & Sensitivity of detectors

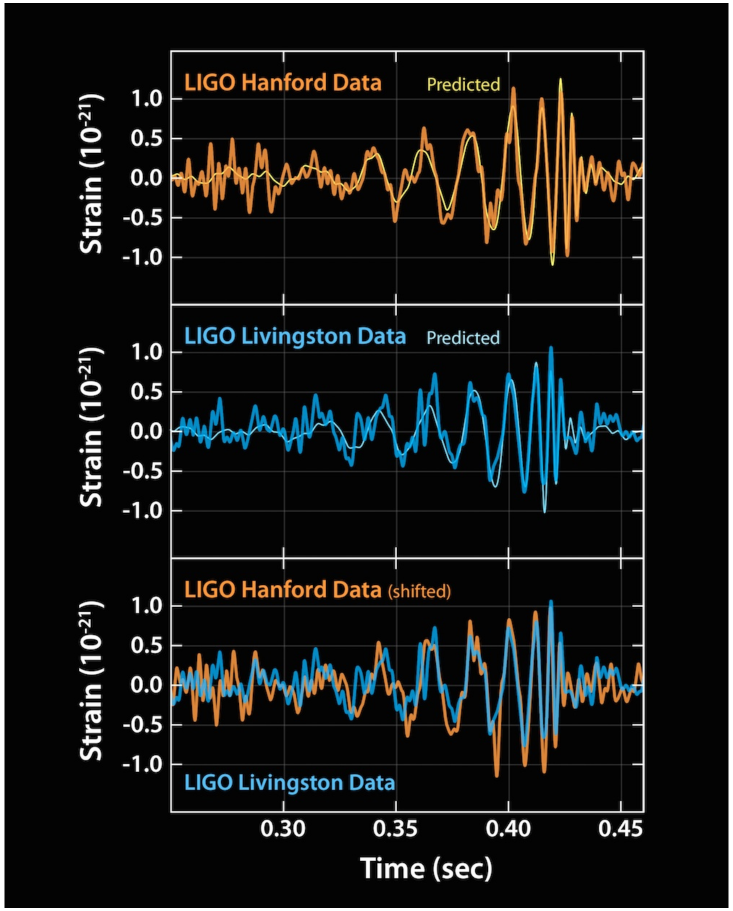


Sources & Sensitivity of detectors



Congratulations on the GW discovery!

GW150914



The next target should be dark matter.
In this talk, I will show you GW detectors are useful for this purpose too.

Axion dark matter?

No evidence of supersymmetry has been found at the LHC.
Hence, there is no reason to stick to neutralinos.

Moreover, the standard ΛCDM Model has a cusp problem, that is,
the structures of sub-galactic scales are overabundant.

An ultralight scalar field (axion) dark matter erases extra structures on sub-galactic scales because of the effective quantum pressure. In fact, de Broglie wavelength of dark matter is given by

$$\lambda_{dB} = \frac{2\pi}{k} = \frac{2\pi}{mv} \approx 3.8\mathrm{kpc}\left(\frac{10^{-3}}{v}\right)\left(\frac{10^{-23}\mathrm{eV}}{m}\right)$$

It is worth investigating the ultralight axion dark matter seriously.

M.R.Baldeschi et al. 1983
M.Membrado et al. 1989
S.J.Sin 1994
J.W.Lee, I.G.Koh 1996
W.Hu et al. 2000
For complete references see J.W.Lee 2016

Contents

- *Axion oscillation*
- *Probing axion DM with PTAs*
- *Probing axion DM with interferometers*
- *Summary*

Axion oscillation

Ultralight Axion

The model

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\,R - \int d^4x\sqrt{-g}\left[\frac{1}{2}\nabla^{\mu}\phi\nabla_{\mu}\phi + \frac{1}{2}m^2\phi^2\right]$$

Occupation number

$$\frac{N}{\Delta x^3 \Delta p^3} \approx \frac{n}{k^3} = \frac{\rho_{DM}}{m k^3} \approx 10^{90}\left(\frac{\rho_{DM}}{0.3\,\mathrm{GeV/cm^3}}\right)\left(\frac{10^{-23}\,\mathrm{eV}}{m}\right)^4$$

Since the occupation number is so high, classical field description is quite good.

Since the typical velocity of the dark matter $v \approx 10^{-3}$

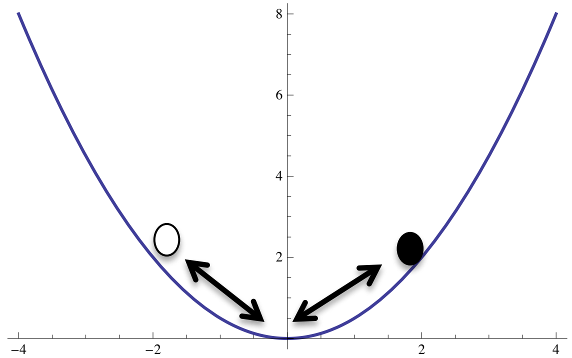
We have a monochromatic frequency

$$E \approx m + \frac{1}{2}mv^2 \approx m$$

Axion oscillation

The axion is an coherently oscillating scalar field

$$\phi = A(\mathbf{x})\cos\left(mt + \alpha(\mathbf{x})\right)$$



The energy density becomes

$$\rho_{DM} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 \approx \frac{1}{2}m^2 A^2$$

Indeed, oscillating part of the energy density $\left(\nabla\phi\right)^2 \approx k^2\phi^2$ is small

$$\rho_{DM}^{osc} \approx \frac{k^2}{m^2}\rho_{DM} \approx v^2\rho_{DM} \approx 10^{-6}\rho_{DM}$$

The pressure is given by

$$p_{DM} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2 \approx -\frac{1}{2}m^2 A^2\cos\left(2mt + 2\alpha(\mathbf{x})\right)$$

The average value of the pressure over the oscillation period is zero.
Hence, the axion can be regarded as the dust matter on cosmological scales.

Time dependent gravitational potential

On the galactic scales, the background spacetime can be regarded as the Minkowski spacetime.

The perturbed metric in the conformal Newtonian gauge takes the form

$$ds^2 = -\left(1+2\Psi\right)dt^2 + \left(1-2\Phi\right)\delta_{ij}dx^i dx^j$$

The spatial component of Einstein equation reads

$$R = -6\ddot{\Phi} + 2\nabla^2\left(2\Phi - \Psi\right) = -T = \rho_{DM}\left[1 + 3\cos\left(2mt + 2\alpha(\mathbf{x})\right)\right]$$

Since the time independent part obeys $\Phi_0 = \Psi_0 \approx -\dfrac{\rho_{DM}}{2k^2}$

We obtain

$$-6\delta\ddot{\Phi} = 3\rho_{DM}\cos\left(2mt + 2\alpha(\mathbf{x})\right)$$

$$\delta\Phi = \frac{\rho_{DM}}{8m^2}\cos\left(2mt + 2\alpha(\mathbf{x})\right) \ll \Phi_0$$

Probing Axion DM with Pulsar Timing Arrays

Effect on the pulsar timing

Khmelnitsky & Rubakov 2014

The timing residual is given by the frequency shift $\Delta t = -\int_0^t \frac{\Omega(t') - \Omega_0}{\Omega_0} dt'$

The geodesic equation gives rise to the formula

$$\frac{\Omega(t) - \Omega_0}{\Omega_0} = \Phi(\mathbf{x}, t) - \Phi(\mathbf{x}_p, t') - \int_{t'}^{t} n^i \partial_i \big(\Phi(\mathbf{x}'', t'') + \Psi(\mathbf{x}'', t''\big) dt''$$

Since the distance is typically larger than the Compton length $D \geq 100\mathrm{pc} \gg \mathrm{m}^{-1}$

$$\frac{\Omega(t) - \Omega_0}{\Omega_0} = \frac{\rho_{DM}}{8m^2}\left[\cos\big(2mt + 2\alpha(\mathbf{x})\big) - \cos\big(2m(t-D) + 2\alpha(\mathbf{x}_p)\big)\right]$$

Pulsar timing residual

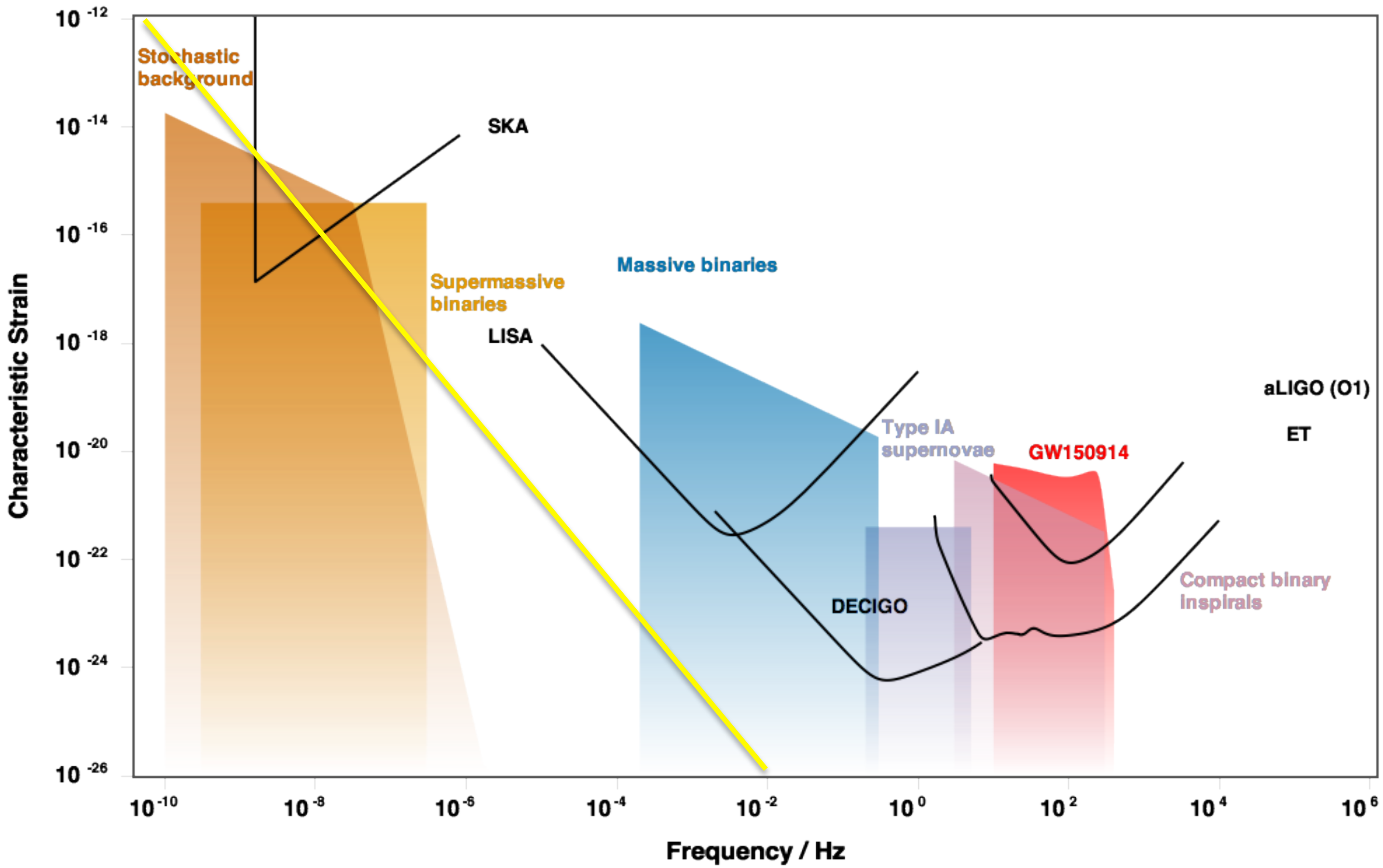
$$\Delta t = \frac{\rho_{DM}}{8m^3} \sin\big(mD + \alpha(\mathbf{x}) - \alpha(\mathbf{x}_p)\big) \cos\big(2mt + \alpha(\mathbf{x}) + \alpha(\mathbf{x}_p) - mD\big)$$

This can be compared with the timing residual due to GWs .
Correspondence is given by

$$h_c = 2\sqrt{3}\,|\delta\Phi| = 2 \times 10^{-15} \left(\frac{\rho_{DM}}{0.3\mathrm{GeV}/\mathrm{cm}^3}\right)\left(\frac{10^{-23}\mathrm{eV}}{m}\right)^2 \qquad f = \frac{\omega}{2\pi} = 5 \times 10^{-9}\,\mathrm{Hz}\left(\frac{m}{10^{-23}\mathrm{eV}}\right)$$

Detectability of axion oscillation with the pulsar timing

10^{-23}eV 10^{-22}eV



Dark energy and f(R) theory

f(R) dark energy model $S=\frac{1}{2}\int d^4x\sqrt{-g}\left[R+f(R)\right]+S_m$

We assume the deviation from Einstein is small

$$f\left(R\right)\ll R \qquad f_R\equiv f'\left(R\right)\ll 1$$

The equations of motion becomes $G_{\mu\nu}-\frac{1}{2}g_{\mu\nu}f+\left(R_{\mu\nu}+g_{\mu\nu}\nabla^\alpha\nabla_\alpha-\nabla_\mu\nabla_\nu\right)f_R=T_{\mu\nu}$

The trace part of Einstein equation $R+2f-\left(R+3\nabla^\alpha\nabla_\alpha\right)f_R=-T$

Since the spatial derivative is small compared to the time derivative $\nabla^\alpha\nabla_\alpha f_R\approx-\ddot{f}_R$

$$3\ddot{f}_R+R=-T \quad \Rightarrow \quad 3f''(R)\ddot{R}+3f'''(R)\dot{R}^2+R=-T$$

Note that we can read off the mass scale $M^2=\frac{1}{3f''\left(R_0\right)}$

Axion oscillation in f(R) theory

Aoki & Soda 2016

A simple solvable model is

$$f(R) = \frac{R^2}{6M^2}$$

The field equation reads

$$\frac{1}{M^2}\ddot{R} + R = \rho_{DM}\left[1 + 3\cos 2mt\right]$$

If we ignore homogeneous solutions, we get

$$R = \rho_{DM} + \frac{3M^2\rho_{DM}}{M^2 - 4m^2}\cos 2mt$$

This gives the equation

$$R = \rho_{DM} + \frac{3\rho_{DM}}{1-\left(2m/M\right)^2}\cos 2mt$$

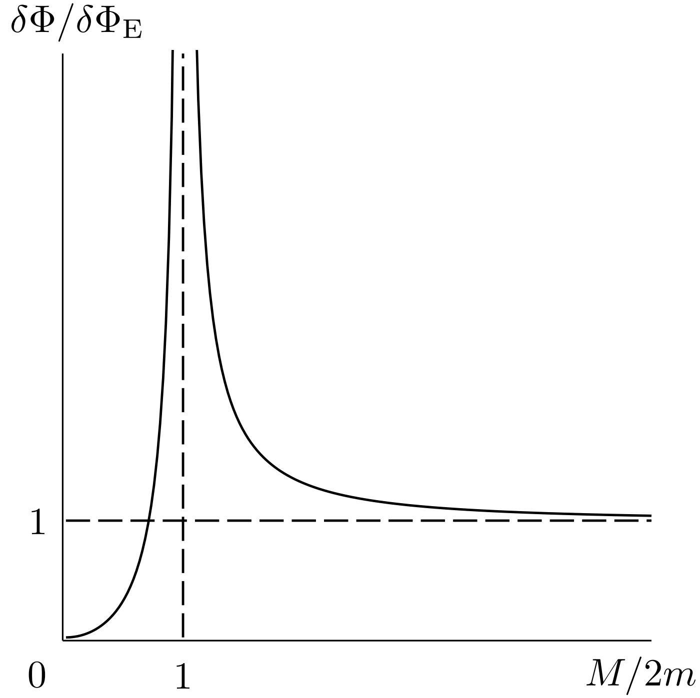
Since we know $R = -6\ddot{\Phi} + 2\nabla^2\left(2\Phi - \Psi\right)$

Time dependent part can be solved as

$$\delta\Phi = \frac{1}{1-\left(2m/M\right)^2}\frac{\pi G\rho_{DM}}{m^2}\cos 2mt$$

Finally, we obtain the resonant behavios

$$\frac{\delta\Phi}{\delta\Phi_E} = \frac{1}{1-\left(2m/M\right)^2}$$



Hu - Sawicki model

A viable model so called Hu-Sawicki model is given by

$$f(R) = -\mu R_c \frac{R^{2n}}{R^{2n} + R_c^{2n}}$$

For $R \geq R_c$, this reduces to

$$f(R) = -\mu R_c \left[1 - \frac{R_c^{2n}}{R^{2n}} \right]$$

$$R_0 = \rho_{DM}$$

$$M^2 = \frac{1}{3f''(R_0)} = \frac{R_c}{6n(2n+1)\mu} \left(\frac{8\pi G \rho_{DM}}{R_c} \right)^{2n+2}$$

Assuming , $R_c \approx \frac{2\Lambda}{\mu}$ $n = 1$ we get an interesting number

$$M \approx 1.5\mu \times 10^{-23} \mathrm{eV}$$

Thus, we have a chance to observe
the oscillation of the gravitational potential in the pulsar timing residual data.

Probing Axion DM
with interferometers

Detecting axion wind

Axion oscillation

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 \\ 0 & -\rho\cos\omega t\,\delta_{ij} \end{pmatrix}$$

Actually, interferometer is moving with the velocity v.
Laser interferometer feels the wind of axion!

Boosted energy momentum tensor

$$t' = \gamma\left(t + \vec{v}\cdot\vec{x}\right)$$

$$\vec{x}' = \vec{x} + \frac{\gamma - 1}{v^2}\left(\vec{v}\cdot\vec{x}\right)\vec{v} + \gamma\vec{v}\,t$$

⇒

$$T_{00} = \rho\gamma^2\left[1 - v^2\cos\omega t'\right]$$

$$T_{0i} = \rho\gamma^2 v_i\left[1 - \cos\omega t'\right]$$

$$T_{ij} = -\rho\cos\omega t'\,\delta_{ij} + \rho\gamma^2 v_i v_j\left[1 - \cos\omega t'\right]$$

This boosted oscillation produces oscillating potentials.

$$\delta g_{\mu\nu} = \begin{pmatrix} -1 - 2\Psi & 0 \\ 0 & (1 - 2\Phi)\delta_{ij} \end{pmatrix} + \delta\tilde{g}_{\mu\nu}$$

$$\delta\Phi\left(t, \vec{x}\right) = \frac{\rho}{8m^2}\cos\omega\gamma\left(t + \vec{v}\cdot\vec{x}\right)$$

$$\delta\Psi\left(t, \vec{x}\right) = -\frac{\rho}{8m^2}\cos\omega\gamma\left(t + \vec{v}\cdot\vec{x}\right)$$

Laser interferometer Signal

Aoki & Soda 2016

In the synchronous gauge, we have

$$\delta g_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & (1-2\delta\Phi)\delta_{ij} + 2\delta B_{,ij} \end{pmatrix} \qquad \delta B_{,ij} = \frac{\rho}{8m^2} v_i v_j \cos\omega\gamma\left(t + \vec{v}\cdot\vec{x}\right)$$

The signal detected by the interferometer should be

$$s = D_{ij} h_{ij} \qquad D_{ij} = \frac{1}{2}\left(m_i m_j - n_i n_j\right)$$

Namely, we obtain

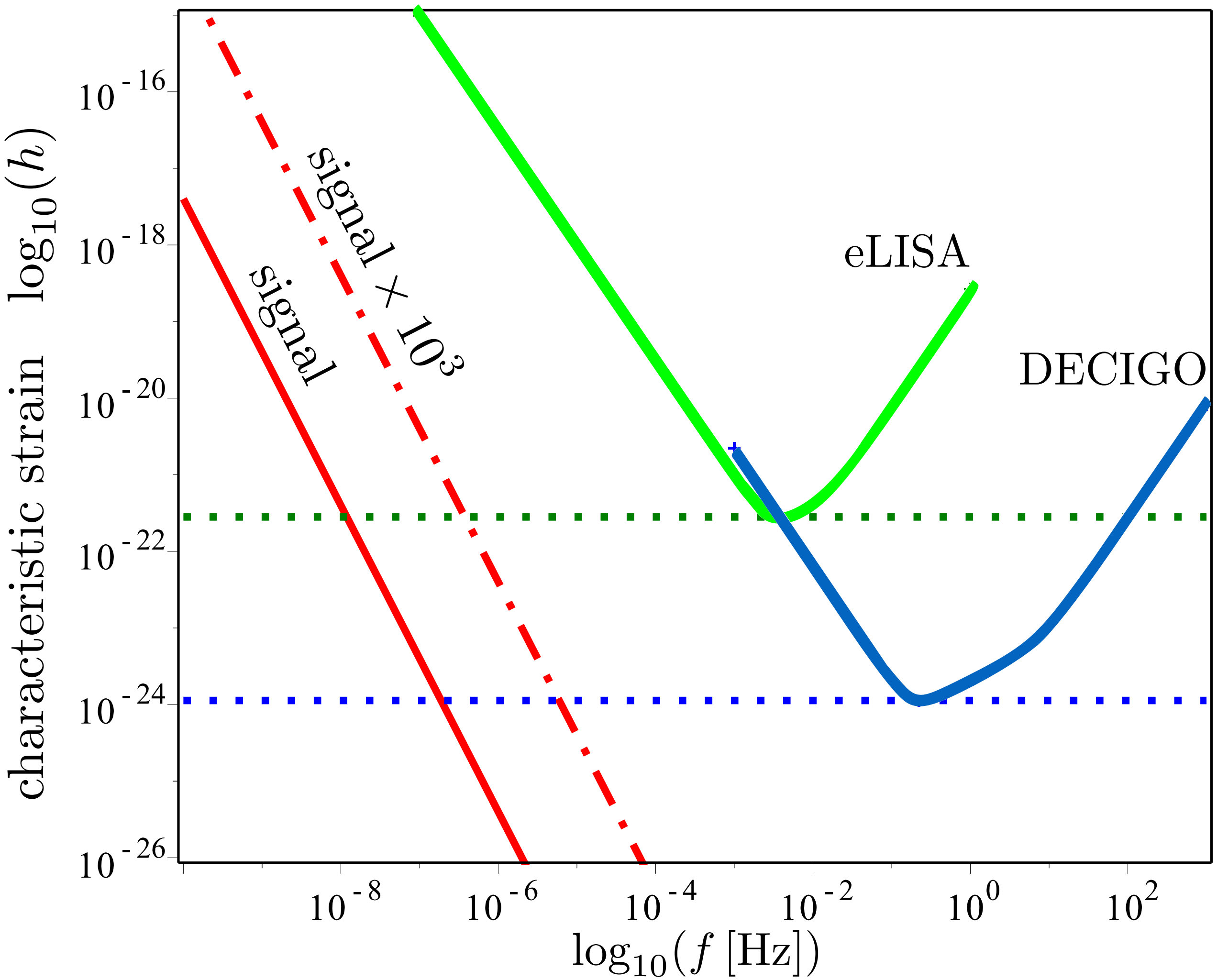
$$s(t) = \left[\left(\vec{v}\cdot\vec{m}\right)^2 - \left(\vec{v}\cdot\vec{n}\right)^2\right]\frac{\rho v^2}{8m^2}\cos\omega\gamma t$$

The amplitude can be estimated as

$$\frac{\rho v^2}{8m^2} = 1.6\times10^{-21}\left(\frac{v}{10^{-3}}\right)^2\left(\frac{10^{-23}\mathrm{eV}}{m}\right)^2$$

This is quite small signal.
However, if the resonance occurs, we have a chance to detect the oscillation.

Detectability of axion oscillation with laser interferometer



Summary

- We have shown that coherent oscillation of axion dark matter can be detected with pulsar timing arrays or space laser interferometers.
- Remarkably, it is sensitive to the dark energy models. The signal could be enhanced significantly.
- We need construct micro-Herz laser interferometer.