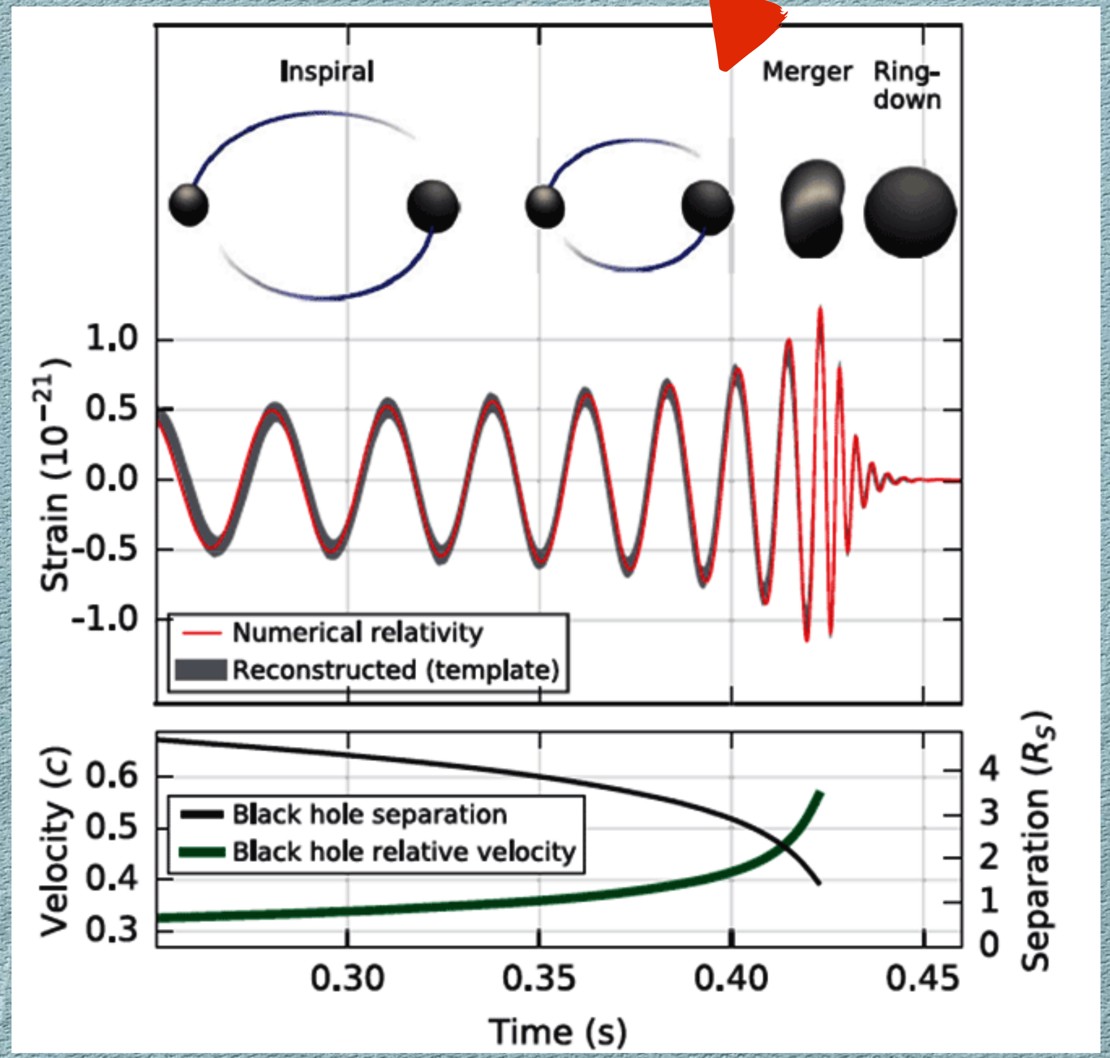

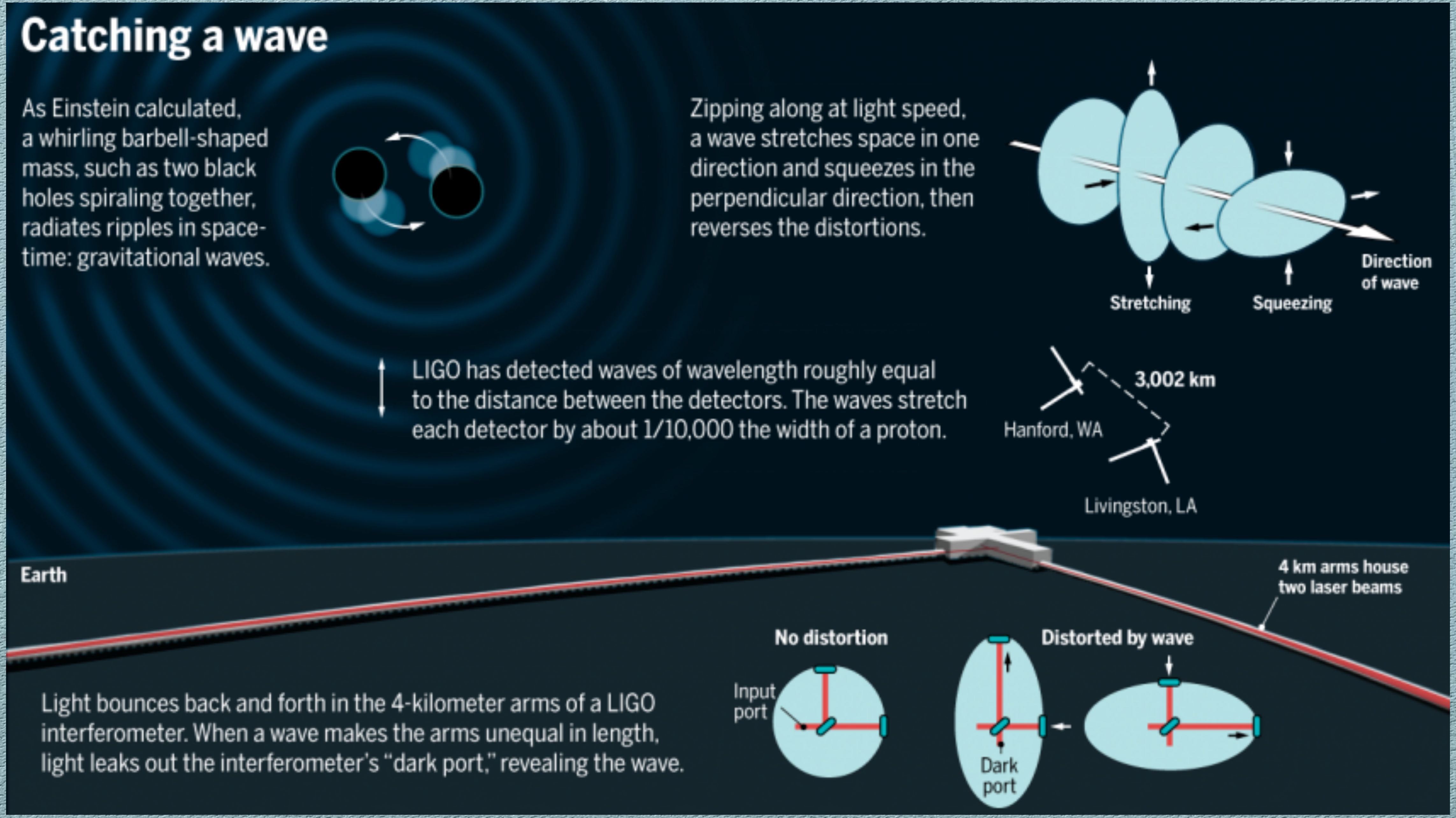

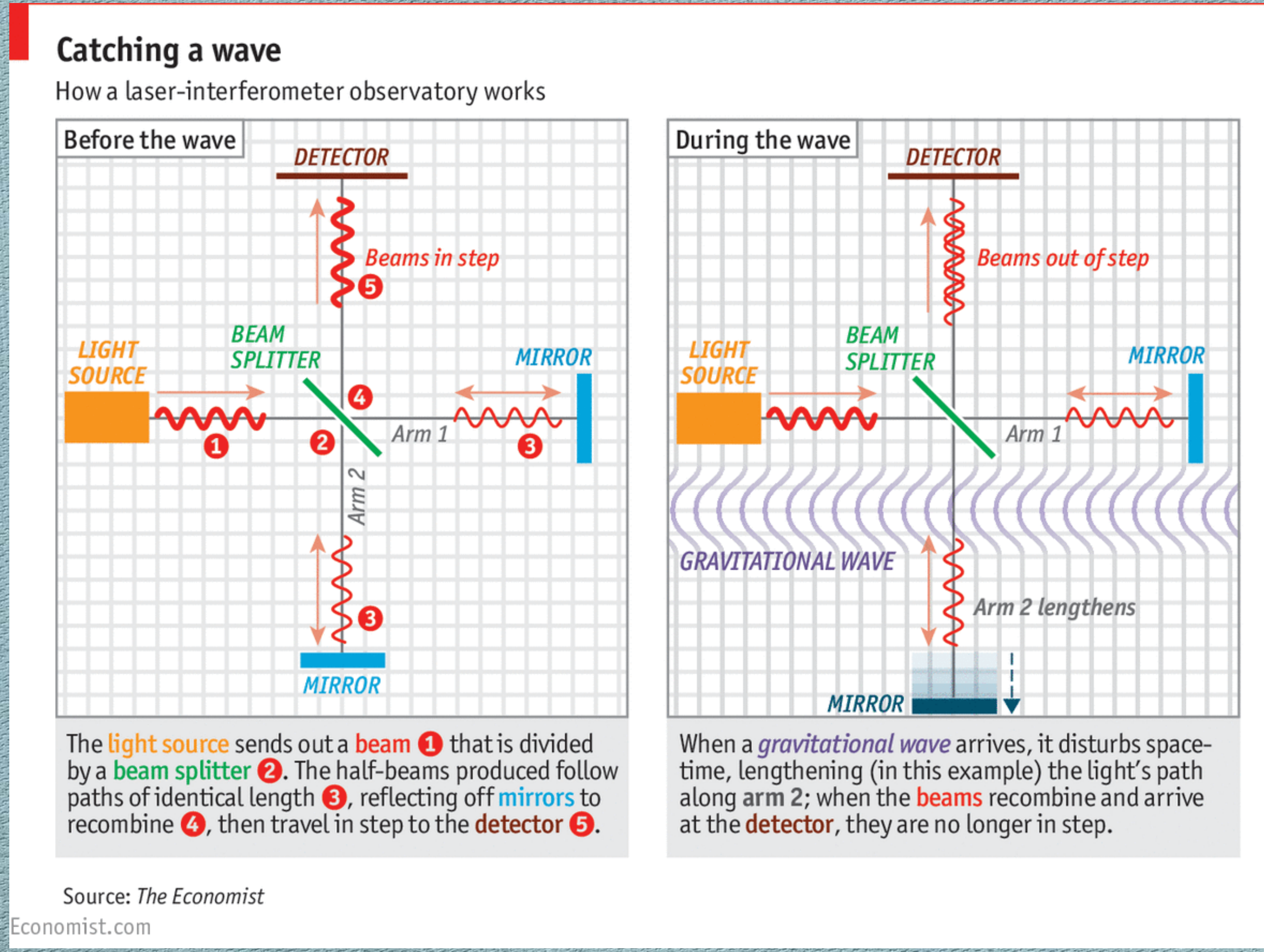

Cosmology with Gravitational Waves

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THE 52ND WORKSHOP ON GRAVITY AND COSMOLOGY APCTP FRP

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Linearized Theory of General Relativity I

- Einstein field equations (EFE) from GR

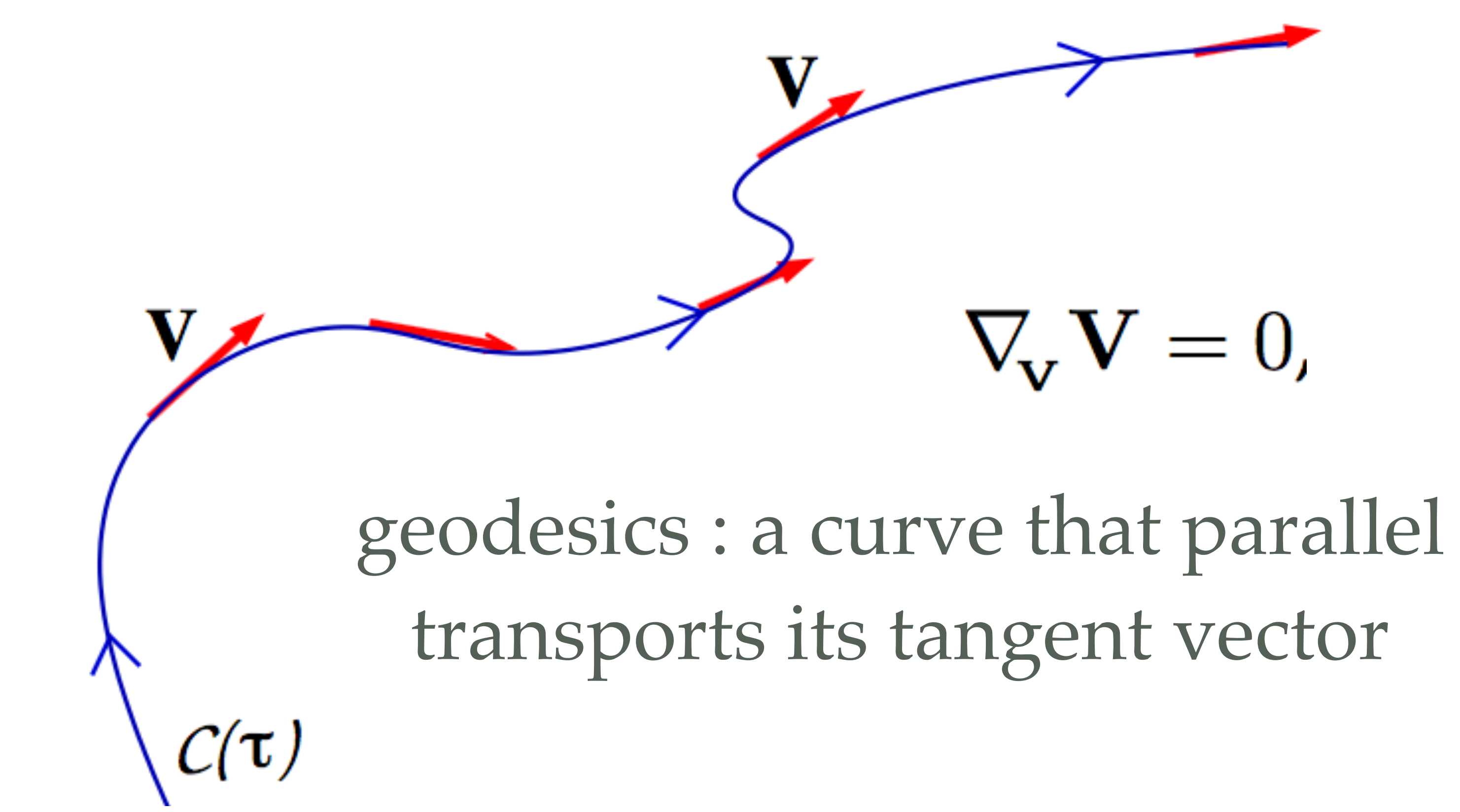
$$G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 8\pi T^{\mu\nu},$$

$$c = G = 1$$

- Geodesic Eq :

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu{}_{\nu\rho} \frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0 \quad \text{for } m \neq 0,$$
$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma^\mu{}_{\nu\rho} \frac{dx^\nu}{d\lambda}\frac{dx^\rho}{d\lambda} = 0 \quad \text{for } m = 0.$$

need for propagation of GW



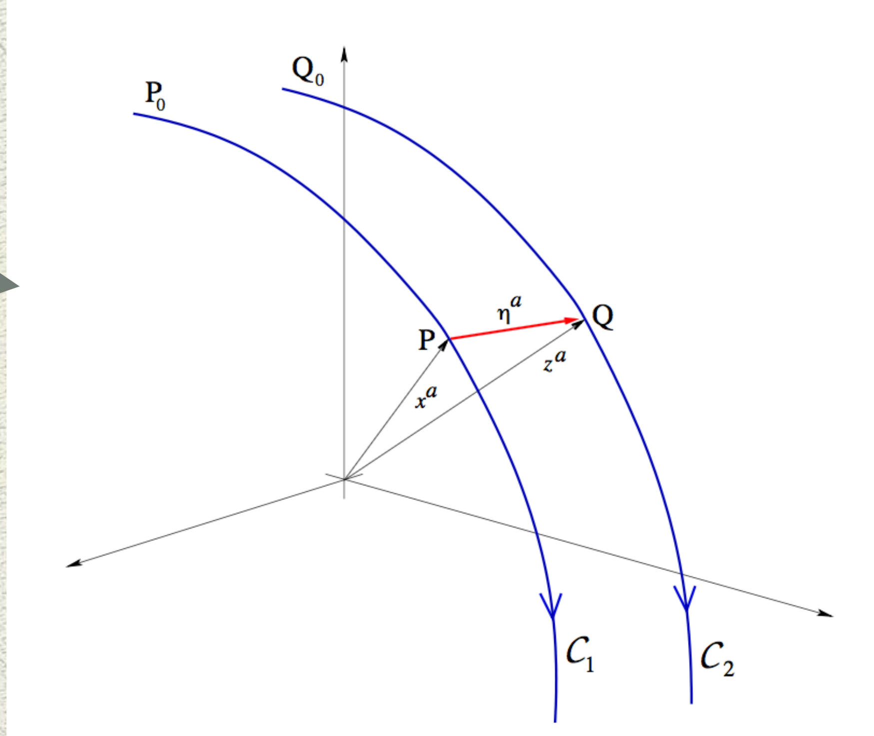
$$V^\mu = dx^\mu / d\tau \qquad \nabla_{\mathbf{v}} \mathbf{V} = 0 = V^\alpha{}_{;\beta} V^\beta = V^\alpha{}_{,\beta} V^\beta + \Gamma^\alpha_{\beta\mu} V^\beta V^\mu$$

$$V^\mu{}_{,\beta} = \frac{\partial}{\partial x^\beta}\left(\frac{dx^\mu}{d\tau}\right) \;;\quad V^\mu{}_{,\beta} V^\beta \equiv \frac{dx^\beta}{d\tau}\frac{\partial}{\partial x^\beta}\left(\frac{dx^\mu}{d\tau}\right) = \frac{d^2 x^\mu}{d\tau^2}$$

- Geodesic deviation Eq (ζ : separation btw two geodesics)

$$\frac{D^2 \zeta^\mu}{D\tau^2} = -R^\mu{}_{\nu\rho\sigma} \zeta^\rho \frac{dx^\nu}{d\tau}\frac{dx^\sigma}{d\tau},$$

need for understanding detector of GW



- Linearized theory of GR (weak field limit)

$$8\pi T_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \text{for } |h_{\mu\nu}| \ll 1$$
$$= \frac{1}{2}\left[h_{\mu\alpha,\nu}{}^{\alpha} + h_{\nu\alpha,\mu}{}^{\alpha} - h_{\mu\nu,\alpha}{}^{\alpha} - h_\alpha{}^\alpha{}_{,\mu\nu} - \eta_{\mu\nu}\left(h_{\alpha\beta}{}^{,\alpha\beta} - h_\alpha{}^\alpha{}_{,\beta}{}^\beta \right)\right]$$

Linearized Theory of General Relativity II

- Gauge transformation (translation)

$x^\mu \to x^{\mu'} = \eta^{\mu'}{}_\nu (x^\nu + \xi^\nu)$ $\quad |\xi_{\mu,\nu}| \lesssim |h_{\mu\nu}|$

$$g_{\mu\nu} \to g_{\mu'\nu'} = x^\rho_{,\mu'} x^\sigma_{,\nu'} g_{\rho\sigma} = \eta_{\mu\nu} + h_{\mu\nu} - \xi_{\mu,\nu} - \xi_{\nu,\mu} + \cdots$$

$$h_{\mu\nu} \to h_{\mu'\nu'} = h_{\mu\nu} - \xi_{\mu,\nu} - \xi_{\nu,\mu}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \text{for } |h_{\mu\nu}| \ll 1$$

- Lorentz transformation (rotation)

$x^\mu \to x^{\mu'} = \Lambda^{\mu'}{}_\nu x^\nu,$

$$g_{\mu\nu} \to g_{\mu'\nu'} = \Lambda^\rho{}_{\mu'} \Lambda^\sigma{}_{\nu'} g_{\rho\sigma} = \eta_{\mu'\nu'} + \Lambda^\rho{}_{\mu'} \Lambda^\sigma{}_{\nu'} h_{\rho\sigma},$$

$$h_{\mu\nu} \to h_{\mu'\nu'} = \Lambda^\rho{}_{\mu'} \Lambda^\sigma{}_{\nu'} h_{\rho\sigma}.$$

$$8\pi T_{\mu\nu} = \frac{1}{2}\Big[h^\alpha_{\mu\alpha,\nu} + h^\alpha_{\nu\alpha,\mu} - h^\alpha_{\mu\nu,\alpha} - h^\alpha_{\alpha,\mu\nu} - \eta_{\mu\nu}\left(h^{,\alpha\beta}_{\alpha\beta} - h^\alpha_\alpha{}_{,\beta}{}^\beta\right)\Big]$$

- Trace-reversed metric perturbation

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h,$$

$16\pi T_{\mu\nu} =$ [redacted] $+$ [redacted] $- \bar{h}_{\mu\nu,\alpha}{}^\alpha$ [redacted]

- harmonic gauge + additional condition (to give same observables)

$$\bar{h}_{\mu\nu} \to \bar{h}_{\mu'\nu'} = \bar{h}_{\mu\nu} - \xi_{\mu,\nu} - \xi_{\nu,\mu} + \eta_{\mu\nu}\xi^\rho{}_{,\rho}.$$

$$\bar{h}_{\mu'\nu',}{}^{\nu'} = \bar{h}_{\mu\nu,}{}^\nu - \xi_{\mu,\nu}{}^\nu$$

$$\xi_{\mu,\nu}{}^\nu = 0$$

$$\bar{h}_{\mu\nu,}{}^\nu = 0.$$

harmonic : pol ⊥ z

need additional condition

Propagation of Gravitational Waves I

- linearized EFE in vacuum

$$\bar{h}_{\mu\nu,\alpha}{}^{\alpha} = 0.$$

- Solution in the form of plane waves

$$\bar{h}_{\mu\nu} = \Re\left[A_{\mu\nu} e^{\imath k_\alpha x^\alpha}\right]$$

- Null vector (from EFE)/ /harmonic gauge

$$k_\alpha k^\alpha = 0, \quad A_{\mu\nu} k^\nu = 0.$$

- A transverse wave

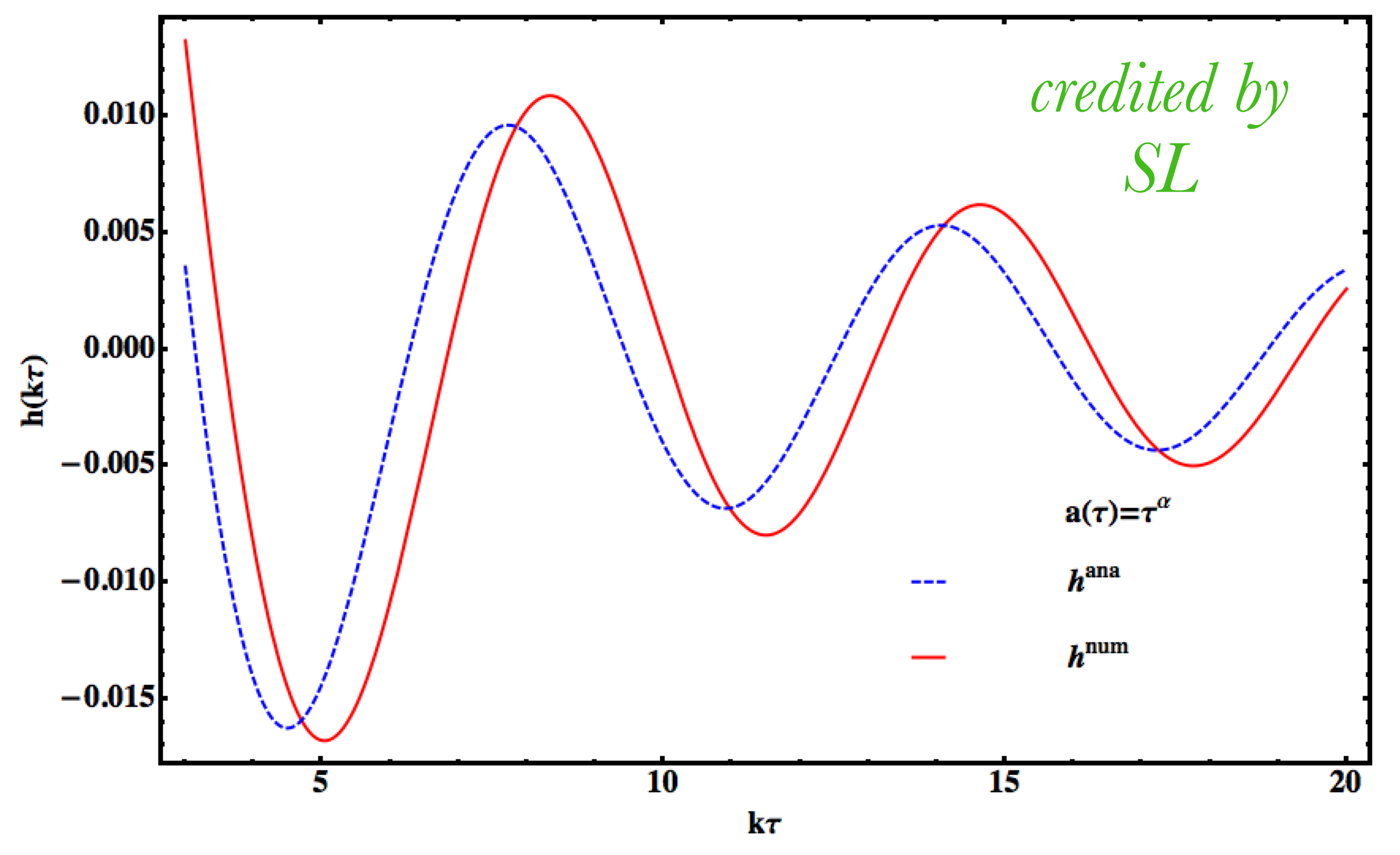
$$\bar{h}^{\alpha}_{\mu\nu,\alpha} = -16\pi T_{\mu\nu} = 0$$

In the expanding Univ

$$ds^2 = a^2(\tau)\left[d\tau^2 - (\eta_{ij} + h_{ij})dx^i dx^j\right]$$

$$h_{ij} \equiv h_k(\tau, z) = h_k(\tau)e^{\pm ikz} \qquad h_k^{''} + 2\frac{a'}{a}h_k^{'} + k^2 h_k = 0$$

$$h(k\tau) = \frac{1}{a(\tau)}\left(\tilde{A}_k\sqrt{k\tau}H^{(1)}_{\alpha-1/2}(k\tau) + \tilde{B}_k\sqrt{k\tau}H^{(2)}_{\alpha-1/2}(k\tau)\right) \qquad a(\tau) \sim \tau^\alpha$$



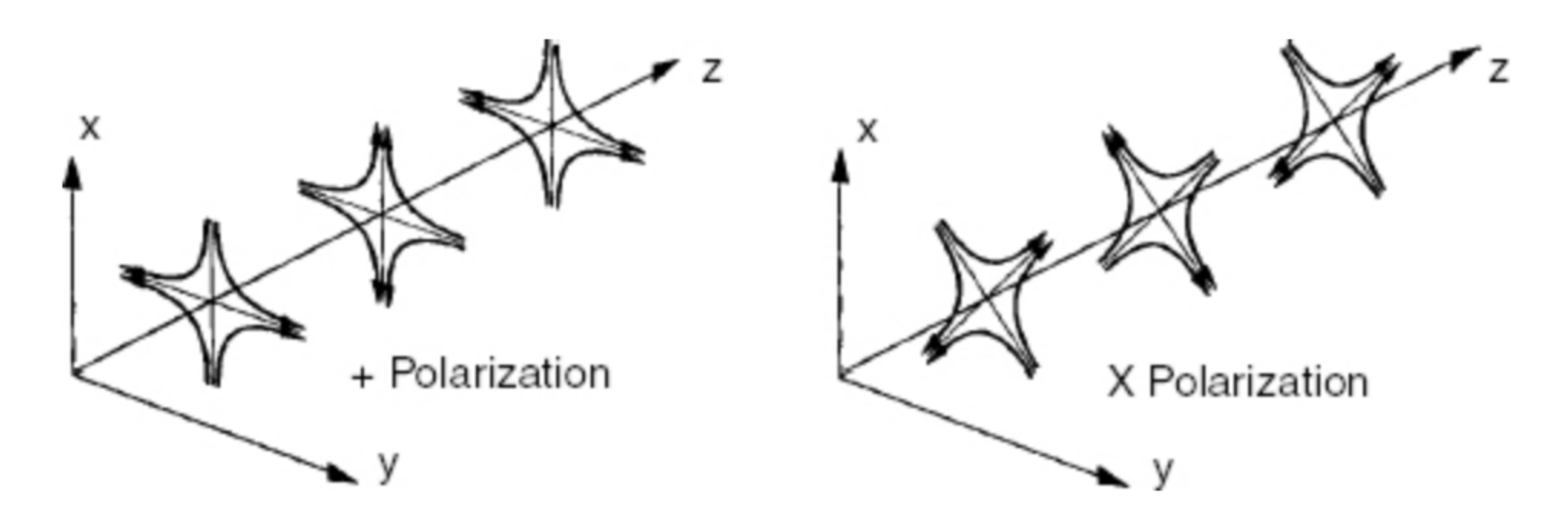
Propagation of Gravitational Waves II

- Residual gauge freedom within harmonic gauge

$$A_{\mu\nu}u^{\nu} = 0.$$

- Residual gauge freedom to set (4th constraint)

$$A_{\mu}{}^{\mu} = 0.$$



- Transverse-Traceless (TT) gauge

$$h_{\mu\nu}^{\mathrm{TT}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cos\left[\omega(t-z)\right]$$

$$ds^2 = -dt^2 + dz^2 + (1 + h_+ \cos[\omega(t-z)])dx^2 + 2h_\times \cos[\omega(t-z)])dxdy + (1 - h_+ \cos[\omega(t-z)])dy^2$$

Interaction with Test Masses (Detectors) I

- Geodesic Eq in TT gauge

$$\frac{d^2x^i}{d\tau^2} = -\Big[\Gamma^i_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau}\Big]_{\tau=0} = -\Big[\Gamma^i_{00}(\frac{dx^0}{d\tau})^2\Big]_{\tau=0} \xrightarrow[\text{TT-gauge}]{\Gamma^i_{00}=0} \frac{d^2x^i}{d\tau^2} = 0$$

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu{}_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0 \quad \text{for } m \neq 0,$$
$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu{}_{\nu\rho}\frac{dx^\nu}{d\lambda}\frac{dx^\rho}{d\lambda} = 0 \quad \text{for } m = 0.$$

- Geodesic deviation Eq in TT gauge

$$\frac{d^2\xi^i}{d\tau^2}\Big|_{\tau=0} = -\Big[2\Gamma^i_{0j}\frac{d\xi^j}{d\tau}\Big]_{\tau=0} = 0$$

$$\frac{D^2\zeta^\mu}{D\tau^2} = -R^\mu{}_{\nu\rho\sigma}\zeta^\rho\frac{dx^\nu}{d\tau}\frac{dx^\sigma}{d\tau}$$

- In TT gauge, GW has no influence on both geodesic and on deviation of it

Interaction with Test Masses (Detectors) II

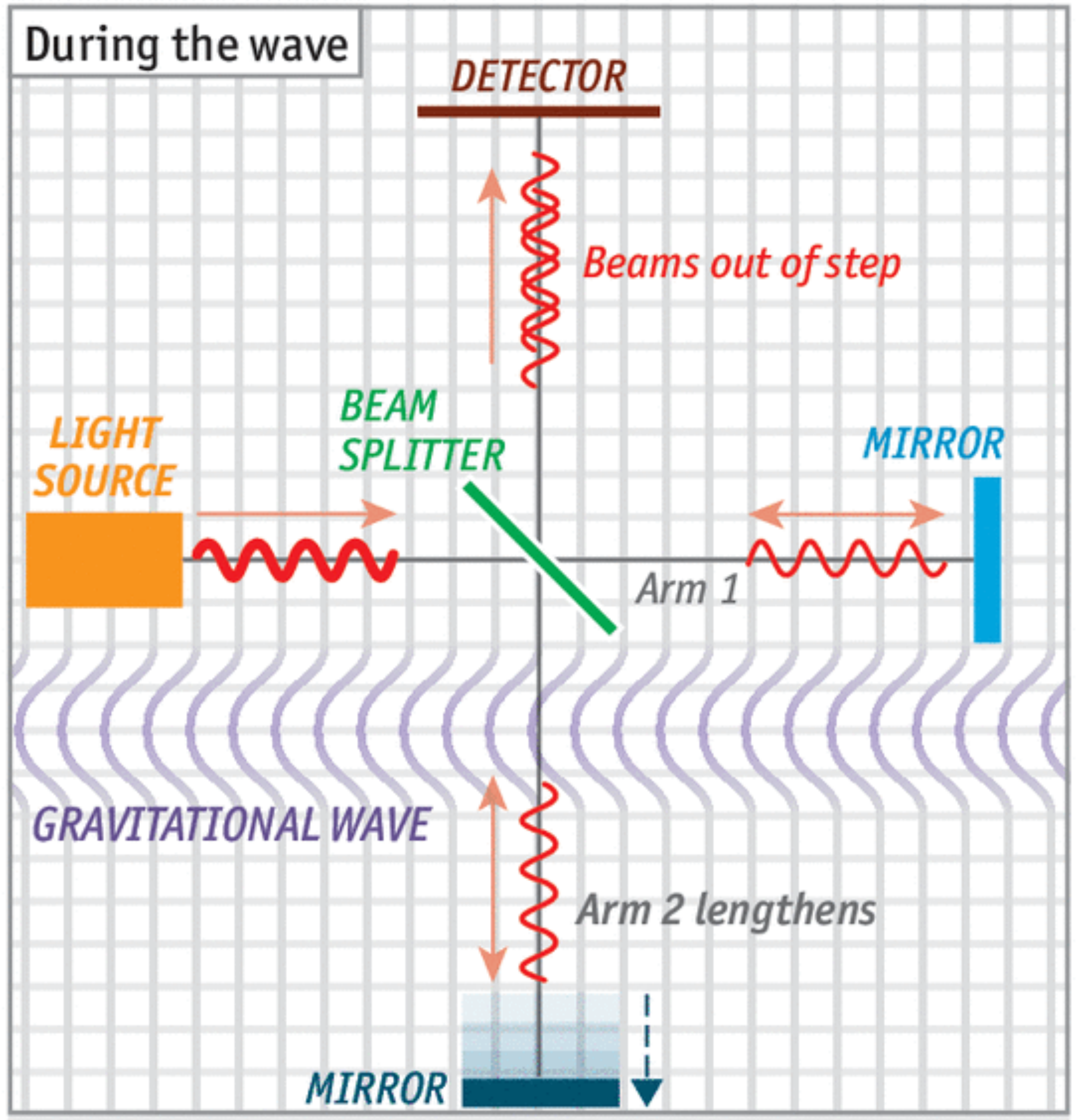
- 2 events in TT frame at $(t,x_1,0,0)$ and $(t,x_2,0,0)$. $x_2-x_1=L$
- Proper distance

$$s = \int ds = \int_{x_1}^{x_2} dx \sqrt{1 + h_+ \cos\omega t} \simeq L\left(1 + \frac{1}{2} h_+ \cos\omega t\right)$$

- In general, proper distance (Li : spatial separation btw 2 test masses)

$$s = \sqrt{L^2 + h_{ij}(t) L_i L_j},$$

- The proper distance expands and shrinks periodically
- light traveling time btw 2 masses ~ proper time
- Interferometer measure the length difference btw 2 arms



When a gravitational wave arrives, it disturbs space-time, lengthening (in this example) the light's path along arm 2; when the beams recombine and arrive at the detector, they are no longer in step.

Interaction with Test Masses (Detectors) III

- Detector capable of measuring changes in the proper distance (an interferometer) with a characteristic size << wavelength of GW
- In this case, the detector in a near local Lorentz frame (freely falling frame) even in the presence of GWs
- The metric in near local Lorentz frame

$$ds^2 \approx -dt^2 + \delta_{ij} dx^i dx^j + \mathcal{O}\left(\frac{x^i x^j}{L_B^2}\right)$$

- L_B : typical variation scale of the metric

From Geodesic deviation Eq, one obtains

$$\frac{d^2\zeta^i}{d\tau^2} + R^i{}_{0j0}\zeta^j \left(\frac{dx^0}{d\tau}\right)^2 = 0$$

$$\ddot{\zeta}^j = -R^i{}_{0j0}\zeta^j,$$

$$R^i{}_{0j0} = R_{i0j0} = -\frac{1}{2}\ddot{h}_{ij}^{\mathrm{TT}}.$$

$$\ddot{\zeta}^i = \frac{1}{2}\ddot{h}_{ij}^{\mathrm{TT}}\zeta^j$$

only valid when $L \ll \lambda_{\mathrm{GW}}/2\pi \equiv \bar{\lambda}$

LIGO : $\bar{\lambda} \sim 10^5$m and $L \sim 10^3$m (valid)
LISA : $\bar{\lambda} \sim 10^5$m and $L \sim 10^3$m (invalid)

Interaction with Test Masses (Detectors) IV

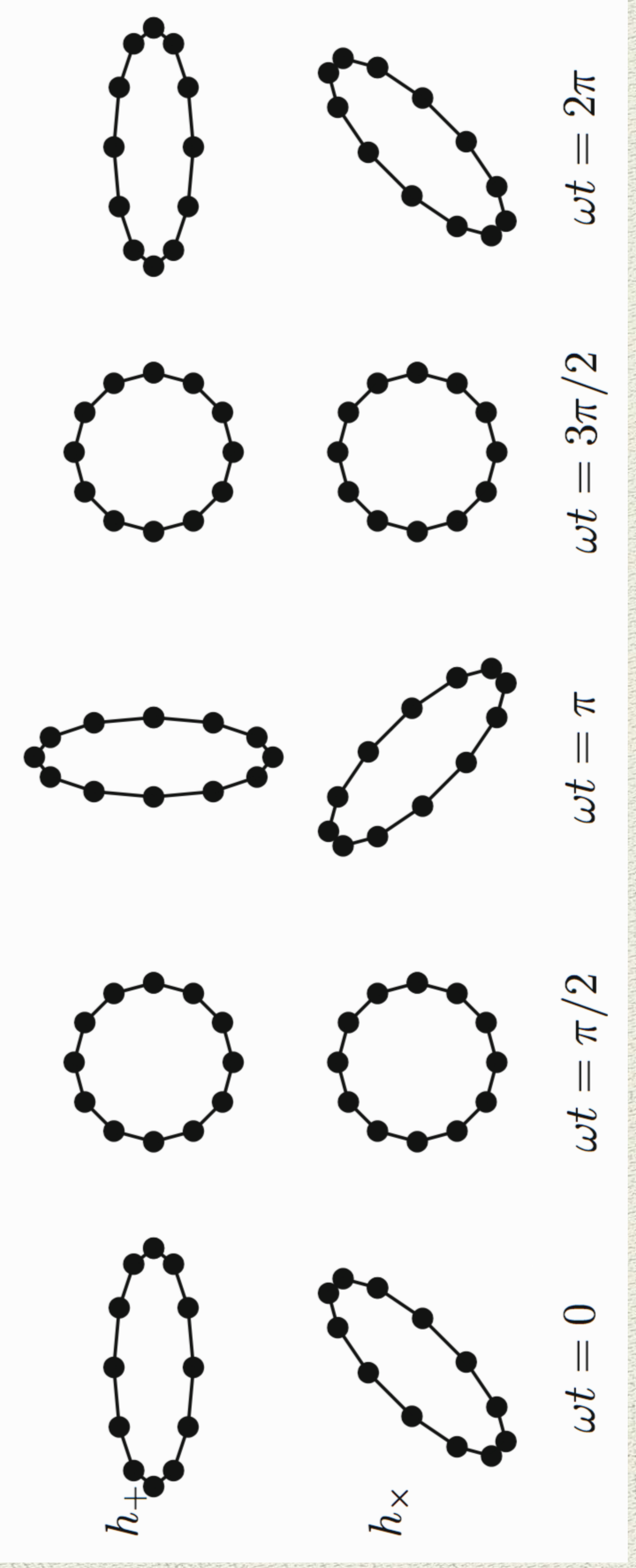
- A ring of test masses in (x,y) plane centered @ z=0 & GW travelling in z-direction
- Two polarizations are independent
- Location of a test mass : $\zeta^a(t) = (x_0 + \delta x(t),\, y_0 + \delta y(t))$
- Evolution of location caused by plus polarization :

$$\begin{pmatrix} \ddot{\delta x} \\ \ddot{\delta y} \end{pmatrix}_+ = -\frac{h_+}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x_0 + \delta x \\ y_0 + \delta y \end{pmatrix} \omega^2 \cos \omega t$$

$$\approx -\frac{h_+}{2} \begin{pmatrix} x_0 \\ -y_0 \end{pmatrix} \omega^2 \cos \omega t,$$

- Deviation caused by plus (cross) polarization :

$$\begin{pmatrix} \delta x \\ \delta y \end{pmatrix}_+ = \frac{h_+}{2} \begin{pmatrix} x_0 \\ -y_0 \end{pmatrix} \cos \omega t \qquad \begin{pmatrix} \delta x \\ \delta y \end{pmatrix}_\times = \frac{h_\times}{2} \begin{pmatrix} y_0 \\ x_0 \end{pmatrix} \cos \omega t$$



SUMMARY SO FAR

PhD TV

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Energy and Momentum of GWs I

- GWs carry energy and momentum (E&M)
- So far, only the propagation of GWs on a flat spacetime
  ⇨ no energy and momentum associated to GWs
- Flat background metric : GWs can't curve the background metric
- To study E&M carried by GWs, one need to extend the metric perturbation to 2nd order

$$g_{\mu\nu} = g^{(\mathrm{B})}_{\mu\nu} + h^{(1)}_{\mu\nu} + h^{(2)}_{\mu\nu}$$

$$R_{\mu\nu} = R^{(\mathrm{B})}_{\mu\nu} + R^{(1)}_{\mu\nu}\left[h^{(1)}_{\mu\nu}\right] + R^{(2)}_{\mu\nu}\left[h^{(1)}_{\mu\nu}\right] + R^{(1)}_{\mu\nu}\left[h^{(2)}_{\mu\nu}\right]$$

2nd order w.r.t. h(1)

$$\begin{aligned} R^{(2)}_{\mu\nu}\left[h_{\mu\nu}\right] \equiv \frac{1}{2}\Big[& \frac{1}{2} h_{\alpha\beta;\mu} h^{\alpha\beta}{}_{;\nu} + h_{\nu}{}^{\alpha;\beta}\left(h_{\alpha\mu;\beta} - h_{\beta\mu;\alpha}\right) \\ & + h^{\alpha\beta}\left(h_{\alpha\beta;\mu\nu} + h_{\mu\nu;\alpha\beta} - h_{\alpha\mu;\nu\beta} - h_{\alpha\nu;\mu\beta}\right) \\ & - \left(h^{\alpha\beta}{}_{;\beta} - \frac{1}{2} h^{;\alpha}\right)\left(h_{\alpha\mu;\nu} + h_{\alpha\nu;\mu} - h_{\mu\nu;\alpha}\right)\Big] \end{aligned}$$

Energy and Momentum of GWs II

- Impact of GWs on the background metric
- See the background EFE with GW source term

$$8\pi T_{\mu\nu}^{(\mathrm{GW})} = R_{\mu\nu}^{(\mathrm{B})} - \frac{1}{2} R^{(\mathrm{B})} g_{\mu\nu}^{(\mathrm{B})}$$

- Stress energy tensor of GWs

$$T_{\mu\nu}^{(\mathrm{GW})} \equiv -\frac{1}{8\pi} \left\langle R_{\mu\nu}^{(2)} - \frac{1}{2} g_{\mu\nu}^{(\mathrm{B})} R^{(2)} \right\rangle_l = \frac{1}{32\pi} \left\langle \bar{h}_{\alpha\beta,\mu} \bar{h}^{\alpha\beta}{}_{,\nu} \right\rangle_l$$

- Energy and momentum radiated per unit time through a sphere of radius r

$$\frac{dE_{(\mathrm{GW})}}{dt} = \frac{r^2}{32\pi} \int d\Omega \left\langle \dot{h}_{ij}^{\mathrm{TT}} \dot{h}_{ij}^{\mathrm{TT}} \right\rangle_l, \quad \frac{dP^k_{(\mathrm{GW})}}{dt} = -\frac{r^2}{32\pi} \int d\Omega \left\langle \dot{h}_{ij}^{\mathrm{TT}} \partial^k h_{ij}^{\mathrm{TT}} \right\rangle_l$$

Generation of Gravitational Waves I

- Recall the linearized EFE in harmonic gauge

$$\bar{h}_{\mu\nu,\alpha}{}^{\alpha} = -16\pi T_{\mu\nu}.$$

- Solution found by using a Green's function with BCs (no incoming radiation) : a function of retarded time and spatial position

$$\bar{h}_{\mu\nu}(t,\vec{x}) = 4\int d^3\vec{y}\,\frac{T_{\mu\nu}(t-|\vec{x}-\vec{y}|,\vec{y})}{|\vec{x}-\vec{y}|}.$$

- Perturbation in TT gauge

$$h^{\mathrm{TT}}_{ij}(t,\vec{x}) = \Lambda_{ijkl}\bar{h}_{kl} = 4\Lambda_{ijkl}\int d^3\vec{y}\,\frac{T_{kl}(t-|\vec{x}-\vec{y}|,\vec{y})}{|\vec{x}-\vec{y}|}$$

Projection operator : project h into TT gauge

$$\Lambda_{ijkl} \equiv P_{ik}P_{jl} - \frac{1}{2}P_{ij}P_{kl},$$

$$P_{ij} \equiv \delta_{ij} - n_i n_j$$

n_i : propagation direction

Generation of Gravitational Waves II

- Simplify stress-energy tensor by imposing non-relativistic

$$\lambda \sim \frac{c}{v} d \gg d.$$

- Then expand it as

$$T_{kl}\left(t-r+\vec{y}\cdot\vec{n},\vec{y}\right) \approx T_{kl}\left(t-r,\vec{y}\right) + y^i n^i T_{kl,0}\left(t-r,\vec{y}\right) + \frac{1}{2} y^i y^j n^i n^j T_{kl,00}\left(t-r,\vec{y}\right) + \cdots$$

- Moments of stress tensor

$$S^{ij}(t) = \int d^3\vec{y}\; T^{ij}\left(t,\vec{y}\right),$$

monopole

$$S^{ijk}(t) = \int d^3\vec{y}\; T^{ij}\left(t,\vec{y}\right) y^k,$$

dipole

$$S^{ijk}(t) = \int d^3\vec{y}\; T^{ij}\left(t,\vec{y}\right) y^k,$$

quadrupole

Generation of Gravitational Waves III

- Perturbation in TT gauge

$$h_{ij}^{\mathrm{TT}}(t,\vec{x}) = \frac{4\Lambda_{ijkl}}{r}\left[S^{kl} + n_m \dot{S}^{klm} + \frac{1}{2} n_m n_p \ddot{S}^{klmp} + \cdots\right]_{\mathrm{ret}}$$

- Moments of energy density (M) and momentum (P)

$$\begin{aligned} M &= \int d^3\vec{y}\, T^{00}(t,\vec{y})\,, & P^i &= \int d^3\vec{y}\, T^{0i}(t,\vec{y})\,, \\ M^i &= \int d^3\vec{y}\, T^{00}(t,\vec{y})\, y^i, & P^{ij} &= \int d^3\vec{y}\, T^{0i}(t,\vec{y})\, y^j, \\ M^{ij} &= \int d^3\vec{y}\, T^{00}(t,\vec{y})\, y^i y^j, & P^{ijk} &= \int d^3\vec{y}\, T^{0i}(t,\vec{y})\, y^j y^k \end{aligned}$$

- Stress tensor in lowest two orders

$$S^{ij} = \frac{1}{2}\ddot{M}^{ij}, \quad \dot{S}^{ijk} = \frac{1}{6}\dddot{M}^{ijk} + \frac{1}{3}\left(\ddot{P}^{ijk} + \ddot{P}^{jik} - 2\ddot{P}^{kij}\right)$$

$$h_{ij}^{\mathrm{TT}}(t,\vec{x}) = \Lambda_{ijkl}\bar{h}_{kl}$$

$$= 4\Lambda_{ijkl}\int d^3\vec{y}\, \frac{T_{kl}(t - |\vec{x}-\vec{y}|, \vec{y})}{|\vec{x}-\vec{y}|}$$

$$T_{kl}(t - r + \vec{y}\cdot\vec{n}, \vec{y}) \approx T_{kl}(t-r,\vec{y}) + y^i n^i T_{kl,0}(t-r,\vec{y})$$

$$+ \frac{1}{2} y^i y^j n^i n^j T_{kl,00}(t-r,\vec{y}) + \cdots$$

$$S^{ij}(t) = \int d^3\vec{y}\, T^{ij}(t,\vec{y})\,,$$

Generation of Gravitational Waves IV

- metric perturbation due to mass quarupole

$$\left[h_{ij}^{\mathrm{TT}}(t,\vec{x})\right]_{\mathrm{quad}} = \frac{2}{r}\Lambda_{ijkl}\ddot{M}^{kl}(u).$$

- reduced quadrupole moment

$$Q^{ij} \equiv M^{ij} - \frac{1}{3}\delta^{ij}M_{kk} \quad \left[h_{ij}^{\mathrm{TT}}(t,\vec{x})\right]_{\mathrm{quad}} = \frac{2}{r}\Lambda_{ijkl}\ddot{Q}^{kl}(u) \equiv \frac{2}{r}\ddot{Q}_{ij}^{\mathrm{TT}}(u).$$

- expression for two polarization

$$h_+ = \frac{\ddot{M}_{11} - \ddot{M}_{22}}{r}, \quad h_\times = \frac{2M_{12}}{r}.$$

$$\frac{dE_{(\mathrm{GW})}}{dt} = \frac{r^2}{32\pi}\int d\Omega \left\langle \dot{h}_{ij}^{\mathrm{TT}}\dot{h}_{ij}^{\mathrm{TT}}\right\rangle_l,$$

$$\frac{dP^k_{(\mathrm{GW})}}{dt} = -\frac{r^2}{32\pi}\int d\Omega \left\langle \dot{h}_{ij}^{\mathrm{TT}}\partial^k h_{ij}^{\mathrm{TT}}\right\rangle_l$$

- radiated energy and momentum

$$\frac{dE}{dt} = \frac{1}{8\pi}\int d\Omega\, \dddot{Q}_{ij}^{\mathrm{TT}}\dddot{Q}_{ij}^{\mathrm{TT}}, \quad \frac{dP^k}{dt} = -\frac{1}{8\pi}\int d\Omega\, \dddot{Q}_{ij}^{\mathrm{TT}}\partial^k\ddot{Q}_{ij}^{\mathrm{TT}}.$$

Generation of Gravitational Waves V

- Two masses m1 and m2 separated by R orbit circular motic with angular velocity ω. Observer at r
- The location of masses

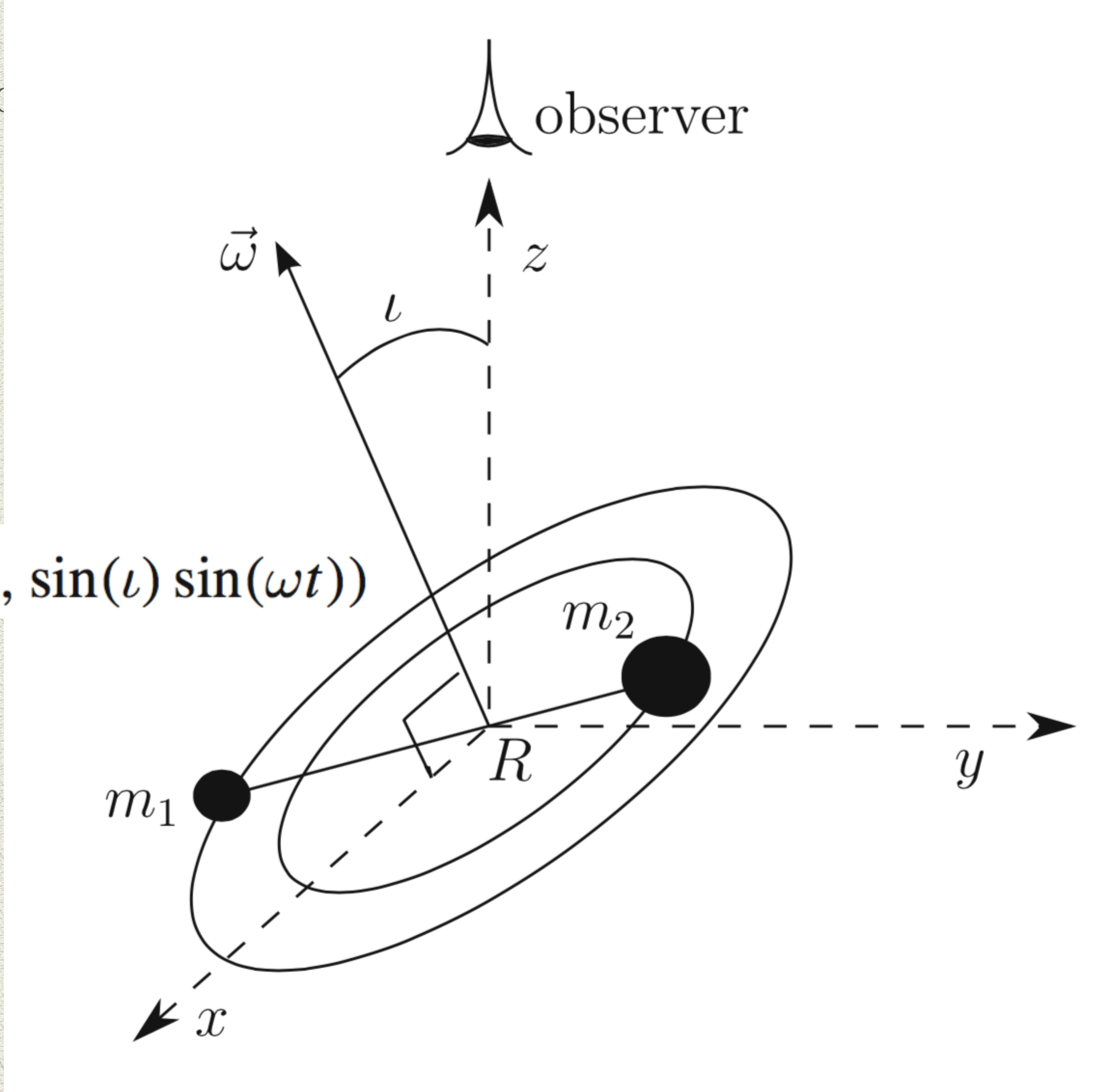
$$\vec{y}_1(t) = \frac{\mu}{m_1} R\hat{e}(t), \quad \vec{y}_2(t) = -\frac{\mu}{m_2} R\hat{e}(t),$$

$$\mu = m_1 m_2/(m_1 + m_2) \quad \hat{e}(t) = (\cos(\omega t), \cos(\iota)\sin(\omega t), \sin(\iota)\sin(\omega t))$$

- Mass quadrupole

$$M^{ij}(t) = \int d^3\vec{y}\; T^{00}(t, \vec{y})\, y^i y^j$$

$$\approx \mu R^2 \begin{pmatrix} \cos^2 \omega t & \cos\iota \cos\omega t \sin\omega t \\ \cos\iota\cos\omega t \sin\omega t & \cos^2\iota \sin^2 \omega t \end{pmatrix}$$



Generation of Gravitational Waves VI

- Metric perturbations

$$h_+ = \frac{4\mu R^2\omega^2}{r}\frac{1+\cos^2\iota}{2}\cos(2\omega t_{\rm ret}) \qquad h_\times = \frac{4\mu R^2\omega^2}{r}\cos\iota\,\sin(2\omega t_{\rm ret})$$

- Assume two masses far enough to use Newtonian centripetal force

$$\frac{m_1(R\mu/m_1)^2\omega^2}{R\mu/m_1} = \frac{m_1 m_2}{R^2}$$

- Using chirp mass to represent metric perturbations

$$h_+ = \frac{4\mathcal{M}_c^{5/3}\omega^{2/3}}{r}\frac{1+\cos^2\iota}{2}\cos(2\omega t)$$

$$h_\times = \frac{4\mathcal{M}_c^{5/3}\omega^{2/3}}{r}\cos\iota\,\sin(2\omega t)$$

$$\mathcal{M}_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

characteristic of radiation :

1) monochromatic
2) a constant amplitude
3) twice the orbital frequency

These statements are only true for quadrupole source

Also masses approach to each other through emission of GWs

Further more, one needs to consider full GR

COSMOLOGY WITH GRAVITATIONAL WAVES I

- Cosmography : Measuring Hubble parameter or Luminosity (Angular diameter) distance to constrain the cosmological parameters

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + (1-\Omega_{m0})(1+z)^{3(1+w)}}$$

$$D_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

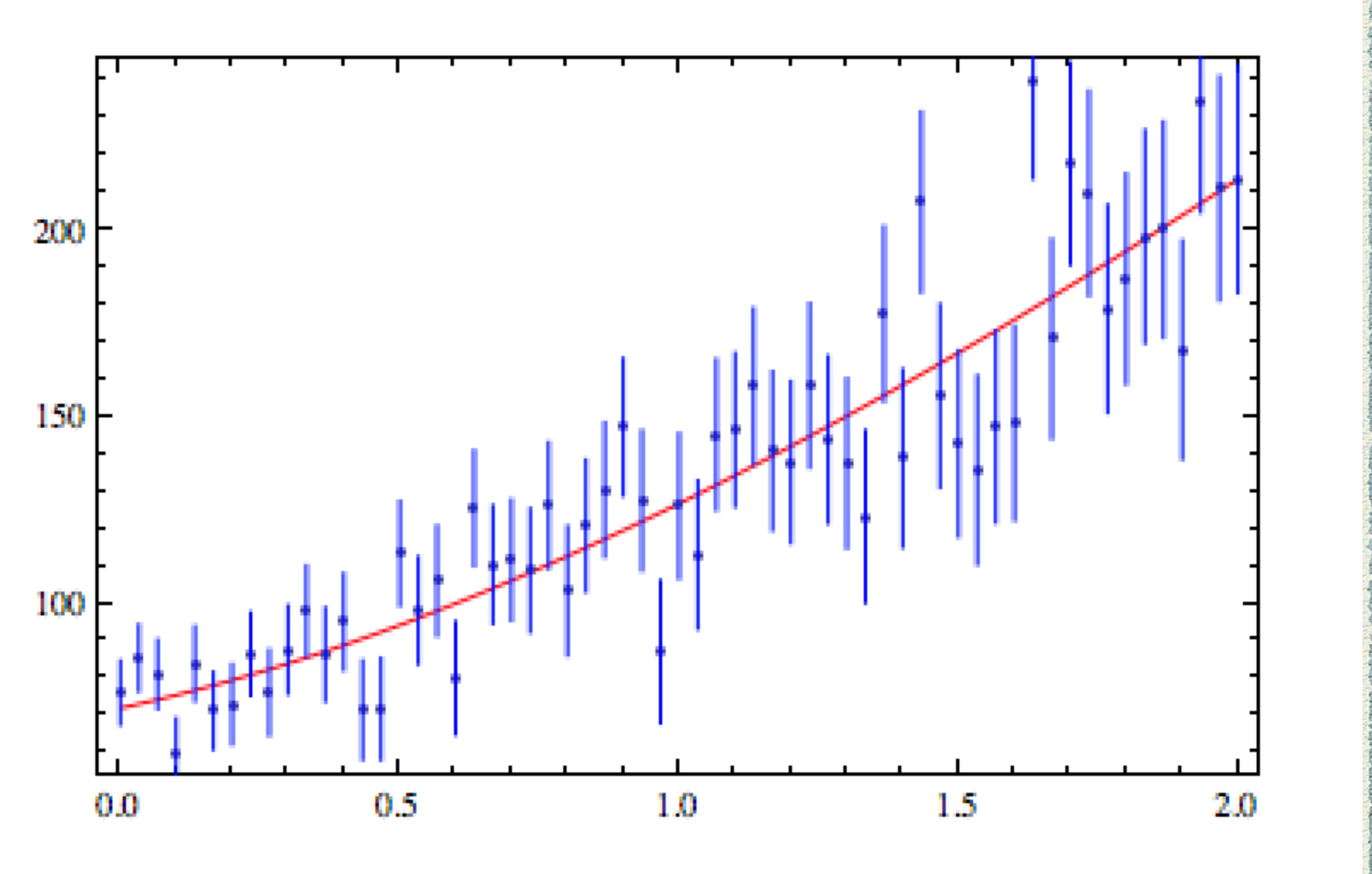
- Perturbation of GWs in the expanding Universe are related to D_L

$$h_c = \frac{4\mathcal{M}_c^{5/3}\omega_{\rm em}^{2/3}}{a(t_0)r} = \frac{4\mathcal{M}_c^{5/3}(1+z)^{2/3}\omega_{\rm obs}^{2/3}}{a(t_0)r}$$

$$= \frac{4\mathcal{M}_c^{5/3}(1+z)^{5/3}\omega_{\rm obs}^{2/3}}{a(t_0)r(1+z)} = \frac{4\mathcal{M}_{c,z}^{5/3}\omega_{\rm obs}^{2/3}}{D_L} \qquad \mathcal{M}_{c,z} \equiv (1+z)\mathcal{M}_c$$

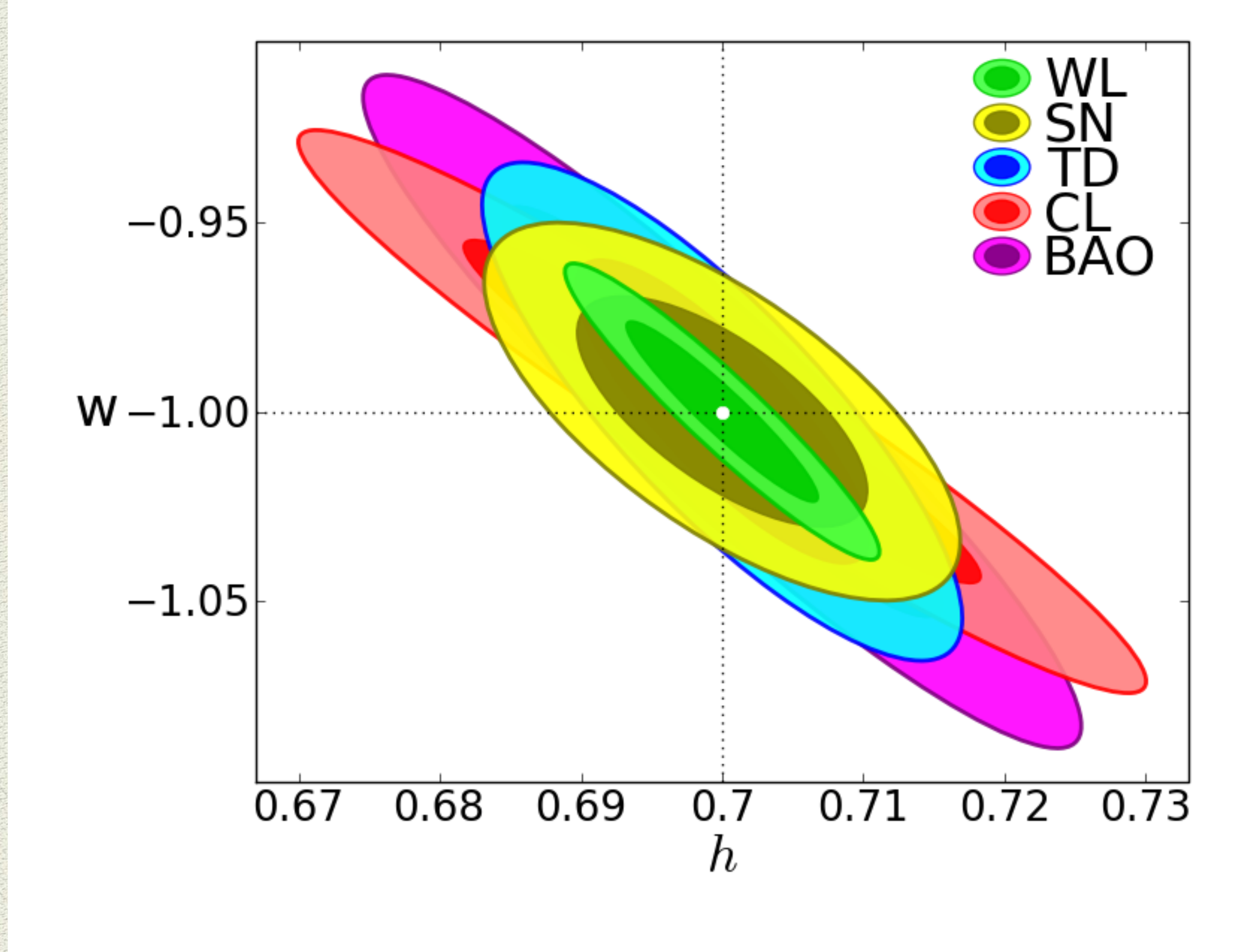
Fractional error in DL is about < 15% for 8% of sources and < 30% for 60% of sources

Simulation for H(z) *SL*



Cosmology with Gravitational Waves II

- Future plan
- Work with GWs Luminosity data to costrain the cosmological parameters
- Use Fisher matrix to forecast the constrain power of GWs data compared to other observations (SNe, BAO, CMB, etc)
- How many data required to have the comparable power of constrain cosmological parameters to others



“Thank You!”

Carpe Diem