# A Pedagogical Review on Various Inflationary Models

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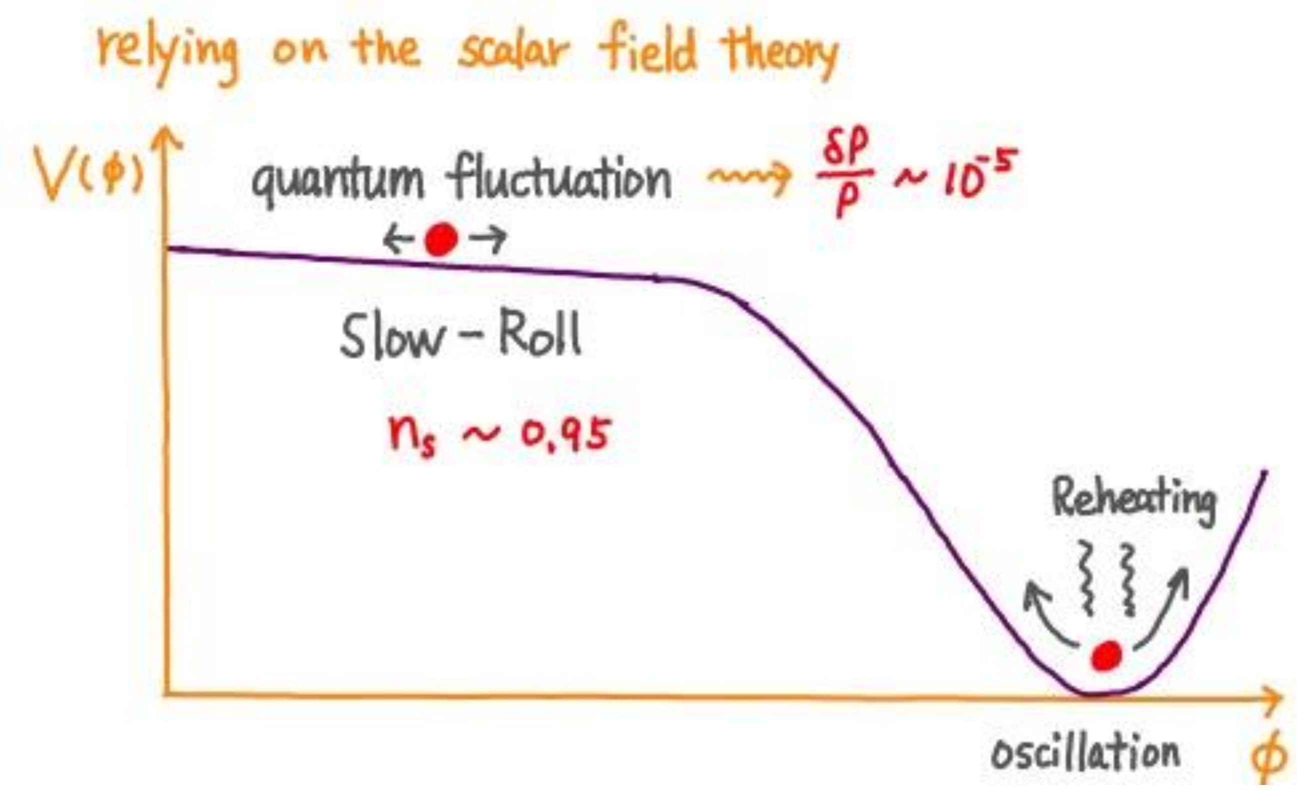
# Inflation (paradigm)

- resolves the Horizon problem and the Flatness problem. [ $d_H = (e^{Ht} - 1)/H$ ] [ $\Omega - 1 = k / H^2 e^{2Ht}$ ]
- provides seeds for galaxy formation.
- dilutes unacceptable topological defects.




Slow Roll conditions:

$$\varepsilon_\varphi \equiv M_{pl}^2 (V'/V)^2 \ll 1$$

$$\eta_\varphi \equiv M_{pl}^2 |V''/V| \ll 1$$

which implies $|m_\varphi^2| \ll 3H^2$.

Employing the scalar field theory with a small mass

→ **SUSY** is helpful,

but NOT enough !!

A Light enough Scalar Field (= inflaton)
with a proper potential is necessary
for successful inflation !!

# How to get a Light Scalar?

- by breaking a continuous (approx.) global symmetry
  $\longrightarrow$ (psuedo-) Goldstone boson

- by introducing Supersymmetry:
  Light fermions by the chiral sym. guarantee Light scalars.

- by introducing a strong interaction
  $\longrightarrow$ composite scalar

# "η Problem" in SUGRA

$$V_F = \sum_I \left|\frac{\partial W}{\partial \phi^I}\right|^2 \equiv |\partial_I W|^2$$

$$D_I W \equiv \frac{\partial W}{\partial \phi^I} + \frac{\partial K}{\partial \phi^I}\frac{W}{M_{pl}^2}$$

↓ local SUSY

$$V_F = e^{K/M_{pl}^2}\left[\overbrace{D_I W (K^{-1})^{I\bar{J}}\,\overline{D_J W}} - \frac{3}{M_{pl}^2}|W|^2\right]$$

$$\equiv e^{K/M_{pl}^2}\,\Lambda \quad > 0$$

For $K = |\phi|^2 + \ldots$ ,  $V_F \approx \Lambda + (\Lambda/M_{pl}^2)|\phi|^2 + \ldots$

$\rightarrow$ **$\eta = 1$ !!**

# ``Chaotic Inflation''

$$V(\varphi) = \lambda \varphi^p \qquad (p > 0)$$

$\varepsilon = (M_P^2/2)\, p^2 \varphi^{-2}$ , $\eta = M_P^2\, p(p-2)\, \varphi^{-2}$

$n - 1 = -\, M_P^2\, p(p+2)\, \varphi^{-2}$ (with $\varphi$ to be evaluated at $t = t_*$ )

$\varphi^2(t_*) - \varphi_e^2 = 2\, N_e\, (\varphi(t_*))\, p\, M_P^2$ ( $\varphi^2(t_*) >> \varphi_e^2$ )

So $n - 1 \approx -\,(p+2)/(2\, N_e) \approx -\,(p+2)/100$ for $N_e \approx 50$.

$p=2$ fits $n \approx 0.96$, but $\varphi(t_*) \approx 14 \times M_P$ ??

$P_R \approx V/(24\pi^2 M_P^4\, \varepsilon) \approx 2.4 \times 10^{-9}$ requires $m \approx 10^{13}$ GeV !!

# ``Natural Inflation"

$$V(a) = \Lambda^4 [\, 1 - \cos(a/f)\, ]$$

[ e.g. by instnaton effect , $\Lambda^4 \approx f_\pi^{\,2} m_\pi^{\,2}$ in QCD]

$f > 3\, M_{pl}$ for $\eta \ll 1$ ,

where $f$ is the $U(1)_{PQ}$ breaking scale.

$U(1)_{PQ}$ above the quantum gravity scale ?

# ``Natural Inflation"

Can be improved with 2 axionic inflatons !!

$$V(a_1,a_2) = \Lambda_1^4 [\, 1 - \cos(\alpha\, a_1/f_1 + \beta\, a_2/f_2)\,] + \Lambda_2^4 [\, 1 - \cos(\gamma\, a_1/f_1 + \delta\, a_2/f_2)\,]$$

[ Kim – Nilles - Peloso]

$f_1$ , $f_2$ ~ $O(M_{GUT})$ for a proper alignment

# Non-SUSY Hybrid Inflation

$$V(\sigma,\varphi) = (M^2 - \lambda\sigma^2)^2/4\lambda + (m^2/2)\varphi^2 + (g^2/2)\,\varphi^2\sigma^2$$

For $\varphi > M/g$, $\sigma = 0$, but

$$n = 1 + (\lambda\, m^2\, M_P{}^2 \,/\, \pi\, M^4) > 1$$

# SUSY Hybrid Inflation

## Introduce $U(1)_R$ sym.

[Copeland etal. '94]

$$W \rightarrow e^{2i\gamma} W \quad ; \quad \varphi \rightarrow e^{2i\gamma} \varphi$$

$$K = |\varphi|^2 \quad ; \quad W = \varphi m^2$$

("K" is the minimal Kahler pot., and "W" is of the "Polonyi" type.)

"$\varphi^2$", "$\varphi^3$", etc. don't appear in W !!

# SUSY Hybrid Inflation

$$K = \phi\phi^* \left( 1 + (0.1\text{-}0.01)\frac{\phi\phi^*}{M_P^2} + \cdots \right) \qquad W = \phi m^2$$

$$D_\phi W = \frac{\partial W}{\partial \phi} + \frac{\partial K}{\partial \phi}\frac{W}{M_{Pl}^2} = m^2 + \phi^* \frac{\phi m^2}{M_{Pl}^2}$$

$$= m^2 \left( 1 + \frac{|\phi|^2}{M_{Pl}^2} \right)$$

$$K_{\phi\phi^*} = 1$$

$$V_F = e^{|\phi|^2/M_{Pl}^2} \left[ m^4 \left( 1 + \frac{|\phi|^2}{M_{Pl}^2} \right)^2 - 3\frac{|\phi|^2 m^4}{M_{Pl}^2} \right]$$

$$\approx \left( 1 + \frac{|\phi|^2}{M_{Pl}^2} + \frac{1}{2}\frac{|\phi|^4}{M_{Pl}^4} + \cdots \right) m^4 \left[ 1 - \frac{|\phi|^2}{M_{Pl}^2} + \frac{|\phi|^4}{M_{Pl}^4} \right]$$

$$= m^4 \left( 1 + 0 + \frac{1}{2}\frac{|\phi|^4}{M_{Pl}^4} + \cdots \right)$$

The **Hubble scale mass term** is accidentally **cancelled**!

Let us introduce also the waterfall fields $\psi$ , $\psi^c$ .

$$W = \varphi( m^2 - \psi\psi^c) ;$$

$$V= | m^2 - \psi\psi^c |^2 + | \varphi |^2( | \psi |^2 + | \psi^c |^2 )$$

At SUSY minimum, $\varphi = 0$ , $\psi\psi^c = m^2$

But if $\varphi \gg m$ , then $\psi = \psi^c = 0$ and

$W_{eff} = \varphi m^2 \longrightarrow V = m^4$ : semi-stable false vacuum

By including the quantum correction, [Dvali, Shafi, Schaefer, '94]

$$V_{inf} = m^4 [ 1 + (1/8\pi^2) \mathrm{Log} (S/\Lambda) ]$$

In this model,

$$\delta T / T \sim 10^{-5} \sim (m / M_{pl})^2$$

So

$$m \sim 10^{16}\ \text{GeV} \ !!$$

Inflation can be associated with GUT breaking mech.!!

# Problems in SUSY Hybrid Infl.

Prediction: $n_\zeta \approx 1 + 2\eta = 1 - 1/N_e = 0.98$

for $N_e = 55\text{-}60$,

while data of WMAP is $n_\zeta \approx 0.96$ .

( $N_e = 25$ for $n_\zeta \approx 0.96$ is not enough.)

# Problems in SUSY Hybrid Infl.

- How about the **Monopole Problem** ?

  → Need higher order terms in W, but the perturbativity?

- $K \supset |S|^4/M_P^2$ gives rise to the **η problem** !!

# D-term Inflation

[ with a gauged U(1) sym. ]

$$V_D = (g^2/2)\,(|\varphi_+|^2 - |\varphi_-|^2 - \xi_{FI}{}^2)^2$$

With $W = \lambda\, \varphi_0 \varphi_+ \varphi_-$ ,

$$V_F/\lambda^2 = |\varphi_+ \varphi_-|^2 + |\varphi_0|^2 (|\varphi_+|^2 + |\varphi_-|^2)$$

For $\varphi_0 >> \xi_{FI}$ , $\varphi_+ = \varphi_- = 0$ and $m_\pm{}^2 = \lambda^2 |\varphi_0|^2 \pm g^2 \xi_{FI}{}^2$

$$V_{inf} = (g^2/2)\, \xi_{FI}{}^4 \,[\, 1 + (g^2/8\pi^2) \,\mathrm{Log}\,(|\lambda\, \varphi_0| / \Lambda)\,]$$

# Problems in D-term Infl.

No Hubble induced mass term for the inflaton !!

But

- $n_\zeta \approx 0.98$
- Cosmic Strings are induced after end of inflation.

# Natural Inflation in SUGRA

Introduce a shift sym.

$\varphi \longrightarrow \varphi + 2\pi i f$ ( i.e. $a \rightarrow a + 2\pi f$, axion )

$$K = K(\varphi + \varphi^*) \quad \text{or} \quad K = K(\mathrm{Re}\,\varphi)$$

$$W = w_0 + m^3 e^{-\varphi/f}$$

"$a$" doesn't appear in K !!

$$V_F \sim \Lambda^4 [\, 1 \pm \cos(a/f)\, ] \,, \quad f > M_P$$

# Chaotic Inflation in SUGRA

**Introduce a shift sym.**

$$\varphi \longrightarrow \varphi + 2\pi i f \qquad (\text{ i.e. } a \rightarrow a + 2\pi f, \text{ axion})$$

$$K = (\varphi + \varphi^*)/2 + |S|^2$$
$$W = m \varphi S$$

softly breaking the shift sym. with a small m

$$V_F = m^2 |\varphi|^2 + \cdots$$

# Higgs Inflation (non-SUSY)

$$L/(-g)^{1/2} = (M_P{}^2/2)\,\Omega^2 R - (\partial h)^2/2 - V$$

where $\Omega^2 = 1 + \xi\, h^2 / M_P{}^2$, $V = (\lambda/4)(h^2 - v_{EW}{}^2)^2$

$$\downarrow \qquad g_{\mu\nu} \longrightarrow \Omega^2\, g_{\mu\nu}$$

$$L/(-g)^{1/2} = (M_P{}^2/2)\,R - [(1+6\,\xi^2 h^2/\,\Omega^2 M_P{}^2)\,/\,\Omega^2](\partial h)^2/2 - V\,/\,\Omega^4$$

# Higgs Inflation (non-SUSY)

$\downarrow$ $h \gg M_P/\xi$ , $\varphi \equiv (3/2)^{1/2} M_P \log \Omega^2$

$$L/(-g)^{1/2} = (M_P{}^2/2)\, R - (\partial\varphi)^2/2$$

$$- (\lambda M_P{}^4/4\,\xi^2)\,(1 - \exp[-(2/3)^{1/2}\,\varphi/M_P]\,)^2$$

- $n_\zeta \approx 0.97$ , $r \approx 0.003$ but
- $\lambda/\xi^2 \approx 4\text{x}10^{-11}$ for $\delta T/T \sim 10^{-5}$
- Vacuum stability, unitarity, perturbativity, etc. ??

# Starobinski type Infl.

$$L/(-g)^{1/2} = (M_P^2/2)\,[R + (\xi^2/2\lambda)\, R^2/M_P^2]$$

$$\downarrow \qquad g_{\mu\nu} \longrightarrow \Omega^2 g_{\mu\nu}\,, \quad \Omega^2 = \exp[(2/3)^{1/2}\varphi/M_P]$$

$$L/(-g)^{1/2} = (M_P^2/2)\, R - (\partial\varphi)^2/2$$

$$- (\lambda M_P^4/4\,\xi^2)\,(1 - \exp[-(2/3)^{1/2}\varphi/M_P])^2$$

- the same as the Higgs infl. So still $\lambda / \xi^2 \approx 4\text{x}10^{-11}$

# Conclusion

Inflation is inevitable
in cosmology,

but still hard to be realized
in field theory.