Exact Holography Relation in finite N

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Introduction

$\mathcal{N}=6\ \ U_k(N)\times U_{-k}(N)$ theory

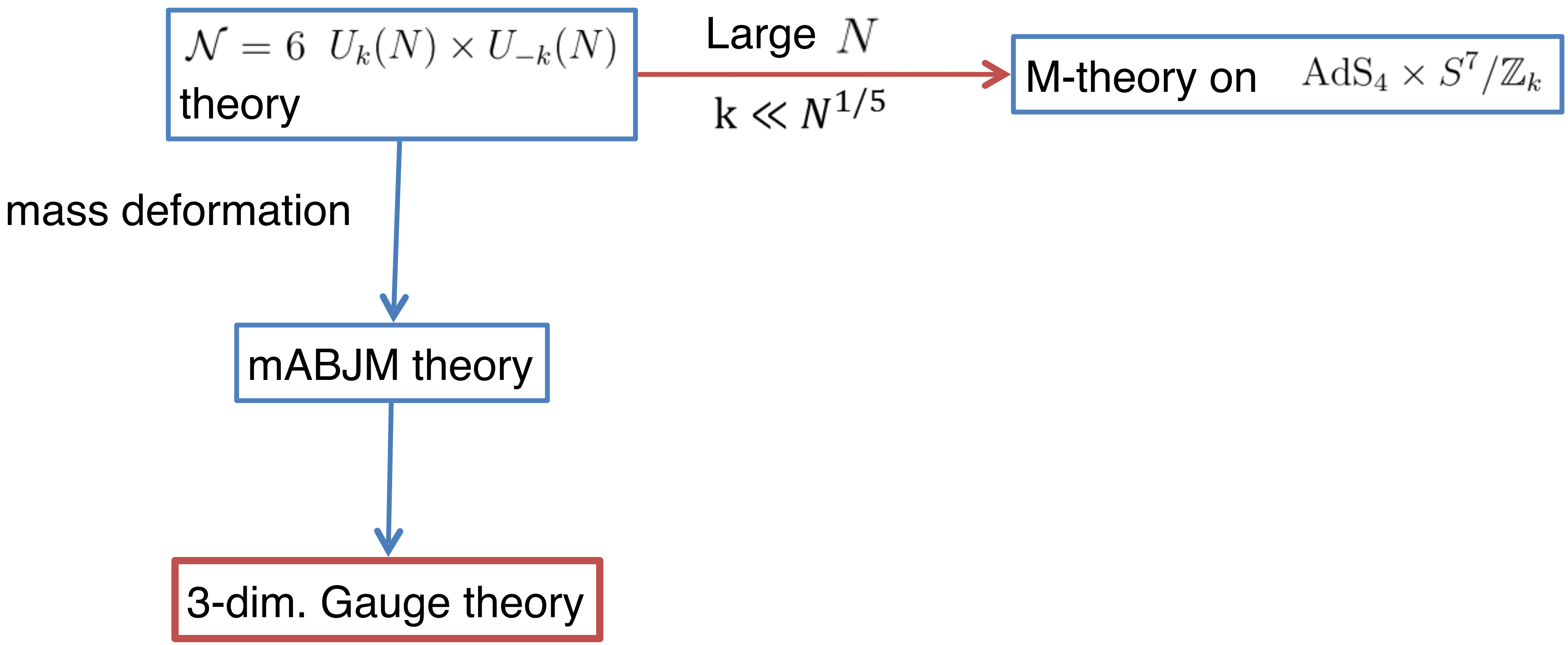
Large N

$\mathrm{k} \ll N^{1/5}$

→

M-theory on $\mathrm{AdS}_4\times S^7/\mathbb{Z}_k$

Introduction



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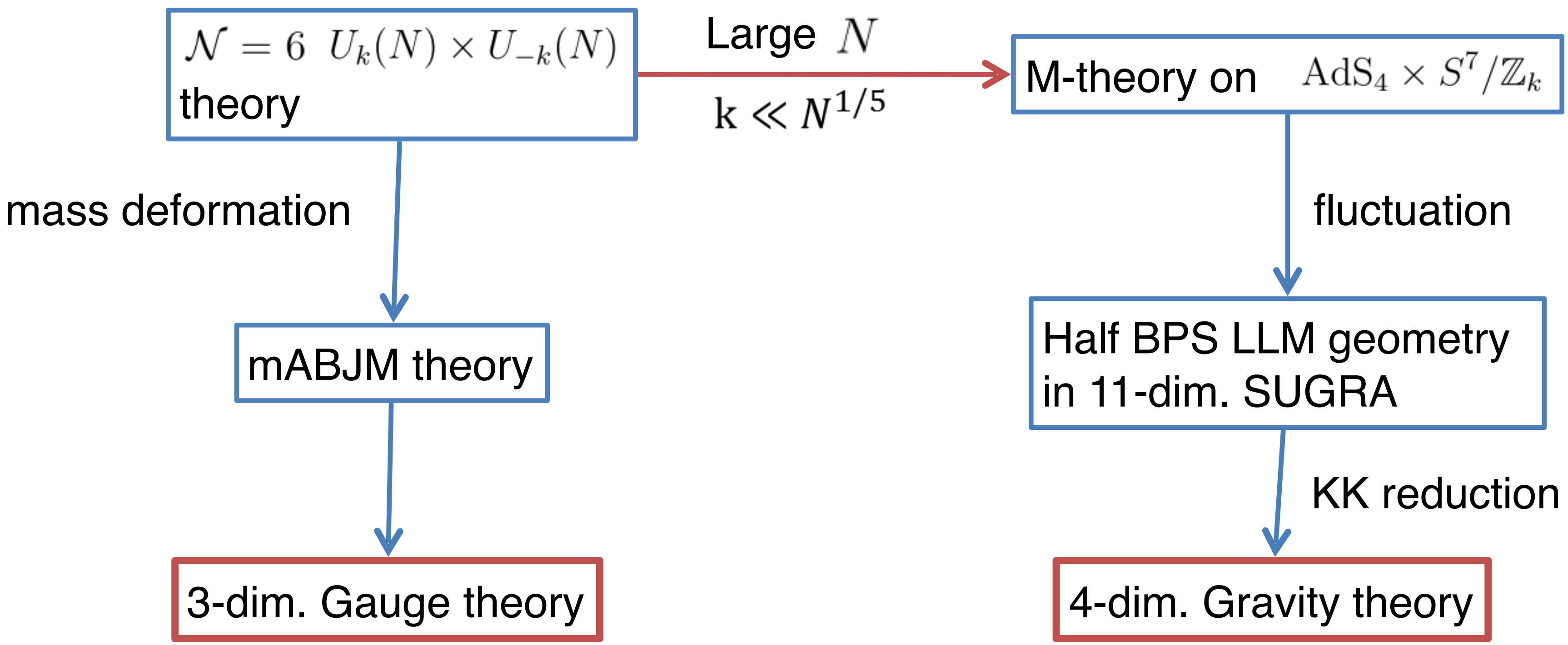

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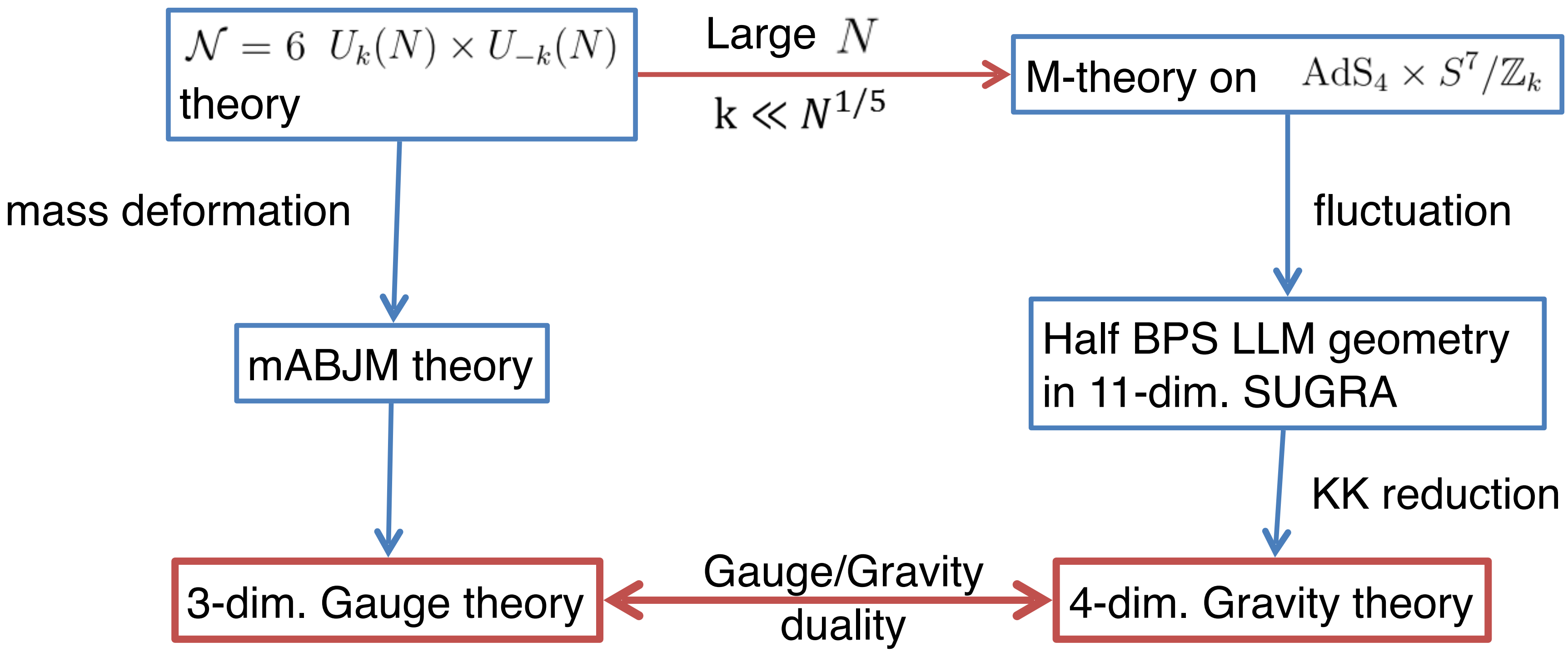

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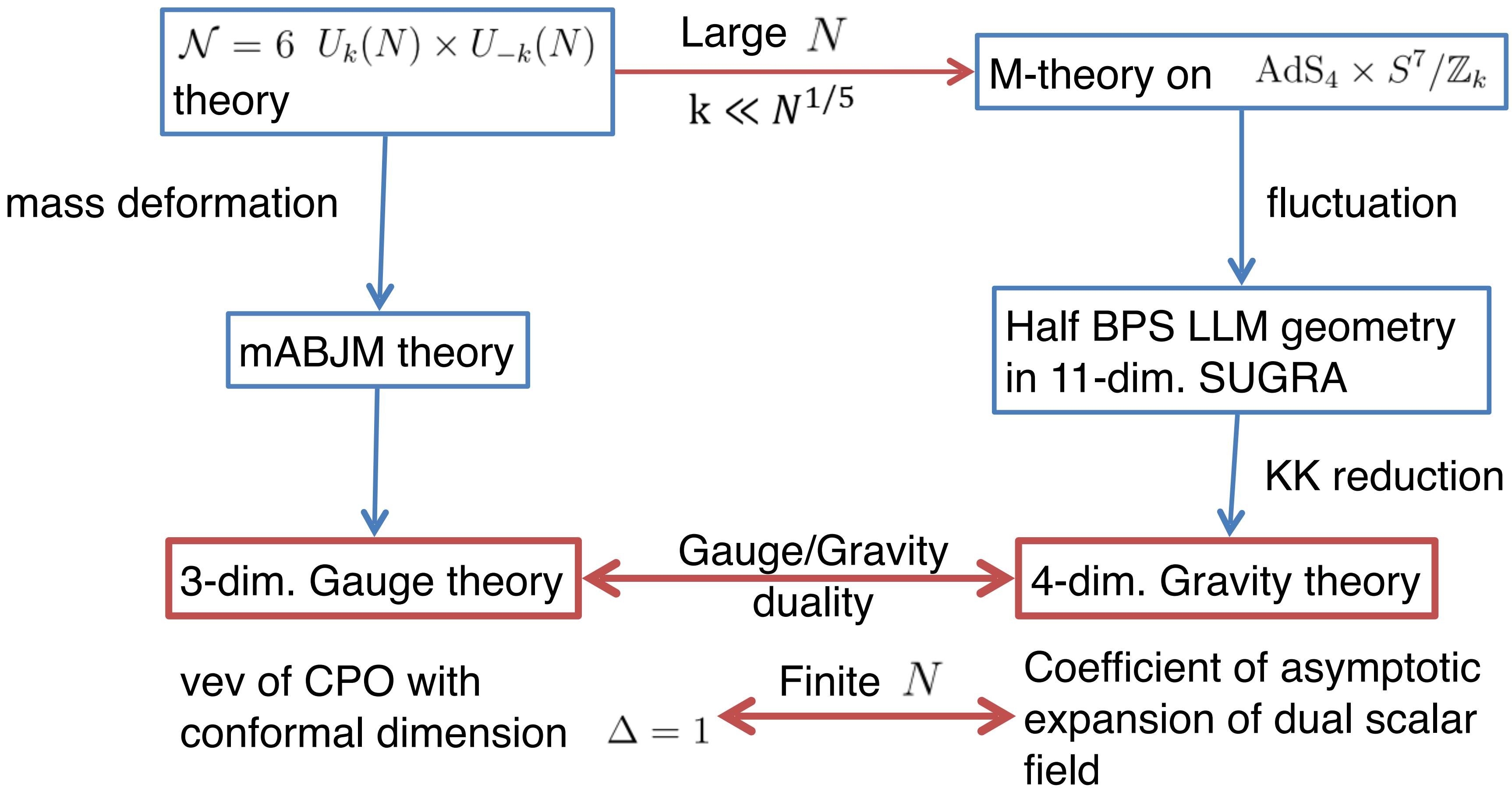

Discrete Higgs vacua of mABJM

[Hosomichi et al 08]
[Gomis et al 08]

ABJM theory
$\mathrm{SU}(4)$ global sym.

→ SUSY preserving mass deformation →

mABJM theory
$\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ …n.

$$Y^A = (Z^a, W^{\dagger a})$$

$$A = 1,2,3,4$$
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$$\mathcal{M}_1^{(n)} = \begin{pmatrix} \sqrt{n} & 0 & & & & \\ & \sqrt{n-1} & 0 & & & \\ & & \ddots & \ddots & & \\ & & & \sqrt{2} & 0 & \\ & & & & 1 & 0 \end{pmatrix}$$

GRVV matrices

$$\mathcal{M}_2^{(n)} = \begin{pmatrix} 0 & 1 & & & & \\ & 0 & \sqrt{2} & & & \\ & & \ddots & \ddots & & \\ & & & 0 & \sqrt{n-1} & \\ & & & & 0 & \sqrt{n} \end{pmatrix}$$

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GRVV matrices → Direct sum

↓

Discrete Higgs vacua

[Gomis et al 08]
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Discrete Higgs vacua of mABJM

- Discrete Higgs vacua (N X N matrices)

$$Z^a = \begin{pmatrix} \mathcal{M}_a^{(n_1)} & & & & & \\ & \ddots & & & & \\ & & \mathcal{M}_a^{(n_i)} & & & \\ & & & 0_{(n_{i+1}+1)\times n_{i+1}} & & \\ & & & & \ddots & \\ & & & & & 0_{(n_f+1)\times n_f} \end{pmatrix} \quad W^{\dagger a} = \begin{pmatrix} 0_{n_1\times(n_1+1)} & & & & & \\ & \ddots & & & & \\ & & 0_{n_i\times(n_i+1)} & & & \\ & & & \bar{\mathcal{M}}_a^{(n_{i+1})} & & \\ & & & & \ddots & \\ & & & & & \bar{\mathcal{M}}_a^{(n_f)} \end{pmatrix}$$

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- Constraints : $\sum_{n=0}^{\infty} (nN_n + (n+1)N'_n) = N$ $\quad \sum_{n=0}^{\infty} ((n+1)N_n + nN'_n) = N$

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- Condition for supersymmetric vacua : $0 \leq N_n, N_n' \leq k$

[Kim,Kim 10]
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- Vacua are clarified by occupation number $\{N_n, N'_n\}$

LLM geometry in 11-dim. SUGRA

- Half-BPS LLM geometry with $SO(2,1)\times SO(4)/\mathbb{Z}_k\times SO(4)/\mathbb{Z}_k$ isometry in 11-dimensional supergravity

$$ds^2 = G_{tt}\left(-dt^2 + dw_1^2 + dw_2^2\right) + G_{xx}\left(d\tilde{x}^2 + d\tilde{y}^2\right) + G_{\theta\theta}ds^2_{S^3/\mathbb{Z}_k} + G_{\tilde{\theta}\tilde{\theta}}ds^2_{\tilde{S}^3/\mathbb{Z}_k}$$

[Lin,Lunin,Maldacena 04]
[Cheon,Kim,Kim 11]

$$G_{tt} = \left(\frac{4\mu_0^2\tilde{y}\sqrt{\frac{1}{4}-Z^2}}{f^2}\right)^{2/3}, \qquad G_{xx} = \left(\frac{f\sqrt{\frac{1}{4}-Z^2}}{2\mu_0\tilde{y}^2}\right)^{2/3},$$

$$G_{\theta\theta} = \left(\frac{f\tilde{y}\sqrt{\frac{1}{2}+Z}}{2\mu_0\left(\frac{1}{2}-Z\right)}\right)^{2/3}, \qquad G_{\tilde{\theta}\tilde{\theta}} = \left(\frac{f\tilde{y}\sqrt{\frac{1}{2}-Z}}{2\mu_0\left(\frac{1}{2}+Z\right)}\right)^{2/3} \qquad f(\tilde{x},\tilde{y}) = \sqrt{1-4Z^2-4\tilde{y}^2V^2}$$

- This solution is determined by two functions

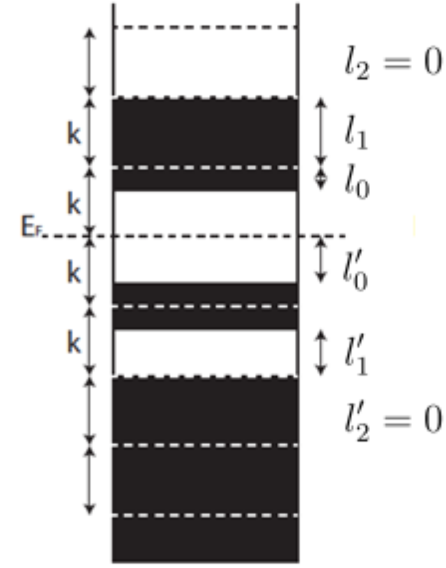
$$Z(\tilde{x},\tilde{y}) = \sum_{i=1}^{2N_B+1}\frac{(-1)^{i+1}(\tilde{x}-\tilde{x}_i)}{2\sqrt{(\tilde{x}-\tilde{x}_i)^2+\tilde{y}^2}} \qquad V(\tilde{x},\tilde{y}) = \sum_{i=1}^{2N_B+1}\frac{(-1)^{i+1}}{2\sqrt{(\tilde{x}-\tilde{x}_i)^2+\tilde{y}^2}}$$

LLM geometry in 11-dim. SUGRA

$\tilde{y} = 0$ boundary

$$Z(\tilde{x}, 0) = +\frac{1}{2}$$: white strip

$$Z(\tilde{x}, 0) = -\frac{1}{2}$$: black strip



Droplet picture

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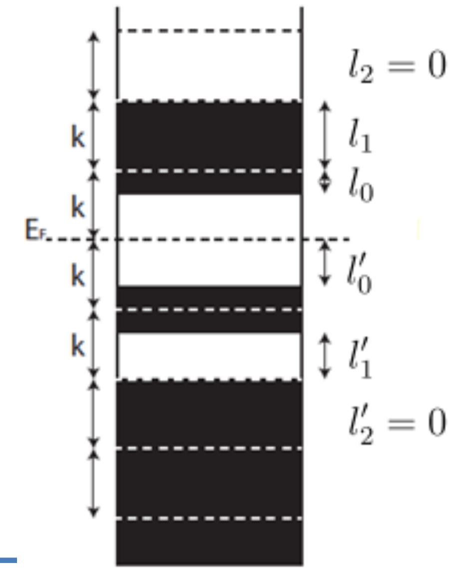

Droplet picture

Discrete torsion $\{l_n, l'_n\}$

Vacuum is identified by the occupation numbers :

$$\{l_n, l'_n\} \Longleftrightarrow \{N_n, N'_n\}$$

[Cheon,Kim,Kim 11]

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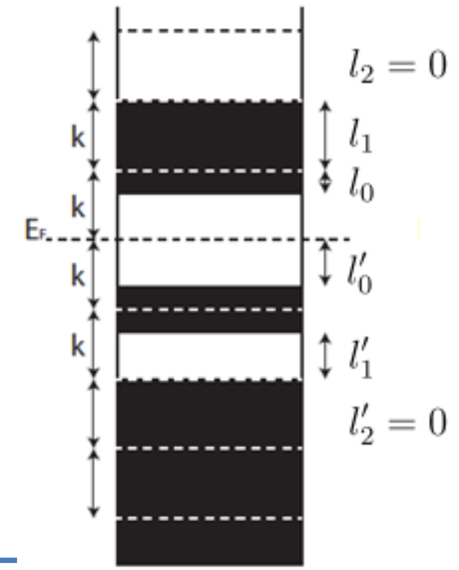

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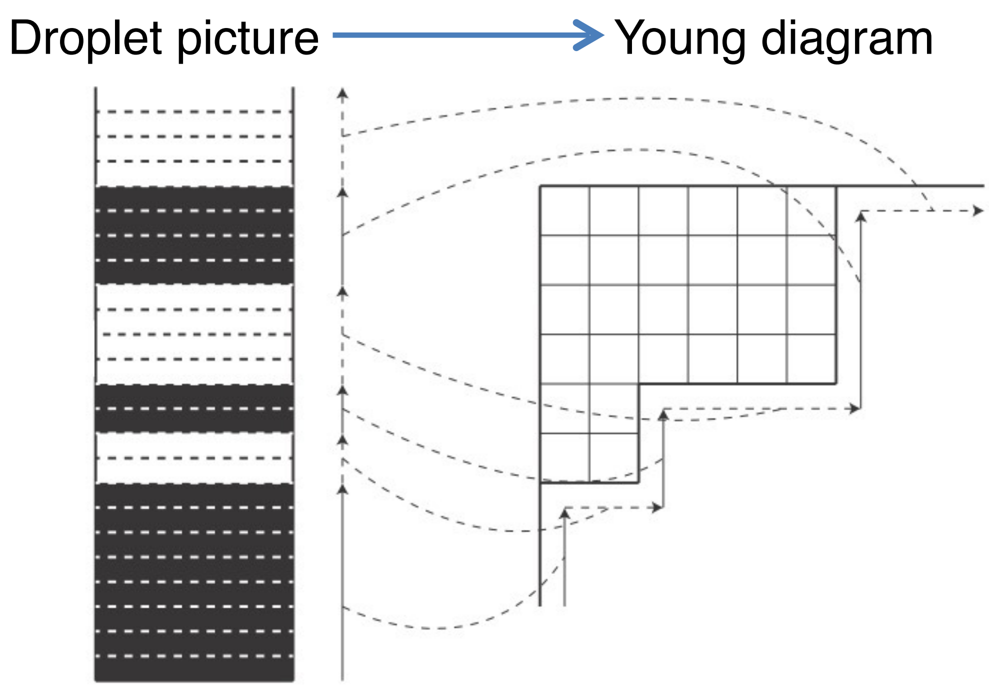
$$C_p = \sum_{i=1}^{2N_B+1} (-1)^{i+1} \left(\frac{\tilde{x}_i}{2\pi l_P^3 \mu_0 \sqrt{A}} \right)^p$$

LLM geometry in 11-dim. SUGRA

- For arbitrary k



LLM geometry in 11-dim. SUGRA

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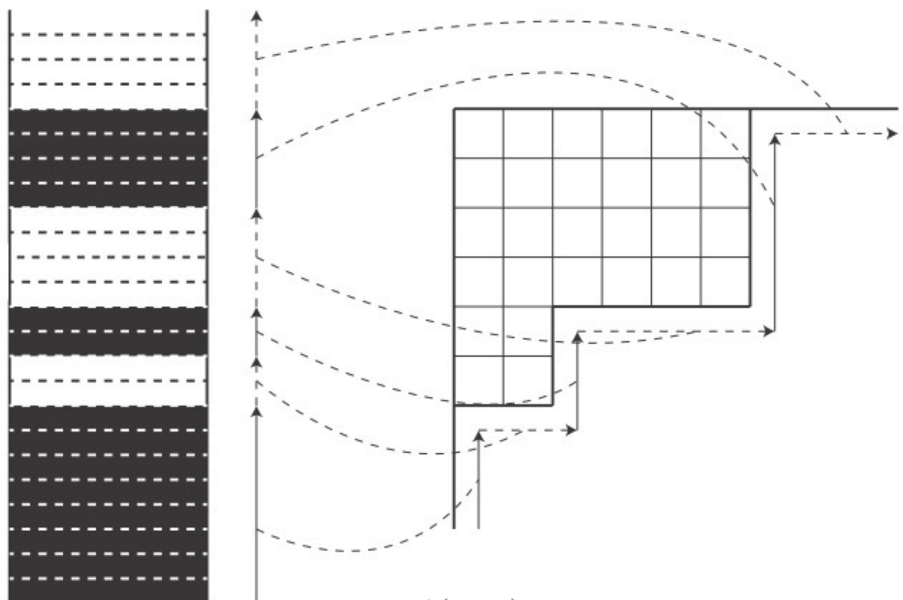
Droplet picture → Young diagram



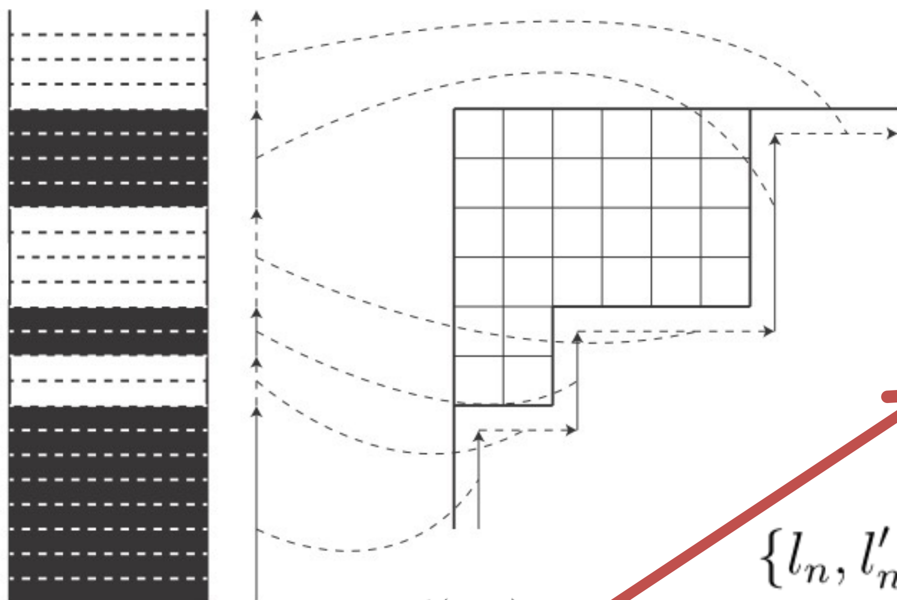
$$l_m^{(k)} = \sum_{n=km}^{k(m+1)-1} l_n$$

Discrete torsion for arbitrary k → Discrete torsion for $k=1$

LLM geometry in 11-dim. SUGRA

- For arbitrary k

Droplet picture → Young diagram



$$N_m^{(k)} = \sum_{n=km}^{k(m+1)-1} N_n$$

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Discrete torsion for arbitrary k → Discrete torsion for $k=1$

vev of CPO with $\Delta = 1$FT side)

- CPO with conformal dim. Δ n ABJM theory

$$\mathcal{O}^{(\Delta)} = C^{B_1 \cdots B_\Delta}_{A_1 \cdots A_\Delta} \,\mathrm{Tr}\left(Y^{A_1} Y^{\dagger}_{B_1} \cdots Y^{A_\Delta} Y^{\dagger}_{B_\Delta} \right)$$

$$\downarrow \quad \Delta = 1 \,, \;\; Y^A = (Z^a, W^{\dagger a})$$

$$\mathcal{O}^{(1)} = \frac{1}{2\sqrt{2}} \,\mathrm{Tr}\left(Z^a Z^{\dagger}_a - W^{\dagger a} W_a \right)$$

vev of CPO with $\Delta = 1$FT side)

- CPO with conformal dim. Δ in ABJM theory

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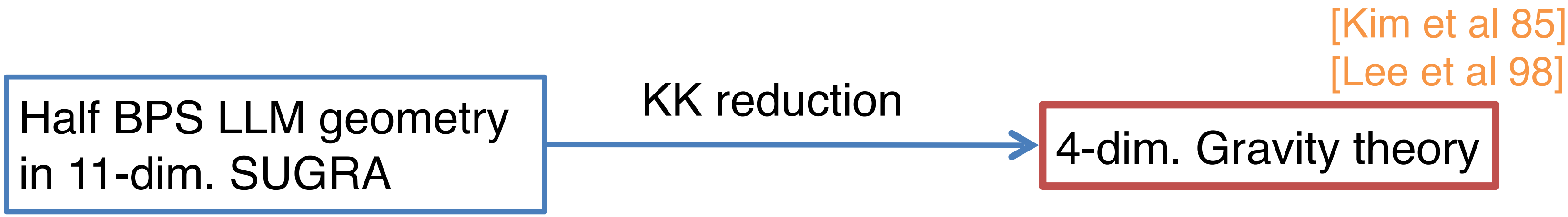
- vev of CPO with $\Delta = 1$ for arbitrary N k

Discrete Higgs vacua $\{N_n, N'_n\}$

$$\langle \mathcal{O}^{(1)} \rangle_0 = \frac{\mu k}{4\sqrt{2}\pi} \sum_{n=0}^{\infty} \Big[k^2 n(n+1)(N_n - N'_n) - kn\left(N_n(k - N_n) - N'_n(k - N'_n)\right) + \frac{1}{3}\left(N_n(N_n^2 - 1) - N'_n(N'^2_n - 1)\right) \Big]$$

Kaluza-Klein reduction



Kaluza-Klein reduction

[Kim et al 85]
[Lee et al 98]

Half BPS LLM geometry in 11-dim. SUGRA → KK reduction → 4-dim. Gravity theory

- Linearlized scalar fields in 4-dim. Gravity theory

$$\left[\nabla_\mu \nabla^\mu - \frac{n(n-6)}{L^2}\right] \Psi^{I_1} = 0$$

$$\left[\nabla_\mu \nabla^\mu - \frac{(n+6)(n+12)}{L^2}\right] \Phi^{I_1} = 0$$

$$\Psi^{I_1} = \frac{n-1}{14(n+3)}\left[-18(n+7)\hat{\phi}^{I_1} + 7\hat{\psi}^{I_1}\right]$$

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$$U^{L_1} = L\epsilon^{\mu\nu\rho\sigma}\nabla_\mu S^{L_1}_{\nu\rho\sigma}$$

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$m^2 L^2_{AdS_4} = \frac{m^2 L^2}{4} = \Delta(\Delta - 3)$ (L radius of S^7

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$$\Psi^{I_1}: \quad \Delta = \frac{I_1}{2}$$

I_1	=	Δ
2		1
4		2

$$\Phi^{I_1}: \quad \Delta = \frac{I_1 + 12}{2}$$

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dual scalar field

$$\Psi^{I_1=2} = -6\sqrt{2}\beta_1\mu_0 z + \mathcal{O}({\mu_0}^3)$$

$$\psi_{(1)} = -6\sqrt{2}\beta_1\mu_0$$

$$\beta_1 = 2C_1^3 - 3C_1C_2 + C_3$$

Exact holography in finite N

- Gauge/Gravity mapping (GKP-W relation)

$$\langle \mathcal{O}^{(1)} \rangle_0 = \mathbb{N}\psi_{(1)} = -6\sqrt{2}\mathbb{N}\beta_1\mu_0 \qquad \beta_1 = 2C_1^3 - 3C_1C_2 + C_3$$

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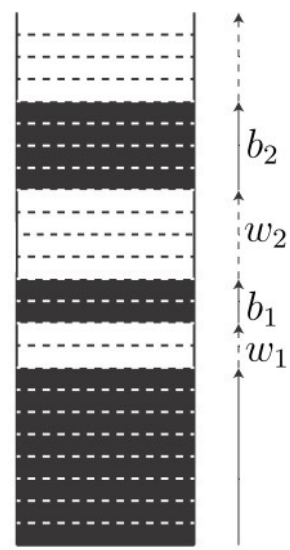

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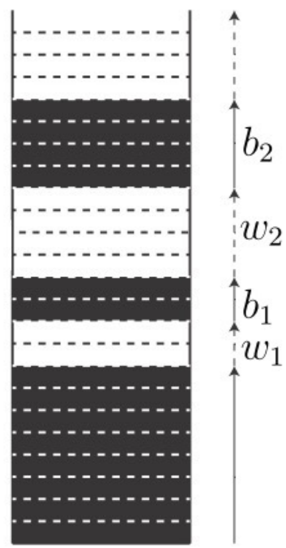

$$\beta_1 = 3\left[\sum_{n=1}^{N_B}\sum_{m=1}^{n} b_n w_m \sum_{l=1}^{N_B}(b_l - w_l) - \sum_{n=2}^{N_B}\sum_{m=1}^{n-1} b_n w_m \left(\sum_{l=1}^{m} b_l - \sum_{l=n}^{N_B} w_l\right)\right]$$

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$$\beta_1 = 2C_1^3 - 3C_1C_2 + C_3$$

$$\{l_n, l'_n\} \Longleftrightarrow \{N_n, N'_n\}$$

Normalization factor : $\mathbb{N} = -\dfrac{N^{\frac{3}{2}}}{36\pi}$

Exact holography in finite N

- Gauge/Gravity mapping (GKP-W relation)

$$\langle \mathcal{O}^{(1)} \rangle_0 = \mathbb{N}\psi_{(1)} = -6\sqrt{2}\mathbb{N}\beta_1\mu_0$$

$$\beta_1 = 2C_1^3 - 3C_1C_2 + C_3$$

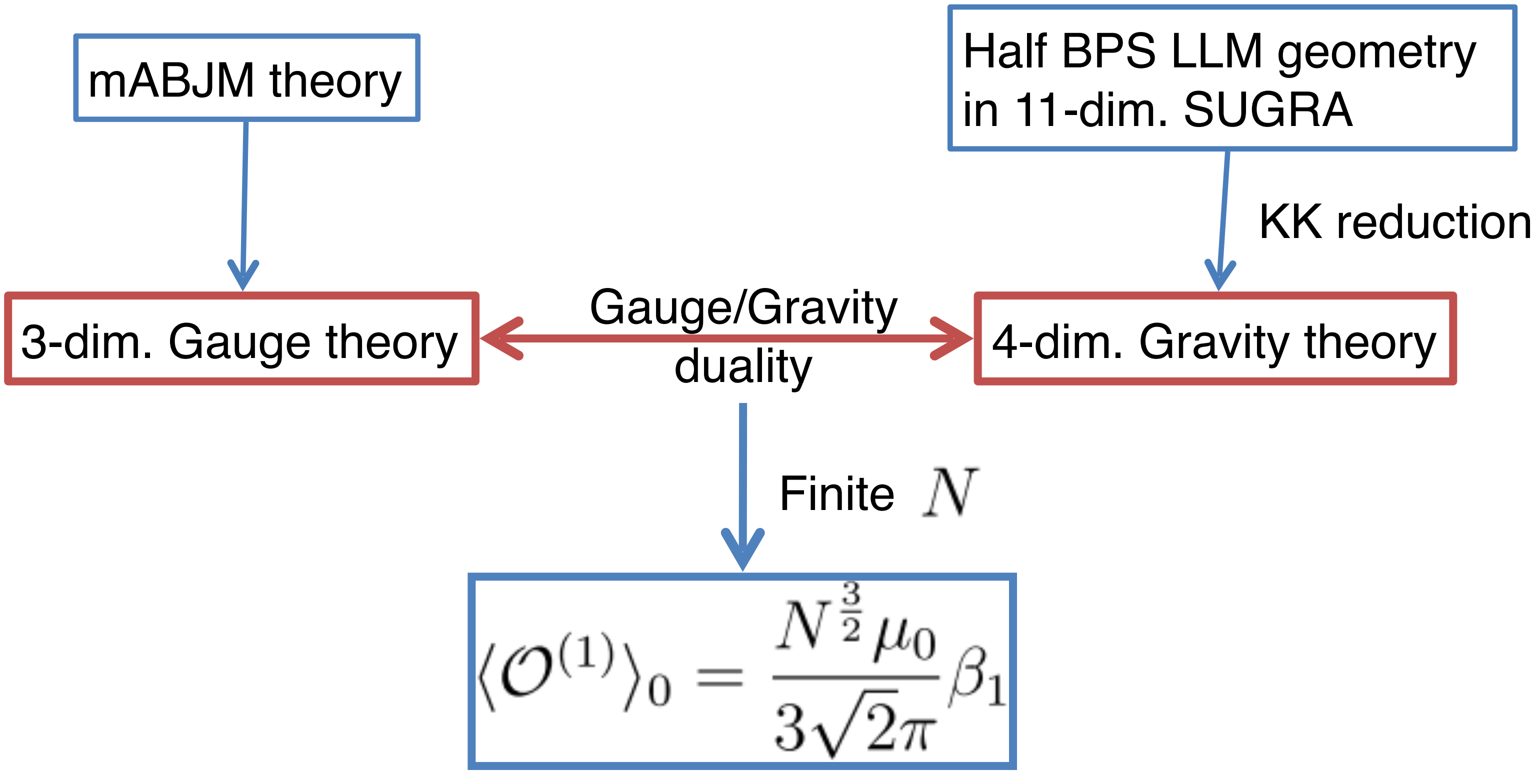
$$\{l_n, l'_n\} \Longleftrightarrow \{N_n, N'_n\}$$

Normalization factor : $\mathbb{N} = -\frac{N^{\frac{3}{2}}}{36\pi}$

- Exact relation in finite N

$$\langle \mathcal{O}^{(1)} \rangle_0 = \frac{N^{\frac{3}{2}}\mu_0}{3\sqrt{2}\pi}\beta_1$$

Summary



$$\langle \mathcal{O}^{(1)} \rangle_0 = \frac{N^{\frac{3}{2}} \mu_0}{3\sqrt{2}\pi} \beta_1$$

$$\beta_1 = 2C_1^3 - 3C_1C_2 + C_3$$

Thank you!