Search for blackhole solutions with anisotropic fluid

Hyeong-Chan Kim
(KNUT)

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Blackhole with anisotropic fluid, (in preparation)

차례

Motivation

The spatial topology of the Universe is an unresolved problem.

Observational Data:

$$\Omega_k = 0.000 \pm 0.005 \ (95\%, \ \textit{Planck} \ \mathrm{TT{+}lowP{+}lensing{+}BAO})$$

It is never manifest if the Universe is flat, closed, or open.

There are trials for inflation models in closed/open universe

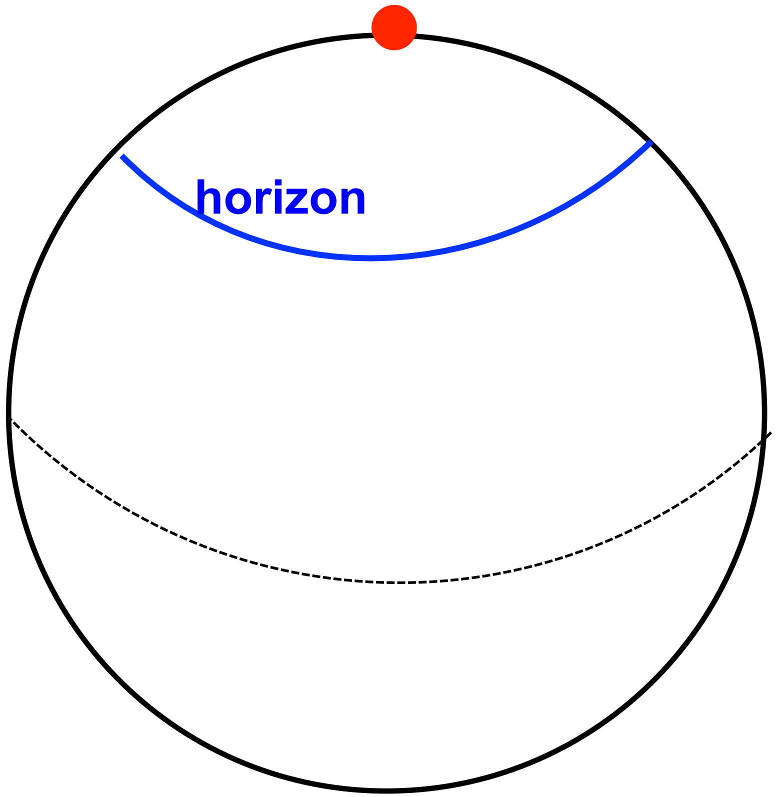
:- predict some peculiar feature distinguishable from FLAT models

:- but, still beyond the current observational resolution

We may consider an object such as a black hole in different spatial topologies.

Curiosity: blackhole in closed spacetime

Geometry?



Introduction (S3 and H3)

Metric

$$x_1^2 + x_2^2 + x_3^2 \pm x_4^2 = \pm R_0^2$$

Spatial Embedding, +: S3, -: H3

(t, r, θ, ϕ) coordinate system,

$$ds^2 = \mp dt^2 + \frac{dr^2}{1 - kr^2/R_0^2} + r^2 d\Omega_2^2$$

k=+1: S3, k=-1: H3

(t, χ, θ, ϕ) coordinate system

S3-I

$$r = R_0 \sin\chi, \qquad (r \leq R_0),$$
$$ds^2 = -dt^2 + R_0^2 d\chi^2 + R_0^2 \sin^2\chi d\Omega_2^2.$$

Closed static S3

S3-II

$$r = R_0 \cosh\chi, \qquad (R_0 \leq r < \infty),$$
$$ds^2 = +dt^2 - R_0^2 d\chi^2 + R_0^2 \cosh^2\chi d\Omega_2^2.$$

Closed time-dependent S3

H3

$$r = R_0 \sinh\chi, \qquad (0 \leq r < \infty),$$
$$ds^2 = -dt^2 + R_0^2 d\chi^2 + R_0^2 \sinh^2\chi d\Omega_2^2.$$

Hyperbolic spacetime

Effective Energy-Momentum Tensor

$$\boxed{\bar{G}^{\mu}_{\nu} = \mp\frac{1}{R_0^2}\mathrm{diag}(3,1,1,1) \qquad \equiv 8\pi\bar{T}^{\mu}_{\nu}}$$

: constant, -: S3 (r>0), +: H3 (r<0)

$$\Rightarrow \boxed{p = -\frac{1}{3}\rho = const.}$$

: Eq. of State

R3: Schwarzschild BH

if M=0, Minkowski Space: spatially flat

$$ds^2 = -\left(1-\frac{2\cancel{M}}{r}\right)dt^2 + \left(1-\frac{2\cancel{M}}{r}\right)^{-1}dr^2 + r^2 d\Omega_2^2$$

Can we have such a black hole parameterized by MASS in S3/H3?

$$\bar{T}^{\mu}_{\nu} = \mp\frac{1}{8\pi R_0^2}\mathrm{diag}(3,1,1,1)$$

with effective EM tensor

$$ds^2 = -f(r)dt^2 + g(r)dr^2 + r^2 d\Omega_2^2.$$

search BH solution

$\Rightarrow$ NO such a solution exists

Metric ansatz and Anisotropic fluid:

Metric ansatz: **Spherically symmetric solution,**

$$ds^2 = -f(r)dt^2 + g(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2.$$

Stress energy tensor

Spherical symmetry

$$T^\mu_\nu = \mathrm{diag}(-\rho, p_1, p_2, p_2).$$

$$\boxed{\begin{aligned} p_1 &= w_1 \rho \\ p_2 &= w_2 \rho, \end{aligned}}$$

The radial pressure may not be the same as the angular one.

Einstein equation:

$$G^0_0 = -\frac{1}{r^2} + \frac{1}{r^2 g} - \frac{g'}{r g^2} = -8\pi\rho(r), \qquad \text{A}$$

$$G^1_1 = -\frac{1}{r^2} + \frac{1}{r^2 g} + \frac{f'}{r f g} = 8\pi p_1(r), \qquad \text{B}$$

$$G^2_2 = \frac{f'}{2r f g} - \frac{f'^2}{4 f^2 g} - \frac{g'}{2 r g^2} - \frac{f' g'}{4 f g^2} + \frac{f''}{2 f g} = 8\pi p_2(r), \qquad \text{C}$$

Bianchi identity

$$\frac{1}{2}\frac{f'}{f}(\rho + p_1) + p_1' + \frac{2(p_1 - p_2)}{r} = 0 \quad \Rightarrow \quad \rho = \rho_0 f^{-\frac{1+w_1}{2w_1}} r^{-\frac{2(w_1-w_2)}{w_1}} \equiv \frac{M'}{4\pi r^2}. \qquad \text{D}$$

Solution to Eq. A $g(r) = \dfrac{1}{1 - 2M(r)/r}; \qquad M(r) = 4\pi \displaystyle\int^r r'^2 \rho(r') dr',$

Eq. B determine relation bet. *f* and *g* by w_1 .

$$f(r) = \frac{(g(r))^{w_1}}{r^{w_1+1}} \exp\left[(1+w_1)\int^r \frac{g(r)}{r} dr\right] = \frac{(r-2M)^{-w_1}}{r} \exp\left[(1+w_1)\int^r \frac{1}{r-2M(r)} dr\right].$$

C equation

$$0 = G_2^2 = \frac{4r^2 w_1 M'}{r-2M}\left[\frac{M''}{M'} + \frac{1+w_1}{r-2M}\left(M' + \frac{1}{2w_1}\right) - \frac{1+w_1+4w_2}{2w_1 r}\right].$$

For $w_1 = w_2 = -1/3$, solutions in S3 and H3.

For $w_1 = w_2 = w>0$, there are many works for perfect fluid stars.
(Usually, the solutions are solved numerically.
The star fails to form a blackhole.
An outer boundary is need to form a stable system.)
example: a photon star $w=1/3$. Sorkin, Wald, Zhang, 1981. etc.

For $w_1 = -1$ another class of exact solutions.

In general, one may solve the equation at least numerically.

Black Holes in S3/H3 with Static Perfect Fluid with w=-1/3

Introduce "Static Perfect Fluid" satisfying S3/H3 Equation of State

(t, r, θ, ϕ) coordinate system,

metric ansatz

$$ds^2 = -f(r)dt^2 + g(r)dr^2 + r^2 d\Omega_2^2.$$

EM tensor

$$T^\mu_\nu = \mathrm{diag}\big[-\rho(r), p(r), p(r), p(r)\big],$$

$$p(r) = -\frac{1}{3}\rho(r)$$

Require S3/H3 Eq. of State

Einstein Eqs

.

$$\begin{aligned}
G^0_0 &= -\frac{1}{r^2} + \frac{1}{r^2 g} - \frac{g'}{rg^2} = -8\pi\rho(r),\\
G^1_1 &= -\frac{1}{r^2} + \frac{1}{r^2 g} + \frac{f'}{rfg} = 8\pi p(r),\\
G^2_2 &= \frac{f'}{2rfg} - \frac{f'^2}{4f^2 g} - \frac{g'}{2rg^2} - \frac{f'g'}{4fg^2} + \frac{f''}{2fg} = 8\pi p(r),
\end{aligned}$$

Solutions

$$\rho(r) = -\frac{3}{8\pi\alpha}\left\{1 \mp \frac{2\alpha|\beta|}{r}\left[\beta(r^2+\alpha)\right]^{1/2}\right\},$$

$$f(r) = \frac{\rho(r)}{\rho_c}, \qquad g^{-1}(r) = -\frac{8\pi}{3}(r^2+\alpha)\rho(r).$$

: 2-parameter solutions

Define new parameters : $R_0^2 \equiv |\alpha|, \qquad K \equiv 2R_0^2|\beta|^{3/2}.$

R_0^2: **3-space curvature scale**

K: **Mass Parameter**

3 types of solutions

(t, χ, θ, ϕ) coordinate system

Class	$\rho(\chi)$	$f(\chi)$	$g(\chi)$
S_3-I	$\frac{3}{8\pi R_0^2}(1 - K\cot\chi)$	$\frac{\rho(\chi)}{\rho_c}, \quad (\rho_c > 0)$	$\frac{3}{8\pi\rho(\chi)}$
S_3-II	$\frac{3}{8\pi R_0^2}(1 \mp K\tanh\chi)$	$\frac{\rho(\chi)}{\rho_c}, \quad (\rho_c < 0)$	$-\frac{3}{8\pi\rho(\chi)}$
H_3	$-\frac{3}{8\pi R_0^2}(1 \mp K\coth\chi)$	$\frac{\rho(\chi)}{\rho_c}, \quad (\rho_c < 0)$	$-\frac{3}{8\pi\rho(\chi)}$

S3-I

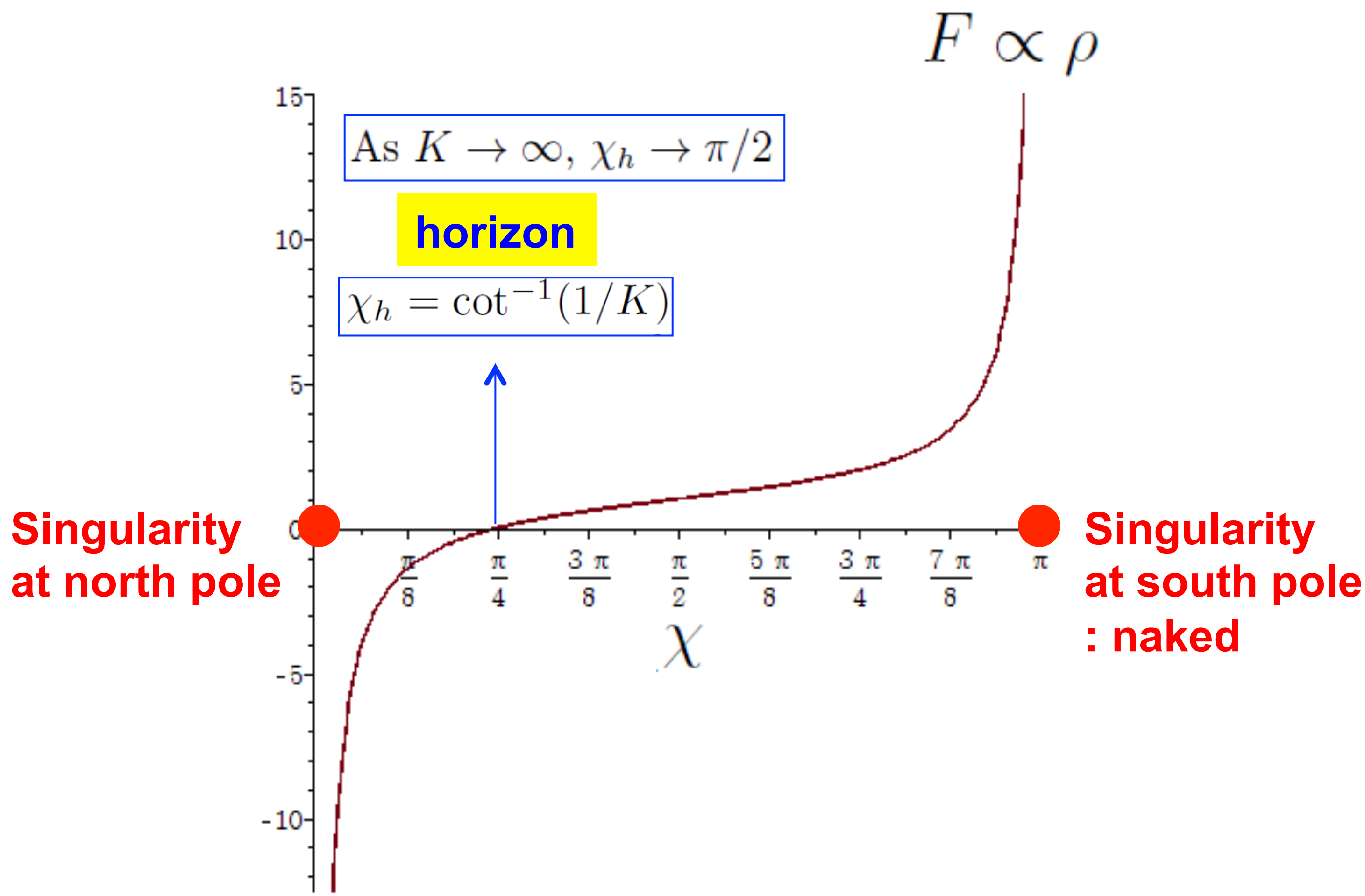
$$r = R_0 \sin\chi \quad (0 \le \chi \le \pi).$$

$$\alpha < 0,\ \beta < 0,\ \text{and}\ 1 - 4\alpha^2\beta^3 > 0$$

$$ds^2 = -\frac{3}{8\pi R_0^2 \rho_c} \underbrace{\boxed{(1 - K\cot\chi)}}_{\boxed{\equiv F}} dt^2 + \frac{R_0^2}{1 - K\cot\chi} d\chi^2 + R_0^2 \sin^2\chi d\Omega_2^2.$$

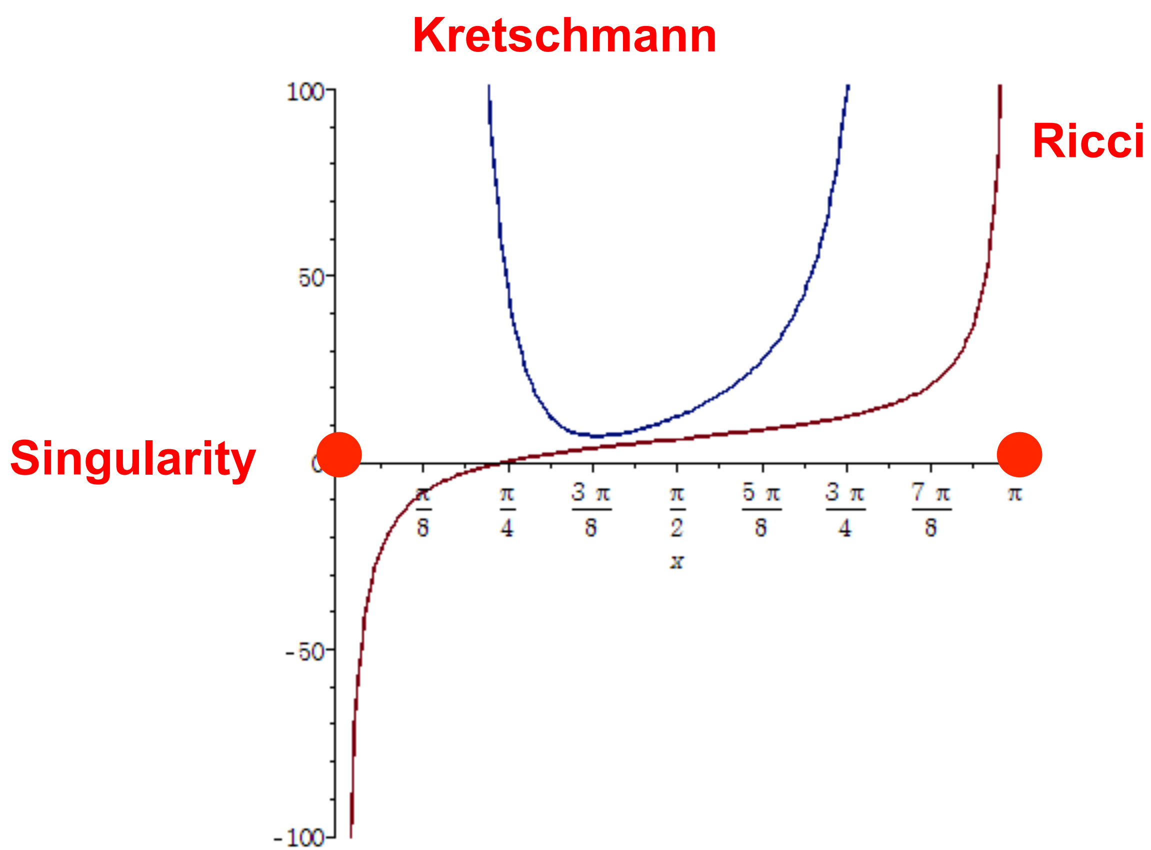
(1) Black Hole Solution

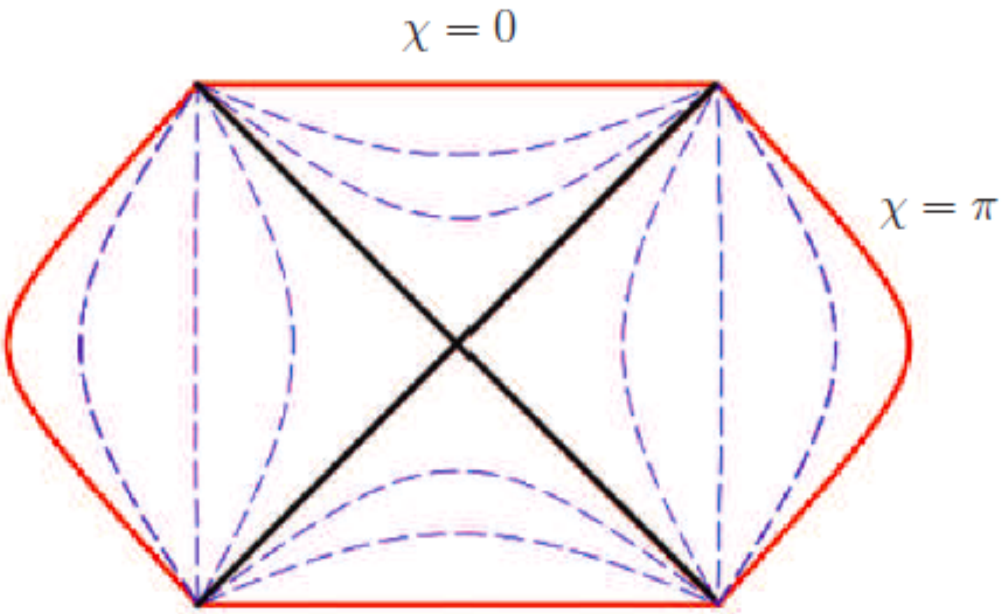
If K=0, it is S3



$$ds^2 = -\frac{3}{8\pi R_0^2 \rho_c}(1 - K\cot\chi)\,dt^2 + \frac{R_0^2}{1 - K\cot\chi}d\chi^2 + R_0^2 \sin^2\chi d\Omega_2^2.$$

Curvature


Kretschmann
Ricci
Singularity
100
50
0
-50
-100
π/8
π/4
3π/8
π/2
5π/8
3π/4
7π/8
π
x



Conformal diagram for S_3-I black hole. There are two singularities at $\chi = 0$ (north pole: center of the black hole) and at $\chi = \pi$ (south pole: naked). The solid diagonal lines represent the horizon at $\chi_h = \cot^{-1}(1/K)$. The dashed lines are the χ-constant lines. The t-constant lines are straight lines passing the center of the diagram (not shown). Outside the horizon, except the outgoing null rays, none of the geodesics can reach the naked singularity at the south pole.

Geodesics

$$ds^2 = -\frac{3}{8\pi R_0^2 \rho_c} F(\chi) dt^2 + g(\chi) d\chi^2 + R_0^2 b^2(\chi) d\Omega_2^2$$

$$t\text{-eq.}: \quad \frac{1}{F(\chi)} \frac{d}{d\lambda} \left[F(\chi) \frac{dt}{d\lambda} \right] = 0,$$

$$\phi\text{-eq.}: \quad \frac{1}{b^2(\chi)} \frac{d}{d\lambda} \left[b^2(\chi) \frac{d\phi}{d\lambda} \right] = 0.$$

$$F(\chi) \frac{dt}{d\lambda} = \text{const.} \equiv E, \qquad b^2(\chi) \frac{d\phi}{d\lambda} = \text{const.} \equiv L.$$

On the $\theta = \pi/2$ plane, the χ-equation becomes

$$g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = -\varepsilon,$$

$\varepsilon = 1, 0$ for timelike and null geodesics,

$$\frac{1}{2} \left(\frac{d\chi}{d\lambda} \right)^2 + V(\chi) = \frac{3E^2}{16\pi R_0^4 |\rho_c|} \equiv \tilde{E}^2,$$

$$V(\phi) = \frac{1}{2} F(\chi) \left[\frac{L^2}{b^2(\chi)} + \frac{\varepsilon}{R_0^2} \right].$$

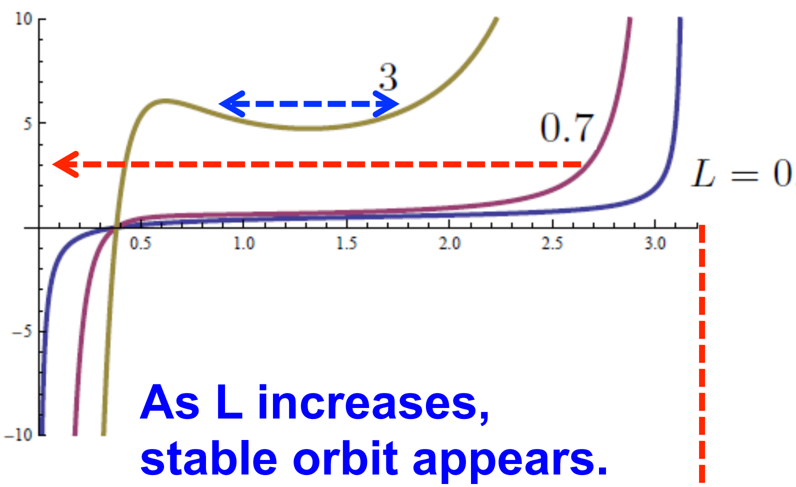
Effective Potential

S3-I

$$F(\chi) = 1 - K\cot\chi, \qquad b(\chi) = \sin\chi,$$

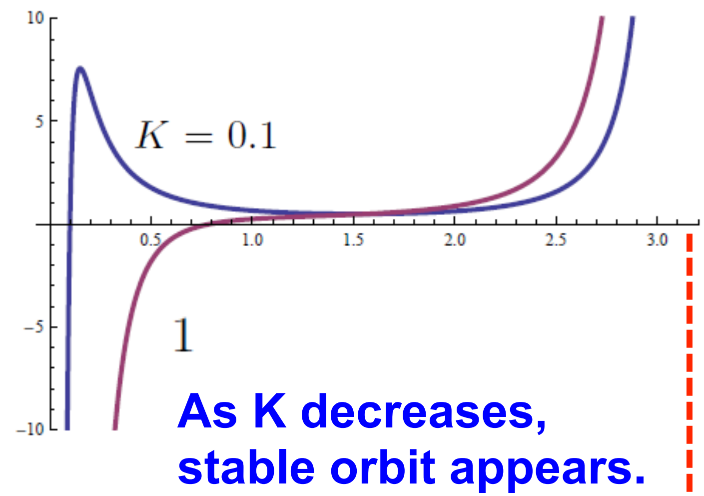
$$V(\phi) = \frac{1}{2}(1 - K\cot\chi)\left(\frac{L^2}{\sin^2\chi} + \frac{\varepsilon}{R_0^2}\right).$$

Timelike



(a) S_3 timelike geodesics for $R_0 = 1$ and $K = 0.4$:

Null



(b) S_3 null geodesics for $R_0 = 1$ and $L = 1$:

NO geodesic can reach the singularity at south pole!!!

Positive energy condition

$$\rho(r) = -\frac{3}{8\pi\alpha}\left\{1 \mp \frac{2\alpha|\beta|}{r}\left[\beta(r^2+\alpha)\right]^{1/2}\right\},$$

$$f(r) = \frac{\rho(r)}{\rho_c}, \qquad g^{-1}(r) = -\frac{8\pi}{3}(r^2+\alpha)\rho(r).$$

$$\boxed{p(r) = -\frac{1}{3}\rho(r)}$$

\rho is proportional to f(r).

To have positive definite energy density,
\rho should change sign at the horizon.

→ The energy density never be absorbed into the horizon.

→ The horizon size may not change.

Solution 2: anisotropic fluid

The case: $w_1 = -1$.

C $$\frac{2w_2}{r} + \frac{M''}{M'} = 0 \quad \Rightarrow \quad M(r) = \begin{cases} \frac{\Lambda}{2} r^{1-2w_2} + M & w_2 \neq \frac{1}{2} \\ \frac{\Lambda}{2} \log \frac{r}{2M} + M & w_2 = \frac{1}{2} \end{cases} .$$

Density and radial pressure:

$$\rho(r) \equiv -p_1(r) = \begin{cases} \frac{(1-2w_2)\Lambda}{8\pi r^{2w_2+2}} & w_2 \neq \frac{1}{2} \\ \frac{\Lambda}{8\pi r^3} & w_2 = \frac{1}{2} \end{cases} .$$

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2; \qquad f(r) = \begin{cases} 1 - \frac{2M}{r} - \frac{\Lambda}{r^{2w_2}}, & w_2 \neq \frac{1}{2} \\ 1 - \frac{2M}{r}\left(1 + \frac{\Lambda}{2M}\log\left|\frac{r}{\Lambda}\right|\right), & w_2 = \frac{1}{2} \end{cases}$$

Singularities:

If $\Lambda = 0$, the metric is nothing but vacuum Schwarzschild spacetime. Therefore, we restrict $\Lambda \neq 0$.

Scalar curvature and Krestchmann invariant:

$$R = \frac{2\Lambda(w_2 - 1)(2w_2 - 1)}{r^{2(w_2+1)}}.$$

Singular at the origin unless $w_2 \leq -1$ or $w_2 = 1/2,\ 1$.

$$R_{abcd}R^{abcd} = \frac{48M^2}{r^6} + \frac{16\Lambda M(w_2+1)(2w_2+1)}{r^{2w_2+5}} + \frac{4\Lambda^2\left(4w_2^4 + 4w_2^3 + 5w_2^2 + 1\right)}{r^{4w_2+4}}.$$

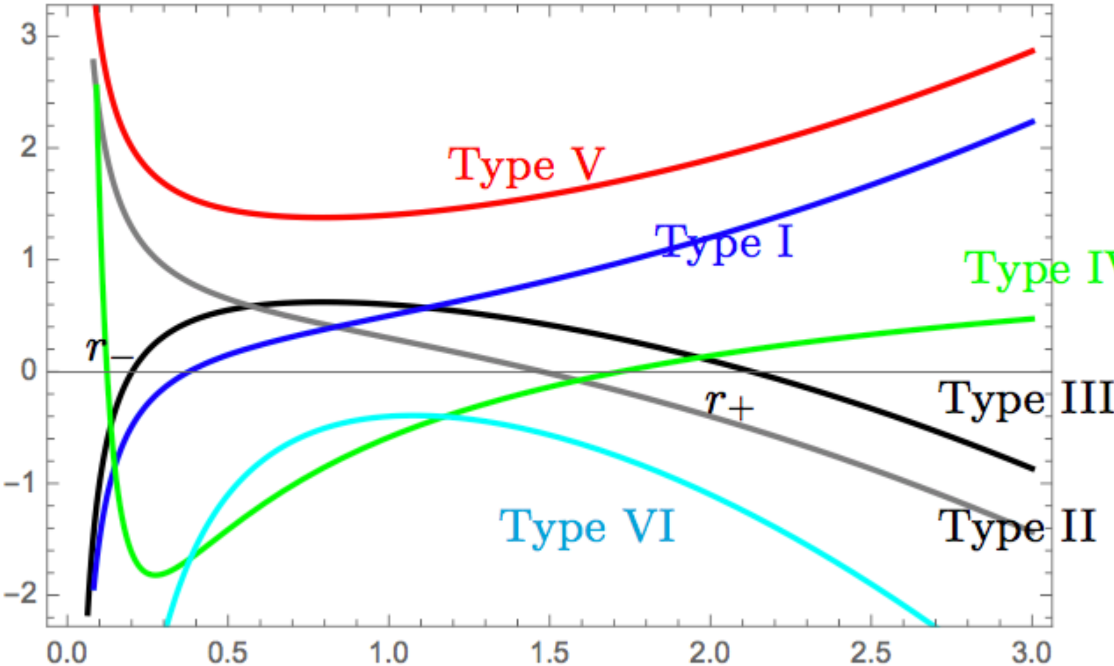
is nonsingular at the origin if $(M = 0 \text{ and } w_2 \leq -1)$.

$\boxed{\text{If } w_2 < -1,\ r \to \infty \text{ is singular.}}$

Only if $(M = 0 \text{ and } w_2 = -1)$. the solution is nonsingular both for the origin and asymptotic region.

→ This is nothing but the vacuum (anti)-de Sitter solution.

Classification of solutions:



Type I: Schwarzschild-like

Type II: de Sitter-like

Type III: Schwarzschild-de Sitter-like

Type IV: 2-horizon blackhole

Type V: Static Naked solution

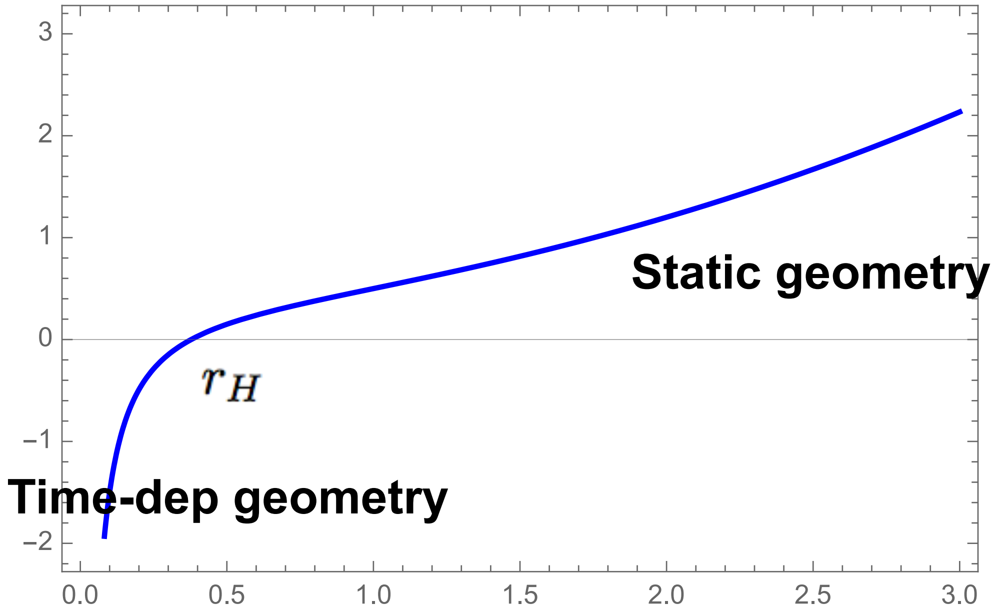
Type VI: Time-dependent Singular

FIG. 1: Typical form of $f(r)$ for type I to VI.

One of the two quantities ρ and p_1 must be negative ($w_1 < 0$).
Therefore, the sign of energy density changes when one crosses an event horizon because t and r exchange their roles as a time and space coordinates.

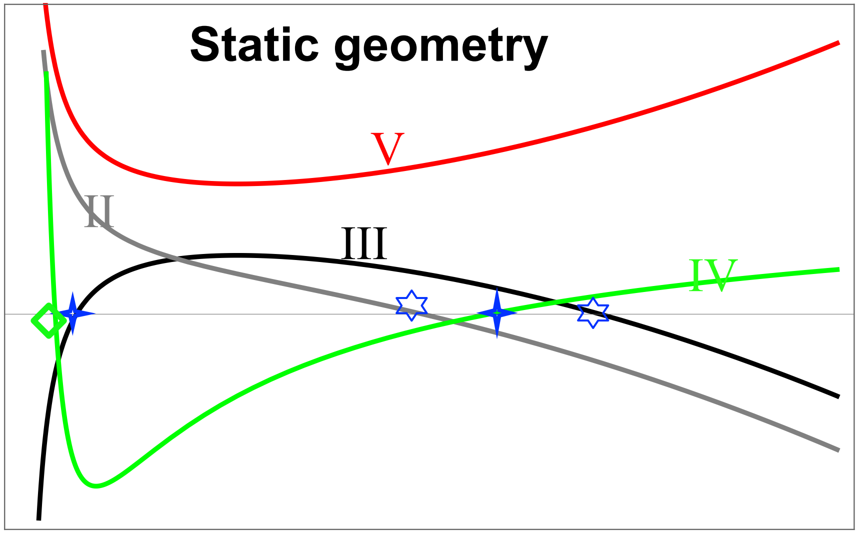
In the presence of a horizon, no metric has a positive definite energy density!

Type having both the static and time-dependent geometries :



- Type I, Modified Schwarzschild geometry.
  - Time-dependent geometry
    $(w_2 > 1/2, \Lambda > 0)$, $(w_2 = 1/2,\ \Lambda < 0)$, $(w_2 \leq 0, M > 0, \Lambda < 0)$
  - Static geometry
    $(0 < w_2 < 1/2, M > 0)$, $(w_2 = 0, M > 0, \Lambda < 1)$.

Type having only the static geometries:



The region with positive $f(r)$ is physical.

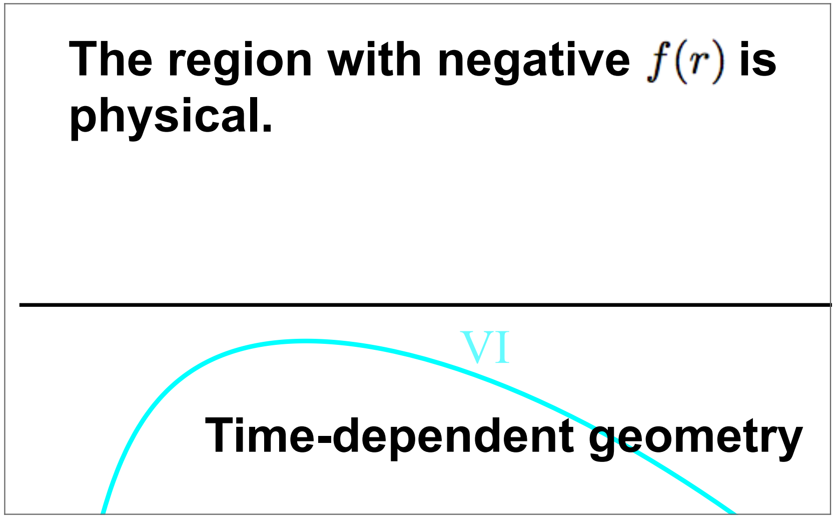
: **Blackhole horizon**

: **Cosmological horizon**

: **inner horizon**

- Type II, Modified de-Sitter geometry.
  $(w_2 = 0, \Lambda > 1, M < 0)_s$, $(w_2 < 0, \Lambda > 0, M > 0)_s$.
- Type III, Modified Schwarzschild-de Sitter geometry.
  $(w_2 < 0, \Lambda > 0,\ M \geq \frac{|w_2|}{|1-2w_2|^{1/|2w_2|-1}} \Lambda^{-1/|2w_2|})_s$.
- Type IV, Blackhole geometry having two horizons.
  $(w_2 > 1/2, \Lambda < 0,\ M \geq w_2/(2w_2 - 1)^{1-1/2w_2} \times |\Lambda|^{1/2w_2})_s$, $(w_2 = 1/2, M > \Lambda/2 > 0)_s$, $(w_2 = 1/2, M = \Lambda/2, \Lambda \geq e)_s$, $(0 < w_2 < 1/2, \Lambda > 0, -w_2(1-2w_2)^{\frac{1-2w_2}{2w_2}} \Lambda^{1/(2w_2)} \leq M < 0)_s$.
- Type V, Static solutions with naked singularities.
  $(w_2 > 1/2, \Lambda < 0, M < \frac{w_2}{(2w_2-1)^{1-1/2w_2}} |\Lambda|^{1/2w_2})_s$, $(w_2 = 1/2, M < \Lambda/2)_s$, $(w_2 = 1/2, M = \Lambda/2, 0 < \Lambda < e)_s$, $(0 < w_2 < 1/2, \Lambda > 0, -w_2(1-2w_2)^{\frac{1-2w_2}{2w_2}} \Lambda^{1/(2w_2)} \leq M < 0)_s$, $(w_2 = 0, M < 0, \Lambda \leq 1)_s$, $(w_2 < 0, M \leq 0, \Lambda < 0)_s$.

Type having only time-dependent geometries:



- Type VI, Time dependent solution having initial or future singularity without horizon. $(w_2 = 0, \Lambda > 1, M > 0)_t$, $(w_2 < 0, \Lambda > 0,\ 0 < M < \frac{|w_2|}{|1-2w_2|^{1/|2w_2|-1}} \Lambda^{-1/|2w_2|})_t$.

Of all, only Type I, III, IV include blackhole solutions.

Type I

Static geometry
$(0 < w_2 < 1/2, M > 0)$, $(w_2 = 0, M > 0, \Lambda < 1)$.

$$f(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{r^{2w_2}},$$

r ~ 0 is governed by (-2M/r) term. (singular)
Asymptotically flat: f(r) → 1.

The energy density for $r > r_H$ is positive definite only if $0 \leq w_2 < 1/2$

Surface gravity: $\kappa = f'(r_H)/2.$

If $w_2 = 0$, **the metric becomes,**

$$ds^2 = -\left(1 - \frac{2M'}{y}\right) d\tau^2 + \frac{dy^2}{1 - 2M'/y} + (\alpha y)^2 (d\theta^2 + \sin^2\theta d\phi^2),$$

$M' = M/\alpha^3.$ $\quad y = r/\alpha \quad \alpha = \sqrt{1-\Lambda},$

Stress energy tensor:

$$T^{\mu\nu} = (\rho + p_2)u^\mu u^\nu + \boxed{(p_1 - p_2)x^a x^b} + p_2 g^{\mu\nu},$$

$$u^\mu = [e^{-\nu/2}\sqrt{1+v^2}, e^{-\lambda/2}v, 0, 0], \qquad x^\nu = [e^{-\nu/2}v, e^{-\lambda/2}\sqrt{1+v^2}, 0, 0].$$

radial unit normal vector

Gauge degree of freedom:

Linearized transform: $t = \tilde{t} + \delta t(\tilde{t}, \tilde{r}), \qquad r = \tilde{r} + \delta r(\tilde{t}, \tilde{r}),$

$$\begin{aligned} ds^2 &= -e^{\nu_0(r)}(1+\delta\nu)dt^2 + e^{\lambda_0(r)}(1+\delta\lambda)dr^2 + e^{\mu_0(r)}(1+\delta\mu)d\Omega^2 \\ &= -e^{\nu_0(\tilde{r})}[1 + \delta\nu + \nu_0'\delta r + \dot{\nu}_0\delta t + 2\dot{\delta t}]d\tilde{t}^2 + 2[e^{\lambda_0}\dot{\delta r} - e^{\nu_0}\delta t']d\tilde{t}d\tilde{r} \\ &\quad + e^{\lambda_0(\tilde{r})}[1 + \delta\lambda + \lambda_0'\delta r + \dot{\lambda}_0\delta t + 2\delta r']d\tilde{r}^2 + e^{\mu_0(\tilde{r})}\left[1 + \delta\mu + \mu_0'\delta r + \dot{\mu}_0\delta t\right]d\Omega^2 \end{aligned}$$

Gauge choice: $\tilde{g}_{0i} = 0 \quad \Rightarrow \quad e^{\lambda_0}\dot{\delta r} = e^{\nu_0}\delta t', \qquad \delta\tilde{\mu} \equiv \delta\mu + \mu_0'\delta r = 0.$

Omitting ~ in $\tilde{r}$ and $\tilde{t}$, this lead to the metric ansatz:

$$\boxed{ds^2 = -e^{\nu_0}(1+\delta\nu)dt^2 + e^{\lambda_0}(1+\delta\lambda)dr^2 + e^{\mu_0}d\Omega^2}$$

First order Einstein equation:

$$\delta\nu' = w_1\delta\lambda' + \Big(\frac{1+3w_1}{2} + 2w_1\frac{\mu_0''}{\mu_0'^2}\Big)\mu_0'\,\delta\lambda.$$

$$\begin{aligned}\delta\ddot{\lambda} \;=\; & f^2(r)\Big\{w_1\delta\lambda'' + \Big(2w_1\big(1+\frac{\mu_0''}{\mu_0'^2}\big) - w_2\Big)\mu_0'\delta\lambda' \\ & + \Big[-\frac{1}{2} - \frac{\mu_0''}{\mu_0'^2} + \Big(\frac{3}{2} + \frac{5\mu_0''}{\mu_0'^2} + \frac{4\mu_0'''}{\mu_0'^3} - \frac{4\mu_0''^2}{\mu_0'^4}\Big)w_1 - \Big(3+\frac{4\mu_0''}{\mu_0'^2}\Big)w_2\Big]\frac{\mu_0'^2}{2}\delta\lambda\Big\}\end{aligned}$$

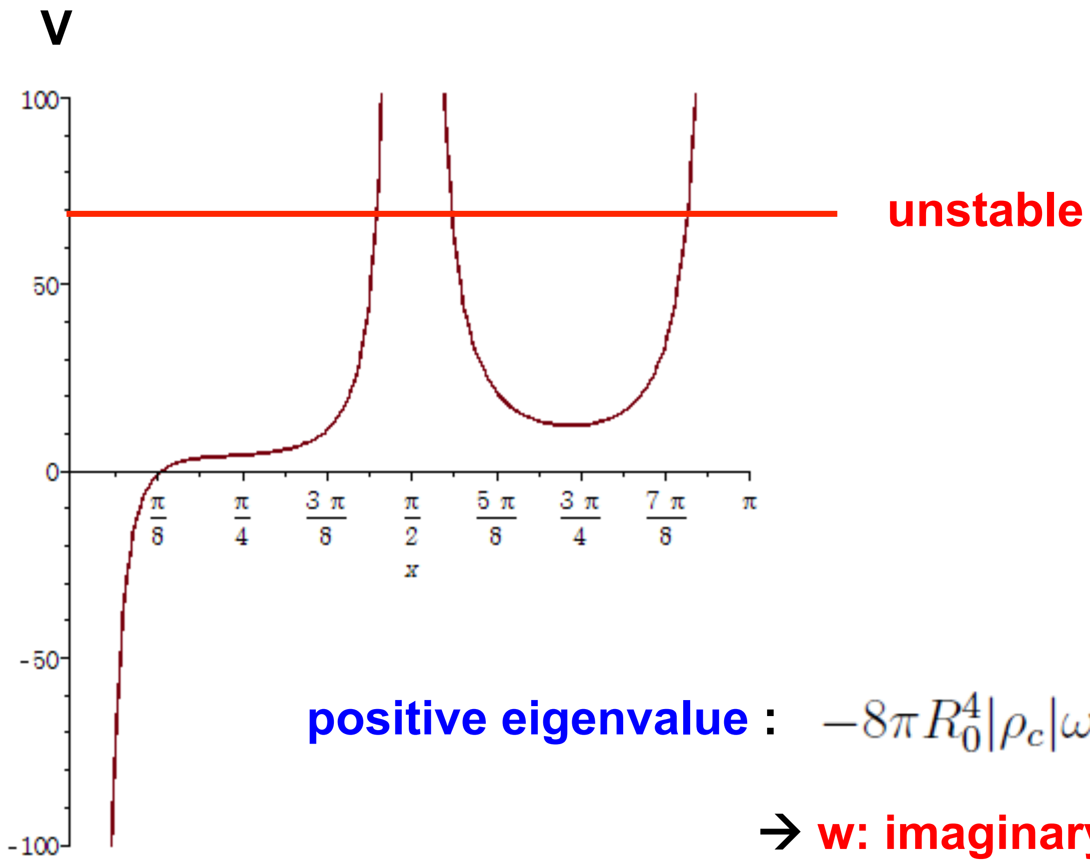
Put $\mu_0(r) = 2\log r,$

Let us set $\delta\lambda = e^{-i\omega t}g_1(r).$

$$-\omega^2 g_1(r) = f^2(r)\left(w_1 g_1''(r) + \frac{2(w_1-w_2)}{r}g_1'(r) - \frac{2w_2}{r^2}g_1(r)\right).$$

Negative sign and negative w_1 implies that the system is unconditionally unstable.

S3-I



positive eigenvalue : $-8\pi R_0^4|\rho_c|\omega^2 > 0$

→ **w: imaginary**

→ **UNSTABLE!!**

$$\left[-\frac{1}{2}\frac{d^2}{dz^2}-\frac{1}{z}\frac{d}{dz}+\boxed{V(z)}\right]\Phi(z)=\boxed{-\frac{\omega^2}{S}}\Phi(z)=\boxed{-8\pi R_0^4|\rho_c|\omega^2}\Phi(z),$$

eigenvalue

since this is negative, there exists always unstable modes for any type of V

Fate of Black Holes

1) BH may collapse: may leave some cosmological remnants

2) BH may remain while BACKGROUND expands
   :- since the b.g. matter is perfect fluid,
      the instability may imply the Friedmann expansion
   :- BH may sustain its nature in expanding b.g. Universe

Other type of metric perturbations

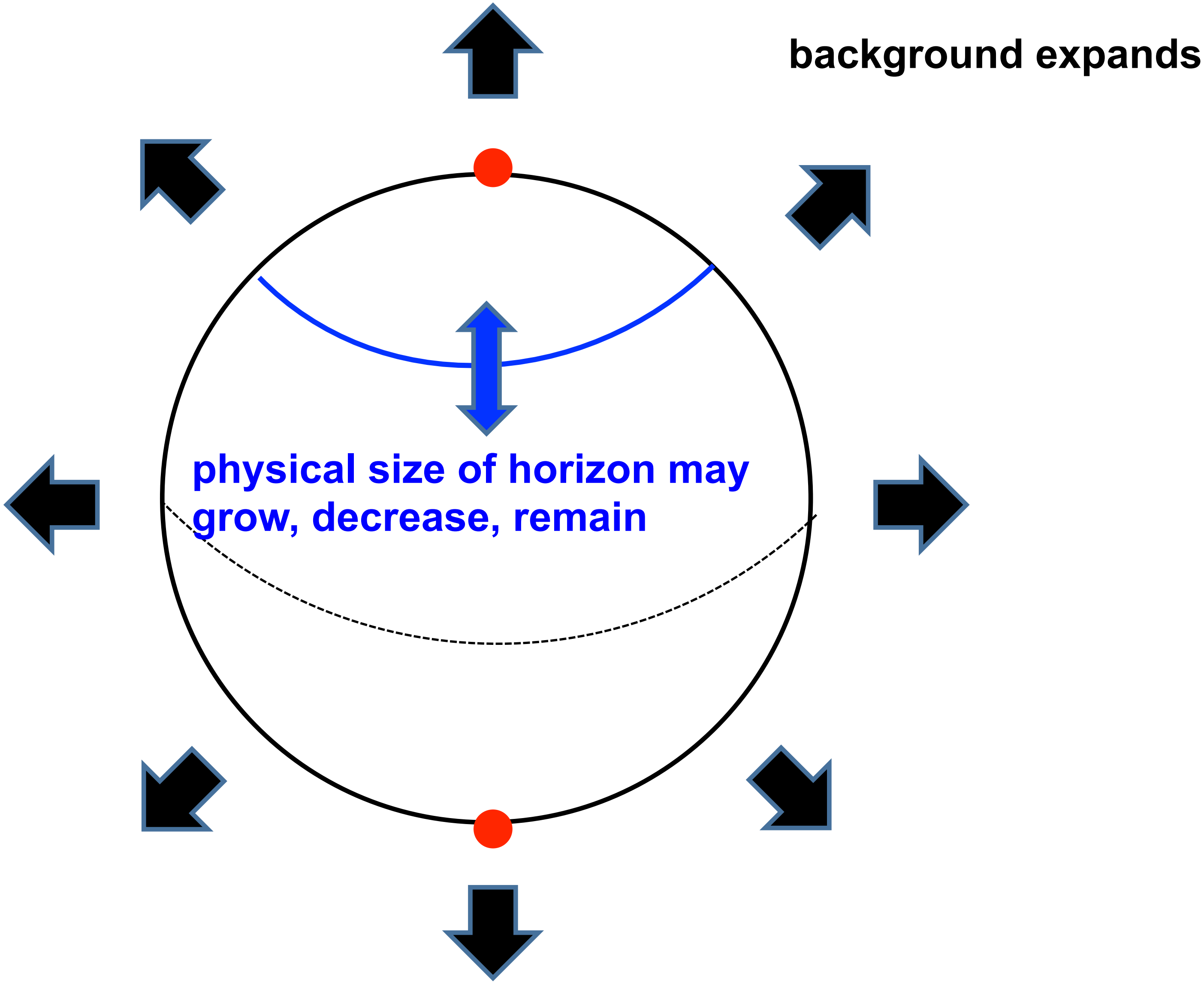
$$\boxed{S^2(t) \equiv 1 + \epsilon a(t)}$$: scale factor

$$\boxed{ds^2 = -f_0[\chi, \kappa(t)]dt^2 + S^2(t)\{g_0[\chi, \kappa(t)]d\chi^2 + R_0^2 b^2(\chi) d\Omega_2^2\}\,.}$$

$$\boxed{K \to K + \epsilon\kappa(t)}$$: horizon location

Solutions: $$\boxed{a(t) = a_1 t + a_0, \qquad \kappa(t) = \kappa_1 t + \kappa_0.}$$

:- Both are linear in “t”
:- Their evolutions are independent
:- Depending on I.C., the Direction of Evolution is determined
e.g.) impose $r_1=0$ at the horizon (S3-I black hole)
→ means NO energy-flow through the horizon: v=0
→ preserves “positivity of energy” density in both regions
→ Coordinate of horizon decreases
→ BUT, “physical size of horizon” is UNCHANGED !!!



background expands
physical size of horizon may grow, decrease, remain

General analysis:

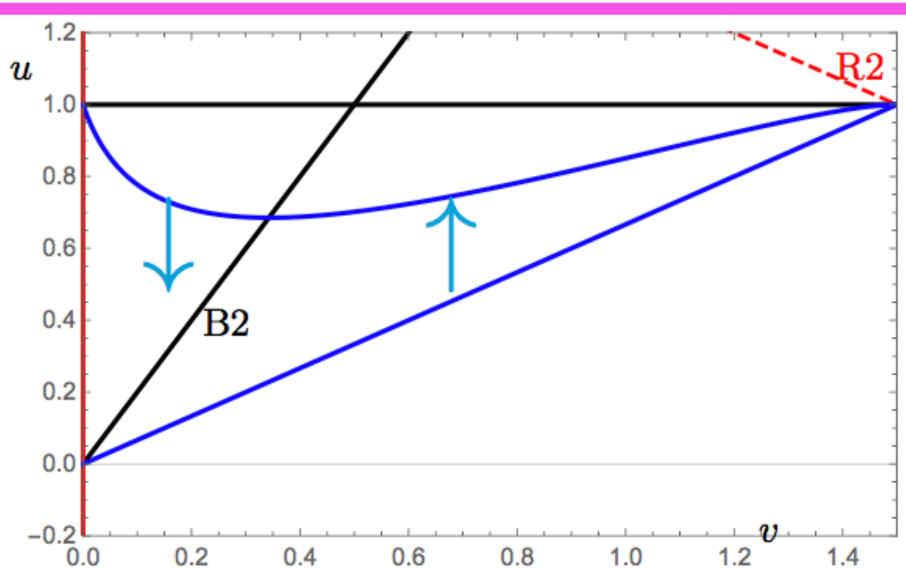
$$0 = G^2_2 = \frac{4r^2 w_1 M'}{r-2M}\left[\frac{M''}{M'} + \frac{1+w_1}{r-2M}\left(M' + \frac{1}{2w_1}\right) - \frac{1+w_1+4w_2}{2w_1 r}\right].$$

Introducing new variables:

$$u = 2M(r)/r \text{ and } \chi = \log(r/r_+),$$

$$v \equiv \frac{dM(r)}{dr} = 4\pi r^2 \rho(r) = (u+u')/2$$

$$\boxed{\frac{du}{dv} = \frac{2w_1(1-u)(2v-u)}{v\left[2w_1(w_1+1)\left(-v+(-2w_1)^{-1}\right)+(1+w_1+4w_2)(1-u)\right]}}.$$



Integral curves represents solutions.

Summary :

1. We have found spherically symmetric blackhole solutions with anisotropic fluids with $w = -1/3$ and $w_1 = -1$, w_2 .

2. All of the solutions appears to be unconditionally unstable.

3. The (unconditional) instability is due to the negativity of w_1.

1. Can we find more general anisotropic blackhole solutions?
   → Yes. At least numerically (the previous slide).

2. Can we find a stable generalization?
   → We should try.


감사합니다.