Bound of Radiation under the Coalesces of Kerr-(anti-)de Sitter Black Holes

Based on B. Gwak(Sejong University) and D. Ro(APCTP), "Spin Interaction under the Collision of Two Kerr-(anti-)de Sitter Black Holes," arXiv:1610.04847.

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Motivation

- Spin-spin interaction in the collision of BHs with positive cosmological constant (especially, particle case).
- Using thermodynamics and MPD equation, relation between the potential of interaction and radiation of the BH collision.
- In BH-particle system, the all of potential energy released in the collision.
- Approximately, radiation may depend on the instability of the black hole.

K(A)dS black hole

- 4-dimensional BH with a positive(negative) cosmological constant.
- Rotating BH with angular momentum J.

$$ds^2 = -\frac{\Delta_r}{\rho^2}\left(dt - \frac{a\sin^2\theta}{\Xi}d\phi\right)^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta \sin^2\theta}{\rho^2}\left(a\,dt - \frac{r^2+a^2}{\Xi}d\phi\right)^2 ,$$

$$\rho^2 = r^2 + a^2\cos^2\theta\,,\ \ \Delta_r = (r^2+a^2)(1-\frac{1}{3}\Lambda r^2) - 2mr\,,\ \ \Delta_\theta = 1 + \frac{1}{3}\Lambda a^2\cos^2\theta\,,\ \ \Xi = 1 + \frac{1}{3}\Lambda a^2\,,$$

- Mass: $M_B = \frac{m}{\Xi^2}$ angular momentum: $J_B = \frac{ma}{\Xi^2}$

- Temperature:

$$T_H = \frac{r_h\left(1 - \frac{\Lambda a^2}{3} - \frac{a^2}{r_h^2} - \Lambda r_h^2\right)}{4\pi(r_h^2 + a^2)}$$

- Bekenstein-Hawking Entropy:

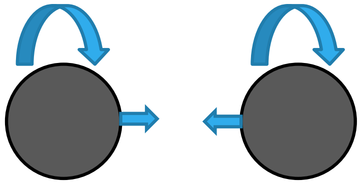
$$S_{BH} = \frac{\pi(r_h^2 + a^2)}{\Xi}$$

- Angular velocity at the outer horizon:

$$\Omega = \Omega_h - \Omega_\infty = \frac{a\Xi}{r_h^2 + a^2} - \frac{\Lambda a}{3} = \frac{a\left(1 - \frac{\Lambda}{3} r_h^2\right)}{r_h^2 + a^2}$$

The collision of two K(A)dS BHs in thermodynamics

- Consider two BHs(M_1, J_1 and M_2, J_2) and $M_2 \ll M_1, J_2 \ll M_1^2$.
- In this particle limit, the interaction is negligible in the initial state.
- Their rotation axes aligned parallel or anti-parallel to their direction of approach
- Low-energy collision.

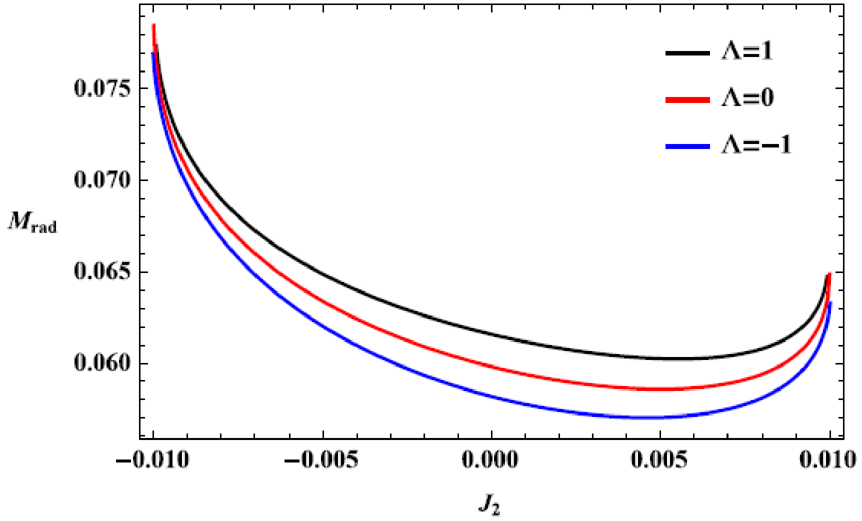


- If two black holes coalesce to form a final one, the final black hole is the same K(A)dS M_f, J_f.

- The final state is related to the initial state by the inequality (the increase of the area): $S_{BH}(M_1, J_1) + S_{BH}(M_2, J_2) \leq S_{BH}(M_f, J_f)$.
- Maximum radiation is at the equality.
- Angular momentum conservation: $J_f = J_1 + J_2$.
- The energy of the radiation is obtained from the energy conservation: $M_{rad} = (M_1 + M_2) - M_f$.

Radiation in the collision

- The upper bound with respect to J_2: $\frac{\partial M_{rad}}{\partial J_2} = -\frac{2m_1 a_1 r_1}{\left(r_1^2+a_1^2\right)^2} + O(J_2)$.
- The alignment of rotating axes:



- Integrate out: This is the energy released by the radiation (it will be approximately spin-interaction potential).
- $U_s = \frac{\left(1-\frac{1}{3}\Lambda r_1^2\right)^2 \Xi_1^2}{2m_1^2 r_1} J_1 J_2$ (>0, parallel rotation, repulsion) or (<0, anti-parallel rotation, attraction).

Spin interaction from Mathisson-Papapetrou-Dixon (MPD) equation

- Spinning particle in the K(A)dS BH (M_1, J_1).
- MPD eqs: $\frac{Dp^\mu}{Ds} = -\frac{1}{2} R^\mu_{\nu\rho\sigma} u^\nu S^{\rho\sigma}$, $\frac{DS^{\mu\nu}}{Ds} = p^\mu u^\nu - p^\nu u^\mu$ $\quad p_\mu S^{\mu\nu} = 0$.
- Particle: $S^2 = \frac{1}{2} S_{\mu\nu} S^{\mu\nu}$, $\mu^2 = -p_\mu p^\mu$.
- Pole to Pole: $v^\mu = \left(\frac{1}{\sqrt{-g_{tt}}}, v^r, 0, 0 \right)$, $\quad$ and $\quad S^\mu = \left(0, \frac{1}{\sqrt{g_{rr}}} S, 0, 0 \right)$,

- The conserved quantities: $C_\xi = p_\mu \xi^\mu + \frac{1}{2} S^{\mu\nu} \nabla_\mu \xi_\nu$
- Energy from ξ_t^μ: $E = -p_t - \frac{1}{2} S^{\mu\nu} \nabla_\mu g_{\nu t}$
- The energy of the particle: $E_s = \frac{2m_1 a_1 r_1}{(r_1^2 + a_1^2)^2} S + \frac{a\Lambda}{3} S$
- The second term is from the rotating boundary of the spacetime:

$$E_0 = \frac{a\Lambda}{3} L \qquad E = E_h - E_0 = \frac{a\left(1 - \frac{\Lambda}{3} r_h^2\right)}{r_h^2 + a^2} L + \frac{r_h Q}{r_h^2 + a^2} e + \frac{\rho_h^2}{r_h^2 + a^2} |p^r|$$

$$\delta M_B = T_H \delta S_{BH} + (\Omega_h - \Omega_0) \delta J_B + \Phi_H \delta Q_B$$

- Then, the potential of the spin-spin interaction is

$$U_s = \frac{\left(1-\frac{1}{3}\Lambda r_1^2\right)^2 \Xi_1^2}{2m_1^2 r_1} J_1 S$$

- The potential is released by the radiation.
- The potential U_s (>o, parallel rotation, repulsion) or (<o, anti-parallel rotation, attraction).

Upper bounds of radiation in the collision of two black holes

- As extrapolation, we just investigate the radiation by the same method.
- In dimensionless coordinates and parameters:

$$\tilde{s} = \frac{s}{\sqrt{\lambda/\Lambda}}\,, \quad \tilde{t} = \frac{t}{\sqrt{\lambda/\Lambda}}\,, \quad \tilde{r} = \frac{r}{\sqrt{\lambda/\Lambda}}\,, \quad \tilde{M} = \frac{M}{\sqrt{\lambda/\Lambda}}\,, \quad \tilde{a} = \frac{a}{\sqrt{\lambda/\Lambda}}\,,$$

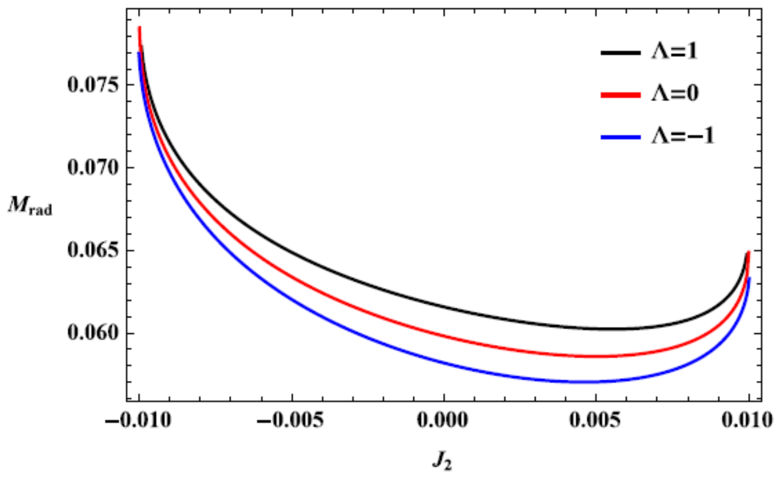
$$\tilde{\rho}^2 = \tilde{r}^2 + \tilde{a}^2\cos^2\theta\,, \;\; \tilde{\Delta}_{\tilde{r}} = (\tilde{r}^2 + \tilde{a}^2)(1 - \frac{1}{3}\lambda\tilde{r}^2) - 2\tilde{m}\tilde{r}\,, \;\; \tilde{\Delta}_\theta = 1 + \frac{1}{3}\lambda\tilde{a}^2\cos^2\theta\,, \;\; \tilde{\Xi} = 1 + \frac{1}{3}\lambda\tilde{a}^2\,,$$

- Many limits such as boundary, reflection, and probability.

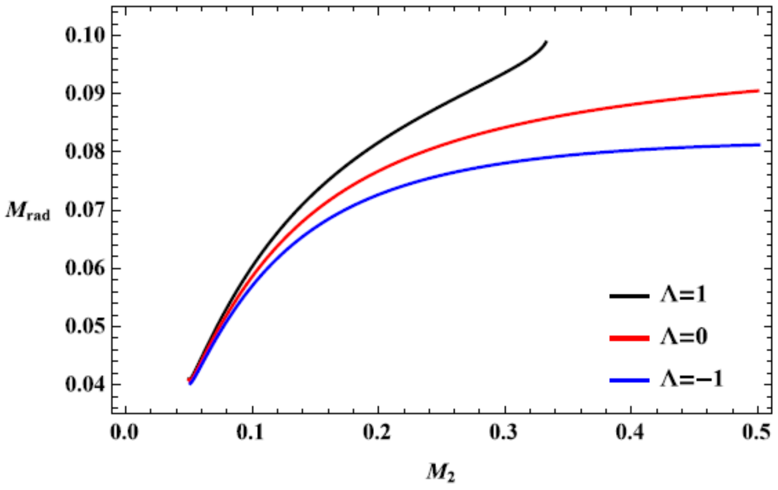
Examples

- The Schwarzchild BHs with $M_1 = M_2$: $\frac{M_{rad}}{M_1+M_2}$=29%.
- The actual radiation is much smaller than results.
- High-energy collision (different method): 19~35%
- LIGO data: GW150914, GW151226 ~ 5.0%

- For different cosmological constant,

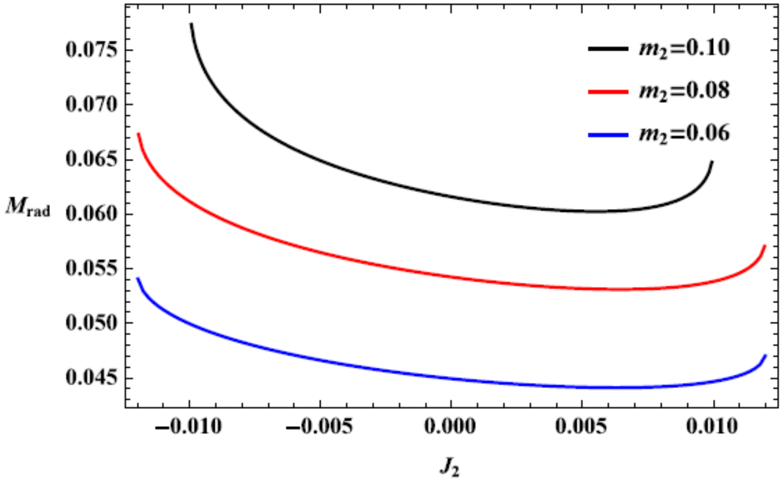


(a) The bounds on the radiation with respect to the angular momentum of the second black hole. The black holes are $m_1 = 0.1$, $a_1 = 0.05$, and $m_2 = 0.1$.

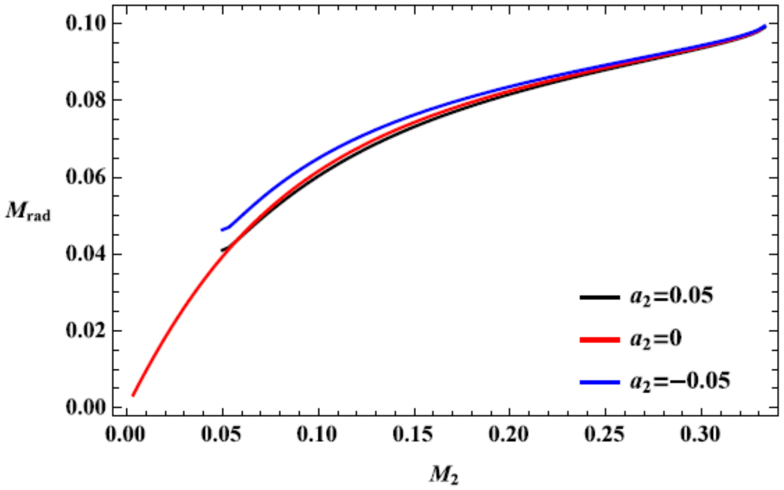


(b) The bounds on the radiation with respect to the mass of the second black hole. The black holes are $m_1 = 0.1$, $a_1 = 0.05$, and $a_2 = 0.05$.

- For positive cosmological constant,



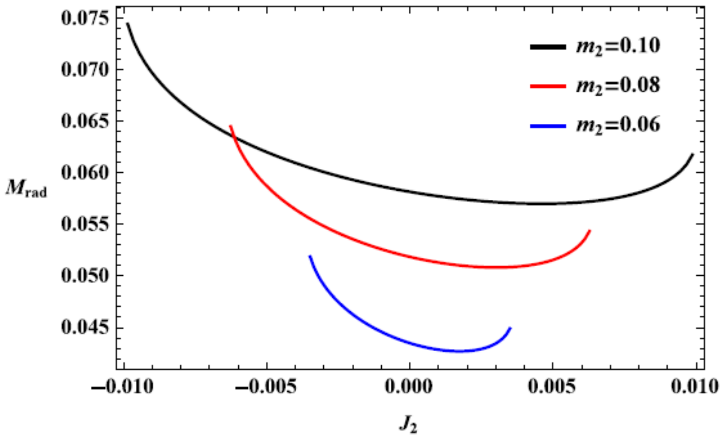
(a) The bounds on the radiation with respect to the angular momentum of the second black hole. The black holes are $m_1 = 0.1$ and $a_1 = 0.05$ with the positive cosmological constant.



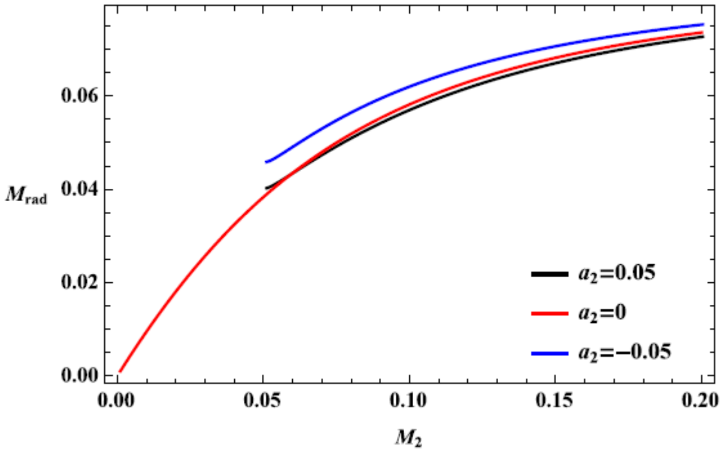
(b) The bounds on the radiation with respect to the mass of the second black hole. The black holes are $m_1 = 0.1$ and $a_1 = 0.05$ with the positive cosmological constant.

Small AdS BHs

- For negative cosmological constant,

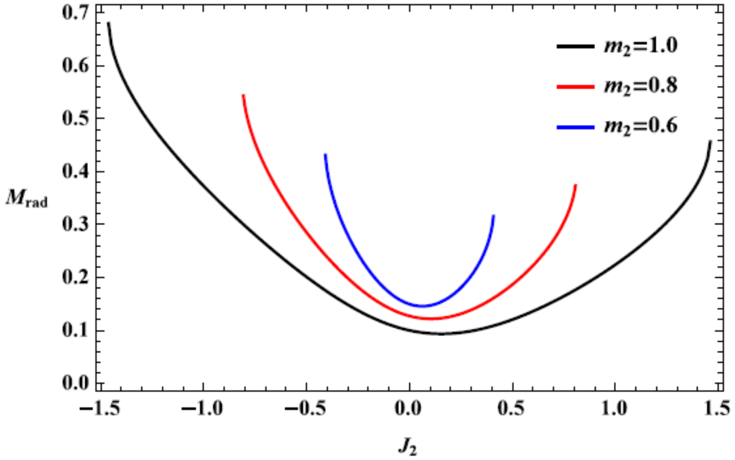


(a) The bounds on the radiation with respect to the angular momentum of the second black hole. The black holes are $m1 = 0.1$ and $a1 = 0.05$ with the negative cosmological constant.

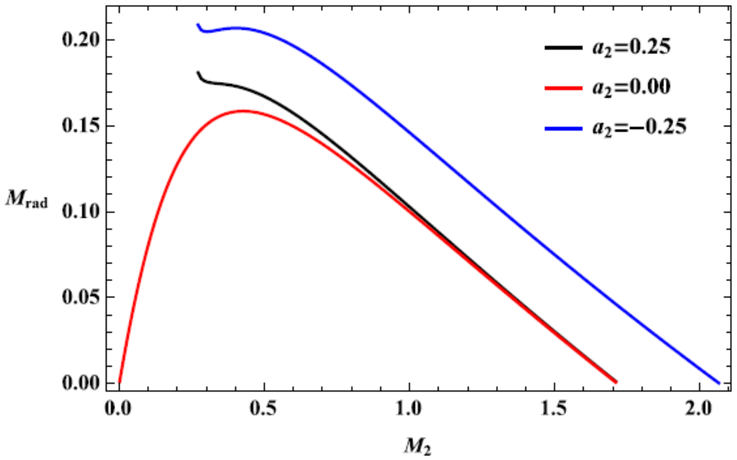


(b) The bounds on the radiation with respect to the mass of the second black hole. The black holes are $m1 = 0.1$ and $a1 = 0.05$ with the negative cosmological constant.

Large AdS BHs



(a) The bounds on the radiation with respect to the angular momentum of the second black hole. The black holes are $m1 = 1.0$ and $a1 = 0.25$ with the negative cosmological constant.



(b) The bounds on the radiation with respect to the mass of the second black hole. The black holes are $m1 = 1.0$ and $a1 = 0.25$ with the negative cosmological constant.

$$M_{rad} = (M_1 + M_2) - M_f = (2 - 2\sqrt{2})m < 0\,.$$

Summary

- Thermodynamics and MDP equations
- The spin-spin interaction: attractive or repulsive with cosmological constant.
- The potential of the interaction can be radiated in the collision of the black holes
- The radiation can be related to the instability of the black holes.

THANK YOU!