Initial Data for an Perturbed AdS Black String

2016-11-18
APCTP Workshop on Gravity and Cosmology

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Gregory-Laflamme instability

- certain black strings and branes are unstable in dimensions higher than four

Final state of GL instability

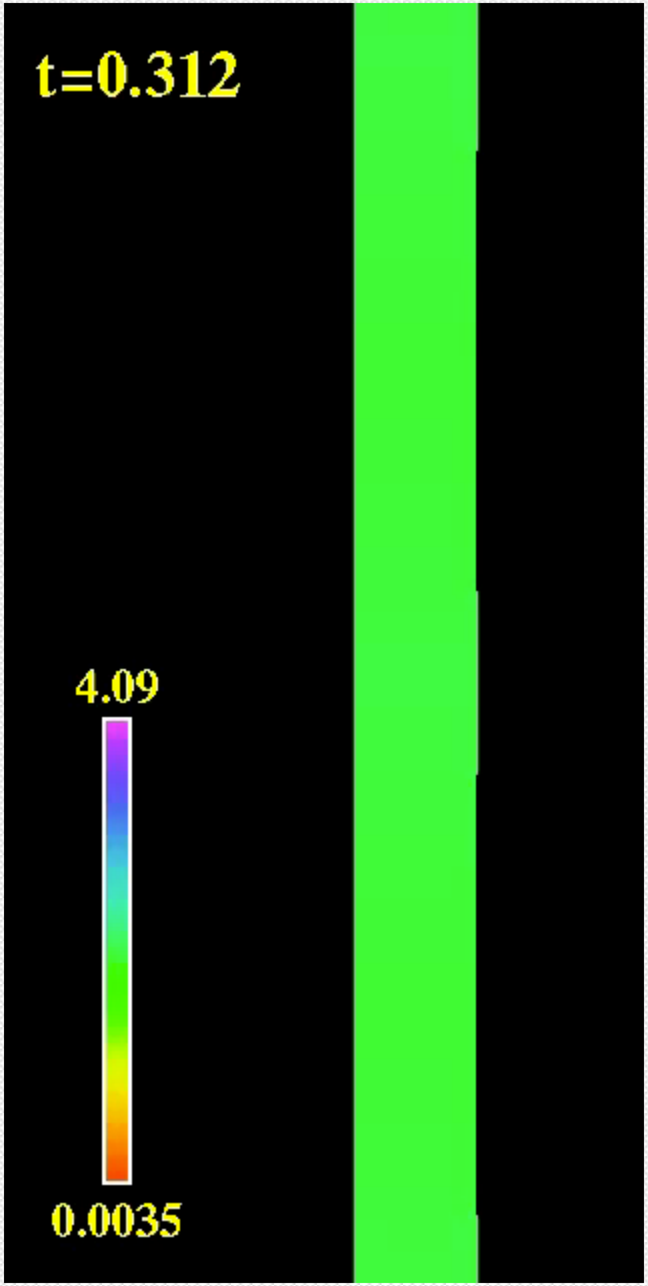
- a perturbed unstable 5D black string
- a numerical relativity simulation
- $R \times S^2$ event horizon topology

$$ds^2 = -\left(1 - \frac{r_0}{r}\right) dt^2 + \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right) + dw^2$$

$$r^2 d\Omega \to \gamma_\Omega r^2 d\Omega$$

$$\gamma_\Omega = 1 + A \sin\left(w \frac{2\pi q}{L} \right) e^{-(r-r_o)^2/\delta_r^2} \, .$$

- harmonic coordinates
  (gauge condition)
- constraint damping terms
- cartoon method
- a fractal structure of hyper-spherical black holes



Stable Black Strings in AdS Space

- AdS black strings
- linearized perturbation analysis
- existence of unstable mode

$$ds^2 = H^{-2}(z)\Big[-f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_2^2 + dz^2\Big],$$

$$f(r) = 1 - \frac{r_0}{r} - \frac{\Lambda_4}{3} r^2.$$

$$\begin{aligned} dS_4(\Lambda_4 > 0): &\qquad H(z) = l_4/l_5 \sinh z/l_4 \\ M_4(\Lambda_4 = 0): &\qquad H(z) = z/l_5 \\ AdS_4(\Lambda_4 < 0): &\qquad H(z) = l_4/l_5 \sin z/l_4, \end{aligned}$$

The AdS Black String Non-perturbative analysis: Numerical Relativity Simulation

Initial Data

- non singular coordinate choice
- hamiltonian and momentum constraints
- iterative methods to find solutions

Evolution

- harmonic gauge conditions
- converting 4+1 variables into metric variables
- evolution equations and code
- apparent horizon finder

The AdS Black String Non-perturbative analysis: Numerical Relativity Simulation

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Non singular coordinate choice

- horizon penetrating coordinates
- Eddington-Finkelstein coordinates in AdS space

$$ds^2 = H^{-2}(z)\Big[-f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_2^2 + dz^2\Big],$$

$$f(r) = 1 - \frac{r_0}{r} + \left(\frac{r}{l_4}\right)^2 \qquad g(r) = \left(\frac{l_4}{r} + \left(\frac{r}{l_4}\right)^2\right)^{-1} \qquad d\tilde{t} = dt + \left(\frac{1}{f(r)} - g(r)\right) dr$$

$$-f dt^2 + \frac{1}{f}dr^2 = -f d\tilde{t}^2 + 2(1 - fg)d\tilde{t}dr + g(2 - fg)dr^2$$

$$\tilde{t} = constant \rightarrow hypersurface \qquad \tilde{t} \rightarrow t, \; as \; r \rightarrow \infty$$

Hamiltonian and momentum constraints

Cauchy $(4+1)$ decomposition of Einstein equations.

$(t=0)$ hypersurface

intrinsic metric (γ_{ij}) and extrinsic curvature (K_{ij})

$$H \equiv {}^{(4)}R + K^2 - K_{ij}K^{ij} = 0\,,$$

$$M_i \equiv D_j(K_i^j - \gamma_i^j K) = 0\,,$$

$$\gamma_\Omega = 1 + A\sin\left(w\frac{2\pi q}{L}\right)e^{-(r-r_o)^2/\delta_r^2}\,.$$

$$F_1\,\partial_r\gamma_{rr} + F_2\,\gamma_{rr}\partial_{ww}\gamma_{rr} + F_3\,\gamma_{rr}\partial_w\gamma_{rr} + F_4\,(\partial_w\gamma_{rr})^2 + F_5\,(\gamma_{rr})^2 + F_6\gamma_{rr} = 0,$$

$$G_1\partial_r K_{\theta\theta} + G_2 K_{\theta\theta} + G_3 = 0\,,$$

$$H_1\partial_w K_{rr} + H_2 K_{rr} + H_3 = 0\,,$$

Hamiltonian and momentum constraints

$$F_1\,\partial_r\gamma_{rr} + F_2\,\gamma_{rr}\partial_{ww}\gamma_{rr} + F_3\,\gamma_{rr}\partial_w\gamma_{rr} + F_4\,(\partial_w\gamma_{rr})^2 + F_5\,(\gamma_{rr})^2 + F_6\gamma_{rr} = 0,$$

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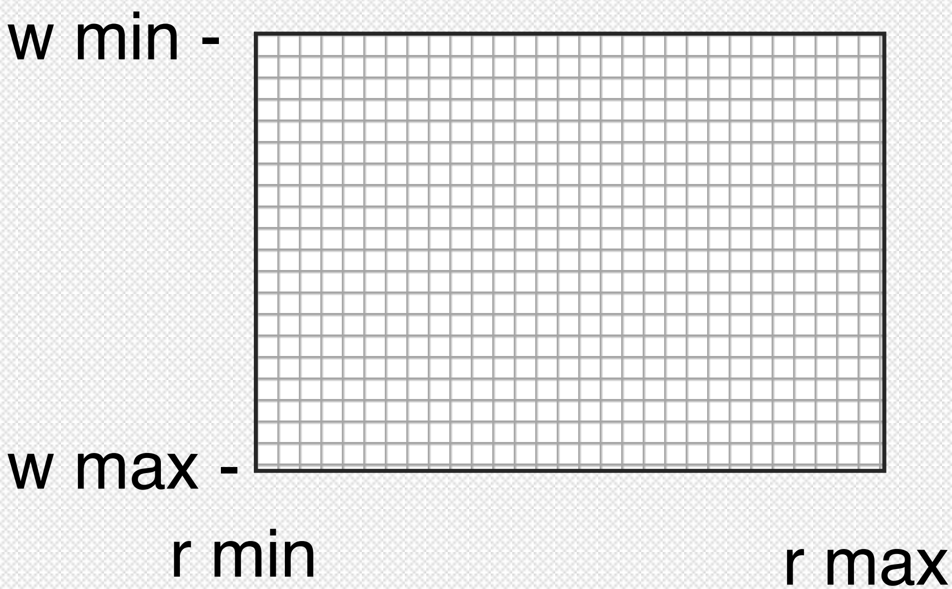
A unperturbed solution as a boundary condition



Iterative methods to find solutions

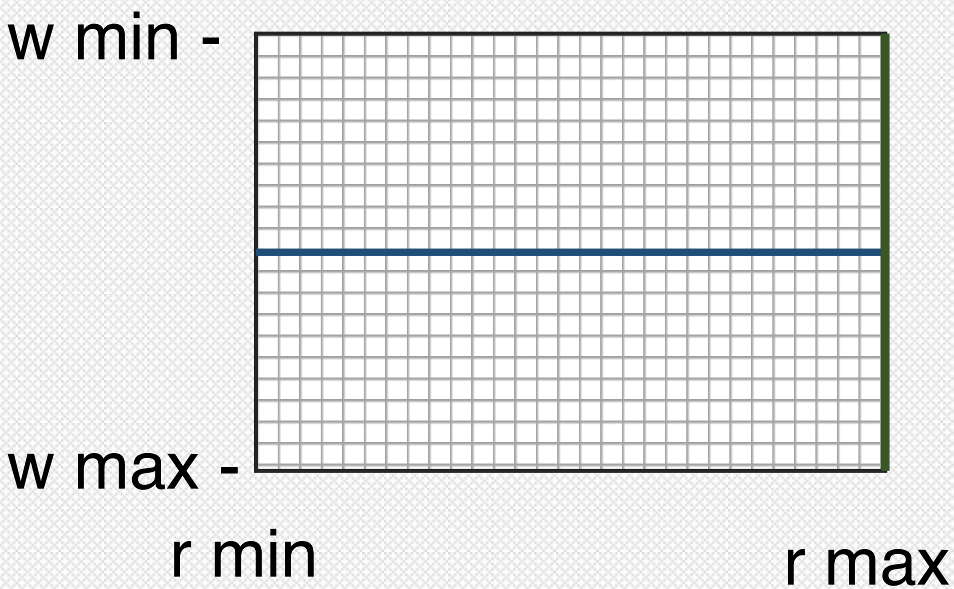
$$G_1 \partial_r K_{\theta\theta} + G_2 K_{\theta\theta} + G_3 = 0\,,$$

$$H_1 \partial_w K_{rr} + H_2 K_{rr} + H_3 = 0\,,$$

Integrated along each lines

$K_{\theta\theta}$: w = constant lines , K_{rr} : r = constant lines

second-order finite differences



Boundary condition

$K_{\theta\theta}$: Green

K_{rr} : Blue

Iterative methods to find solutions

$$F_1\,\partial_r\gamma_{rr} + F_2\,\gamma_{rr}\partial_{ww}\gamma_{rr} + F_3\,\gamma_{rr}\partial_w\gamma_{rr} + F_4\,(\partial_w\gamma_{rr})^2 + F_5\,(\gamma_{rr})^2 + F_6\gamma_{rr} = 0,$$

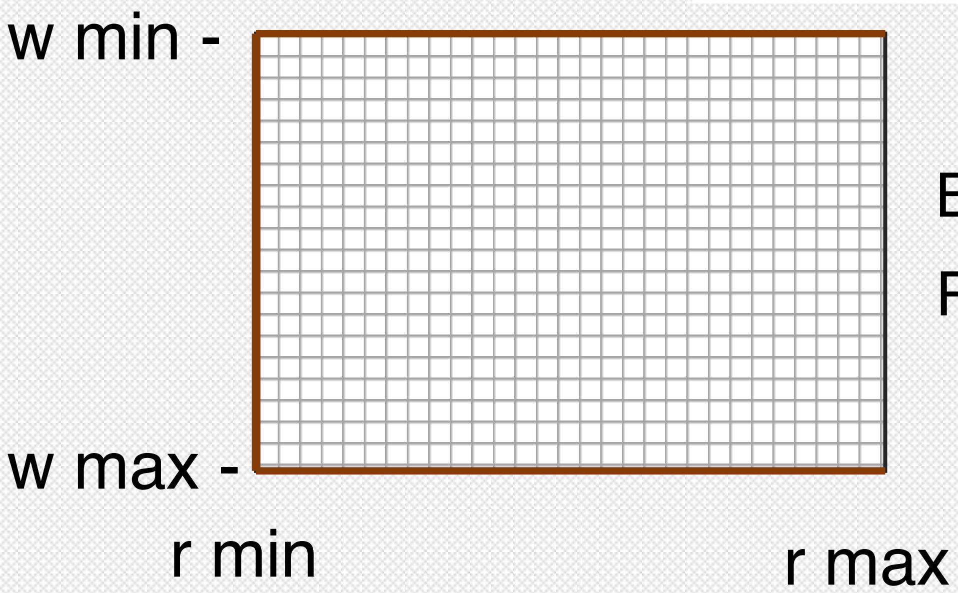
Relaxation scheme for each constant r layers

Nonlinear Gauss-Seidel

shcemes (Newton iteration)

$$L_i(u_0, \dots, u_{N-1}) = f_i, \qquad i = 0, \dots, N-1$$

$$u_i^{\text{new}} = u_i^{\text{old}} - \frac{L_i(u_i^{\text{old}}) - f_i}{\partial L_i(u_i^{\text{old}})/\partial u_i}$$



Boundary condition

Red lines

References

GL instability
arxiv.org/abs/1107.5821

Final state of GL instability
arxiv.org/abs/1106.5184

Stable Black Strings in AdS Space
T. Hirayama and G. Kang, Phys. Rev. D64, 064010 (2001)
https://arxiv.org/pdf/hep-th/0104213v2.pdf

$$\Delta_L h_{\mu\nu}(x) \equiv \Box h_{\mu\nu}(x) + 2R_{\mu\rho\nu\tau}h^{\rho\tau}(x) = m^2 h_{\mu\nu}(x),$$

$$[-\partial_z^2 + V(z)]\xi(z) = m^2\xi(z), \quad V(z) = -\frac{3}{2}\frac{H''}{H} + \frac{15}{4}\Big(\frac{H'}{H}\Big)^2.$$

- 논문결과물