Thick Brane World

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- Multidimensional theory

1. Why are the **physically observed** dimensions of our Universe $= 3 + 1$ (space + time)?
2. If the **real** dimensions are still more than four, why are extra dimensions not observable, and what about any consequences of multidimensionality for a four-dimensional observer?
3. Many problems of elementary particles physics could be solved by introducing strings and supersymmetry, and by **increasing the number of spacetime dimensions**.
4. In increasing the number of dimensions, a legitimate question arises: "Why are the extra dimensions unobservable in our world?"
5. Two solutions to this question were suggested, namely a mechanism of spontaneous compactification and brane world models.

- Spontaneous compactification

1. compactified extra dimensions, it is supposed that these dimensions are rather small and unobservable.

2. There are various mechanisms of spontaneous compactification:
   - Freund-Rubin compactification :
     $\rightarrow$ special ansatz for antisymmetric tensors
   - Englert compactification :
     $\rightarrow$ setting a gauge field in an internal space equal to the spin connection, suitable embedding in a gauge group
   - monopole or instanton mechanism compactification :
     $\rightarrow$ using scalar chiral fields
   - compactification using radiative corrections

- Brane world scenario

1. This approach is quite different from the traditional compactification approach and allows even **non-compact** extra dimensions.

2. It is supposed that our Universe is such a brane-like object.

3. particles corresponding to **electromagnetic**, **weak** and **strong** interactions are confined on some hypersurface (called a brane) which, in turn, is embedded in some multidimensional space (called a bulk).

4. Only **gravitation** and some exotic matter (e.g. the dilaton field) could propagate in the bulk.

Introduction

- Brane world scenario

1. The idea about **non-compactified extra dimensions** was suggested.
   - Akama K 1983 Pregeometry ed K Kikkawa et al(arXiv:hep-th/0001113) :
     $\rightarrow$ our four-dimensional world is the interior of a vortex
   - Rubakov V A and Shaposhnikov M E 1983 Phys. Lett. B 125 136–8 :
     $\rightarrow$ our four-dimensional world is a domain wall
2. In many works, it was assumed that the brane is infinitely **thin**.

- Thick Brane

1. It is widely considered that the most fundamental theory would have a **minimal length scale**.
2. From a realistic point of view, a brane should have a **thickness**.
3. The inclusion of brane thickness gives us new possibilities and new problems.
4. Definition: for 5D

$$ds^2 = a^2(y) g_{\mu\nu} dx^\mu dx^\nu + dy^2$$

where $a(y)$ is warp function which is regular, has a peak at the brane and falls off rapidly away from the brane.

Introduction

1. The normalizability of the graviton **zero mode** gives the condition that

$$\int_{-\infty}^{\infty} a(y)^4 dy = \text{finite}\,.$$

2. There is some arbitrariness in the definition of what the effective 4D quantities should be.

3. How to identify as a four-dimensional observable quantity?

4. The simplest prescription one can envisage is to define the 4D effective quantity associated to a 5D quantity as its spatial average over the brane thickness.

   - P. Mounaix and D. Langlois, Phys. Rev. D **65**, 103523 (2002)

Review : Thin Brane Model

1. The simplest action of the thin brane model is

$$S = \int d^5x\sqrt{-g_5}\left(\frac{1}{2}R - \Lambda_5\right) - \sigma\int d^4x\sqrt{-g}$$

where σ denotes the tension of the brane.

$$ds^2 = a^2(y)g_{\mu\nu}dx^\mu dx^\nu + dy^2$$

2. Solving the Einstein equations, we find

$$a(y) = e^{-|y|}, \quad (g_{\mu\nu} = \eta_{\mu\nu})$$

Review : **Thick** Brane Model

1. The simplest action of the **thick** brane model is

$$S = \int d^5x\sqrt{-g_5}\left(\frac{1}{2}R - \frac{1}{2}(\partial\varphi)^2 - V(\varphi)\right)$$

where φ is the five-dimensional scalar field, $\varphi = \varphi(z)$.

$$ds^2 = a^2(y)g_{\mu\nu}dx^\mu dx^\nu + dy^2$$

2. $g_{\mu\nu}$ denotes the metric of the **maximally symmetric** four-dimensional spacetimes

$$R^{(4)}_{\mu\nu} = 3Kg_{\mu\nu}\,, \quad R^{(4)} = 12K\,,$$

3. From the equations of motion, we get the following equations:

$$(\varphi')^2 = -3\mathcal{H}' + 3\mathcal{H}^2 - 3K\,,$$

$$V(\varphi) = -\frac{1}{2a^2}\left(3\mathcal{H}' + 9\mathcal{H}^2 - 9K\right)\,,$$

we can construct a thick brane model starting from a given warp factor.

Review : **Thick** Brane Model

1. Soluions : Minkowski case ($g_{\mu\nu} = \eta_{\mu\nu}$)

$$a^2 = \left(\frac{1}{e^{2ny} + e^{-2ny}} \right)^{1/n} ,$$

and

$$\varphi = \pm\sqrt{\frac{6}{n}} \tan^{-1}(e^{2ny}) , \quad V = -6 + 3(n+2)\sin^2\left(\sqrt{\frac{6n}{3}}\varphi\right)$$

When we take the limit $n \to \infty$, this warp factor approaches $e^{-2|y|}$ which is the warp factor in the thin brane model.

2. The parameter n controls the 'thickness' of the brane.

Review : **Thick** Brane Model

1. Warp Factor : Thin and Thick

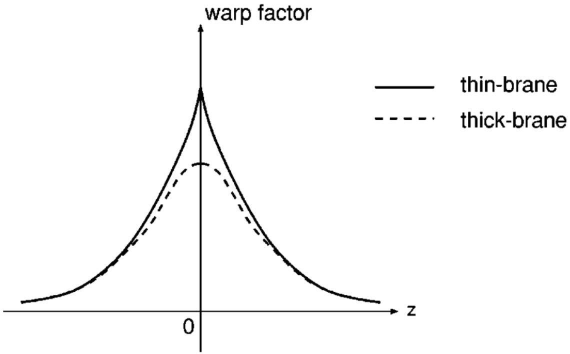


FIG. 1. The warp factors of the thin and the thick brane are shown. The solid line denotes the warp factor of the thin brane and the dashed line denotes that of the thick brane.

Figure: S. Kobayashi, K. Koyama and J. Soda, Phys. Rev. D **65**, 064014 (2002)

The Model

1. The thick branes are described by the following 5D action:

$$S = \int d^5x\sqrt{-g}\left[\frac{M_5^3}{2}R + \frac{1}{2}\xi\varphi^2 R - \frac{1}{2}\omega(\varphi)\nabla_M\varphi\nabla^M\varphi - V(\varphi)\right],$$

where M_5 is the 5D Planck mass.

2. Einstein equations

$$\left(M_5^3 + \xi\varphi^2\right)G_{MN} + 2\xi g_{MN}\varphi\Box\varphi - 2\xi\varphi\nabla_M\nabla_N\varphi + g_{MN}\left(\frac{\omega}{2} + 2\xi\right)\nabla_A\varphi\nabla^A\varphi$$
$$-(\omega + 2\xi)\nabla_M\varphi\nabla_N\varphi + g_{MN}V = 0\,.$$

3. Scalar equation

$$\omega\Box\varphi + \frac{1}{2}\omega_\varphi\nabla_A\varphi\nabla^A\varphi + \xi\varphi R - V_\varphi = 0\,,$$

The Model

1. To start with, let us to assume a regular 5D background metric

$$ds^2 = a^2(z)\hat{g}_{\mu\nu}dx^\mu dx^\nu + dz^2\,.$$

2. Now we can write down the Einstein tensor

$$(\mu,\nu):\; G_{\mu\nu} = \hat{G}_{\mu\nu} + a^2(6\mathcal{H}^2 + 3\mathcal{H}')\hat{g}_{\mu\nu}\,,$$
$$(z,z):\; G_{zz} = 6\mathcal{H}^2 - \frac{1}{2a^2}\hat{R}\,,$$

where $\mathcal{H} \equiv a'(z)/a(z)$.

3. Here, we assume that 4D Einstein equations are as

$$\hat{G}_{\mu\nu} = -\Lambda^{(4)}\hat{g}_{\mu\nu}\,.$$

Here, $\Lambda^{(4)} = 0$: Minkowski
$\Lambda^{(4)} > 0$: de Sitter
$\Lambda^{(4)} = 0$: anti-de Sitter

Minkowski brane solutions ($\Lambda^{(4)} = 0$ case)

1. In the simplest case, $\Lambda^{(4)} = 0$, the solutions which are obtained from the following potential

$$V(\phi) = -6\frac{\xi\alpha^4}{M_5^5}\varphi^4 + \left(-\frac{6\alpha^4}{M_5^2} + \frac{17\xi\alpha^2}{2}\right)\varphi^2 + \frac{3\alpha^2}{2}M_5^3 - M_5^5\xi\,,$$

$$\omega(\varphi) = \frac{\xi\alpha^2}{M_5^5}\varphi^2 + \frac{3\alpha^2}{M_5^2} - 2\xi\,,$$

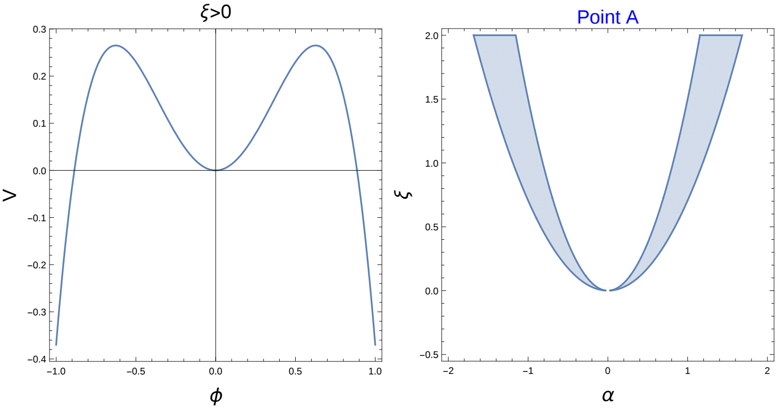
2. The solution is given by

$$\varphi(z) = M_5^{5/2}z\,, \quad a(z) = e^{-\frac{1}{2}\alpha^2 z^2}\,.$$

where $\mathcal{H} \equiv a'(z)/a(z)$.

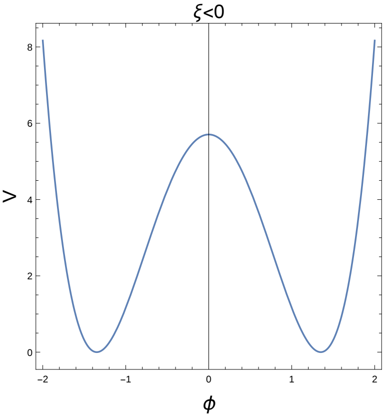
Minkowski brane solutions ($\Lambda^{(4)} = 0$ case)

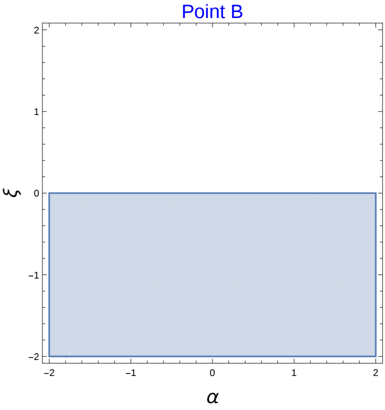
1. Potential and Stable regions: $\xi > 0$



Minkowski brane solutions ($\Lambda^{(4)} = 0$ case)

1. Potential and Stable regions: $\xi < 0$





Gravity fluctuations ($\Lambda^{(4)} = 0$ case)

1. We are interested in study the stability of the gravity sector of a braneworld scenario. Hence, we introduce the small perturbation $h_{\mu\nu}(x^\mu, z)$ as

$$ds^2 = a^2(z)(\hat{g}_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu + dz^2 \,.$$

2. Imposing the transverse-traceless gauge, the graviton equation of motion is

$$-\frac{1}{2}h''_{\mu\nu} + (\beta - 2)\mathcal{H}h'_{\mu\nu} - \frac{1}{2a^2}\hat{\Box}h_{\mu\nu} = 0$$

where $\beta \equiv \xi M_5^2/\alpha^2$.

3. Assuming the Kaluza-Klein (KK) decomposition

$$h_{\mu\nu}(x^\mu, z) = \sum_m h^{(m)}_{\mu\nu}(x^\mu)\phi_m(z)\,, \quad \hat{\Box}h^{(m)}_{\mu\nu}(x^\mu) = m^2 h^{(m)}_{\mu\nu}(x^\mu)$$

where m is four-dimensional KK mass of the fluctuation.

Gravity fluctuations ($\Lambda^{(4)} = 0$ case)

1. the gravitational KK modes in the extra dimension is described by the following Sturm-Liouville equation

$$\phi(z)'' - 2(\beta - 2)\mathcal{H}\phi(z)' = -\frac{m^2}{a^2}\phi(z)$$

2. To deal with a conformal metric, we change the coordinate to $dz^2 = a^2 dy^2$. Further, defining $\phi = \Psi / a^{3/2-\beta}$, the Sturm-Liouville equation reduces to a Schrödinger-like form

$$\left[-\partial_y^2 + U_{eff}\right]\Psi(y) = m^2\Psi(y) \,.$$

where

$$U_{eff} \equiv \frac{(3-2\beta)^2}{4} H^2 + \frac{(3-2\beta)}{2}\frac{dH}{dy} \,,$$

and

$$H \equiv \frac{da(y)/dy}{a(y)} \,, \quad a(y) = e^{-\mathrm{erfi}^{-1}\left(\sqrt{\frac{2}{\pi}}\alpha y\right)^2} \,.$$

Gravity fluctuations ($\Lambda^{(4)} = 0$ case)

1. Schrodinger-like potential U_{eff}

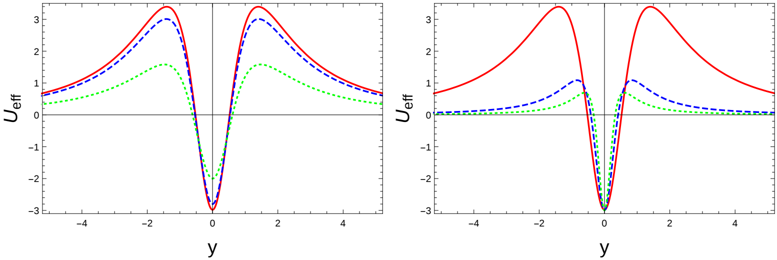


Figure: • **Left**: $(\alpha, \xi) = (1, 0.01)$-red solid, $(1, 0.1)$-blue dashed, $(1, 0.5)$-green dotted. • **Right**: $(1, 0.01)$-red solid, $(2, 0.01)$-blue dashed, $(3, 0.01)$-green dotted.

Gravity fluctuations ($\Lambda^{(4)} = 0$ case)

1. The gravitational zero-mode ($m = 0$ normalizable state) is trapped in the brane and is given by

$$\Psi_0(y) \simeq a^{3/2-\beta}(y)$$

2. Numerical procedure is needed to solve the equations of the massive modes.

Gravity fluctuations ($\Lambda^{(4)} = 0$ case)

1. We solved Schrodinger-like equation for mass eigenvalues

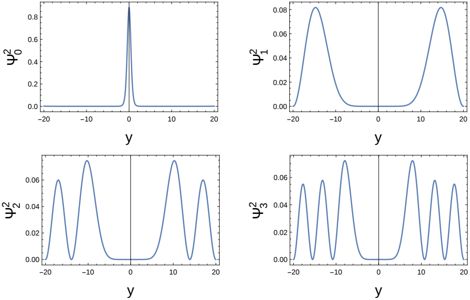


Figure: Probability density profile for the gravitational states.

2. The **massless** graviton is localized on the brane.

Gravity fluctuations ($\Lambda^{(4)} = 0$ case)

1. The equation is given by

$$-\Psi''(y) + \frac{A(A+1)}{y^2}\Psi(y) = m^2\Psi(y)$$

where

$$A \simeq 3/2 - \beta$$

The solution is as following

$$\Psi_m(y) = a_m\sqrt{y}Y_{A+1/2}(my) + b_m\sqrt{y}J_{A+1/2}(my)$$

Gravity fluctuations ($\Lambda^{(4)} = 0$ case)

1. $mz \gg 1$

$$\Psi_m(y) \simeq a_m \sqrt{\frac{2}{\pi m}} \sin\left(mz - \frac{\pi}{2}A - \frac{\pi}{2}\right) + b_m \sqrt{\frac{2}{\pi m}} \cos\left(mz - \frac{\pi}{2}A - \frac{\pi}{2}\right)$$

2. $mz \ll 1$

$$\Psi_m(y) \simeq -\frac{a_m z^{1/2}\Gamma(A+1/2)}{\pi}\left(\frac{2}{mz}\right)^{A+1/2}\left[1 + \frac{1}{A-1/2}\left(\frac{mz}{2}\right)^2\right]$$
$$\frac{b_m z^{1/2}}{\Gamma(A+3/2)}\left(\frac{mz}{2}\right)^{A+1/2}$$

Therefore,

$$a_m = m^{A+1/2}, \quad b_m = m^{-A+3/2}$$

3. Overall factor m^{A-1}

$$\Psi_m(0) \simeq m^{A-1}$$

Corrections to the Newtonian Potential ($\Lambda^{(4)} = 0$ case)

1. A solution of the Schrodinger-like equation, Ψ_m with mass m contributes with a Yukawa-like correction to the Newton's law.
2. Hence, the gravitational potential between two point-like sources of mass M_1 and M_2 located at the origin $(y = 0)$ in the transverse space, will be exponentially suppressed as

$$U_{\text{corr}}(r) = G\frac{M_1 M_2}{r} + \int_{\epsilon}^{\infty} dm M_5^3 \frac{M_1 M_2}{r} e^{-mr} \left|\Psi_m(0)\right|^2$$
$$= G\frac{M_1 M_2}{r}\left(1 + \frac{C}{(Ar)^{2A-1}}\right)$$

Corrections to the Newtonian Potential ($\Lambda^{(4)} = 0$ case)

1. Newtonian potential with the correction

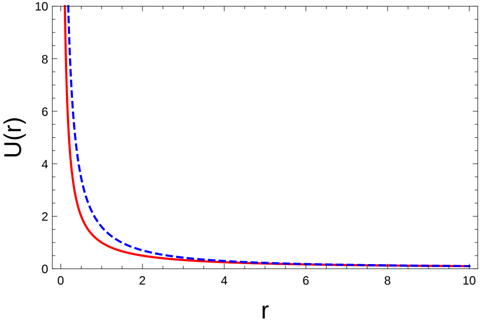


Figure: Newton's law(Red) and Newton's law with KKcorrection(Blue).

2. The gravitational force is slightly **increased at short distances** due to the massive modes

Conclusions

Thick Brane Model

- We obtained stable thick brane solutions.
- The study of the gravity fluctuations showed that the zero-mode is trapped in the brane.
- The gravitational force is slightly increased at short distances.
- This work is still ongoing : analysis of non-zero $\Lambda^{(4)}$.