Cosmic Inflation from Yang-Mills Instantons in Extra Dimensions

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This talk is based on the ongoing collaboration with
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Warning!

1. Very preliminary

2. No helpful references

3. My collaborators are busy

☺ Any comments are welcome.

All is forgiven ?

Physical cosmology has been formulated in terms of effective field theories coupled to general relativity.

Everything is allowed in cosmology.

No genetic origin of inflaton(s) and ad hoc inflation potential for cosmic inflation

Alternative, predictive model for the cosmic inflation?

Advertisement: Recently a background-independent formulation of cosmic inflation was formulated in terms of matrix quantum mechanics.

Emergent Spacetime and Cosmic Inflation I & II, arXiv:1503.00712
Cosmic Inflation from Emergent Spacetime Picture, arXiv:1610.00712

Cosmic Inflation from Yang-Mills Instantons

We consider an eight-dimensional Einstein-Yang-Mills theory to explore whether Yang-Mills instantons formed in extra dimensions can trigger a cosmic inflation in our four-dimensional spacetime. We first observe that the Yang-Mills instantons in extra dimensions and isometric in four-dimensional spacetime acts as a (quantized) cosmological constant for the four-dimensional Einstein gravity. As a result, the cosmic inflation in our four-dimensional spacetime can be triggered by the Yang-Mills instantons whereas the extra dimensions are dynamically compactified since the eight-dimensional spacetime must be Ricci-flat. Furthermore we want to examine whether the back-reaction from Yang-Mills instantons in extra dimensions can be used for the graceful exit from the inflation a.k.a. reheating mechanism.

2 Yang-Mills instantons and quantized cosmological constant

Consider an eight-dimensional spacetime $\mathcal{M}_8$ whose metric is given by

$$ds^2 = G_{MN} dX^M dX^N = E^A \otimes E^A, \tag{2.1}$$

where $X^M = (x^\mu, y^\alpha),\ M, N = 0, 1, \cdots, 7; \mu, \nu = 0, 1, 2, 3; \alpha, \beta = 4, 5, 6, 7$, are local coordinates on $\mathcal{M}_8$ and $E^A = (E^m, E^a)\ A, B = 0, 1, \cdots, 7; m, n = 0, 1, 2, 3; a, b = 4, 5, 6, 7$, are orthonormal vielbeins in $\Gamma(T^*\mathcal{M}_8)$. Let $\pi : E \to \mathcal{M}_8$ be a $\mathfrak{g}$-bundle over $\mathcal{M}_8$ whose curvature is defined by

$$\begin{aligned} F &= dA + A \wedge A = \frac{1}{2} F_{MN}(X) dX^M \wedge dX^N && (2.2) \\ &= \frac{1}{2}\Big(\partial_M A_N - \partial_N A_M + [A_M, A_N]\Big) dX^M \wedge dX^N && (2.3) \end{aligned}$$

where $A = A^i_M(X)\tau^i dX^M = \big(A_\mu(x,y)dx^\mu, A_\alpha(x,y)dy^\alpha\big)$ is a connection one-form of the $\mathfrak{g}$-bundle E and τ^i $\big(i = 1, \cdots, \mathrm{rank}(\mathfrak{g})\big)$ are Lie algebra generators obeying the commutation relation

$$[\tau^i, \tau^j] = f^{ijk}\tau^k. \tag{2.4}$$

We choose a normalization $\mathrm{Tr}\tau^i\tau^j = -\delta^{ij}$. The action for the eight-dimensional Yang-Mills theory on a curved manifold $\mathcal{M}_8$ is then defined by

$$S_{YM} = \frac{1}{4G^2_{YM}} \int_{\mathcal{M}_8} d^8X\sqrt{-G}\,\mathrm{Tr}G^{MP}G^{NQ}F_{MN}F_{PQ}. \tag{2.5}$$

In order to appreciate whether Yang-Mills instantons formed in extra dimensions give rise to a vacuum energy which triggers a cosmic inflation in the four-dimensional spacetime, let us consider a simple geometry $\mathcal{M}_8 = \mathcal{M}_{3,1} \times X_4$ with a product metric

$$\begin{aligned} ds^2 &= G_{MN}dX^MdX^N = g_{\mu\nu}(x)dx^\mu dx^\nu + h_{\alpha\beta}(y)dy^\alpha dy^\beta \\ &= E^m \otimes E^m + E^a \otimes E^a. \end{aligned} \tag{2.6}$$

For this product geometry, the action (2.5) takes the form

$$S_{YM} = \frac{1}{4G^2_{YM}} \int_{\mathcal{M}_{3,1}} d^4x\sqrt{-g} \int_{X_4} d^4y\sqrt{h}\,\mathrm{Tr}\Big(g^{\mu\nu}g^{\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + 2g^{\mu\nu}h^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta} + h^{\alpha\gamma}h^{\beta\delta}F_{\alpha\beta}F_{\gamma\delta}\Big). \tag{2.7}$$

We are interested in the gauge field configuration given by

$$A_\mu(x,y) = 0, \qquad A_\alpha(x,y) = A_\alpha(y), \tag{2.8}$$

for which the above action reduces to

$$S_{YM} = \frac{1}{4G^2_{YM}} \int_{\mathcal{M}_{3,1}} d^4x\sqrt{-g} \int_{X_4} d^4y\sqrt{h}\,\mathrm{Tr}h^{\alpha\gamma}h^{\beta\delta}F_{\alpha\beta}F_{\gamma\delta}. \tag{2.9}$$

The problem is if there exists any gauge field configuration for which the four-dimensional action along the internal space X_4 becomes a nonzero constant, i.e.,

$$I_n \equiv -\int_{X_4} d^4y\sqrt{h}\,\mathrm{Tr}h^{\alpha\gamma}h^{\beta\delta}F_{\alpha\beta}F_{\gamma\delta} = \mathrm{constant}. \tag{2.10}$$

It is well-known that the four-dimensional gauge fields satisfying the condition (2.10) are precisely Yang-Mills instantons obeying the self-duality equation

$$F_{\alpha\beta} = \pm\frac{1}{2}\frac{\varepsilon^{\xi\eta\gamma\delta}}{\sqrt{h}}h_{\alpha\xi}h_{\beta\eta}F_{\gamma\delta}. \tag{2.11}$$

In this case, $I_n = 32\pi^2 n$ with $n \in \mathbb{N}$ and the action (2.9) can be written as

$$S_{YM} = -\frac{1}{4G^2_{YM}} \int_{\mathcal{M}_{3,1}} d^4x\sqrt{-g}I_n. \tag{2.12}$$

Since $[G_{YM}^2] = L^4$ in eight dimensions, it may be instructive to rewrite (2.9) as

$$S_\Lambda = -\frac{1}{8\pi G_4} \int_{\mathcal{M}_{3,1}} d^4x \sqrt{-g}\Lambda \tag{2.13}$$

where G_4 is the four-dimensional Newton constant and

$$\Lambda = \frac{2\pi G_4}{G_{YM}^2} I_n \tag{2.14}$$

has the correct dimension of the cosmological constant in four dimensions, i.e., $[\Lambda] = L^{-2}$.[1] Therefore we see that the Yang-Mills instantons in Eq. (2.10) generate the coupling with the *quantized* cosmological constant Λ in the four-dimensional spacetime.

In conclusion, if Yang-Mills instantons are formed in X_4, their instanton number behaves like a (quantized) cosmological constant in $\mathcal{M}_{3,1}$. Hence it is reasonable to expect that the Yang-Mills instantons in the internal space generate a cosmic inflation in our four-dimensional spacetime. In next section we will examine this idea.

3 Cosmic inflation from Yang-Mills instantons

In order to investigate whether Yang-Mills instantons in the internal space X_4 can trigger the cosmic inflation in the four-dimensional spacetime $\mathcal{M}_{3,1}$, let us consider the eight-dimensional Yang-Mills theory (2.5) coupled to Einstein gravity. It is described by the Einstein-Yang-Mills theory with the total action

$$S = \frac{1}{16\pi G_8} \int_{\mathcal{M}_8} d^8X \sqrt{-G} R + S_{YM} \tag{3.1}$$

where G_8 is the eight-dimensional gravitational constant. The gravitational field equations read as

$$R_{MN} - \frac{1}{2} G_{MN} R = 8\pi G_8 T_{MN} \tag{3.2}$$

with the energy-momentum tensor given by

$$T_{MN} = \frac{1}{G_{YM}^2} \mathrm{Tr}\Big(G^{PQ} F_{MP} F_{NQ} - \frac{1}{4} G_{MN} F_{PQ} F^{PQ} \Big). \tag{3.3}$$

The action (2.5) leads to the equations of motion for Yang-Mills gauge fields

$$G^{MN} D_M F_{NP} = 0, \tag{3.4}$$

where the covariant derivative is defined with respect to both the Yang-Mills and gravitational connections, i.e.,

$$D_M F_{NP} = \partial_M F_{NP} - \Gamma_{MN}{}^Q F_{QP} - \Gamma_{MP}{}^Q F_{NQ} + [A_M, F_{NP}] \tag{3.5}$$

[1] If $[G_{YM}^2] = M_{GUT}^{-4}$ where $M_{GUT} \sim 10^{16}$ GeV is the energy scale of grand unified theory (GUT), $\sqrt{\Lambda} \gtrsim 10^{14}$ GeV. Hence the Yang-Mills instantons in the GUT scale are eligible for a source of the cosmic inflation.

and $\Gamma_{MN}{}^P$ is the Levi-Civita connection. Therefore we need to show that the cosmic inflation triggered by the Yang-Mills instantons satisfies both (3.2) and (3.4).

In order to find a solution, consider an ansatz for an eight-dimensional metric

$$\begin{aligned} ds^2 &= G_{MN}dX^M dX^N = g_{\mu\nu}(x)dx^\mu dx^\nu + e^{2f(x)}h_{\alpha\beta}(y)dy^\alpha dy^\beta \\ &\equiv g_{\mu\nu}(x)dx^\mu dx^\nu + \widetilde{h}_{\alpha\beta}(x,y)dy^\alpha dy^\beta. \end{aligned} \tag{3.6}$$

We also denote the corresponding vielbeins by

$$ds^2 = G_{MN}dX^M dX^N = \widetilde{E}^m \otimes \widetilde{E}^m + \widetilde{E}^a \otimes \widetilde{E}^a, \tag{3.7}$$

where

$$\widetilde{E}^m = E^m, \qquad \widetilde{E}^a = e^{f(x)}E^a. \tag{3.8}$$

Although we are considering a warped product metric (4.1), the separation such as Eq. (2.7) is still valid and the action for the configuration (2.8) reduces to

$$S_{YM} = \frac{1}{4G_{YM}^2}\int_{\mathcal{M}_{3,1}} d^4x\sqrt{-g}\int_{X_4} d^4y\sqrt{h}\mathrm{Tr}h^{\alpha\gamma}h^{\beta\delta}F_{\alpha\beta}F_{\gamma\delta}, \tag{3.9}$$

where we used the fact that the action (2.10) is invariant under the Weyl transformation $h_{\alpha\beta} \to e^{2f(x)}h_{\alpha\beta}$. Then one can see that the equations of motion (3.4) take the simple form

$$h^{\alpha\beta}D_\alpha F_{\beta\gamma} = 0. \tag{3.10}$$

It is easy to show that Eq. (4.5) is automatically satisfied as far as the gauge fields obey the self-duality equation (2.11). In consequence, the Yang-Mills instantons satisfy the equations of motion (3.4) even in a warped spacetime with the metric (4.1).

The energy-momentum tensor (3.3) is determined by the Yang-Mills instantons and one finds that

$$\begin{aligned} T_{\mu\nu} &= -\frac{1}{4G_{YM}^2}\widetilde{g}_{\mu\nu}\mathrm{Tr}F_{\alpha\beta}F^{\alpha\beta}, \\ T_{\alpha\beta} &= \frac{e^{-2f(x)}}{G_{YM}^2}\mathrm{Tr}\Big(h^{\gamma\delta}F_{\alpha\gamma}F_{\beta\delta} - \frac{1}{4}h_{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}\Big) = 0, \\ T_{\mu\alpha} &= 0, \end{aligned} \tag{3.11}$$

where $\widetilde{g}_{\mu\nu}(x) = e^{-4f(x)}g_{\mu\nu}(x)$ and all indices are raised and lowered with the original metric (2.6). We used the fact that the energy momentum tensor identically vanishes for an instanton solution satisfying Eq. (2.11). Note that the energy-momentum tensor $T_{\mu\nu}$ can be written as the form

$$T_{\mu\nu} = \widetilde{g}_{\mu\nu}(x)\rho_n(y), \tag{3.12}$$

where $\rho_n(y)$ is the instanton density in X_4 which is uniform along the four-dimensional spacetime $\mathcal{M}_{3,1}$. Hence $T_{\mu\nu}$ effectively acts as a cosmological constant in $\mathcal{M}_{3,1}$ as we observed in the previous section. In the end, the gravitational field equations (3.2) read as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_8 T_{\mu\nu}, \tag{3.13}$$

$$R_{\mu\alpha} = 0, \qquad R_{\alpha\beta} = 0. \tag{3.14}$$

For the warped product geometry (3.6), the Ricci tensor is given by

$$\begin{aligned} R_{\mu\nu} &= R^{(0)}_{\mu\nu} - 4(\nabla^{(g)}_\mu \partial_\nu f + \partial_\mu f \partial_\nu f), \\ R_{\alpha\beta} &= R^{(0)}_{\alpha\beta} - (\nabla^2_{(g)} f + 4 g^{\mu\nu} \partial_\mu f \partial_\nu f) e^{2f(x)} h_{\alpha\beta}, \\ R_{\mu\alpha} &= 0, \end{aligned} \tag{3.15}$$

where $R^{(0)}_{\mu\nu}$ and $R^{(0)}_{\alpha\beta}$ are the Ricci tensors when $f = 0$ and $\nabla^{(g)}_\mu$ is a covariant derivative with respect to the metric $g_{\mu\nu}(x)$. And the Ricci scalar is

$$R = R_{(g)} + e^{-2f(x)} R_{(h)} - 8\nabla^2_{(g)} f - 20 g^{\mu\nu} \partial_\mu f \partial_\nu f, \tag{3.16}$$

where $R_{(g)}$ and $R_{(h)}$ are the Ricci scalars of the metrics $g_{\mu\nu}$ and $h_{\alpha\beta}$ when $f = 0$, respectively.

For the ansatz (3.6), the Einstein equations can thus be written as the form

$$\begin{aligned} & R^{(0)}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R_{(g)} - 4\big(\nabla^{(g)}_\mu \partial_\nu f + \partial_\mu f \partial_\nu f\big) \\ & + 2\big(\nabla^2_{(g)} f + g^{\rho\sigma} \partial_\rho f \partial_\sigma f - 4\pi G_8 e^{-4f(x)} \rho_n(y)\big) g_{\mu\nu} = 0, \end{aligned} \tag{3.17}$$

$$R^{(0)}_{\alpha\beta} = (\nabla^2_{(g)} f + 4 g^{\mu\nu} \partial_\mu f \partial_\nu f) e^{2f(x)} h_{\alpha\beta}. \tag{3.18}$$

If the four-dimensional metric is given by

$$g_{\mu\nu} = \mathrm{diag}\big(-1, e^{2Ht}, e^{2Ht}, e^{2Ht}\big), \tag{3.19}$$

the Einstein tensor for the metric (3.19) is given by

$$R^{(0)}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R_{(g)} = -3H^2 g_{\mu\nu}. \tag{3.20}$$

Instead, if we consider a different ansatz for an eight-dimensional metric

$$ds^2 = G_{MN} dX^M dX^N = e^{2f(y)} g_{\mu\nu}(x) dx^\mu dx^\nu + h_{\alpha\beta}(y) dy^\alpha dy^\beta, \tag{3.21}$$

the energy-momentum tensor (3.3) is given by

$$\begin{aligned} T_{\mu\nu} &= -\frac{1}{4G^2_{YM}} \widetilde{g}_{\mu\nu} \mathrm{Tr} F_{\alpha\beta} F^{\alpha\beta}, \\ T_{\alpha\beta} &= \frac{1}{G^2_{YM}} \mathrm{Tr}\Big(h^{\gamma\delta} F_{\alpha\gamma} F_{\beta\delta} - \frac{1}{4} h_{\alpha\beta} F_{\gamma\delta} F^{\gamma\delta}\Big) = 0, \\ T_{\mu\alpha} &= 0, \end{aligned} \tag{3.22}$$

where $\widetilde{g}_{\mu\nu}(x) = e^{2f(y)}g_{\mu\nu}(x)$. For the warped geometry with the metric (3.21), the corresponding Ricci tensor is given by

$$\begin{aligned} R_{\mu\nu} &= R^{(0)}_{\mu\nu} - (\nabla^2_{(h)}f + 4h^{\alpha\beta}\partial_\alpha f \partial_\beta f)e^{2f(y)}g_{\mu\nu}, \\ R_{\alpha\beta} &= R^{(0)}_{\alpha\beta} - 4(\nabla^{(h)}_\alpha \partial_\beta f + \partial_\alpha f \partial_\beta f), \\ R_{\mu\alpha} &= 0, \end{aligned} \tag{3.23}$$

where $R^{(0)}_{\mu\nu}$ and $R^{(0)}_{\alpha\beta}$ are the Ricci tensors when $f = 0$ and $\nabla^{(h)}_\alpha$ is a covariant derivative with respect to the metric $h_{\alpha\beta}(y)$. And the Ricci scalar is

$$R = e^{-2f(y)}R_{(g)} + R_{(h)} - 8\nabla^2_{(h)}f - 20h^{\alpha\beta}\partial_\alpha f \partial_\beta f, \tag{3.24}$$

where $R_{(g)}$ and $R_{(h)}$ are the Ricci scalars of the metrics $g_{\mu\nu}$ and $h_{\alpha\beta}$ when $f = 0$, respectively.

For the latter ansatz, the Einstein equations can be written as the form

$$R^{(0)}_{\mu\nu} - \frac{1}{2}R_{(g)}g_{\mu\nu} + \big(\nabla^2_{(h)}f + 4h^{\alpha\beta}\partial_\alpha f \partial_\beta f - 8\pi G_8 \rho_n(y)\big)e^{2f(y)}g_{\mu\nu} = 0, \tag{3.25}$$

$$R^{(0)}_{\alpha\beta} = 4(\nabla^{(h)}_\alpha \partial_\beta f + \partial_\alpha f \partial_\beta f). \tag{3.26}$$

If the four-dimensional metric is given by

$$g_{\mu\nu} = \mathrm{diag}\big(-1, e^{2Ht}, e^{2Ht}, e^{2Ht}\big), \tag{3.27}$$

the first equation (3.25) is reduced to the following differential equation

$$\nabla^2_{(h)}f + 4h^{\alpha\beta}\partial_\alpha f \partial_\beta f - 8\pi G_8 \rho_n(y) - 3H^2 e^{-2f(y)} = 0. \tag{3.28}$$

4 Different Ansatz

In order to find a solution, let us examine a different ansatz for an eight-dimensional metric

$$\begin{aligned} ds^2 &= G_{MN}dX^M dX^N = k(x,y)^2\Big(g_{\mu\nu}(x)dx^\mu dx^\nu + e^{2f(x)}h_{\alpha\beta}(y)dy^\alpha dy^\beta\Big) \\ &\equiv \widetilde{g}_{\mu\nu}(x,y)dx^\mu dx^\nu + \widetilde{h}_{\alpha\beta}(x,y)dy^\alpha dy^\beta. \end{aligned} \tag{4.1}$$

We also denote the corresponding vielbeins by

$$ds^2 = G_{MN}dX^M dX^N = \widetilde{E}^m \otimes \widetilde{E}^m + \widetilde{E}^a \otimes \widetilde{E}^a, \tag{4.2}$$

where

$$\widetilde{E}^m = k(x,y)E^m, \qquad \widetilde{E}^a = k(x,y)e^{f(x)}E^a. \tag{4.3}$$

Although we are considering a general ansatz (4.1), the separation such as Eq. (2.7) is still valid and the action for the configuration (2.8) reduces to

$$S_{YM} = \frac{1}{4G_{YM}^2}\int_{\mathcal{M}_{3,1}} d^4x \int_{X_4} d^4y \sqrt{-\widetilde{g}}\sqrt{h}\mathrm{Tr}h^{\alpha\gamma}h^{\beta\delta}F_{\alpha\beta}F_{\gamma\delta}, \tag{4.4}$$

where we used the fact that the action (2.10) is invariant under the Weyl transformation $h_{\alpha\beta} \to k(x,y)^2 e^{2f(x)}h_{\alpha\beta}$. Then one can see that the equations of motion (3.4) take the simple form

$$h^{\alpha\beta}D_\alpha F_{\beta\gamma} + \frac{1}{2}F_{\gamma\beta}\partial^\beta k^{-2} = 0. \tag{4.5}$$

It is easy to show that Eq. (4.5) is automatically satisfied as far as the gauge fields obey the self-duality equation (2.11). In consequence, the Yang-Mills instantons satisfy the equations of motion (3.4) even in a general spacetime with the metric (4.1).

The energy-momentum tensor (3.3) is determined by the Yang-Mills instantons and one finds that

$$\begin{aligned}
T_{\mu\nu} &= -\frac{1}{4G_{YM}^2}k(x,y)^{-4}e^{-4f(x)}\widetilde{g}_{\mu\nu}(x,y)\mathrm{Tr}F_{\alpha\beta}F^{\alpha\beta}, \\
T_{\alpha\beta} &= \frac{k(x,y)^{-2}e^{-2f(x)}}{G_{YM}^2}\mathrm{Tr}\Big(h^{\gamma\delta}F_{\alpha\gamma}F_{\beta\delta} - \frac{1}{4}h_{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}\Big) = 0, \\
T_{\mu\alpha} &= 0,
\end{aligned} \tag{4.6}$$

where all indices of gauge fields are raised and lowered with the original metric (2.6). We used the fact that the energy momentum tensor identically vanishes for an instanton solution satisfying Eq. (2.11). Note that the energy-momentum tensor $T_{\mu\nu}$ can be written as the form

$$T_{\mu\nu} = k(x,y)^{-2}e^{-4f(x)}\rho_n(y)g_{\mu\nu}(x), \tag{4.7}$$

where $\rho_n(y)$ is the instanton density in X_4 which is uniform along the four-dimensional spacetime $\mathcal{M}_{3,1}$. In the end, the gravitational field equations (3.2) read as

$$R_{\mu\nu} - \frac{1}{2}\widetilde{g}_{\mu\nu}R = 8\pi G_8 T_{\mu\nu}, \tag{4.8}$$

$$R_{\mu\alpha} = 0, \qquad R_{\alpha\beta} = 0. \tag{4.9}$$

Let us denote the metric (4.1) by $G_{MN} = k(x,y)^2\overline{G}_{MN}$. Then we have the relation for the Weyl transformation:

$$\begin{aligned}
\Gamma_{MN}{}^P &= \overline{\Gamma}_{MN}{}^P + \frac{1}{k}\big(\delta^P_M\partial_N k + \delta^P_N\partial_M k - \overline{G}_{MN}\partial^P k\big), \\
R_{MN} &= \overline{R}_{MN} + \frac{1}{k^2}\big(12\partial_M k\partial_N k - 5\overline{G}_{MN}\partial^P k\partial_P k\big) - \frac{1}{k}\big(6\overline{\nabla}_M\partial_N k + \overline{G}_{MN}\Box k\big), \\
R &= \frac{1}{k^2}\overline{R} - \frac{14}{k^4}\big(2\partial^M k\partial_M k + k\Box k\big),
\end{aligned} \tag{4.10}$$

where the barred quantities are evaluated with the metric $\overline{G}_{MN}$, e.g., $\overline{\Box}k = \frac{1}{\sqrt{-\overline{G}}}\partial_M\big(\sqrt{-\overline{G}}\,\overline{G}^{MN}\partial_N k\big)$. Since the metric $\overline{G}_{MN}$ takes the form of a warped product geometry, the corresponding Ricci tensor is given by

$$\begin{aligned} \overline{R}_{\mu\nu} &= R^{(0)}_{\mu\nu} - 4(\nabla^{(g)}_\mu \partial_\nu f + \partial_\mu f \partial_\nu f), \\ \overline{R}_{\alpha\beta} &= R^{(0)}_{\alpha\beta} - (\nabla^2_{(g)} f + 4 g^{\mu\nu}\partial_\mu f \partial_\nu f) e^{2f(x)} h_{\alpha\beta}, \\ \overline{R}_{\mu\alpha} &= 0, \end{aligned} \tag{4.11}$$

where $R^{(0)}_{\mu\nu}$ and $R^{(0)}_{\alpha\beta}$ are the Ricci tensors when $f = 0$ and $\nabla^{(g)}_\mu$ is a covariant derivative with respect to the metric $g_{\mu\nu}(x)$. And the Ricci scalar is

$$\overline{R} = R_{(g)} + e^{-2f(x)} R_{(h)} - 8\nabla^2_{(g)} f - 20 g^{\mu\nu}\partial_\mu f \partial_\nu f, \tag{4.12}$$

where $R_{(g)}$ and $R_{(h)}$ are the Ricci scalars of the metrics $g_{\mu\nu}$ and $h_{\alpha\beta}$ when $f = 0$, respectively. With this notation, the Laplacian in Eq. (4.10) is expressed as

$$\overline{\Box}k = \nabla^2_{(g)} k + e^{-2f(x)} \nabla^2_{(h)} k + 4 g^{\mu\nu}\partial_\mu f \partial_\nu k. \tag{4.13}$$

5 Discussion

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