Double Field Theory and Applications

Sung Moon Ko

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Based on

This presentation is directly based on the publications listed below

1. **Sung Moon Ko**, Charles Melby-Thompson, Rene Meyer, Jeong-Hyuck Park. "Dynamics of Perturbations in Double Field Theory & Non-Relativistic String Theory". JHEP 1512 (2015) 144, arXiv:1508.01121 [hep-th]
2. **Sung Moon Ko**, Jeong-Hyuck Park, Minwoo Suh. "The Rotation Curve of a Point Particle in Stringy Gravity" arXiv:1606.09307v1 [hep-th]

1. Motivation
2. Basic Structures of DFT
3. Dynamics of Perturbations in Double Field Theory
4. The Rotation Curve of a Point Particle in Stringy Gravity
5. Conclusion

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- Basic structures of DFT was established
- While DFT is based on string theory, it contains bunch of stringy features
- DFT has potential to describe Stringy Gravity
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Doubled Spacetime

- $Y^A = (\tilde{x}_\mu, x^\nu), \quad \partial_A = (\tilde{\partial}^\mu, \partial_\nu)$

Field contents in DFT Hull-Zweibach-Hohm[2010]

- $\mathcal{H}_{AB} = \mathcal{H}_{BA} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}, \qquad e^{-2d} = \sqrt{g}e^{-2\phi}$

$\mathrm{O}(D, D)$ indices can be raise or lowered by metric

- $\mathcal{J}_{AB} = \mathcal{H}_{AC}\mathcal{H}^C{}_B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Section condition

- $\partial_A \partial^A \Phi \equiv 0$

Strong constraints can be derived from assumption $\Phi = \phi_1 \phi_2$

- $\partial_A \phi_1 \partial^A \phi_2 = 0$

Conventional choice

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Projector can be defined Jeon-Park-Lee[2011]

- $P_{AB} = \frac{1}{2}(\mathcal{J} + \mathcal{H})_{AB}\,, \qquad \bar{P}_{AB} = \frac{1}{2}(\mathcal{J} - \mathcal{H})_{AB}$
- $P_{AB} = P_{BA}\,, \qquad P_{AC}P^{C}{}_{B} = P_{AB}\,, \qquad \bar{P}_{AC}P^{C}{}_{B} = 0$

Properties of derivative of projections operators

- $$\delta P_{AB} = -\delta P_{AB} = \tfrac{1}{2}\delta\mathcal{H}_{AB}\,, \qquad P_{A}{}^{C}\delta P_{CD}P^{D}{}_{B} = \bar{P}_{A}{}^{C}\delta P_{CD}\bar{P}^{D}{}_{B} = 0$$
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DFT connection

- $$\begin{aligned}\Gamma_{CAB} = \;& 2\left(P\partial_{C}P\bar{P}\right)_{[AB]} + 2\left(\bar{P}_{[A}{}^{D}\bar{P}_{B]}{}^{E} - P_{[A}{}^{D}P_{B]}{}^{E}\right)\partial_{D}P_{EC} \\ & -\tfrac{4}{D-1}\left(\bar{P}_{C[A}\bar{P}_{B]}{}^{D} + P_{C[A}P_{B]}{}^{D}\right)\left(\partial_{D}d + (P\partial^{E}P\bar{P})_{[ED]}\right)\end{aligned}$$

Semi-covariant derivative

- $\nabla_{C}T_{A_1\cdots A_n} = \partial_{C}T_{A_1\cdots A_n} - \omega\Gamma^{B}{}_{BC}T_{A_1\cdots A_n} + \sum_{i=1}^{n}\Gamma_{CA_i}{}^{B}T_{A_1\cdots A_{i-1}BA_{i+1}\cdots A_n}$

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Semi-covriant derivative satisfies metric compatability

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Useful to define six-indexed projector Jeon-Park-Lee[2011]

- $\mathcal{P}_{CAB}{}^{DEF} = P_C{}^D P_{[A}{}^{[E} P_{B]}{}^{F]} + \frac{2}{D-1} P_{C[A} P_{B]}{}^{[E} P^{F]D}$
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Satisfying following relations

- $\mathcal{P}_{CABDEF} = \mathcal{P}_{DEFCAB} = \mathcal{P}_{C[AB]D[EF]}$
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Defining gauge parameter

- $X^A = (\Lambda_\mu, \delta x^\nu)$

Variation of the fields under the doubled gauge parameter

- $$\delta_X \mathcal{H}_{AB} = X^C \partial_C \mathcal{H}_{AB} + 2\partial_{[A} X_{C]} \mathcal{H}^C{}_B + 2\partial_{[B} X_{C]} \mathcal{H}_A{}^C$$
  $$\delta_X (e^{-2d}) = \partial_A (X^A e^{-2d})$$

Generalized Lie-derivative is defined

- $\hat{\mathcal{L}}_X T_{A_1 \ldots A_n} :=$
  $$X^B \partial_B T_{A_1 \ldots A_n} + \omega \partial_B X^B T_{A_1 \ldots A_n} + \sum_{i=1}^{n} (\partial_{A_i} X_B - \partial_B X_{A_i}) T_{A_1 \ldots A_{i-1}}{}^B{}_{A_{i+1} \ldots A_n}$$

Generalized Lie-derivatives are closed under the C-bracket up to section condition

- $[\hat{\mathcal{L}}_X, \hat{\mathcal{L}}_Y] = \hat{\mathcal{L}}_{[X,Y]_C} + \hat{\mathcal{O}}_{X,Y}$

Where C-bracket is defined as follows

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  $$\delta_X (e^{-2d}) = \partial_A (X^A e^{-2d})$$

Generalized Lie-derivative is defined

- $\hat{\mathcal{L}}_X T_{A_1 \dots A_n} :=$
  $$X^B \partial_B T_{A_1 \dots A_n} + \omega \partial_B X^B T_{A_1 \dots A_n} + \sum_{i=1}^{n} (\partial_{A_i} X_B - \partial_B X_{A_i}) T_{A_1 \dots A_{i-1}}{}^B{}_{A_{i+1} \dots A_n}$$

Generalized Lie-derivatives are closed under the C-bracket up to section condition

- $[\hat{\mathcal{L}}_X, \hat{\mathcal{L}}_Y] = \hat{\mathcal{L}}_{[X,Y]_C} + \hat{\mathcal{O}}_{X,Y}$

Where C-bracket is defined as follows

- $[X, Y]^A_C := X^B \partial_B Y^A - Y^B \partial_B X^A + \frac{1}{2} Y^B \partial^A X_B - \frac{1}{2} X^B \partial^A Y_B$

Anomalous term

- $(\delta_X - \hat{\mathcal{L}}_X)\Gamma_{CAB} \equiv 2[(\mathcal{P}+\bar{\mathcal{P}})_{CAB}{}^{FDE} - \delta_C{}^F\delta_A{}^D\delta_B{}^E]\partial_F\partial_{[D}X_{E]}$
- $(\delta_X - \hat{\mathcal{L}}_X)\nabla_C T_{A_1\cdots A_n} = \sum_{i=1}^n 2(\mathcal{P}+\bar{\mathcal{P}})_{CA_i}{}^{BFDE}\partial_F\partial_{[D}X_{E]}T_{A_1\cdots A_{i-1}BA_{i+1}\cdots A_n}$

Covariantization process

- $$P_C{}^D\bar{P}_{A_1}{}^{B_1}\bar{P}_{A_2}{}^{B_2}\cdots\bar{P}_{A_n}{}^{B_n}\nabla_D T_{B_1\cdots B_n}$$
  $$\bar{P}_C{}^D P_{A_1}{}^{B_1}P_{A_2}{}^{B_2}\cdots P_{A_n}{}^{B_n}\nabla_D T_{B_1\cdots B_n}$$
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If we set

- $R_{CDAB} := \partial_A \Gamma_{BCD} - \partial_B \Gamma_{ACD} + \Gamma_{AC}{}^{E}\Gamma_{BED} - \Gamma_{BC}{}^{E}\Gamma_{AED}$

Satisfying properties

- $\mathcal{R}_{CDAB} = \mathcal{R}_{[CD][AB]}\,, \qquad P_C{}^I \bar{P}_D{}^J \mathcal{R}_{IJAB} = 0$

Riemann-like DFT curvature can be defined

- $S_{ABCD} := \frac{1}{2}(R_{ABCD} + R_{CDAB} - \Gamma^{E}{}_{AB}\Gamma_{ECD})$

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- $\mathcal{L}_{\mathrm{DFT}} = e^{-2d}(P^{AB}P^{CD} - \bar{P}^{AB}\bar{P}^{CD})S_{ACBD}$

With proper Riemannian parametrization

- $\int \mathrm{d}x^4 \,\sqrt{-g}e^{-2\phi}\Big(R + 4\partial_\mu\phi\partial^\mu\phi - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} \Big)$

E.O.M also take form as follows

- $R_{\mu\nu} + 2\nabla_\mu\partial_\nu\phi - \frac{1}{4}H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma} = 0$
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Basic Structures of DFT

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Dynamics of Perturbations in Double Field Theory
&
Non-Relativistic String Theory

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- Non-Riemannian solutions lead to new landscape of string theory
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Motivation

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From the section condition [Lee-Park 2013]

- $\partial_A \partial^A \Phi = 0$

Coordinate gauge symmetry can be read

- $$\Phi(x + \triangle) = \Phi(x)\,, \qquad \triangle^A = \phi \partial^A \varphi$$
  $$x^A \simeq x^A + \phi(x) \partial^A \varphi(x)$$

Above relation hints us to assume one-form transformation rule

- $dx'^M = dx^M + d(\phi \partial^M \varphi)$

Hence to construct covariant vector, gauge connection is required

- $\mathcal{A}'^M = \mathcal{A}^M + \mathrm{d}(\Phi_1 \partial^M \Phi_2)$

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The action of DFT sigma model [Lee-Park 2013]

- $S = \frac{1}{4\pi\alpha'} \int d^2\sigma \mathcal{L}_{\rm sig}$

  $\mathcal{L}_{sig} := -\frac{1}{2}(-h)^{\frac{1}{2}} h^{ab} D_a X^M D_b X^N \mathcal{H}_{MN}(X) - \varepsilon^{ab} D_a X^M A_{bM}$

There are two types of generalized metric : Rimannian

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There are two types of generalized metric : Non-Riemannian

With the background $g_{\alpha\beta} = G\eta_{\alpha\beta}$, $B_{\alpha\beta} = (G - \mu)\varepsilon_{\alpha\beta}$, and taking the limit $G \to \infty$, we obtain flat non-Riemannian background

- $\mathcal{H}_{AB} = \begin{pmatrix} 0 & \varepsilon^{\alpha}{}_{\beta} \\ -\varepsilon_{\alpha}{}^{\beta} & 2\mu\eta_{\alpha\beta} \end{pmatrix}$

With this metric, DFT sigma model reduced to G-O string model

- $S_{G-O} = \frac{1}{2\pi\alpha'} \int d^2z \left(\beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} + \frac{\mu}{2}\partial\gamma\bar{\partial}\bar{\gamma} + \partial X^i \bar{\partial} X^i \right)$

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DFT lagrangian for the NS-NS sector

- $\mathcal{L} = \frac{1}{8}[(P^{AC}P^{BD} - \bar{P}^{AC}\bar{P}^{BD})S_{ABCD} - 2\Lambda]$

Main Result

DFT fluctuations satisfy the following completely covariant E.O.M

- $(P^{AB} - \bar{P}^{AB})\nabla_A \partial_B \delta d - \frac{1}{2}\nabla_A \nabla_B \delta P^{AB} \equiv 0$

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These E.O.M can be drived from effective lagrangian

- $\mathcal{L}_{\text{eff}} := e^{-2d}\left[\frac{1}{2}(P-\bar{P})^{AB}\partial_A \delta d \partial_B \delta d - \frac{1}{2}\partial_A \delta d \nabla_B \delta P^{AB} + \frac{1}{8}\delta P^{AB}(\bar{\triangle}_A{}^C P_B{}^D - \triangle_A{}^C \bar{P}_B{}^D)\delta P_{CD}\right]$

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DFT fluctuations satisfy the following completely covariant E.O.M

- $(P^{AB} - \bar{P}^{AB})\nabla_A\partial_B\delta d - \frac{1}{2}\nabla_A\nabla_B\delta P^{AB} \equiv 0$

  $P_A{}^C\bar{P}_B{}^D\nabla_C\partial_D\delta d + \frac{1}{4}(P_A{}^C\bar{\triangle}_B{}^D - \triangle_A{}^C\bar{P}_B{}^D)\delta P_{CD} \equiv 0$

These E.O.M can be drived from effective lagrangian

- $$\mathcal{L}_{\text{eff}} := e^{-2d}\left[\tfrac{1}{2}(P - \bar{P})^{AB}\partial_A\delta d\partial_B\delta d - \tfrac{1}{2}\partial_A\delta d\nabla_B\delta P^{AB} + \tfrac{1}{8}\delta P^{AB}(\bar{\triangle}_A{}^C P_B{}^D - \triangle_A{}^C\bar{P}_B{}^D)\delta P_{CD}\right]$$

Here $\triangle_A{}^B, \bar{\triangle}_A{}^B$ are the novel second order differntial operator

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DFT lagrangian for the NS-NS sector

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Linearized gauge symmetries are

- $\delta_\xi(\delta P_{AB}) = \mathcal{H}_{AC}\partial_B\xi^C + \mathcal{H}_{CB}\partial_A\xi^C - \mathcal{H}_{AC}\partial^C\xi_B - \mathcal{H}_{CB}\partial^C\xi_A$

Specific equations for linearized gauge symmetry

- $$\begin{aligned} &\delta h_{\alpha\beta} = 2\sigma^{\gamma}_{(\alpha}\partial_{\beta)}\tilde{\lambda}_\gamma - 2\partial_\gamma\tilde{\lambda}_{(\alpha}\sigma^{\gamma}_{\beta)} + 2f\eta_{\gamma(\alpha}\partial_{\beta)}\lambda^\gamma\,, && \delta h^{\alpha\beta} = 0\,,\\ &\delta h_\alpha{}^\beta = \partial_\alpha\lambda^\gamma\sigma^\beta_\gamma - \sigma^\gamma_\alpha\partial_\gamma\lambda^\beta\,, && \delta h^{\alpha i} = -g^{ij}\partial_j\lambda^\alpha\,,\\ &\delta h_\alpha{}^i = g^{ij}(\partial_\alpha\tilde{\lambda}_j - \partial_j\tilde{\lambda}_\alpha) - \sigma^\gamma_\alpha\partial_\gamma\lambda^i\,, && \delta h^\alpha{}_i = \sigma^\alpha_\gamma\partial_i\lambda^\gamma\,,\\ &\delta h_{\alpha i} = g_{ij}\partial_\alpha\lambda^j + f\eta_{\alpha\gamma}\partial_i\lambda^\gamma + \sigma_\alpha{}^\beta(\partial_i\tilde{\lambda}_\beta - \partial_\beta\tilde{\lambda}_i)\,. \end{aligned}$$

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Fluctuation E.O.M take the form

- $(P_A{}^C \bar{\triangle}_B{}^D - \triangle_A{}^C \bar{P}_B{}^D)\delta P_{CD} + 8P_A{}^C \bar{P}_B{}^D \partial_C \partial_D \delta d = 0$
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Planewave form

- $\delta P_{AB}(x) = h_{AB} e^{ip_+ x^+ + ip_- x^- + ik_i x^i}$

Consider following form of fluctuation

- $$\begin{aligned}
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- $\hat{\mathcal{L}}_\xi \mathcal{H}_{AB} = \xi^C \partial_C \mathcal{H}_{AB} + \mathcal{H}_{AC}(\partial_B \xi^C - \partial^C \xi_B) + \mathcal{H}_{CB}(\partial_A \xi^C - \partial^C \xi_A) = 0$

Bargmann algebra

- $$[B_i, H] = P_i \qquad [B_i, P_j] = \delta_{ij} N \qquad [M_{ij}, P_k] = \delta_{ik} P_j - \delta_{jk} P_i$$
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Splitting coordinates $x^i = (x^m, u)$, we introduce novel background

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The rotation curve of a point particle in stringy gravity

Motivation

- DFT has potential to describe Stringy Gravity
- There are definite evidences for Dark Matter
- But still no clear understanding of underlying mechanics of Dark Matter
- DFT might have a chance to modify convensional gravity
- Hints for the fundamental frame between String and Einstein frame

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For point-like particle in DFT

- $S_{\text{particle}} = \int \mathrm{d}\tau\ e^{-1}\,\mathrm{D}_\tau x^A \mathrm{D}_\tau x^B \mathcal{H}_{AB}(x) - \frac{1}{4}m^2 e$

With conventional gauge choice

- $\mathrm{D}_\tau x^A \equiv (\dot{\tilde{x}}_\mu - A_\mu\,,\,\dot{x}^\nu)$

Further with Riemannian parametrized DFT-metric and dilaton

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$\equiv \dot{x}^\mu \dot{x}^\nu g_{\mu\nu} + \left(\dot{\tilde{x}}_\mu - A_\mu + \dot{x}^\rho B_{\rho\mu}\right)\left(\dot{\tilde{x}}_\nu - A_\nu + \dot{x}^\sigma B_{\sigma\nu}\right) g^{\mu\nu}$

Each fields are coupled with String frame metric

For point-like particle in DFT

- $S_{\text{particle}} = \int \mathrm{d}\tau \; e^{-1} \mathrm{D}_\tau x^A \mathrm{D}_\tau x^B \mathcal{H}_{AB}(x) - \frac{1}{4} m^2 e$

With conventional gauge choice

- $\mathrm{D}_\tau x^A \equiv \left(\dot{\tilde{x}}_\mu - A_\mu \,,\, \dot{x}^\nu\right)$

Further with Riemannian parametrized DFT-metric and dilaton

- $\mathcal{H}_{AB} \equiv \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}, \qquad e^{-2d} \equiv \sqrt{-g}e^{-2\phi}$

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First, we consider the most general spherical symmetric ansatz

- $\mathrm{d}s^2 = e^{2\phi(r)}\left[-A(r)\mathrm{d}t^2 + A^{-1}(r)\mathrm{d}r^2 + A^{-1}(r)C(r)\,\mathrm{d}\Omega^2\right]$
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These ansatz stisfies Killing equation Park-Rey-Rim-Yuho[2015]

- $\hat{\mathcal{L}}_{V_a}\mathcal{H}_{AB} = 0$
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Recall the E.O.M of DFT in Riemannian parametrization

- $R_{\mu\nu} + 2\nabla_\mu \partial_\nu \phi - \frac{1}{4} H_{\mu\rho\sigma} H_\nu{}^{\rho\sigma} = 0$
- $\nabla^\lambda H_{\lambda\mu\nu} - 2(\partial^\lambda \phi) H_{\lambda\mu\nu} = 0$
- $R + 4\Box\phi - 4\partial_\mu \phi \partial^\mu \phi - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} = 0$

The rotation symmetric solution can be fixed

- $A(r) = \left(\frac{r-\alpha}{r+\beta}\right)^{\frac{a}{\sqrt{a^2+b^2}}}$
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- $\mathrm{d}s^2 = e^{\phi(r)}\left[-f(r)\mathrm{d}t^2 + f^{-1}(r)\mathrm{d}r^2 + h^2(r)\,\mathrm{d}\Omega^2\right]$

Where each functions are written explicitly as following

- $f(r) = (1-\frac{l}{r})^{-\delta}, \qquad h^2(r) = r^2(1-\frac{l}{r})^{1-\delta}$
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One can find the mapping between our notation

- $\delta = \frac{a}{\sqrt{a^2+b^2}}, \quad \gamma = \frac{b}{\sqrt{a^2+b^2}}$
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Most General Spherical Symmetric Ansatz

Introducing the proper radius

- $R := \sqrt{g_{\vartheta\vartheta}(r)} = \sqrt{C(r)/A(r)}\, e^{\phi(r)}$

The solution is now tweaked

- $\mathrm{d}s^2 = -e^{2\phi} A\, \mathrm{d}t^2 + e^{2\phi} A^{-1} \left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)^{-2} \mathrm{d}R^2 + R^2 \mathrm{d}\Omega^2\,.$

From geodesic equation of radial direction

- $\frac{\mathrm{d}^2 r}{\mathrm{d}\tau^2} + \Gamma^r_{tt} \left(\frac{\mathrm{d}t}{\mathrm{d}\tau}\right)^2 + \Gamma^r_{\varphi\varphi} \left(\frac{\mathrm{d}\varphi}{\mathrm{d}\tau}\right)^2 = 0\,.$

Rotation velocity can be extracted

- $$\begin{aligned} V_{\text{orbit}} &= \left| R \frac{\mathrm{d}\varphi}{\mathrm{d}t} \right| = \left[-\frac{1}{2} R \frac{\mathrm{d}g_{tt}}{\mathrm{d}R} \right]^{\frac{1}{2}} \\ &= \left[\frac{\left(\gamma_+ \left(\frac{r-\alpha}{r+\beta}\right)^{\frac{a+b}{\sqrt{a^2+b^2}}} + \gamma_- \left(\frac{r-\alpha}{r+\beta}\right)^{\frac{a-b}{\sqrt{a^2+b^2}}} \right) \left(\gamma_+ (a+b) \left(\frac{r-\alpha}{r+\beta}\right)^{\frac{2b}{\sqrt{a^2+b^2}}} + \gamma_- (a-b) \right)}{\gamma_+ (2r-\alpha+\beta-a+b) \left(\frac{r-\alpha}{r+\beta}\right)^{\frac{2b}{\sqrt{a^2+b^2}}} + \gamma_- (2r-\alpha+\beta-a-b)} \right] \end{aligned}$$

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DFT global conserved charge is defined as Park-Rey-Rim-Yuho[2015]

- $Q[X] = \oint_{\partial\mathcal{M}} \mathrm{d}^2 x_{AB}\ e^{-2d} \left(K^{AB} + 2X^{[A}B^{B]}\right)$
- $K^{AB} = 4(\bar{P}\nabla)^{[A}(PX)^{B]} - 4(P\nabla)^{[A}(\bar{P}X)^{B]}$
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The global charge is conserved if the following meets

- $\partial_A \partial_{[B} X_{C]} = 0$

Only surviving component of K^{AB} is K^{tr}

- $K^{tr} + 2X^{[t}\tilde{B}^{r]} = g^{rr}g^{tt}\partial_r g_{tt} X^t + 4X^t g^{rr}\partial_r d - X^t \partial_r g^{rr}$

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- $K^{tr} + 2X^{[t}\tilde{B}^{r]} = g^{rr}g^{tt}\partial_r g_{tt} X^t + 4X^t g^{rr}\partial_r d - X^t \partial_r g^{rr}$

With DFT dilaton $d = \phi - \frac{1}{4}\ln(-g)$

- $K^{tr} + 2X^{[t}\tilde{B}^{r]} = -2X^t g^{rr} g^{\theta\theta}\partial_r g_{\theta\theta} + 4X^t g^{rr}\partial_r \phi$

Substituting specific metric solution

- $e^{-2d}K^{tr} = e^{2\phi}\left(2C\partial_r\phi + \frac{C}{A}\partial_r A\right)\sin\theta X^t$
- $2X^{[t}B^{r]} = 4X^t g^{rr}\partial_r\phi - g^{rr}g^{tt}\partial_r g_{tt}X^t - 2g^{rr}g^{\theta\theta}\partial_r g_{\theta\theta}X^t$

The global conserved charge

- $Q_K[\partial_t] := -\oint_{\partial\mathcal{M}} d^{D-2}x_{AB}e^{-2d}K^{[tr]} = \frac{1}{4}(a + \sqrt{b^2 - h^2})$
- $Q_{XB} = \frac{1}{2}(a + \frac{a-b}{a+b}\sqrt{a^2+b^2}) - \frac{1}{4}(a + \sqrt{b^2+h^2})$
- $Q[\partial_t] = \frac{1}{2}(a + \frac{a-b}{a+b}\sqrt{a^2+b^2})$

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Schwarzschild solution $(b = h = 0)$

- $\mathrm{d}s^2 = -(1 - a/R)\mathrm{d}t^2 + \dfrac{\mathrm{d}R^2}{1 - a/R} + R^2\mathrm{d}\Omega^2$
- $V_{\text{orbit}} = \sqrt{\dfrac{a}{2R}}$

Hernquist model $(a = h = 0)$

- $\mathrm{d}s^2 = \dfrac{-\mathrm{d}t^2 + \mathrm{d}R^2}{1 + b/R} + R^2\mathrm{d}\Omega^2$
- $e^{2\phi} = \dfrac{1}{1 + b/R}$
- $V_{\text{orbit}} = \sqrt{\dfrac{bR}{2(R+b)^2}}$

F-JNW solution $(h = 0)$

- $\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + A(r)^{-1}\left[\mathrm{d}r^2 + r\left(r - \sqrt{a^2+b^2}\right)\mathrm{d}\Omega^2\right]$
- $A(r) := \left[\mathrm{d}r^2 + r\left(r - \sqrt{a^2+b^2}\right)\mathrm{d}\Omega^2\right]$
- $e^{2\phi} = \left(1 - \dfrac{\sqrt{a^2+b^2}}{r}\right)^{\frac{b}{\sqrt{a^2+b^2}}}$

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Hernquist model $(a = h = 0)$

- $\mathrm{d}s^2 = \dfrac{-\mathrm{d}t^2 + \mathrm{d}R^2}{1 + b/R} + R^2\mathrm{d}\Omega^2$
- $e^{2\phi} = \dfrac{1}{1 + b/R}$
- $V_{\mathrm{orbit}} = \sqrt{\dfrac{bR}{2(R + b)^2}}$

F-JNW solution $(h = 0)$

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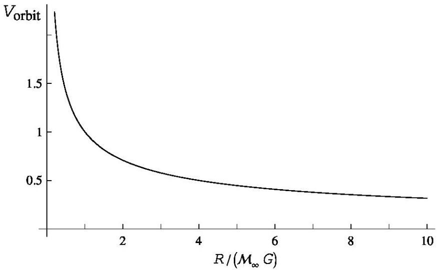

Figure: Schwarzschild

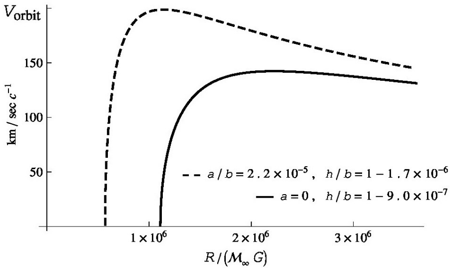

Figure: Main Result

Conclusion

- DFT introduces non-Riemannian background very naturally
- DFT linearized E.O.M had been obtained
- Target space formulation of DFT also involve non-relativistic string theory
- Galilean and Schödinger algebra are induced from DFT under certain background
- Corresponding global conserved charge was computed
- The solution can reproduce series of well-known solutions
- The shape of plot is similar to that of dark matter
- Scalar dilaton and two form field can be treated as candidates of dark matter
- The global conserved charge of DFT might admits negative values
- Gravitational lensing effect is also a very attractive subject

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