More on Rotating Black Holes in D=3 Horava Gravity

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We heard about **LIGO**'s detection of **gravitational waves**.

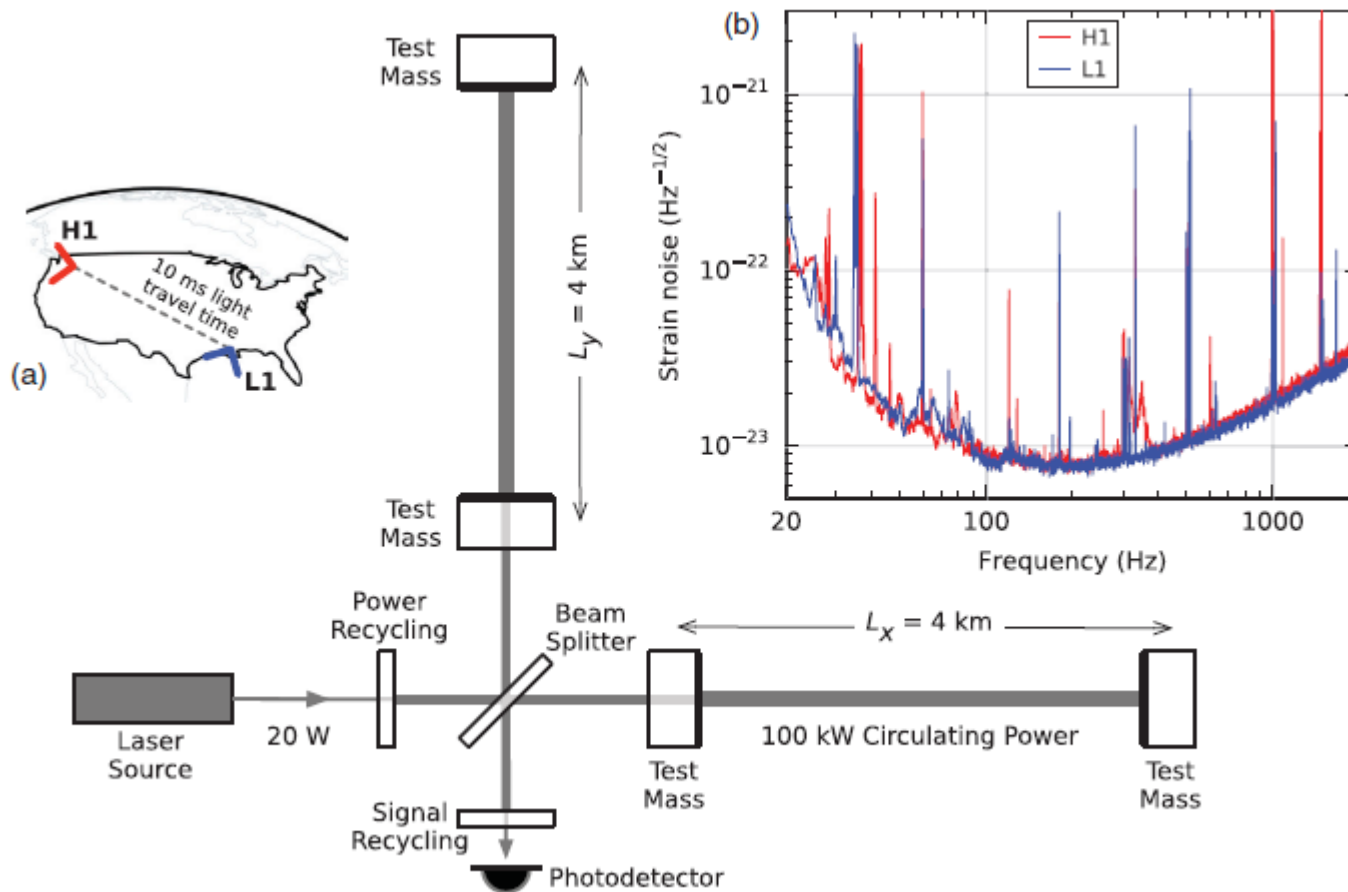## Observation of Gravitational Waves from a Binary Black Hole Merger
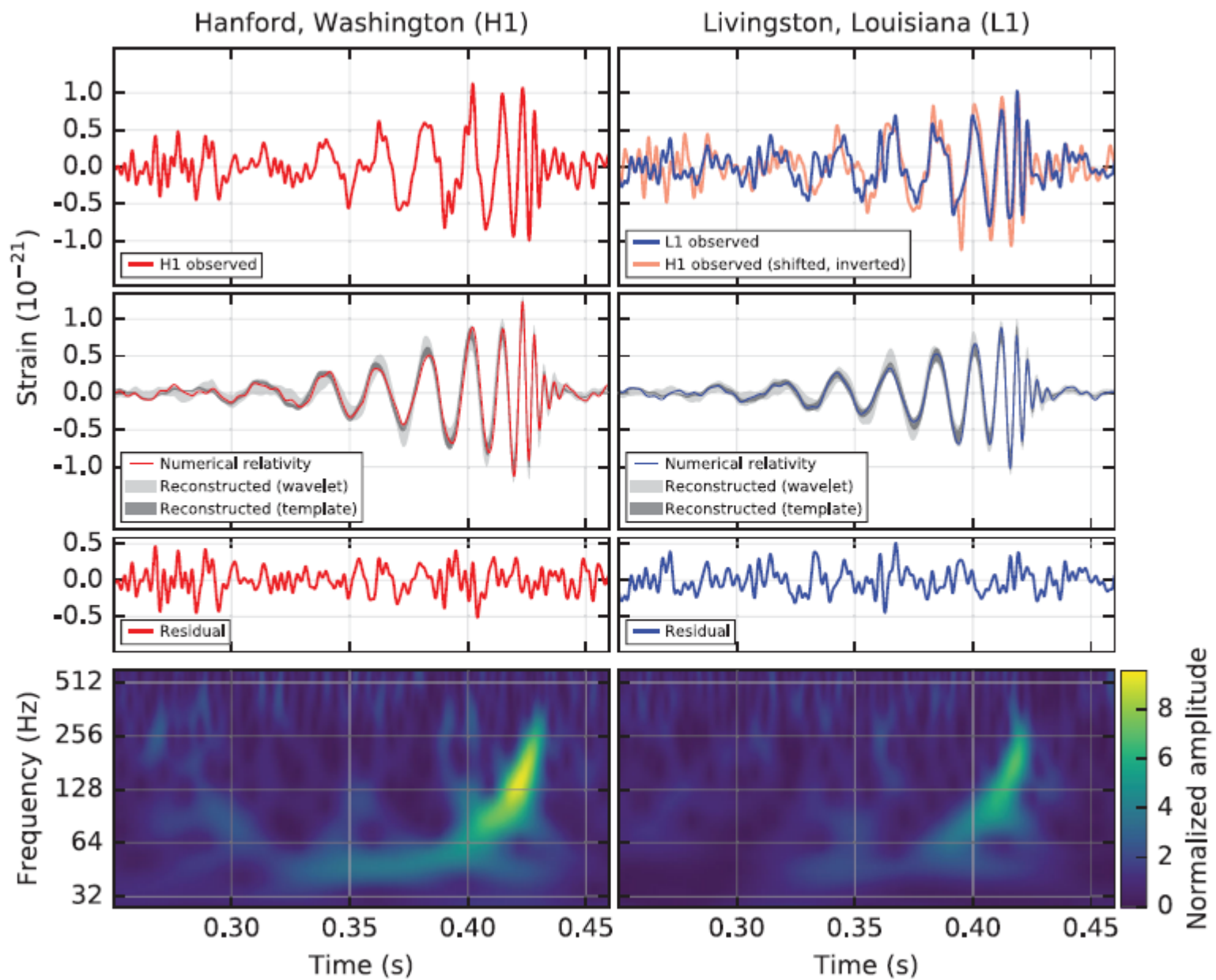
B. P. Abbott et al.[*]

(LIGO Scientific Collaboration and Virgo Collaboration)

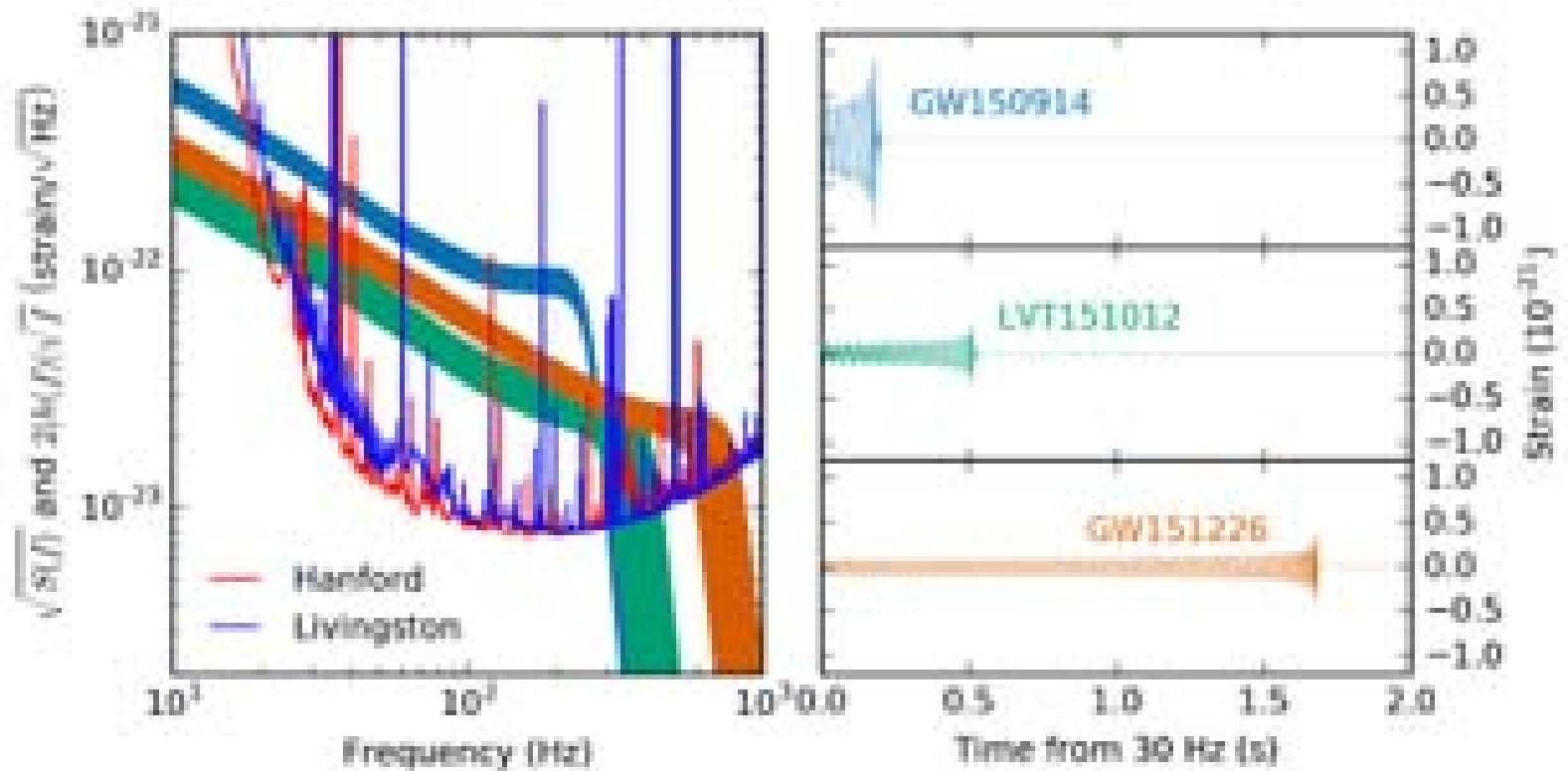(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4}M_{\odot}$ and $29^{+4}_{-4}M_{\odot}$, and the final black hole mass is $62^{+4}_{-4}M_{\odot}$, with $3.0^{+0.5}_{-0.5}M_{\odot}c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

(a)

H1

10 ms light travel time

L1

(b)

Test Mass

$L_y = 4$ km

Test Mass

Power Recycling

Beam Splitter

$L_x = 4$ km

Laser Source

20 W

Signal Recycling

100 kW Circulating Power

Test Mass

Test Mass

Photodetector

Strain noise (Hz$^{-1/2}$)

Frequency (Hz)

H1
L1

Hanford, Washington (H1) — Livingston, Louisiana (L1)

More Detections
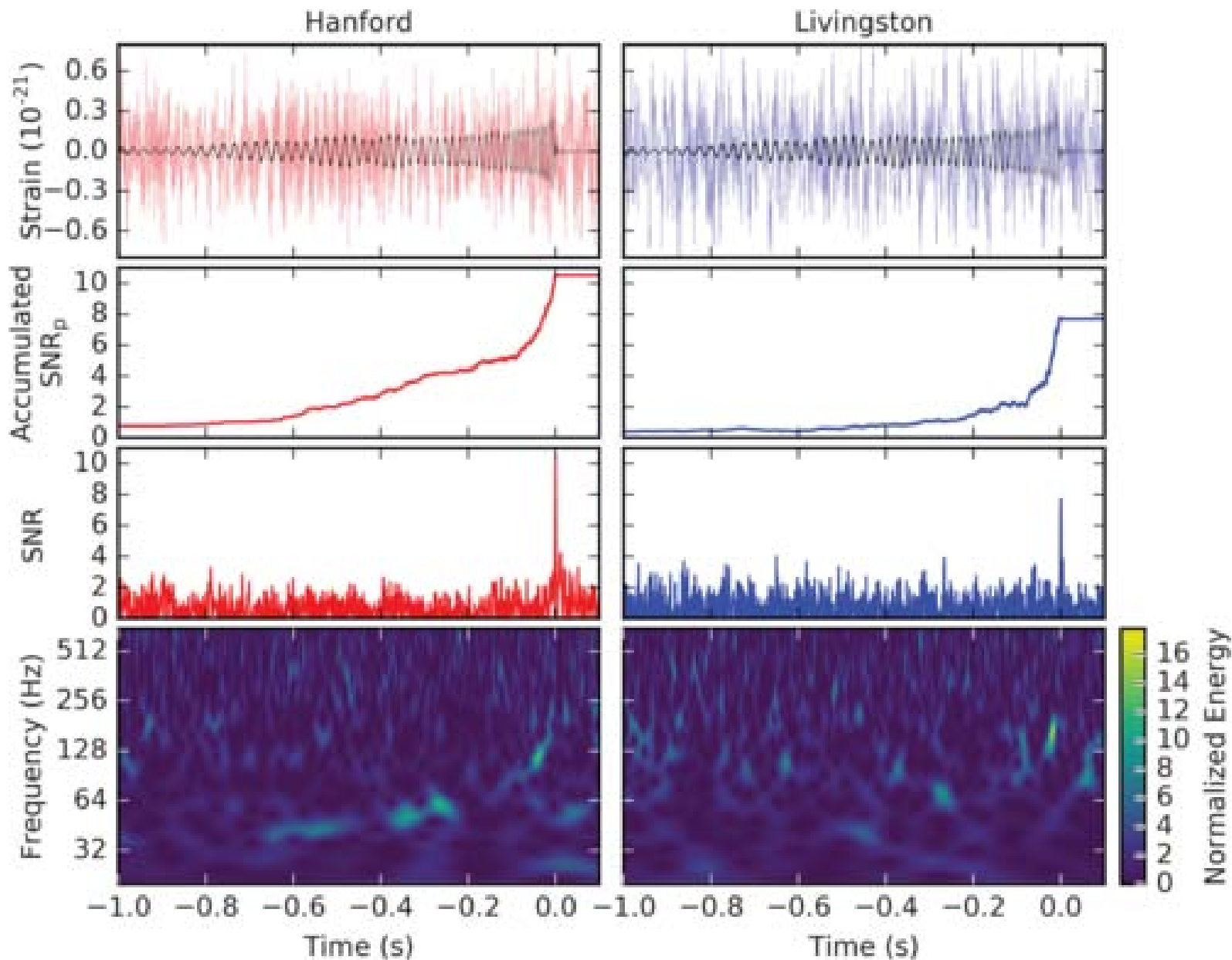
They conclude that these are the results of a ``merger" of two spinning black holes.

There remains some open issues, though.

- **1. Can we distinguish the black holes with other horizon-less compact objects ? [Cardoso. et al., PRL 116, 171101 (2016]**

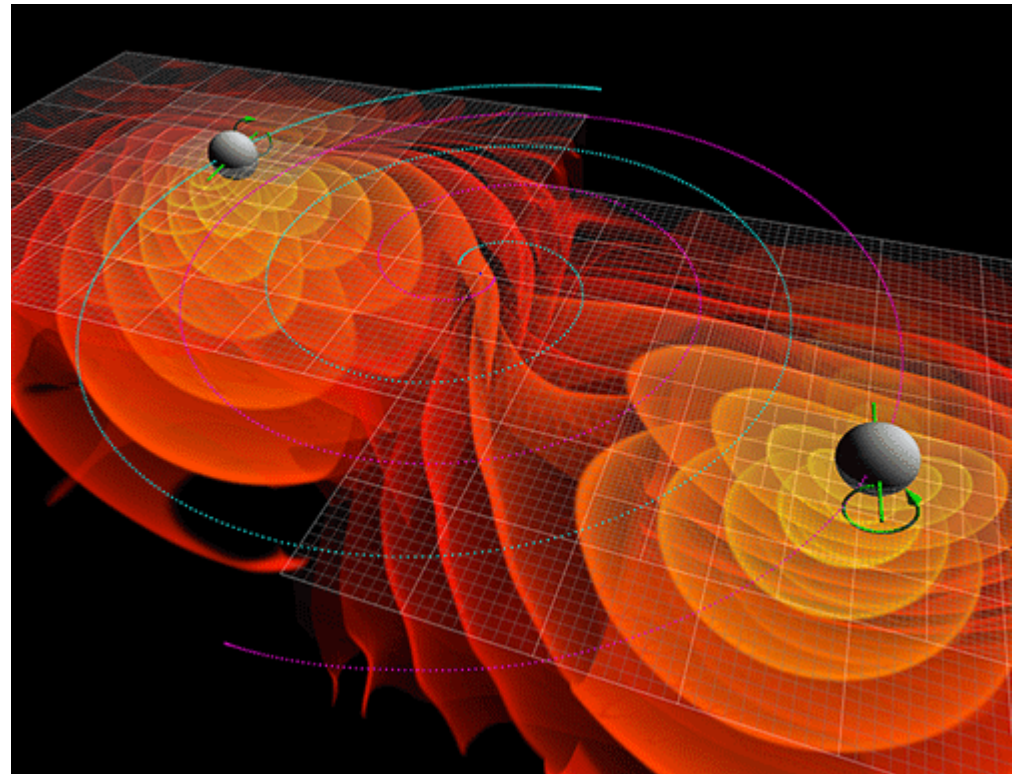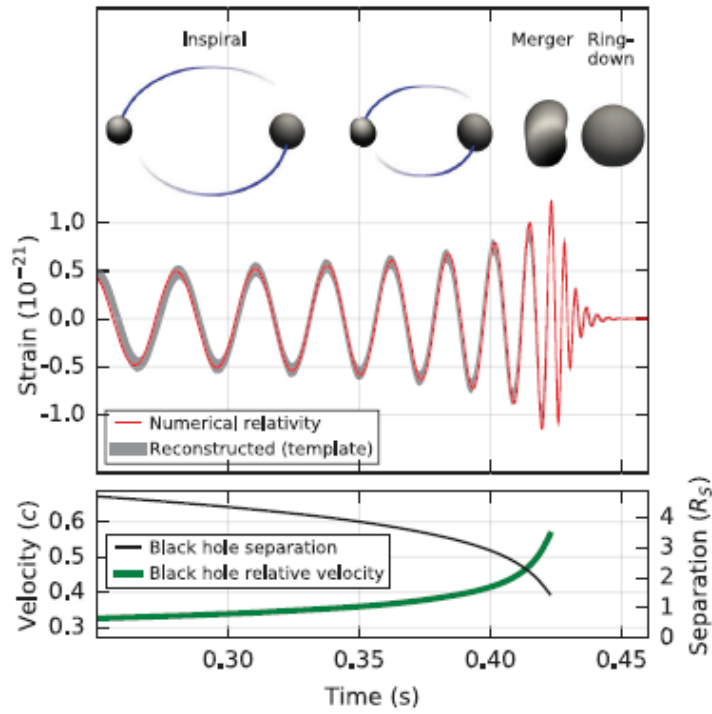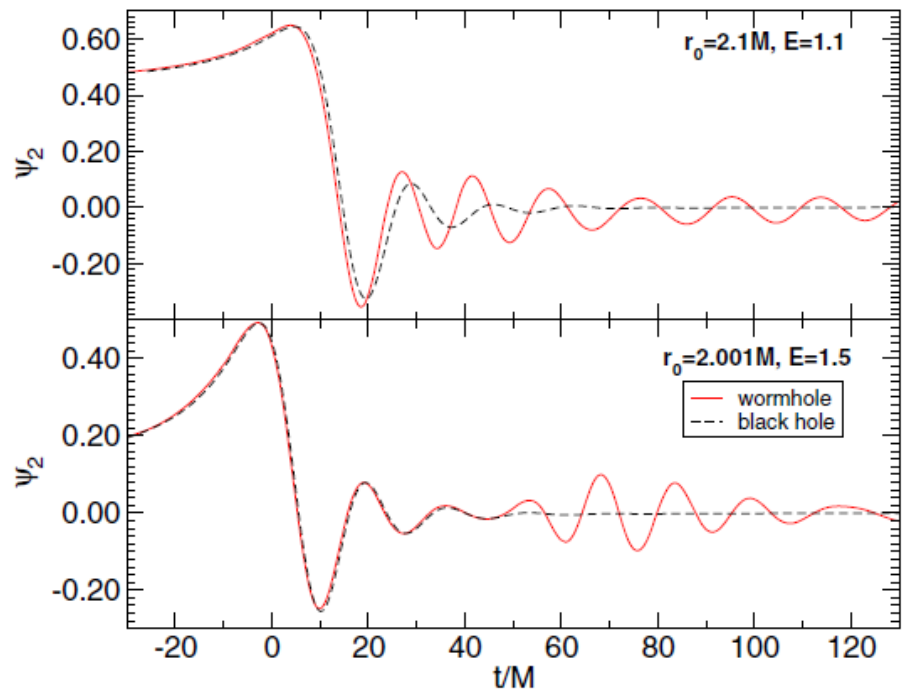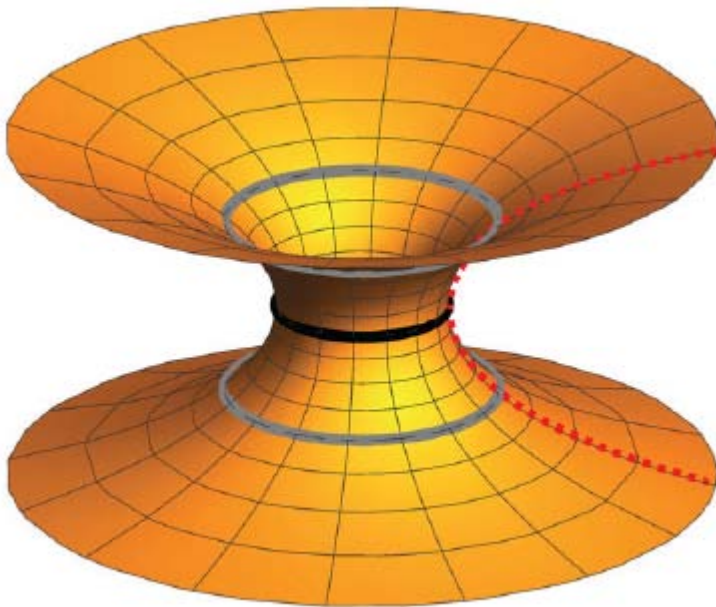- **2. Was there EM radiations from the merging black holes ?**

[V. Connaughton, et.al. APJ. 826 (2016)L6 [arXiv:1602.03920l Cited by 93 records]
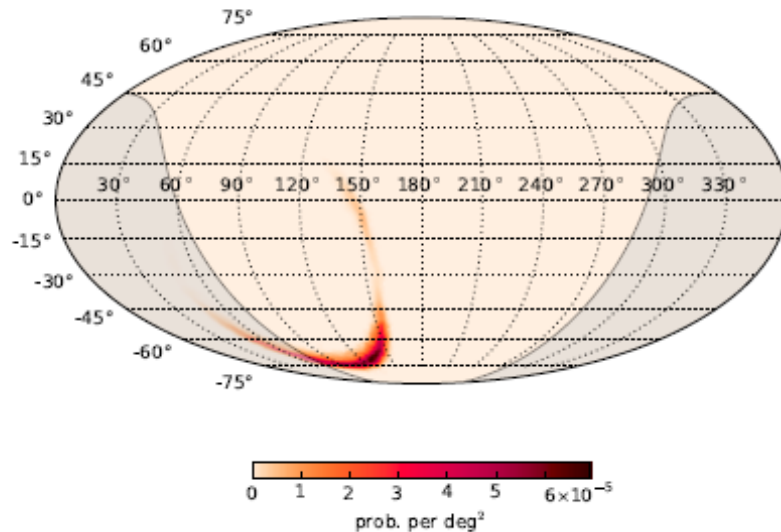
Fig. 1.— Localization map for GW150914, the GW event reported in Abbott et al. (2016). The grey shaded region indicates the region of sky occulted to *Fermi* by the Earth at the time of GW150914. The region not occulted by the Earth contains 75% of the probability of the localization map, with all but 6% of the probability contained in the lower lobe. The entire region was visible to *Fermi* GBM 25 minutes after the GW event was detected.
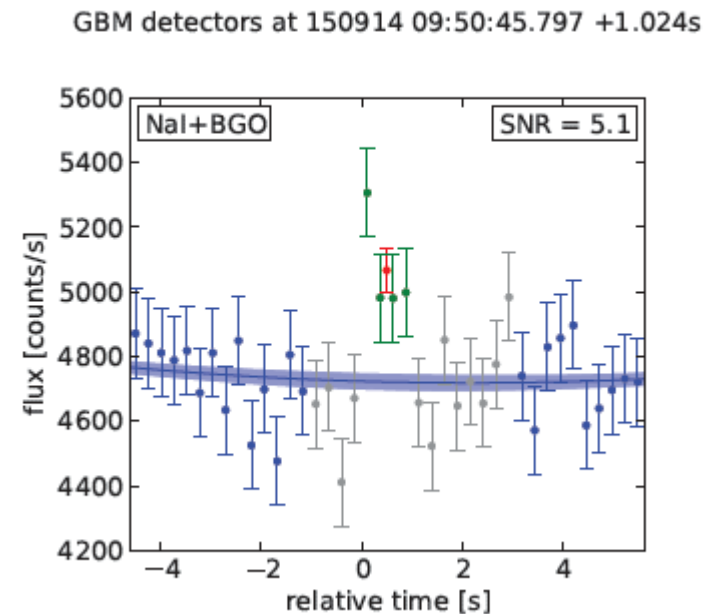
Fig. 2.— Count rates detected as a function of time relative to the start of GW150914-GBM, ~0.4 s after the GW event GW150914, weighted and summed to maximize signal-to-noise for a modeled source. CTIME time bins are 0.256 s wide. The blue data points are used in the background fit.

Date updates

- **But, the Fermi data was not confirmed in later analysis by INTEGRAL telescope and AGILE telescope !!**
  [J. Greiner, et. al. APJ. 827 (2016) L38 [arXiv:1606.00314]]

- **Is there similar signal for GW151226 ?**

But, everyone will agree that this is the **strongest gravity** event that we have observed !

- Actually, LIGO got **two birds in one stone** !

GW150914/151226

- **1. One is, of course, about the first, "direct" detection of gravitational waves.**

- **Cf. Indirect evidence was found in**

Hulse-Taylor's neutron star binary (1975-2005): Agrees with GW radiation in GR !

$$-\left\langle \frac{dE}{dt} \right\rangle = \frac{32G^4 m_1^2 m_2^2 (m_1 + m_2)}{5c^5 a^5 (1 - e^2)^{7/2}} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4 \right)$$
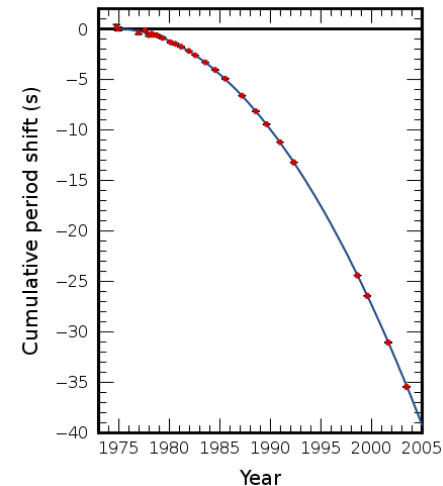
GW150914/151226

- **2. The other is about the first, "direct" detection of (spinning) black holes (if they are).**

- **Cf. Indirect evidences have been known for many years: supermassive black holes at the galaxy centers.**
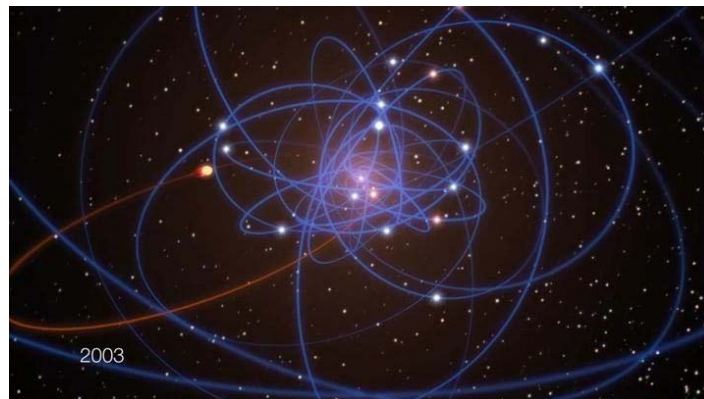
Messages of LIGO

- **1. We open GW astronomy era, beyond EM Wave astronomy !**

The Gravitational Wave Spectrum
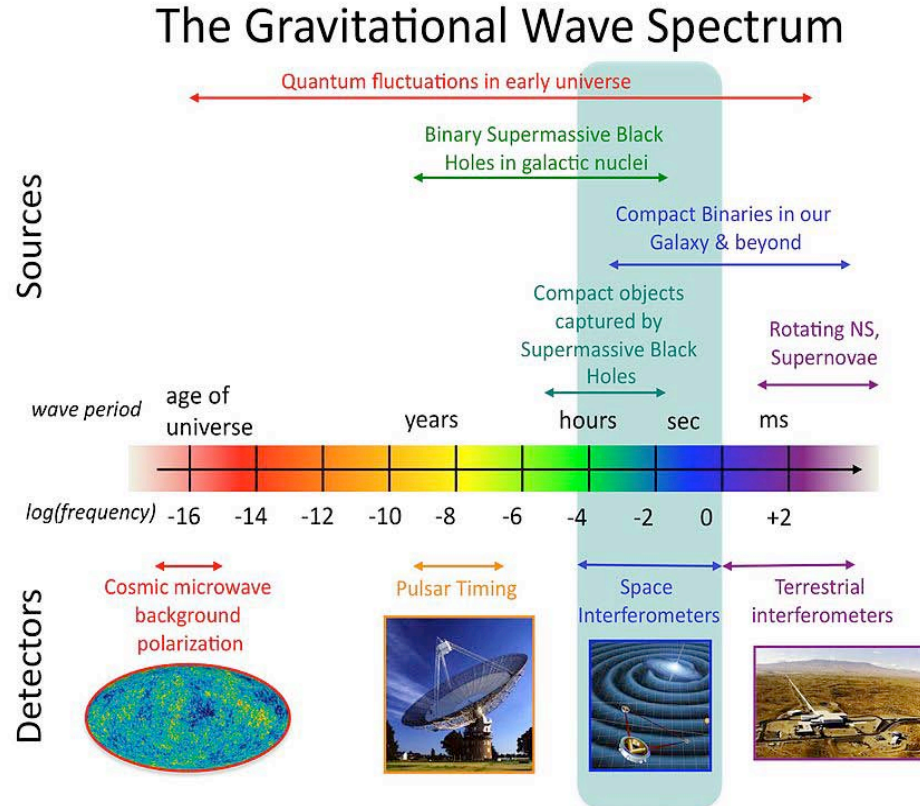
Messages of LIGO

- 2. We open **the strong gravity test era of GR**, beyond **the weak gravity tests in solar system !!**

- 2′. In particular, we open the new era of testing the **black hole physics**.

- Before **LIGO**'s detection, black hole physics has been just an **academic subject.**

- **Now, it is the time we should consider it more seriously !!**

Now, we may ask that "do we understand BH completely from GR ?"

- **Maybe NOT ! Why ??**

GR predicts the existence of black holes.

- **Black holes have horizons which divide the casually connected two regions.**

- **Usually, black holes have singularities inside horizons (Cosmic censorship conjecture).**

- **The metric outside a collapsed object settles down to Kerr sol. with (positive mass) m and angular momentum a (D=4, asymptotically flat): Uniqueness theorem; No hair theorem.**

- When "ordinary" matters (i.e., positive energy density, pressure (weak energy condition), accrete to a black hole, its "horizon area A" increases: Not so strange !

- When one black hole (BH1) falls into another black hole (BH2), the resulting horizon area (BH3) is greater than the sum of areas of BH1 and BH2 (Hawking's area theorem):

- $$A\_3 > A\_1 + A\_2$$

- This is quite non-trivial result !

- **For example, for collision of two rotating black holes, this gives**

$$m_3(m_3 + (m_3{}^2 - a_3{}^2)^{\frac{1}{2}}) > m_1(m_1 + (m_1{}^2 - a_1{}^2)^{\frac{1}{2}}) + m_2(m_2 + (m_2{}^2 - a_2{}^2)^{\frac{1}{2}}).$$

- **If we consider a_1=a_2=0 for simplicity, the energy emitted in gravitational (or other forms of radiation if there is),**

- **m_1+m_2-m_3**

- **is limited by the efficiency (1-2^(-1/2)) ~30% (maximum for m_1=m_2) [Hawking, PRL 1971].**

- **With non-vanishing a_1, a_2, generally, the efficiency of radiation is limited by**
- **(1-2^(-3/2)) ~65%.[Hawking, CMP 1972; I don't know how to prove !]**

- **These all are based on GR.**
- **These seems to be consistent with GW150914/151226 (~5%), within the accuracy.**
- **But, ...**

J. Bekenstein and S. Hawking found that

- **Black holes satisfy the thermodynamics laws. For example,**

- **1st Law:** $dE = \dfrac{\kappa}{8\pi}\, dA + \Omega\, dJ + \Phi\, dQ,$

- **2nd Law:** $\dfrac{dA}{dt} \geq 0.$

- **This is similar to the usual thermodynamics of ordinary thermal systems with**

$$T_{\mathrm{H}} = \dfrac{\kappa}{2\pi}. \qquad S_{\mathrm{BH}} = \dfrac{A}{4}.$$

There was no way to understand this result until

- **Hawking found that BH radiates with the temperature,**

$$T = \frac{\hbar c^3}{8\pi G M k_{\mathrm{B}}} \quad \left(\approx \frac{1.227 \times 10^{23} \text{ kg}}{M} \text{ K} = 6.169 \times 10^{-8} \text{ K} \times \frac{\mathrm{M}_\odot}{M} \right)$$

- **by considering "quantum fields" living on the black hole background (non-dynamical, classical): Hawking radiation.**

- **So, now black hole has entropy !!**

$$S_{\mathrm{BH}} = \frac{kA}{4\ell_{\mathrm{P}}^2} \quad \ell_{\mathrm{P}} = \sqrt{G\hbar/c^3}$$

- So, black hole is a classical (exact) solution of Einstein equation of GR.
- But, it satisfies a thermodynamics-like law which can not be understood at classical level, without quantum effects.
- This means that "the law is originated from classical solutions, but it is also a precursor of quantum theory of gravity (quantum gravity)!! "
- Actually, we do not know how to compute the black hole entropy ! This may be one strong motivation for studying quantum gravity.

Do we know about the quantum gravity, then ?

- Since Einstein's gravity theory can be considered as a field theory, we need to consider the quantization for gravity fields.

- But from usual experience in other field theories, like QED, Standard model, QCD, we need ``renomalizability'' in order that divergences do not appear in physical observables and theory predictions can be compared with experiments.

- So, the better question may be...

Do we know about renormalizable gravity, then ?

- **It is known that Einstein gravity can not be the renormalizable (quantum) gravity.**

- **It is known also that Einstein gravity with higher-curvatures may be renormalizable but there are ghost problems !**

- **Recently(2009 Jan.), Horava gravity was proposed as a renormalizable quantum gravity without ghost problems.**

How much do we know about black hole solutions in Horava gravity ?

- Many black **hole** and black **string** solutions have been found.

- But, **no rotating** solutions have been found, except some special cases (**slowly rotating**).

Now, how to get the (fully) rotating black holes in Horava gravity ?

- **Today, I will show a step towards to the goal.**

- **But, in D=2+1 dimensions, instead of D=3+1 dimensions.**
- **WHY ??**

Why D=2+1 dimensions ?

- **It is simple enough to get some results with less labors than D=3+1.**

- **But it is not too simple to get some trivial results:**

- **There exists black hole (for negative cosmo. const.) even though no gravitons (BTZ black hole).**

This suggests that D=2+1 is a good labaratory of D=3+1 rotating Horava black holes.

- **Actually, it turns out that this is the case.**
- **And one can get the exact D=2+1 rotating black holes which is Horava gravity generalization of BTZ solution.**

Q:Can rotating solution exists ? (inspired by G. Kang)

- **In 4D Kerr black hole, no naked curvature singularity needs (Energy theorem)**

$$M^2 \geq J,$$

- **This is analogous to particle energy bound**

$$E = \sqrt{p^2 + m^2} \geq p$$

  for sub-luminal particle $v < c$. **or equivalently** $m^2 > 0$

Q:Can rotating solution exists (cont'd) ?

- In D=2+1 BTZ black hole, we have similar mass bound, in order that the (conical) singularity is not naked,

$$M \geq J\sqrt{-\Lambda},$$

- But in Horava gravity, there is no absolute speed limit but depends on the momentum (or energy).

- Does this mean that there is no rotating black holes in Horava gravity ?

Actually, there was even a (wrong) no-go theorem ! :

A no-go theorem for slowly rotating black holes in Hořava-Lifshitz gravity
Enrico Barausse (Guelph U.), Thomas P. Sotiriou (SISSA, Trieste & INFN, Trieste). Jul 2012. 5 pp.
Published in **Phys.Rev.Lett. 109 (2012) 181101,**

- **Erratum:** **Phys.Rev.Lett. 110 (2013) no.3, 039902**

- e-Print: **arXiv:1207.6370** **[gr-qc]** Cited by 36 records

Plan

1. Horava gravity: Introduction [P. Horava, arXiv: 0901.3775[PRD]].

2. Non-rotating black holes: Summary

3. Rotating black holes in D=2+1 Horava gravity, revisited

4. Future directions and open problems.

1. Horava gravity: Introduction
[P. Horava, arXiv: 0901.3775[PRD]].

Background:

- The string theory may be a promising candidate for quantum gravity.

- Yet, it is also a rather "large" theory with a huge landscape of the universe.

- Q: Is there any "smaller" framework for the quantum gravity ?

- For example, Yang-Mills theory is complete in QFT already (renormalizable; UV complete): String theory embedding is not necessary.

- **Likewise, is there any (perturbatively) renormalizable quantum gravity theory ?**
- **This idea is, of course, very old, but this can not be realized in Einstein's gravity or its (relativistic) higher-derivative generalizations: There are ghosts, in addition to massless gravitons, and unitarity violation:**

$$\frac{1}{k^2} + \frac{1}{k^2} G_N k^4 \frac{1}{k^2} + \frac{1}{k^2} G_N k^4 \frac{1}{k^2} G_N k^4 \frac{1}{k^2} + \ldots = \frac{1}{k^2 - G_N k^4} = \frac{1}{k^2} - \frac{1}{k^2 - 1/G_N}.$$

Massless gravitons

Ghosts (!)

- **But, for anisotropic (mom.) dimensions,**

$$[\mathbf{x}] = -1, \qquad [t] = -z,$$

the propagator becomes(?)

$$\frac{1}{\omega^2 - \mathbf{k}^2 - G(\mathbf{k}^2)^z}$$

G: Dimensionless coupling

At high mom. k with (z>1), this expands as,

$$\frac{1}{\omega^2 - c^2\mathbf{k}^2 - G(\mathbf{k}^2)^z} = \frac{1}{\omega^2 - G(\mathbf{k}^2)^z} + \frac{1}{\omega^2 - G(\mathbf{k}^2)^z} c^2\mathbf{k}^2 \frac{1}{\omega^2 - G(\mathbf{k}^2)^z} + \ldots$$

Improved UV divergences but no ghosts, i.e., no unitarity problem.

- **Whereas at low momentum k,**

$$\frac{1}{\omega^2 - \mathbf{k}^2 - G(\mathbf{k}^2)^z} = \frac{1}{\omega^2 - \mathbf{k}^2} + \frac{1}{\omega^2 - \mathbf{k}^2} G(\mathbf{k}^2)^z \frac{1}{\omega - \mathbf{k}^2} + \ldots$$

Flow to z=1

The Action Construction:

- **Einstein-Hilbert** action:

$$S_{EH} = \frac{1}{16\pi G_N} \int dx^4 \sqrt{-g^{(4)}} \left(R^{(4)} - 2\Lambda \right)$$

Lorentz invariant !

Lorentz scalars

$$= \frac{1}{16\pi G_N} \int d^4x \sqrt{g}\, N \left\{ \left(K_{ij} K^{ij} - K^2 \right) + R - 2\Lambda \right\}$$

$$R^{(4)}$$

in ADM decomposition

$$ds^2 = -N^2 c^2 dt^2 + g_{ij} \left(dx^i + N^i dt \right) \left(dx^j + N^j dt \right)$$

- **Here, we have used the Gauss-Godacci relation (up to boundary terms)**

$$R^{(4)} = (K_{ij}K^{ij} - K^2) + R$$

Extrinsic curvature of t=constant hypersurface

Intrinsic curvature : 3 curvature

t_final

t

t_initial
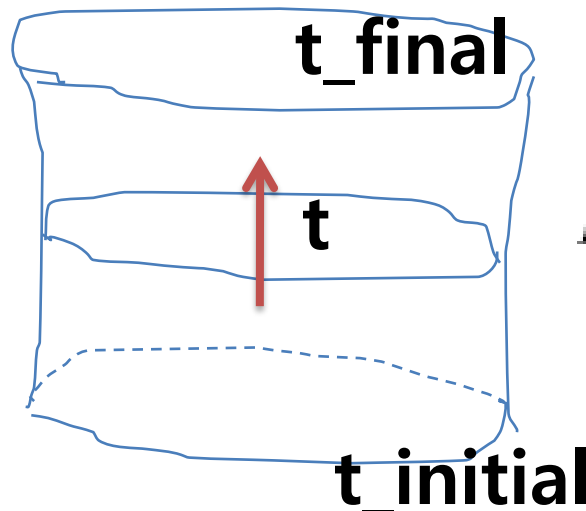
$$K_{ij} = \frac{1}{2N}\left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$$

- **In the an**isotropic (momentum) dimensions,
$$[\mathbf{x}] = -1, \qquad [t] = -z,$$

  we do **not need to** keep the **Lorentz invariant** combinations only. **(Planck unit)**

- For example, we may consider
$$\left(K_{ij} K^{ij} - \lambda K^2 \right) + \beta R$$

, in which the Lorentz symmetry is **explicitly broken** for
$$\lambda \neq 1, \, \beta \neq 1$$

but there is still **Foliation Preserving** diffeomorphisms **(FPDiff)**.

- **However, in order not to introduce higher-time derivatives to avoid the ``possible'' ghost problems, we do not consider "simply" the following terms**

$$(K_{ij}K^{ij})^2, K^4 \cdots$$

but only consider

$$R^2, \quad R_{ij}R^{ij}, \quad \nabla_k R_{ij} \nabla^k R^{ij}, \quad \cdots$$

So, the action can be written as

$$S_{\text{Horava}} = \frac{2}{\kappa^2} \int dt \, d^D\mathbf{x} \, \sqrt{g} N \left(K_{ij}K^{ij} - \lambda K^2 \right)$$

Kinetic term

$$+ \int dt \, d^D\mathbf{x} \, \sqrt{g} N \, V[g_{ij}]$$

Potential term

Dimension counting

- **For an arbitrary spatial dimension D,**

$$[g_{ij}] = 0, \qquad [N_i] = z - 1, \qquad [N] = 0.$$

$$[dt \, d^D \mathbf{x}] = -D - z,$$

$$[\kappa] = \frac{z - D}{2}.$$

Dimensionless coupling for z=D: Power counting renormalizable

$$S_V = \int dt \, d^D \mathbf{x} \, \sqrt{g} N \, V[g_{ij}]$$

(Planck unit)

-D-z

D+z

- So, in **D=3** (3+1 space-time), we need the potential V with **[V]=6**: **6'th**-order **spatial** derivatives with "**dimension-less**" couplings !

- From

$$[\nabla_k] = [\nabla^k] = 1 \quad [R_{ij}] = [R^{ij}] = 2,$$

  we have **large** numbers of possible terms, which are invariant by themselves, like

$$\nabla_k R_{ij} \nabla^k R^{ij}, \qquad \nabla_k R_{ij} \nabla^i R^{jk}, \qquad R\Delta R, \qquad R^{ij} \Delta R_{ij};$$

$$R^3, \qquad R^i_j R^j_k R^k_i, \qquad R R_{ij} R^{ij},$$

  In **D=2** (2+1 dimensions), we need **[V]=4**: 4'th-order spatial derivatives with "**dimension-less**" couplings !

- There are **too many couplings** for explicit computations, though some of them may be constrained by the **stability** and **unitarity**. We need some **pragmatic** way of reducing the number of couplings, in a reliable manner.

- So, Horava adopted **"detailed balance" condition** from the **critical phenomena.**

- **Horava required the potential to be of**

$$S_V = \frac{\kappa^2}{8} \int dt\, d^D x \sqrt{g} N\, E^{ij} \mathcal{G}_{ijk\ell} E^{k\ell},$$

by demanding

D-dimensional Euclidean action

$$\sqrt{g} E^{ij} = \frac{\delta W[g_{k\ell}]}{\delta g_{ij}}$$

for some action W **and** $\mathcal{G}_{ijk\ell}$, **the inverse of De Witt metric**

$$G^{ijk\ell} = \frac{1}{2}\left(g^{ik}g^{j\ell} + g^{i\ell}g^{jk}\right) - \lambda g^{ij}g^{k\ell}$$

Cf. Kinetic part is also given by

$$S = \frac{1}{2} \int dt\, d^D x \sqrt{g} \left\{ \frac{1}{\kappa^2 N} \left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right) G^{ijk\ell} \left(\dot{g}_{k\ell} - \nabla_k N_\ell - \nabla_\ell N_k\right)\right.$$

- **For D=3, W is 3-dimensional Euclidean action.**

- **First, we may consider Einstein-Hilbert action,**

$$W = \frac{1}{\kappa_W^2} \int d^D \mathrm{x} \sqrt{g}(R - 2\Lambda_W).$$

then, this gives 4'th-order spatial derivative potential, with a dimensionful coupling,

$$S_V = \frac{\kappa^2}{8\kappa_W^4} \int dt\, d^D \mathrm{x} \sqrt{g} N \left(R^{ij} - \frac{1}{2} R g^{ij} + \Lambda_W g^{ij} \right) \mathcal{G}_{ijk\ell} \left(R^{k\ell} - \frac{1}{2} R g^{k\ell} + \Lambda_W g^{k\ell} \right).$$

- **So, this is not enough to get 6'th order !!**

- **In 3-dim, we also have a peculiar, 3'rd-derivative-order action, called (gravitational) Chern-Simons action.**

$$W = \frac{1}{w^2} \int_\Sigma \omega_3(\Gamma).$$

$$\omega_3(\Gamma) = \text{Tr}\left(\Gamma \wedge d\Gamma + \frac{2}{3}\Gamma \wedge \Gamma \wedge \Gamma\right) \equiv \varepsilon^{ijk}\left(\Gamma_{i\ell}^m \partial_j \Gamma_{km}^\ell + \frac{2}{3}\Gamma_{i\ell}^m \Gamma_{jm}^\ell \Gamma_{kn}^m\right) d^3\text{x}$$

- **This produces the potential**

$$-\frac{\kappa^2}{2w^4} C_{ij} C^{ij}$$

with the Cotton tensor $C^{ij} = \varepsilon^{ikl}\nabla_k\left(R_\ell^j - \frac{1}{4}R\delta_\ell^j\right)$

- **Then, in total, he got the 6'th-order action**

$$S = \int dt\, d^3\mathrm{x}\, \sqrt{g}\, N \left\{ \frac{2}{\kappa^2} K_{ij} G^{ijk\ell} K_{k\ell} - \frac{\kappa^2}{2} \left[\frac{1}{w^2} C^{ij} - \frac{\mu}{2} \left(R^{ij} - \frac{1}{2} R g^{ij} + \Lambda_W g^{ij} \right) \right] \right.$$
$$\left. \times\, \mathcal{G}_{ijk\ell} \left[\frac{1}{w^2} C^{k\ell} - \frac{\mu}{2} \left(R^{k\ell} - \frac{1}{2} R g^{k\ell} + \Lambda_W g^{k\ell} \right) \right] \right\}.$$

or

$$S = \int dt\, d^3\mathrm{x}\, \sqrt{g}\, N \left\{ \frac{2}{\kappa^2} \left(K_{ij} K^{ij} - \lambda K^2 \right) - \frac{\kappa^2}{2w^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu}{2w^2} \varepsilon^{ijk} R_{i\ell} \nabla_j R_k^\ell \right.$$
$$\left. - \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} + \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \left(\frac{1-4\lambda}{4} R^2 + \Lambda_W R - 3\Lambda_W^2 \right) \right\}.$$

from

$$W = \frac{1}{w^2} \int \omega_3(\Gamma) + \mu \int d^3\mathrm{x}\, \sqrt{g}\, (R - 2\Lambda_W).$$

- Some **improved UV** behaviors, **without ghosts**, are expected, i.e., **renormalizability**

  ⟹ **Predictable** Quantum Gravity **!!(?)**

- But, it seems that the **detailed balance** condition is **too strong** to get **general spacetimes** with an **arbitrary** cosmological constant.

- For example, there is **no Minkowski** , i.e., **vanishing c.c.** vacuum solution ! **(Lu, Mei, Pope '09):** There is **no Newtonian** gravity limit **!!**

- **We need to break the detailed balance but without altering UV behaviors:** It is called, **soft breaking** in IR or **IR modification.**

- **A "soft" breaking of the detailed balance is given by the action :**

$$S_g = \int dt d^3x \sqrt{g} N \left[\frac{2}{\kappa^2} \left(K_{ij}K^{ij} - \lambda K^2 \right) - \frac{\kappa^2}{2\nu^4} C_{ij}C^{ij} + \frac{\kappa^2 \mu}{2\nu^2} \epsilon^{ijk} R^{(3)}_{i\ell} \nabla_j R^{(3)\ell}_{k} \right.$$

$$\left. - \frac{\kappa^2 \mu^2}{8} R^{(3)}_{ij} R^{(3)ij} + \frac{\kappa^2 \mu^2}{8(3\lambda - 1)} \left(\frac{4\lambda - 1}{4} (R^{(3)})^2 - \Lambda_W R^{(3)} + 3\Lambda_W^2 \right) + \frac{\kappa^2 \mu^2 \omega}{8(3\lambda - 1)} R^{(3)} \right]$$

IR modification term

- **It is found that there does exit the black hole which converges to the usual Schwarzschild solution in Minkowski limit, i.e., $\Lambda_W \to 0$ for $\lambda = 1$ (s.t. Einstein-Hilbert in IR) (Kehagias-Sfetsos '09) .**

2. Non-rotating black holes:
Summary

- **The general action with IR-modification is**

$$S = \int dt d^3x \sqrt{g} N \left[\frac{2}{\kappa^2} \left(K_{ij} K^{ij} - \lambda K^2 \right) - \frac{\kappa^2}{2\nu^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu}{2\nu^2} \epsilon^{ijk} R^{(3)}_{i\ell} \nabla_j R^{(3)\ell}_{k} \right.$$

$$\left. - \frac{\kappa^2 \mu^2}{8} R^{(3)}_{ij} R^{(3)ij} + \frac{\kappa^2 \mu^2}{8(3\lambda - 1)} \left(\frac{4\lambda - 1}{4} (R^{(3)})^2 - \Lambda_W R^{(3)} + 3\Lambda_W^2 \right) + \frac{\kappa^2 \mu^2 \omega}{8(3\lambda - 1)} R^{(3)} \right]$$

- **From the ansatz (N^i =0)**

$$ds^2 = -N(r)^2 c^2 dt^2 + \frac{dr^2}{f(r)} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right)$$

- **The equations of motion are**

$$(2\lambda - 1) \frac{(f-1)^2}{r^2} - 2\lambda \frac{f-1}{r} f' + \frac{\lambda - 1}{2} f'^2 - 2(\omega - \Lambda_W)(1 - f - rf') - 3\Lambda_W^2 r^2 = 0 \,,$$

$$\left(\frac{N}{\sqrt{f}} \right)' \left((\lambda - 1) f' - 2\lambda \frac{f-1}{r} + 2(\omega - \Lambda_W) r \right) + (\lambda - 1) \frac{N}{\sqrt{f}} \left(f'' - \frac{2(f-1)}{r^2} \right) = 0$$

- **Let's consider $\lambda = 1$, then I obtain**

$$N^2 = f = 1 + (\omega - \Lambda_W)r^2 - \sqrt{r[\omega(\omega - 2\Lambda_W)r^3 + \beta]}$$

- **For $\omega = 0$, this reduces to LMP's solution (with $\beta = -\alpha^2/\Lambda_W$)**

$$f = 1 - \Lambda_W r^2 - \frac{\alpha}{\sqrt{-\Lambda_W}}\sqrt{r}$$

- **For $\Lambda_W = 0$, this reduces to KS's solution (with $\beta = 4\omega M,$) with an arbitrary paremeter ω ,**

$$f = 1 + \omega r^2 - \sqrt{r[\omega^2 r^3 + 4\omega M]}$$

- **Black hole solution for $\Lambda_W \to 0$ limit ($\lambda = 1$):**

$$ds^2 = -N(r)^2 c^2 dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$
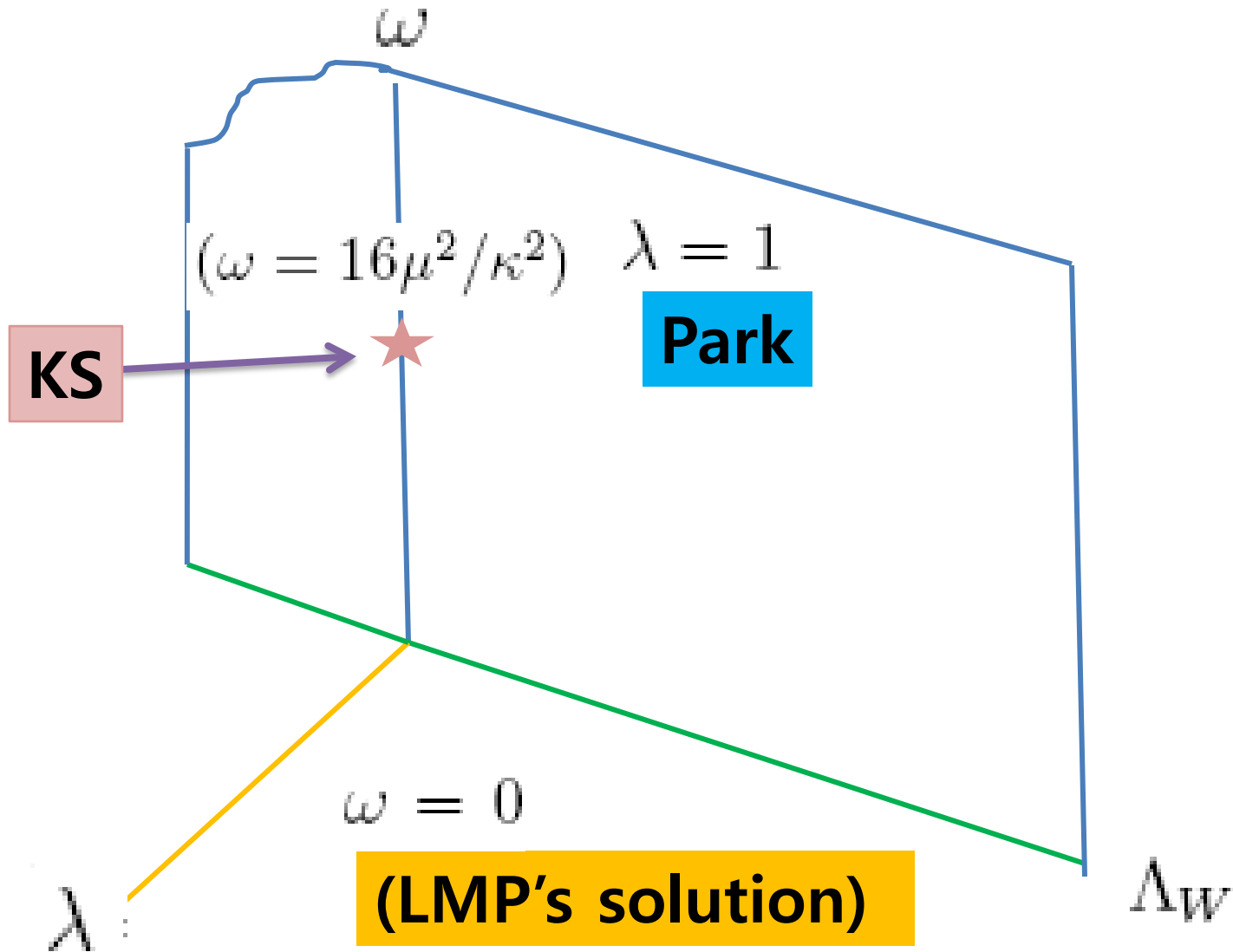
$$N^2 = f = 1 + \omega r^2 - \sqrt{r[\omega^2 r^3 + 4\omega M]}$$

$$\approx 1 - \frac{2M}{r} + \mathcal{O}(r^{-4})$$

~ **Schwarzshild Solution**

: Independently of ω **!!**

$$(G = c \equiv 1)$$

- **So, we obtained the general solution for** $\lambda = 1$ **.**

$(\omega = 16\mu^2/\kappa^2)$ $\quad \lambda = 1$

Park

KS

$\omega = 0$

(LMP's solution)

λ

Λ_W

Remarks

- **There are more general solutions with arbitrary $\lambda > 1/3$ which reduce to LMP's solution for $\omega = 0$. But, there is no "explicit", analytic solution but only in "implicit" forms. (See Kiritsis-Kofinas ('09))**

Other Known Exact Solutions (D=4)

(1) LMP-type:

- **Topological (Charged) B.H.**(Cai,Cao,Ohta; Kim,Kim,Kim),

- **Dyonic (B.H.) Solution** (Colgain, Yavartanoo),

- **Slowly Rotating B.H.** (Ghodsi;Aliev,Senturk)

- **...**

(2) KS-type:

- **Slowly Rotating B.H.** (Lee,Kim,Myung)

Unknown Exact Solutions (D=4)
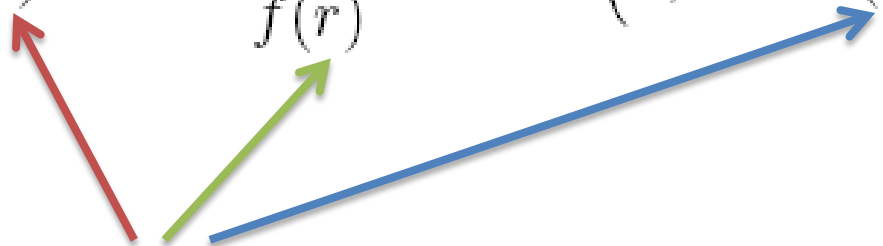
In IR-modified Horava gravity,

- **Rotating Solution ?**
- **Topological (Charged/Dyonic) B.H. ? (Cao, Park)**
- **Black String ? (Cho, Kang !?)**
- **...**

3. **Rotating** black holes in D=2+1 Horava gravity, **revisited**

- **The D=2+1 renormalizable action is**

$$S = \frac{1}{\kappa} \int dt\, d^2x \sqrt{g}\, N \left(K_{ij}K_{ij} - \lambda K^2 + \xi R + \alpha R^2 - 2\Lambda \right)$$

- **From the ansatz**

$$ds^2 = -N^2(r)c^2 dt^2 + \frac{1}{f(r)} dr^2 + r^2 \left(d\phi + N^\phi(r)dt \right)^2$$

Due to circular symmetry in 2-dim. space

Cf. In higher than D=2+1, this simplicity does not happen.

Cf. Renormalizability in 3D Lorentz invariant actions.

- **Topologically massive gravity (TMG):** (1) may be renormalizable but **no proof** beyond one-loop **(Deser,Yang('90))** (2) no analogue in 4D.

- **New massive gravity (NMG, BHT):** ghost (unitarity) problem and renormalization are not compatable **(Muneyuki, Ohta (2012))**

The reduced action:

$$\mathcal{L} = \frac{2\pi}{\kappa}\frac{N}{\sqrt{f}}\left[\frac{fr^3\left(N^{\phi'}\right)^2}{2N^2} - \xi f' + \alpha\frac{f'^2}{r} - 2\Lambda r\right]$$

By varying $N, N^\phi, \text{and } f,$ **one can obtain the equations of motions.**

Eqs. of motions.

$$-\frac{fr^3(N^{\phi\prime})^2}{2N^2} - \xi f' + \alpha\frac{f'^2}{r} - 2\Lambda r = 0,$$

$$\left(\frac{\sqrt{f}}{N}r^3 N^{\phi\prime}\right)' = 0,$$

$$\left(\frac{N}{\sqrt{f}}\right)'\left(2\alpha\frac{f'}{r} - \xi\right) + 2\alpha\frac{N}{\sqrt{f}}\left(\frac{f''}{r} - \frac{f'}{r^2}\right) = 0.$$

- **These coupled Eqs. can be exactly solved !**

Rotating Black Hole Solutions
(Corrected [2016])

$$f = -\mathcal{M} + \frac{br^2}{2}\left[1 - \sqrt{a + \frac{c}{r^4}} + \sqrt{\frac{c}{r^4}}ln\left(\sqrt{\frac{c}{ar^4}} + \sqrt{1 + \frac{c}{ar^4}}\right)\right],$$

$$\frac{N}{\sqrt{f}} \equiv W = 1/\sqrt{1 + \frac{c}{ar^4}},$$

$$N^\phi = -\frac{\mathcal{J}}{2}\sqrt{\frac{a}{c}}ln\left[\sqrt{\frac{c}{ar^4}} + \sqrt{1 + \frac{c}{ar^4}}\right]$$

Two integration constants

- **with**

$$a = 1 + \frac{8\alpha\Lambda}{\xi^2},$$

$$b = \frac{\xi}{2\alpha},$$

$$c = \frac{2\alpha\bar{J}^2}{\xi^2}.$$

Allowed ranges of parameters:

- **For the real-valued metric functions**

$$f, \ N, \text{ and } N^\phi$$

$$\Longrightarrow \qquad a, c \geq 0,$$

- $\qquad$ **or** $\quad \dfrac{8\alpha\Lambda}{\xi^2} \geq -1 \qquad \alpha \geq 0,$

For large r and small α

$$f = -\frac{\Lambda}{\xi}r^2\left(1 - \frac{2\alpha\Lambda}{\xi^2}\right) - \mathcal{M} + \frac{\mathcal{J}^2}{4r^2\xi}\left(1 - \frac{4\alpha\Lambda}{\xi^2}\right) - \frac{\alpha\mathcal{J}^4}{24\xi^3}\frac{1}{r^6} + \mathcal{O}(\alpha^2, r^{-10}),$$

$$W = 1 - \frac{\alpha\mathcal{J}^2}{\xi^2}\frac{1}{r^4} + \mathcal{O}(\alpha^2, r^{-8}),$$

$$N^\phi = -\frac{\mathcal{J}}{2r^2} + \frac{\alpha\mathcal{J}^3}{6\xi^2}\frac{1}{r^6} + \mathcal{O}(\alpha^2, r^{-10}).$$

- **In the limit $\alpha \to 0$, the solution reduces to BTZ black hole sol (with $\xi = 1$) :**

$$N^2_{BTZ} = f_{BTZ} = -\frac{\Lambda}{\xi}r^2 - \mathcal{M} + \frac{\mathcal{J}^2}{4r^2\xi}, \quad N^\phi_{BTZ} = -\frac{\mathcal{J}}{2r^2}.$$

Curvature Invariants:

$$R = -\frac{f'}{r} = -b\left(1 - \sqrt{a + \frac{c}{r^4}}\right)$$

$$K^{ij}K_{ij} = \frac{r^2}{2W^2}\left(N^{\phi'}\right)^2$$

$$= \frac{\mathcal{J}^2}{2r^4}$$

(Point) singularity at r=0 !

No Ring singularity !! (cf. 4D Kerr sol.)

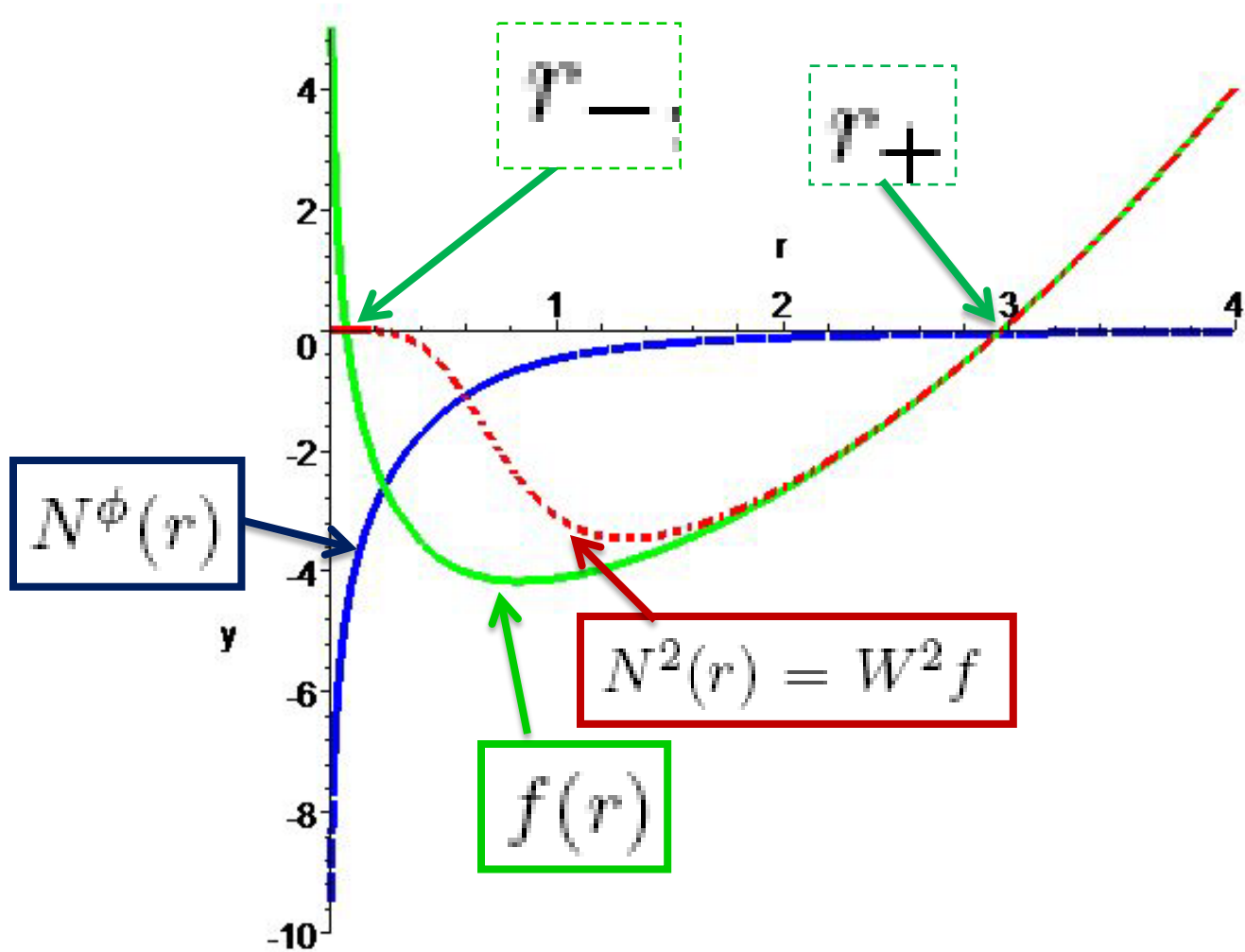# Cf. BTZ in Einstein gravity.

- **No** curvature singularity:

$$R^{(3)} = R + K^{ij}K_{ij} - K^2 \underbrace{- f'/r - f''} = 6\Lambda.$$

Boundary term in **Einstein** action.

But, important in curvature invariants to cancel the **unphysical** singularity in R and $K^{ij}K_{ij}$!

Horizon Structure

$r-$
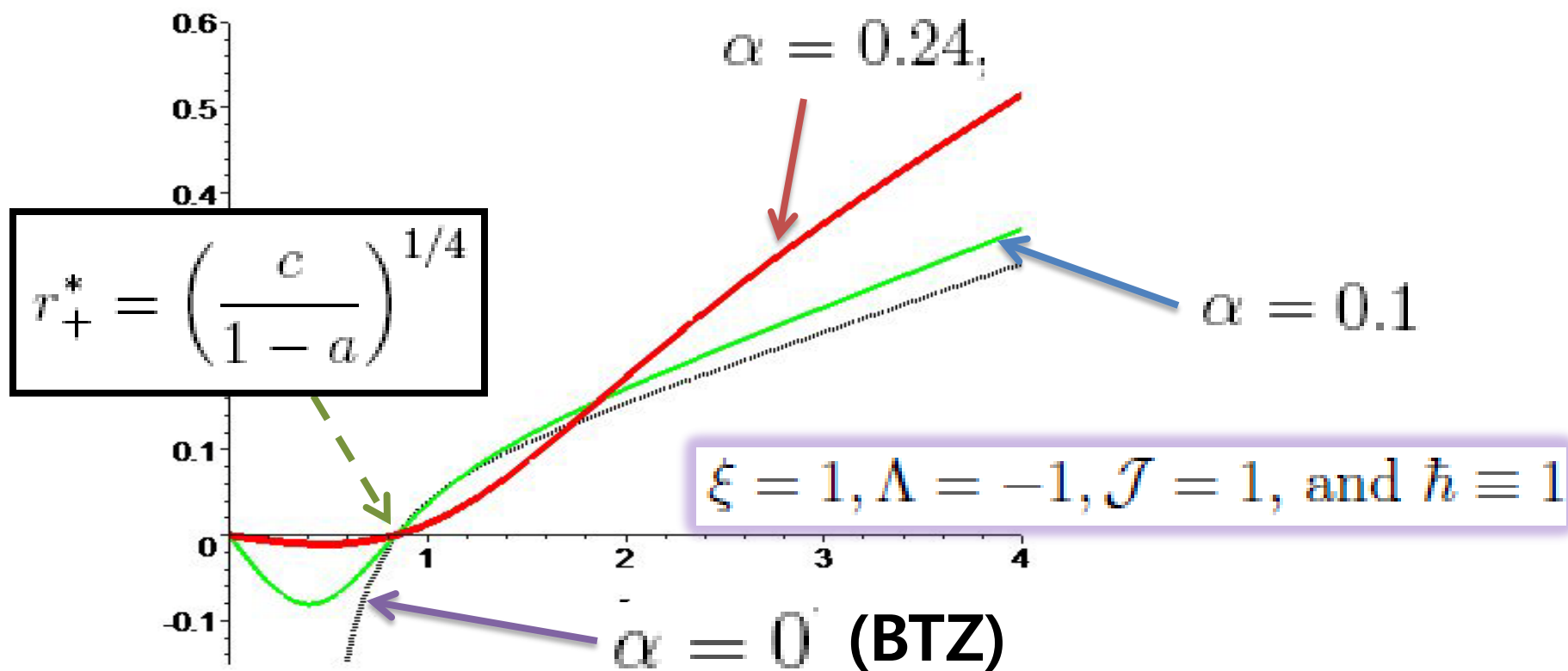
r_+

$N^\phi(r)$

$N^2(r) = W^2 f$

$f(r)$

$(\xi = 1, \Lambda = -1, \mathcal{M} = 5, \mathcal{J} = 1, \alpha = 0.1)$

Hawking Temperature

$$T_+ = \frac{\hbar (Wf')|_{r_+}}{4\pi}$$

$$= \frac{\hbar}{4\pi} b r_+ \left(1 - \sqrt{a + \frac{c}{r_+^4}} \right)$$

$$r_+^* = \left(\frac{c}{1-a} \right)^{1/4}$$

$\alpha = 0.24,$

$\alpha = 0.1$

$\xi = 1, \Lambda = -1, \mathcal{J} = 1,$ and $\hbar \equiv 1$

$\alpha = 0$ **(BTZ)**

Mass and Angular momentum

- **Canonical** mass and angular momentum are defined as the **time translation and angular rotation generators** at **the infinite boundary**, by the boundary action B

$$S_{total} = S + B,$$
$$B = (t_2 - t_1)(-W(\infty)M + N^\phi(\infty)J)$$

- **Such that**

$$\delta S_{total} = (EOM) + (\delta S)(\infty) + \delta B$$

$$M = \frac{2\pi\xi\sqrt{a}}{\kappa}\bar{M}, \qquad J = \frac{2\pi\xi}{\kappa}\bar{J}.$$
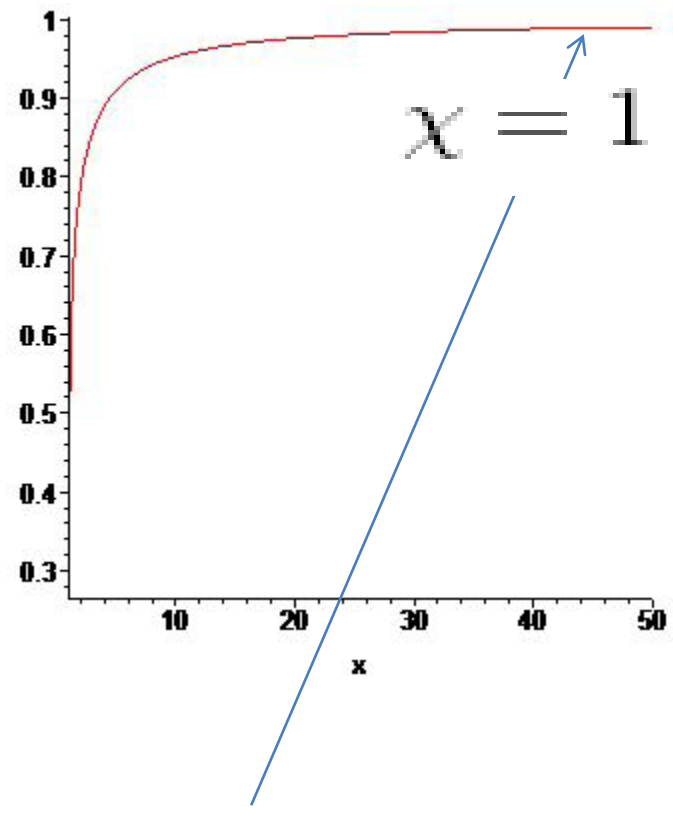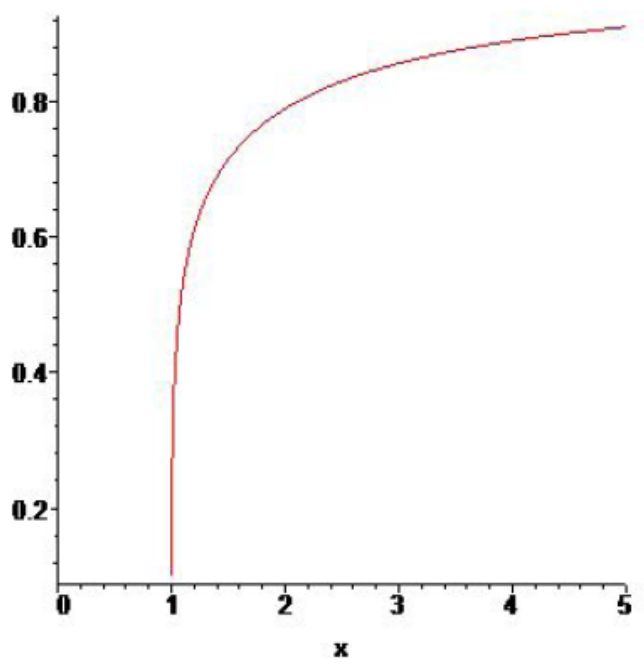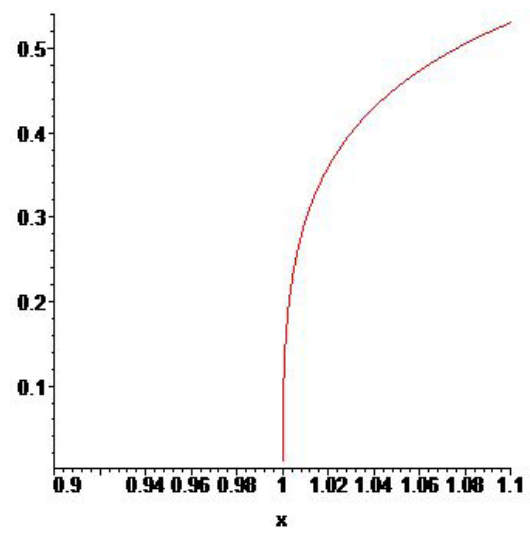
Mass bounds (Energy theorem)

$$M \geq J\sqrt{-\Lambda}\,\chi(x),$$

$$\chi(x) = \sqrt{x^2 - 1}\, ln\left(\frac{1}{\sqrt{x^2 - 1}} + 1\right)$$

$$x = \xi/(\sqrt{-8\Lambda\alpha})$$

Mass Bounds work still for each x !!

$\chi(x)$

$x = 1$

$$M \geq J\sqrt{-\Lambda}$$

BTZ black hole

The first law of thermodynamics ?

- **Let's consider**

$$dM = AdJ + Bdr_+$$

- **With**

$$A = \Omega_+ = -N^\phi|_+,$$

$$B = \frac{\pi\xi^2}{\kappa\alpha}r_+\sqrt{a}\left(1 - \sqrt{a + \frac{c}{r_+^4}}\right).$$

The first law of thermodynamics ?

- **But, one can not get the usual first law (!!)**

$$dM = T_+ dS + \Omega_+ dJ$$

- **with the usual Hawking temperature and the chemical potential**

$$T_+ = \frac{\hbar (W f')|_{r_+}}{4\pi}$$

$$= \frac{\hbar}{4\pi} b r_+ \left(1 - \sqrt{a + \frac{c}{r_+^4}} \right)$$

$$\Omega_+ = -N^\phi|_+$$

Non-integrability of Entropy.

$$dS \equiv \partial_{r_+} S \ dr_+ + \partial_J S \ dJ$$

$$\partial_J \partial_{r_+} S - \partial_{r_+} \partial_J S = \frac{8\pi\alpha J}{\sqrt{a}r_+^4 \sqrt{1 + c/ar^4}}.$$

Entropy is not integrable by non-relativistic higher curvature corrections, in company with J !!

;Entropy is integrable asympotically only !

Summary of Properties

- 1. **Point** **curvature singularity at r=0:** **Contrast to Ring singularity in 4D Kerr in Einstein gravity.**

- 2. **The usual first law does "not" work: This seems to signal the absence of absolute horizon due to non-relativistic nature of Horava gravity in UV.**

- 3. **In the limit** $\alpha \to 0$ **, the solution reduces to BTZ black hole:**

$$N^2_{BTZ} = f_{BTZ} = -\frac{\Lambda}{\xi}r^2 - \bar{M} + \frac{\bar{J}^2}{4r^2\xi}, \qquad N^\phi_{BTZ} = -\frac{\bar{J}}{2r^2}$$

Summary of Properties, cont'd

- **4. Mass bounds still work for each theory (** $x = \xi/(\sqrt{-8\Lambda\alpha})$ **), but in a modified form.**

Discussion

- **The Hawking temperature implies the Hawking radiation: So we have "Hawking radiation without black hole entropy" !**

- **cf. Similar situation in analogue black hole (Visser('98))**

- **In our case, this seems to be a genuine effect of Lorentz-violating gravity.**

4D rotating Horava black hole ?

- **Work in progress...**

4. Future directions and open problems.

- We need to know about "4D rotating black hole solutions" in order to compute something which can be compared with LIGO or future data.

- On the theory side, we need to understand the "full" symmetry of Horava theory and how GR's Diff symmetry are recovered in IR.

- Also, we need to understand the concept of the horizons or universal horizons in our Lorentz violating gravity.

Future directions and open problems.

- **We need the rigorous proof of renomalizability: This is in slow progress (3D, …)**

- **Cf. Yang-Mills theory, Weinberg-Salam model, …**

Thank you !!