Hadron structure inferred from local QCD current

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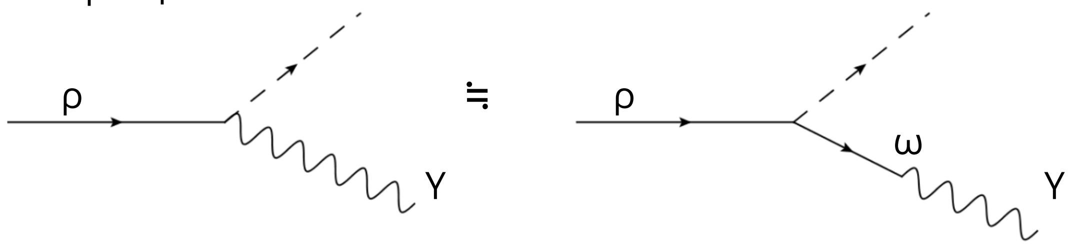
Photon as final state

- External photon can be replaced with vector meson

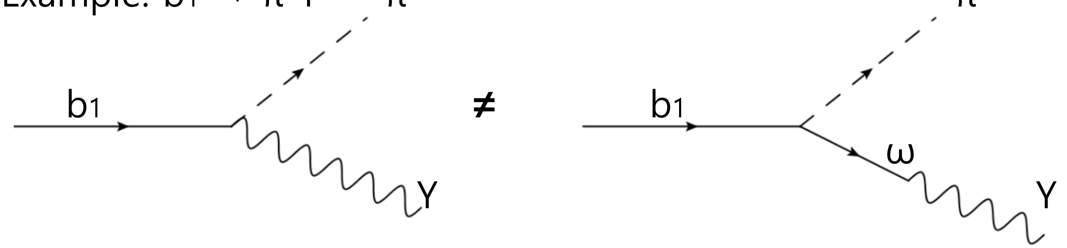
Example: $\rho \to \pi$ Y



Effective Lagrangian in VMD hypothesis explains well in low energy regime

- Exceptional phenomenon

Example: $b_1 \to \pi$ Y



$\Gamma(b_1 \to \pi Y)$ = 230 KeV (experiment) $\Gamma(b_1 \to \pi Y)$ = 30 KeV (VMD scenario)

For b_1 decay, VMD hypothesis does not work well

Current-field identity

- Preliminary: charged vector meson (ρ) Lagrangian

$$\mathcal{L} = -\frac{1}{4} f_{\mu\nu}^2 - \frac{1}{2} m_0 \rho^2 + \mathcal{L}_m(\psi, D_\nu \psi, f_{\mu\nu}) - \frac{1}{4} F_{\mu\nu}^2$$

$$f_{\mu\nu}^a = \partial_\mu \rho_\nu^a - \partial_\nu \rho_\mu^a + g_0 f^{abc} \rho_\mu^b \rho_\nu^c$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$\hat{\rho}_\mu^0 \equiv \rho_\mu^3 + (e_0/g_0) A_\mu, \; \hat{\rho}_\mu^\pm \equiv \rho_\mu^\pm$

ρ can be coupled with U(1) gauge

Equations of motion

$$\partial^\mu F_{\mu\nu} = -\frac{\delta \mathcal{L}}{\delta \hat{\rho}_\mu^0} \frac{\delta \hat{\rho}_\mu^0}{\delta A_\nu} = -\frac{e_0}{g_0} \frac{\delta \mathcal{L}}{\delta \hat{\rho}_\nu^0} = -e_0 \left(\frac{m_0^2}{g_0} \right) \rho_\nu^0,$$

$$\partial^\mu \hat{f}_{\mu\nu}^a = g_0 J_\nu^{\rho;a} + m_0^2 \hat{\rho}_\nu^a,$$

With field redefinition $\phi_\mu^a \equiv (m_0^2/g_0) \hat{\rho}_\mu^a = (1/g_0) \partial^\nu \hat{f}_{\nu\mu}^a - J_\mu^{\rho,a}$

$$[\phi_i^a(r,t), \phi_j^b(r',t)] = 0,$$

$$[\phi_0^a(r,t), \phi_0^b(r',t)] = i f^{abc} \delta^3(r - r') \phi_0^c(r,t),$$

$$[\phi_0^a(r,t), \phi_j^b(r',t)] = i f^{abc} \delta^3(r - r') \phi_j^c(r,t) + i \left(m_0^2/g_0^2 \right) \delta^{ab} \partial_{r_j} \delta^3(r - r'),$$

ρ can be regarded as external source of E.M. field
→ Photon can receive effective mass from intermediate ρ states

Axial anomaly

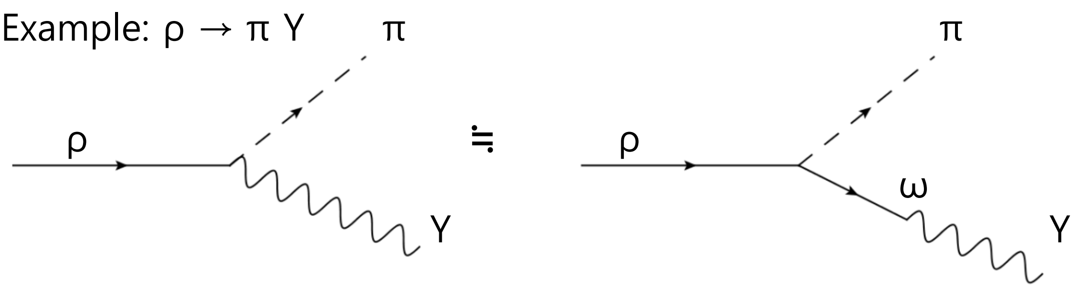
- Nonzero axial divergence (Wess-Zumino)

$$-G_i = \frac{1}{4\pi^2}\epsilon_{\mu\nu\sigma\tau}\,\mathrm{tr}\left[\frac{\lambda_i}{2}\left\{\frac{1}{4}V_{\mu\nu}V_{\sigma\tau} + \frac{1}{12}A_{\mu\nu}A_{\sigma\tau} + \frac{2}{3}i(A_\mu A_\nu V_{\sigma\tau} + A_\mu V_{\nu\sigma}A_\tau + V_{\mu\nu}A_\sigma A_\tau) - \frac{8}{3}A_\mu A_\nu A_\sigma A_\tau\right\}\right]$$

This nontrivial divergence satisfies QCD symmetry and allows anomalous interaction

$$\mathscr{L}_{VV\phi} = -g_{VV\phi}\epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}(\partial^\mu V^\nu \partial^\alpha V^\beta \tilde{\phi}) \quad (\epsilon_{0123} = +1)$$

Example: ρ → π Y



All the external legs are vector, axial-vector, and pesudo scalar
Parity-even spin-1 mesonic state can not described in chiral representation

$\mathbf{b_1}$ in $\left(\frac{1}{2},\frac{1}{2}\right) \oplus \left(\frac{1}{2},\frac{1}{2}\right)$ representation

- Interpolating current in local tensor representation

$$b_1[1^+(1^{+-})] \rightarrow \frac{1}{2}\epsilon_{ijk}\langle 0|\bar{q}T^a\sigma_{ij}q|b_1(p,\lambda)\rangle = if^T_{b_1^a}\epsilon_{ijk}(-\epsilon_k^{(\lambda)}p_0).$$

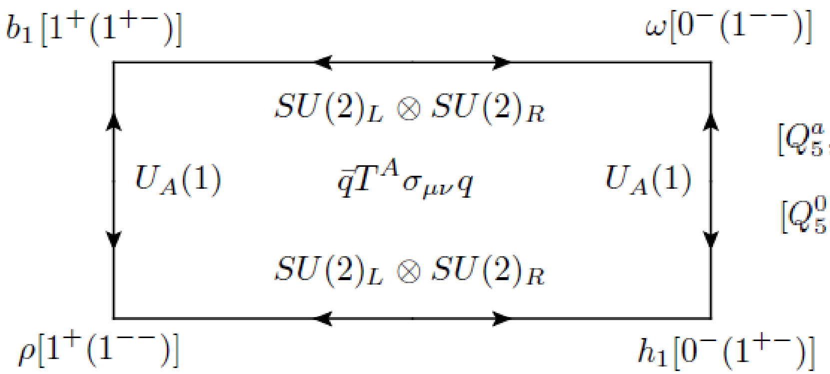
The other vector mesons in tensor bilinear

$$\rho[1^+(1^{--})] \rightarrow \langle 0|\bar{q}T^a\sigma_{0k}q|\rho(p,\lambda)\rangle = if^T_{\rho^a}(-\epsilon_k^{(\lambda)}p_0),$$

$$\omega[0^-(1^{--})] \rightarrow \langle 0|\bar{q}T^0\sigma_{0k}q|\omega(p,\lambda)\rangle = if^T_{\omega}(-\epsilon_k^{(\lambda)}p_0),$$

$$h_1[0^-(1^{+-})] \rightarrow \frac{1}{2}\epsilon_{ijk}\langle 0|\bar{q}T^0\sigma_{ij}q|h_1(p,\lambda)\rangle = if^T_{h_1}(-\epsilon_k^{(\lambda)}p_0),$$

If $U(1)_A$ and $SU(2)_L \times SU(2)_R$ symmetries exist, all the vector mesons are in $U(2)_L \times U(2)_R$



$$[Q_5^a, J_k^{b_1,b}(x)] = -i\delta^{ab}J_k^{\omega}(x),\ [Q_5^a, J_k^{\omega}(x)] = iJ_k^{b_1,a}(x),$$

$$[Q_5^0, J_k^{\omega}(x)] = -iJ_k^{h_1}(x),\ [Q_5^a, J_k^{\rho,b}(x)] = i\delta^{ab}J_k^{h_1}(x),$$

Soft pion breaking

- Example: field transformation and leaking charge

$$\psi_A(\vec{x},t) \to \psi_A(\vec{x},t) - i\Lambda_i[Q^i(t), \psi_A(\vec{x},t)]$$
$$= \psi_A(\vec{x},t) - i\Lambda_i M^i_{AB}\psi_B(x,t),$$

$$\mathcal{L} \to \mathcal{L} - (\partial^\alpha \Lambda_i) J^i_\alpha(\vec{x},t) - \Lambda_i(\partial^\alpha J^i_\alpha(\vec{x},t))$$

If the symmetry is broken

$$\partial^\alpha J^i_\alpha(\vec{x},t) = i[Q^i(t), u(\vec{x},t)] = -i[Q^i(t), \mathcal{H}(\vec{x},t)]$$

Leaking charge flow

$$\frac{dQ^i(t)}{dt} = \int d^3x \partial^\alpha J^i_\alpha(\vec{x},t) = -i[Q^i(t), \int d^3x \mathcal{H}(\vec{x},t)]$$

- Pion corresponds to leaking chiral charge flow

$$\langle \pi^a(q)B|C(0)|A\rangle = i\int d^4x e^{iqx}(\partial^2 + m_\pi^2)\langle B|\mathrm{T}[\pi^a(x)C(0)]|A\rangle$$
$$= i\lim_{q\to 0}\left(\frac{q^2 - m_\pi^2}{m_\pi^2 f_\pi}\right)\int d^4x e^{iqx}\langle B|\mathrm{T}[\partial^\mu J^a_{5\mu}(x)C(0)]|A\rangle$$

$$\pi^a(x) \simeq -(1/m_\pi^2 f_\pi)\partial^\alpha J^a_{5\alpha}(x)$$

$$\lim_{q\to 0}\langle \pi^a(q)B|C(0)|A\rangle = \frac{i}{f_\pi}\langle B|Q^a_5, C(0)]|A\rangle - \frac{q^\mu M_\mu}{f_\pi} \simeq \frac{i}{f_\pi}\langle B|Q^a_5, C(0)]|A\rangle$$

→ b_1 decay process can be understood in aspects of chiral symmetry breaking

$[Q^a_5, J^{b_1,b}_k(x)] = -i\delta^{ab}J^\omega_k(x),\ [Q^a_5, J^\omega_k(x)] = iJ^{b_1,a}_k(x),$ (in tensor representation)

$\mathbf{b_1}$ decay process

- Correlation function from b_1 to ω via pion breaking

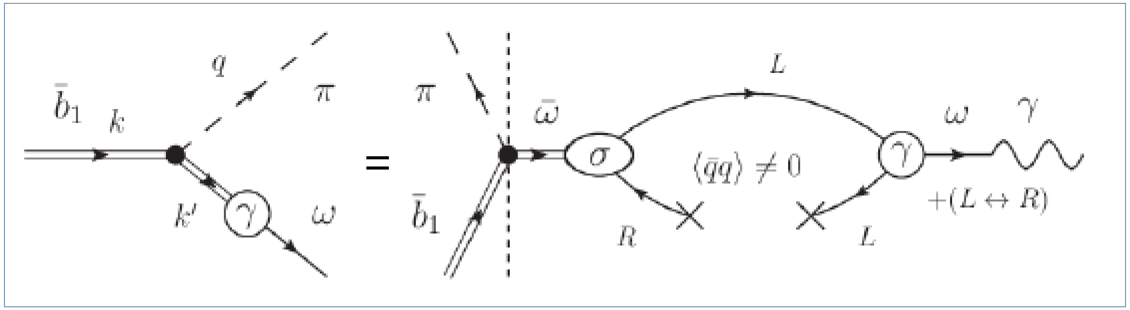
$$\left\langle \pi^a(q)\omega(k') \left| i\tilde{J}^a_{\mu\bar{\mu}}(k) \right| 0 \right\rangle \simeq f^T_{b_1} \sum_\lambda \langle \pi^a(q)\omega(k')|\bar{b}^a_1(k,\lambda)\rangle \left(\bar{\epsilon}^{(\lambda)^*}_\mu k_{\bar{\mu}} - \bar{\epsilon}^{(\lambda)^*}_{\bar{\mu}} k_\mu\right)$$

$$- f^T_\rho \sum_\lambda \langle \pi^a(q)\omega(k')|\bar{\rho}(k,\lambda)\rangle \frac{\bar{\epsilon}_{\mu\bar{\mu}\alpha\bar{\alpha}}}{2} \left(\bar{\epsilon}^{(\lambda)^*\alpha} k^{\bar{\alpha}} - \bar{\epsilon}^{(\lambda)^*\bar{\alpha}} k^\alpha\right) + \cdots$$

$$= \frac{i}{f_\pi} \int d^3x e^{-i\vec{q}\cdot\vec{x}} \left\langle \omega(k') \left| \left[J^a_{50}(x), i\tilde{J}^a_{\mu\bar{\mu}}(k) \right] \right| 0 \right\rangle$$

$$= \frac{i}{f_\pi} \left[-3i \left\langle \omega(k) \left| iJ^0_{\mu\bar{\mu}}(k) \right| 0 \right\rangle + R_1(q) \right]$$

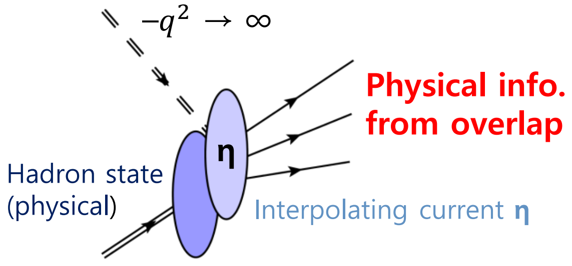
$$\simeq \boxed{\frac{3f^T_\omega}{f_\pi} \sum_\lambda \langle \omega(k)|\bar{\omega}(k,\lambda)\rangle} \left(\bar{\epsilon}^{(\lambda)^*}_\mu k_{\bar{\mu}} - \bar{\epsilon}^{(\lambda)^*}_{\bar{\mu}} k_\mu\right) + \cdots$$



Double line denotes vector meson in tensor representation

Interpolating current

- To obtain physical information



a. Quasi-particle state will be extracted from the overlap
b. We need to construct proper interpolating current which can be strongly overlapped with object hadron state
c. Our object: **ω meson in tensor representation**

- Projection operator

Covariant interpolation

$$\omega[0^-(1^{--})] \to \langle 0 \left| \bar{q} T^0 \sigma_{\mu\nu} q \right| \omega(p,\lambda) \rangle = i f_\omega^T \left(\epsilon_\mu^{(\lambda)} p_\nu - \epsilon_\nu^{(\lambda)} p_\mu \right)$$

$$b_1[1^+(1^{+-})] \to -\frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \langle 0 \left| \bar{q} T^a \sigma^{\alpha\beta} q \right| b_1(p,\lambda) \rangle = i f_{b_1^a}^T \left(\epsilon_\mu^{(\lambda)} p_\nu - \epsilon_\nu^{(\lambda)} p_\mu \right)$$

Projection of parity eigenmodes

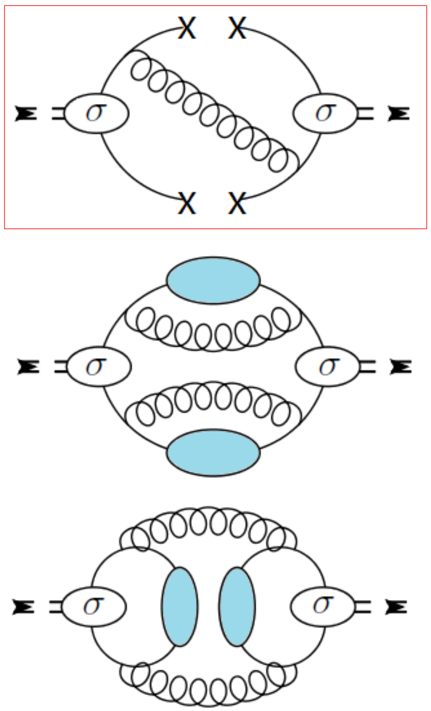
$$\sum_\lambda \langle (1^{--})_{\mu\bar{\mu}}(p,\lambda) | (1^{--})_{\nu\bar{\nu}}(p,\lambda) \rangle \simeq p^2 {f_-^T}^2 P^{(-)}_{\mu\bar{\mu},\nu\bar{\nu}}, \qquad P^{(-)}_{\mu\bar{\mu};\nu\bar{\nu}} = g_{\mu\nu} \frac{p_{\bar{\mu}} p_{\bar{\nu}}}{p^2} + g_{\bar{\mu}\bar{\nu}} \frac{p_\mu p_\nu}{p^2} - g_{\bar{\mu}\nu} \frac{p_\mu p_{\bar{\nu}}}{p^2} - g_{\mu\bar{\nu}} \frac{p_{\bar{\mu}} p_\nu}{p^2},$$

$$\sum_\lambda \langle (1^{+-})_{\mu\bar{\mu}}(p,\lambda) | (1^{+-})_{\nu\bar{\nu}}(p,\lambda) \rangle \simeq p^2 {f_+^T}^2 P^{(+)}_{\mu\bar{\mu},\nu\bar{\nu}}, \qquad P^{(+)}_{\mu\bar{\mu};\nu\bar{\nu}} = P^{(-)}_{\mu\bar{\mu},\nu\bar{\nu}} + (g_{\mu\bar{\nu}} g_{\bar{\mu}\nu} - g_{\mu\nu} g_{\bar{\mu}\bar{\nu}}),$$

Four-quark condensates

- Four-quark pieces determine spectral structure of invariant

$$\mathcal{W}_M^{\text{subt.}}[\Pi^{\mp}_{\bar{q}T^A\sigma q}(k^2)] = -\frac{1}{16\pi^2}(M^2)^2 E_1(s_0) - \frac{1}{48}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle \mp \frac{16\pi\alpha_s}{M^2}\left(\langle\bar{q}T^A\tau^{\bar{a}}q\bar{q}T^A\tau^{\bar{a}}q\rangle + \langle\bar{q}T^A\tau^{\bar{a}}\gamma_5 q\bar{q}T^A\tau^{\bar{a}}\gamma_5 q\rangle\right) - \frac{8\pi\alpha_s}{9M^2}\langle\bar{q}T^0\tau^{\bar{a}}\gamma_\eta q\bar{q}T^0\tau^{\bar{a}}\gamma^\eta q\rangle$$



Usual vacuum saturation hypothesis gives same factorization

$$\langle\bar{q}T^A\tau^{\bar{a}}q\bar{q}T^A\tau^{\bar{a}}q\rangle \to -\frac{a_A}{18}\langle\bar{q}T^0 q\rangle^2$$

$$\langle\bar{q}T^A\tau^{\bar{a}}\gamma_5 q\bar{q}T^A\tau^{\bar{a}}\gamma_5 q\rangle \to -\frac{b_A}{18}\langle\bar{q}T^0 q\rangle^2$$

In Bank-Casher formula, only Dirac zero-mode contributes

$$\langle\bar{q}q\rangle = -\int d^4x \left\langle \sum_\lambda \frac{\psi_\lambda(x)^\dagger\psi_\lambda(x)}{V}\frac{1}{m-i\lambda}\right\rangle = -\pi\langle\mathrm{Tr}[J_{\lambda=0}(0,0)]\rangle$$

Dirac zero-mode correlation is on the gauge orbit
→ colored pieces can make non-zero contribution

In vacuum, all the topological configuration is possible
→ parity-odd pieces (*b_A* type) can contribute as alternating s eries of winding number

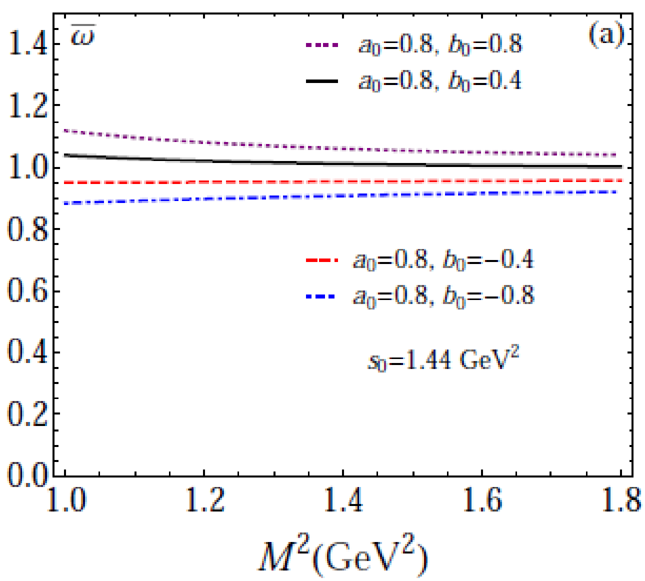
[a_0=0.8, b_0=0.4] has been used for the isoscalar mode

Borel sum rules for $[0^-(1^{--})]$ state

- Four-quark pieces determine spectral behavior of invariant

$$\mathcal{W}_M^{\rm subt.}[\Pi^{\mp}_{\bar{q}T^A\sigma q}(k^2)] = \frac{1}{\pi}\int_0^{s_0} ds e^{-s/M^2}\,\mathrm{Im}\left[-\frac{f_{\mp}^{T\,2}}{s-m_{\mp}^2+i\epsilon}\right] = f_{\mp}^{T\,2} e^{-m_{\mp}^2/M^2}$$

$$= -\frac{1}{16\pi^2}(M^2)^2 E_1(s_0) - \frac{1}{48}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle \mp \frac{16\pi\alpha_s}{M^2}\left(\langle\bar{q}T^A\tau^{\bar{a}}q\bar{q}T^A\tau^{\bar{a}}q\rangle + \langle\bar{q}T^A\tau^{\bar{a}}\gamma_5 q\bar{q}T^A\tau^{\bar{a}}\gamma_5 q\rangle\right)$$
$$-\frac{8\pi\alpha_s}{9M^2}\langle\bar{q}T^0\tau^{\bar{a}}\gamma_\eta q\bar{q}T^0\tau^{\bar{a}}\gamma^\eta q\rangle$$



$$m_{\mp}^2 = (M^2)^2\left(\frac{\partial}{\partial M^2}\mathcal{W}_M^{\rm subt.}[\Pi^{(\mp)}_{\bar{q}T^A\sigma q}(k^2)]\right)\Big/\mathcal{W}_M^{\rm subt.}[\Pi^{(\mp)}_{\bar{q}\sigma q}(k^2)]$$

Mass number ranges from 900 MeV ~ 1000 MeV
→ higher than mass of ω(782)
→ there is no ω resonance in mass number 1 GeV

consider anomalous coupling

$$\mathcal{L}^{\epsilon 1}_{\omega\pi\rho} = \frac{g_{\omega\pi\rho}}{2}\epsilon^{\mu\bar{\mu}\alpha\bar{\alpha}}\omega_{\mu\bar{\mu}}\partial_\alpha\pi^a\rho^a_{\bar{\alpha}}$$

π-ρ hybrid state can be suggested

Corresponding phenomenological current

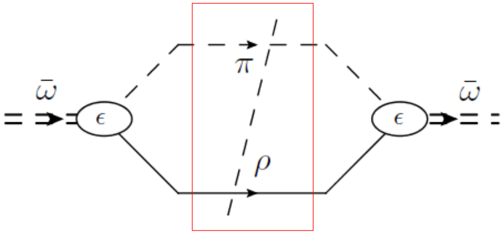
$$J^{\bar{\omega}}_{\mu\bar{\mu}}(x) \equiv \epsilon_{\mu\bar{\mu}\alpha\bar{\alpha}}\mathrm{Tr}\left[\partial^\alpha\pi(x)\rho^{\bar{\alpha}}(x)\right]$$

Borel sum rules for $[0^-(1^{--})]$ state

- Spectral sum rules for hybrid state

Imaginary part is changed as

$$\mathrm{Im}[\Pi^{(-)}_{\bar{q}T^0\sigma q}(k^2)] = \pi f_-^{T^2}\delta(k^2 - m_-^2) \Rightarrow c^2_{\bar{\omega}\pi\rho}\mathrm{Im}[\Pi^{\bar{\omega}}_{(-)}(k^2)]$$



Phenomenological correlator

$$\Pi^{\bar{\omega}}_{\mu\bar{\mu};\nu\bar{\nu}}(k) = i\int d^4x e^{ikx}\left\langle \mathrm{T}\left[\epsilon_{\mu\bar{\mu}\alpha\bar{\alpha}}\mathrm{Tr}[\partial^\alpha\pi(x)\rho^{\bar{\alpha}}(x)]\epsilon_{\nu\bar{\nu}\beta\bar{\beta}}\mathrm{Tr}[\partial^\beta\pi(0)\rho^{\bar{\beta}}(0)]\right]\right\rangle$$

Weighted invariant for parity-odd mode

$$\begin{aligned}
\mathcal{W}^{\mathrm{subt.}}_M[\Pi^{\bar{\omega}}_{(-)}(k^2)] =& \frac{3}{4(4\pi)^2}\int_{m_\rho^2}^{s_0} ds e^{-s/M^2}\left[\left(-\frac{s}{6} + \frac{m_\rho^2}{2} - \frac{1}{2}\frac{(m_\rho^2)^2}{s} + \frac{1}{6}\frac{(m_\rho^2)^3}{s^2}\right)\right] \\
=& \frac{3}{4(4\pi)^2}\Bigg[-\frac{1}{6}\left[-M^2\left(s_0 e^{-s_0/M^2} - m_\rho^2 e^{-m_\rho^2/M^2}\right) - (M^2)^2\left(e^{-s_0/M^2} - e^{-m_\rho^2/M^2}\right)\right] \\
&+ \frac{m_\rho^2}{2}\left[-M^2\left(e^{-s_0/M^2} - e^{-m_\rho^2/M^2}\right)\right] - \frac{(m_\rho^2)^2}{2}\left[\Gamma\left(0, m_\rho^2/M^2\right) - \Gamma\left(0, s_0/M^2\right)\right] \\
&+ \frac{(m_\rho^2)^3}{6}\left[-\frac{1}{s_0}e^{-s_0/M^2} + \frac{1}{m_\rho^2}e^{-m_\rho^2/M^2} - \frac{1}{M^2}\left(\Gamma\left(0, m_\rho^2/M^2\right) - \Gamma\left(0, s_0/M^2\right)\right)\right]\Bigg]
\end{aligned}$$

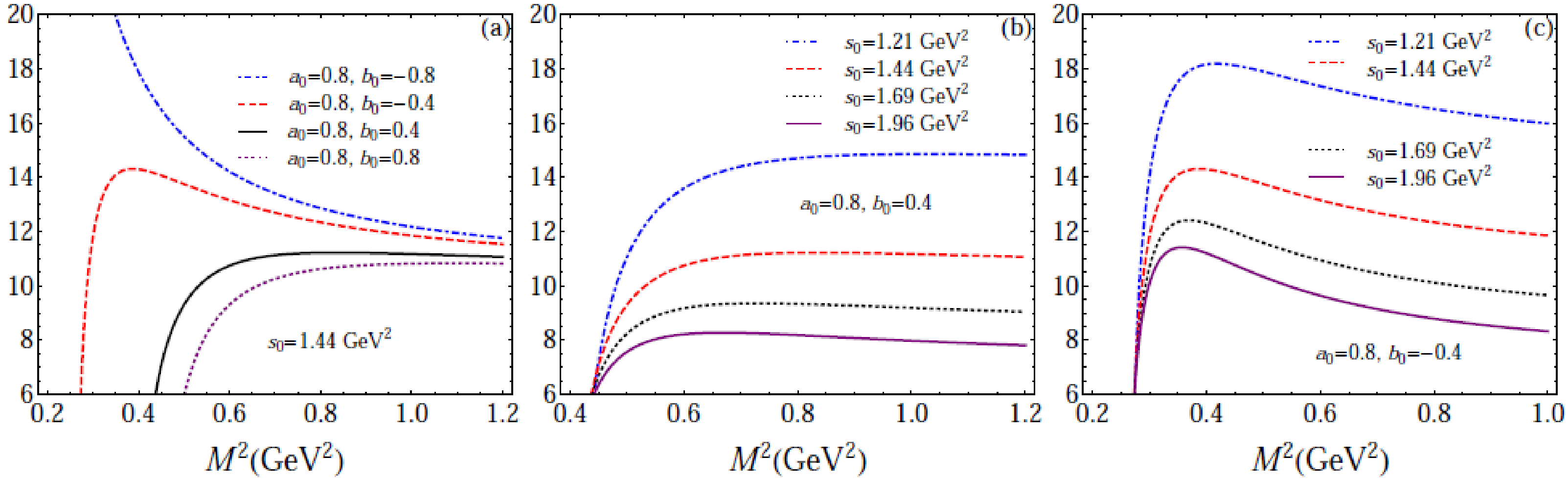
Coupling between tensor current and hybrid state

$$|c_{\bar{\omega}\pi\rho}| = \left[\mathcal{W}^{\mathrm{subt.}}_M[\Pi^{\bar{q}T^0\sigma q}_{(-)}(k^2)]/\mathcal{W}^{\mathrm{subt.}}_M[\Pi^{\bar{\omega}}_{(-)}(k^2)]\right]^{\frac{1}{2}}$$

Borel sum rules for $[0^{-}(1^{--})]$ state

- Spectral sum rules for size of coupling

$$|c_{\bar{\omega}\pi\rho}| = \left[\mathcal{W}_M^{\text{subt.}}[\Pi_{(-)}^{\bar{q}T^0\sigma q}(k^2)]/\mathcal{W}_M^{\text{subt.}}[\Pi_{(-)}^{\bar{\omega}}(k^2)]\right]^{\frac{1}{2}}$$



In proper parameter set **[a₀=0.8 b₀=0.4]**, Borel curve for coupling is stable

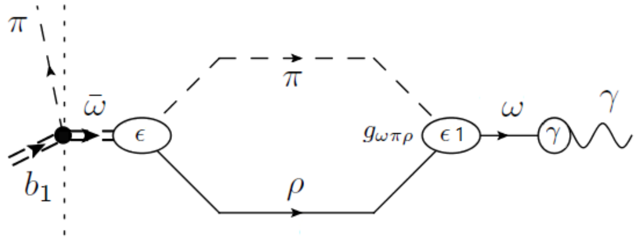
$\omega[0^{-}(1^{--})]$ like state after pion breaking from b1 is π-ρ hybrid state

This intermediate hybrid state has loop structure → off-shell contribution can be important

$\Gamma(\mathbf{b_1} \rightarrow \pi\text{-[hybrid]} \rightarrow \pi\text{-}\Upsilon)$

- Two possible final Y state

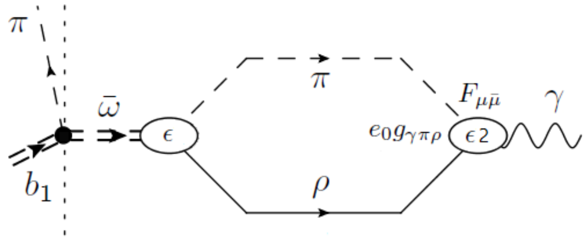
Final photon state via ω(782) (VMD channel)



$$\mathcal{L}^{\epsilon 1}_{\omega\pi\rho} = \frac{g_{\omega\pi\rho}}{2}\epsilon^{\mu\bar{\mu}\alpha\bar{\alpha}}\omega_{\mu\bar{\mu}}\partial_\alpha\pi^a\rho^a_{\bar{\alpha}}$$
$$= -g_{\omega\pi\rho}\epsilon^{\mu\bar{\mu}\alpha\bar{\alpha}}\omega_{\bar{\mu}}\partial_\alpha\pi^a\partial_\mu\rho^a_{\bar{\alpha}}$$

If the whole legs are on mass-shell, the vertex would become similar with ε1 (VMD)

Final photon state from the hybrid direct after pion breaking (direct channel)



$$\mathcal{L}^{\epsilon 2}_{\gamma\pi\rho} = \frac{e_0 g_{\gamma\pi\rho}}{2m_\rho}\epsilon^{\mu\bar{\mu}\alpha\bar{\alpha}}F_{\mu\bar{\mu}}\partial_\alpha\pi^a\rho^a_{\bar{\alpha}} \simeq \frac{e_0 g_{\gamma\pi\rho}}{m_\rho}F_{\mu\bar{\mu}}\bar{\omega}^{\mu\bar{\mu}}$$

If the whole legs are on mass-shell, the vertex would become similar with ε1 (VMD)

However in the goldstone pion limit, the phase space in loop is off mass-shell
→ direct channel would be important

Controversial points

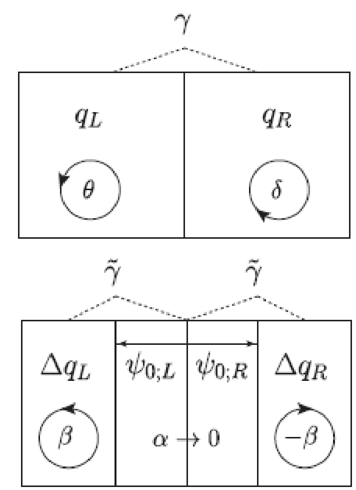
- Is it accidental coincidence?

$$
\begin{aligned}
\bar{q}T^0\gamma_k q =\ & \psi^\dagger_{0;L}T^0\bar{\sigma}_k\psi_{0;L} + \psi^\dagger_{0;L}T^0\bar{\sigma}_k\Delta q_L + \Delta q_L^\dagger T^0\bar{\sigma}_k\psi_{0;L} + \Delta q_L^\dagger T^0\bar{\sigma}_k\Delta q_L \\
& + \psi^\dagger_{0;R}T^0\sigma_k\psi_{0;R} + \psi^\dagger_{0;R}T^0\sigma_k\Delta q_R + \Delta q_R^\dagger T^0\sigma_k\psi_{0;R} + \Delta q_R^\dagger T^0\sigma_k\Delta q_R \\
=\ & \omega_{0;k} + \Delta\omega_k - \bar{\phi}_L T^0\tilde{\gamma}_5\tilde{\gamma}_0\tilde{\gamma}_k\phi_L + \bar{\phi}_R T^0\tilde{\gamma}_5\tilde{\gamma}_0\tilde{\gamma}_k\phi_R,
\end{aligned}
$$

$$
\Delta\bar{q}T^0\gamma_k q = i\beta^a\left(-\bar{\phi}_L T^a\tilde{\gamma}_0\tilde{\gamma}_k\phi_L + \bar{\phi}_R T^a\tilde{\gamma}_0\tilde{\gamma}_k\phi_R\right) = \beta^a\left(-\tilde{\rho}^a_{k;L} + \tilde{\rho}^a_{k;R}\right)
$$

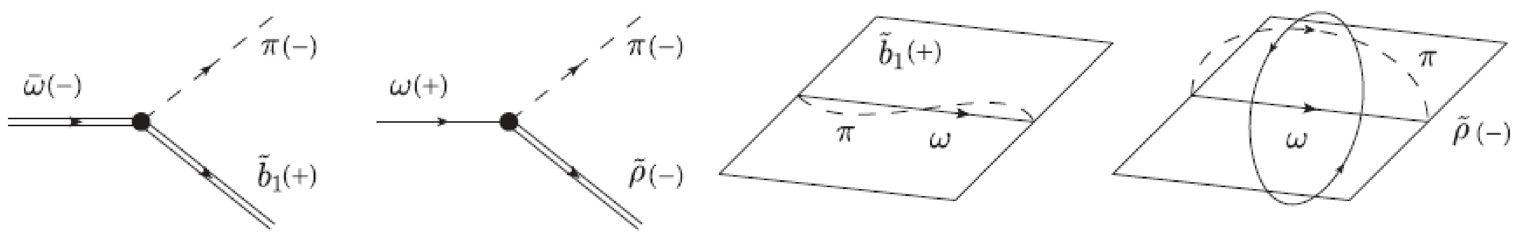
$$
\begin{aligned}
\bar{q}T^0\sigma_{0k} q =\ & i\big(\psi^\dagger_{0;R}T^0\bar{\sigma}_k\psi_{0;L} + \psi^\dagger_{0;R}T^0\bar{\sigma}_k\Delta q_L + \Delta q_R^\dagger T^0\bar{\sigma}_k\psi_{0;L} + \Delta q_R^\dagger T^0\bar{\sigma}_k\Delta q_L \\
& + \psi^\dagger_{0;L}T^0\sigma_k\psi_{0;R} + \psi^\dagger_{0;L}T^0\sigma_k\Delta q_R + \Delta q_L^\dagger T^0\sigma_k\psi_{0;R} + \Delta q_L^\dagger T^0\sigma_k\Delta q_R\big) \\
=\ & \bar{\omega}_{0;k} + \Delta\bar{\omega}_k + i\bar{\varphi}_L T^0\tilde{\gamma}_0\tilde{\gamma}_k\varphi_L + i\bar{\varphi}_R T^0\tilde{\gamma}_0\tilde{\gamma}_k\varphi_R,
\end{aligned}
$$

$$
\begin{aligned}
\Delta\bar{q}T^0\sigma_{0k} q =\ & -\beta^a\Delta\bar{q}T^a\gamma_5\gamma_0\gamma_k\Delta q - \beta^a\left(\bar{\varphi}_L T^a\tilde{\gamma}_5\tilde{\gamma}_0\tilde{\gamma}_k\varphi_L + \bar{\varphi}_R T^a\tilde{\gamma}_5\tilde{\gamma}_0\tilde{\gamma}_k\varphi_R\right) \\
=\ & -2\beta^a\Delta\bar{b}^a_{1k} - \beta^a\left(\tilde{b}^a_{1k;L} + \tilde{b}^a_{1k;R}\right),
\end{aligned}
$$



$$
\mathcal{A}\begin{pmatrix}\Delta q_L \\ \psi_0\end{pmatrix} \equiv \begin{pmatrix}\psi_0 \\ \Delta q_L\end{pmatrix}
$$

$$
\mathcal{A}\begin{pmatrix}\Delta q_R \\ \psi_0\end{pmatrix} \equiv \begin{pmatrix}\psi_0 \\ \Delta q_R\end{pmatrix}
$$



(a) (b) (c) (d)