Research Topics in KNU nuclear/hadron physics

오용석 (경북대)

Topics

- Energy density functional
- Nuclear alpha decay
- Neutron star and the hyperon puzzle
- Neutrino scattering in matter
- Hadron production: high-spin formalism
- b1 meson decay
- Generalized parton distribution functions

Effective Lagrangian for interactions of high-spin baryons

Sang-Ho Kim (APCTP), Yongseok Oh (Kyungpook National Univ.)

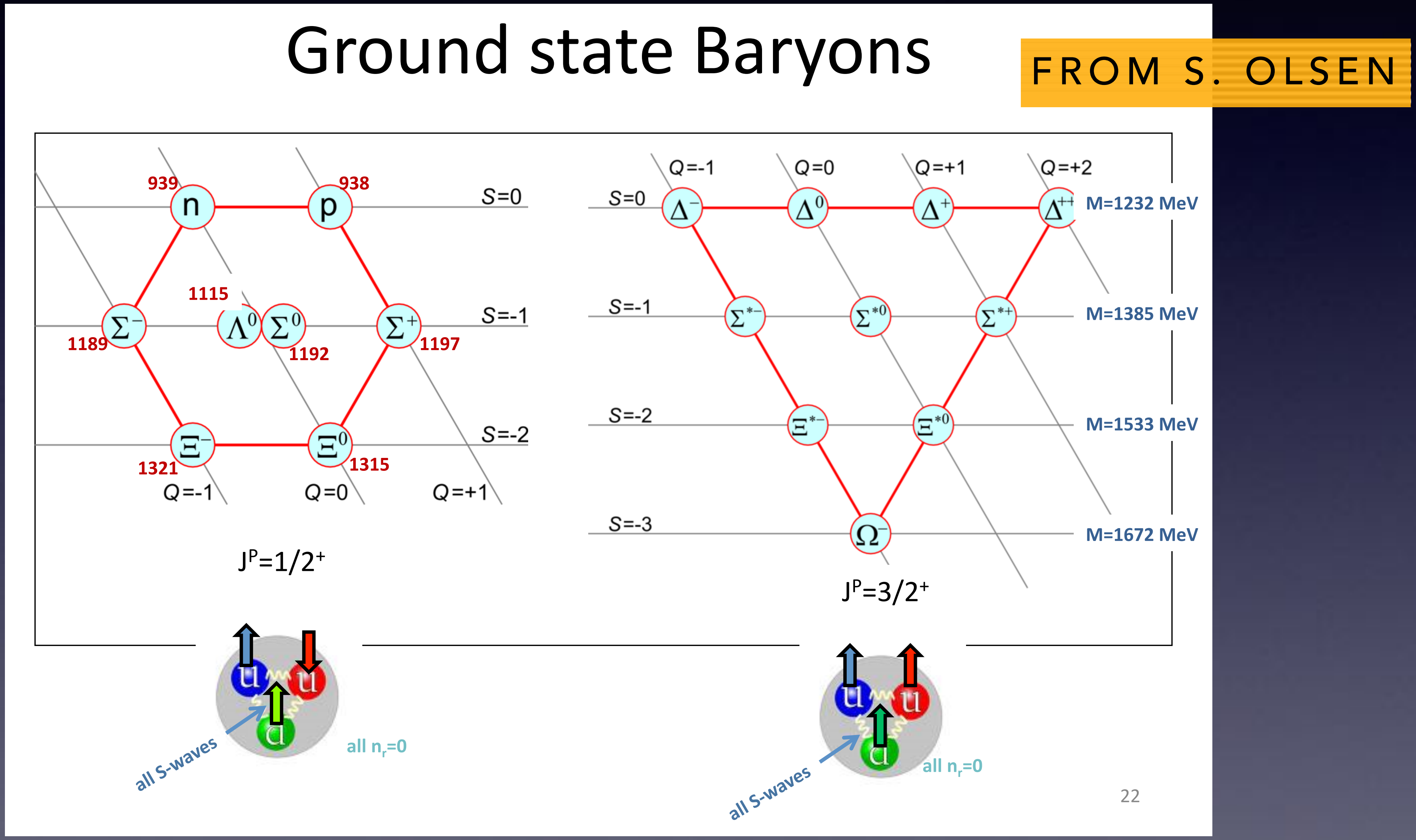
Exploring Hadrons and Electromagnetic Probes:
Structure, Excitations, Interactions
Jefferson Lab. Nov. 2-3, 2017

CONTENTS

- Introduction & Motivation
- Formalism
- Decay of baryons: $R \longrightarrow \pi N$, $R \longrightarrow V N$ or γN
- Summary and Outlook

Introduction & Motivation

- Ground state baryons



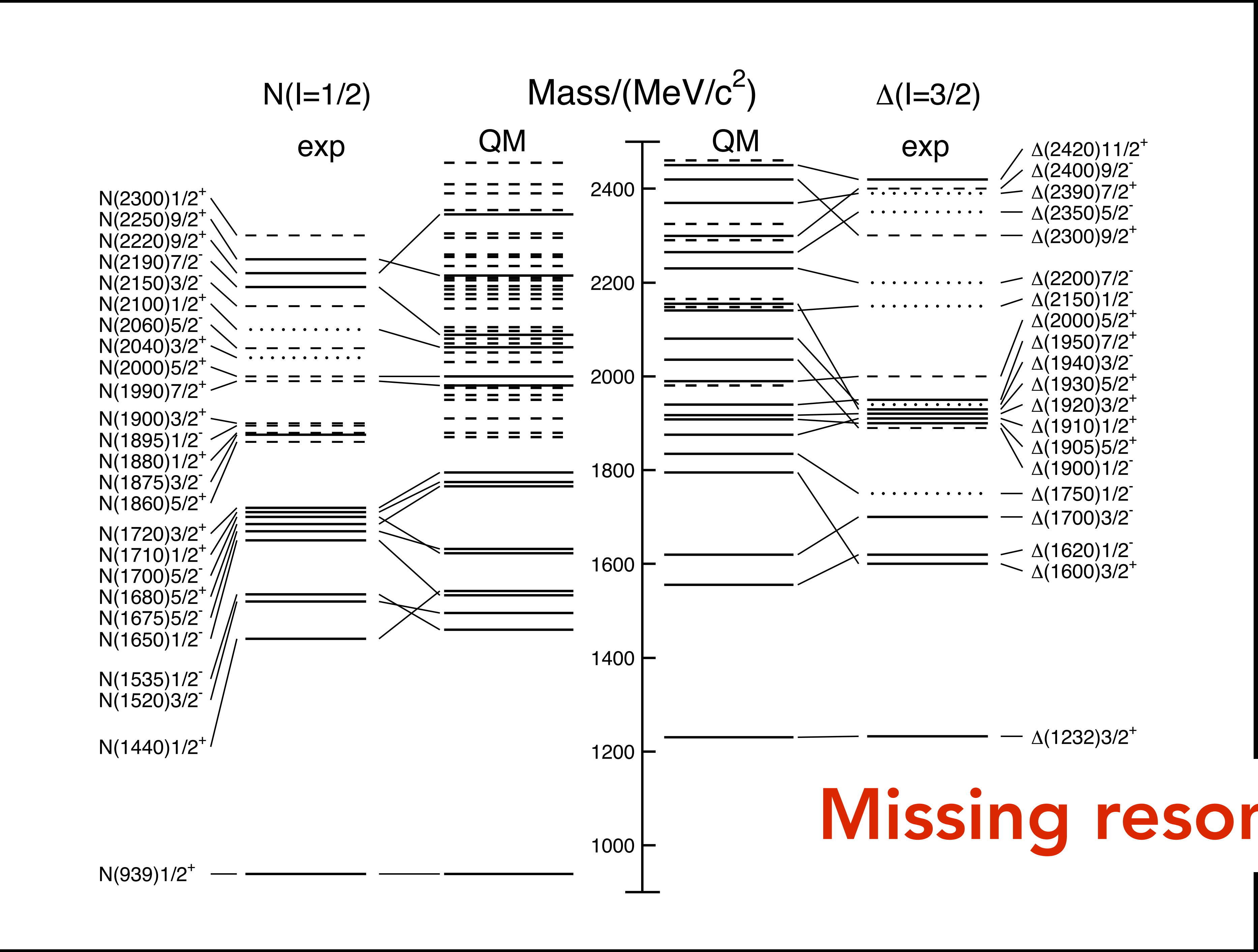
- Most static properties of the ground state baryons are governed by the group structure.
- How can we get information on the dynamics of the constituents of hadrons?

Introduction & Motivation

orbital excitations, radial excitations

$$\boldsymbol{J} = \boldsymbol{S} + \boldsymbol{L}$$

Excitation Spectrum of the nucleon

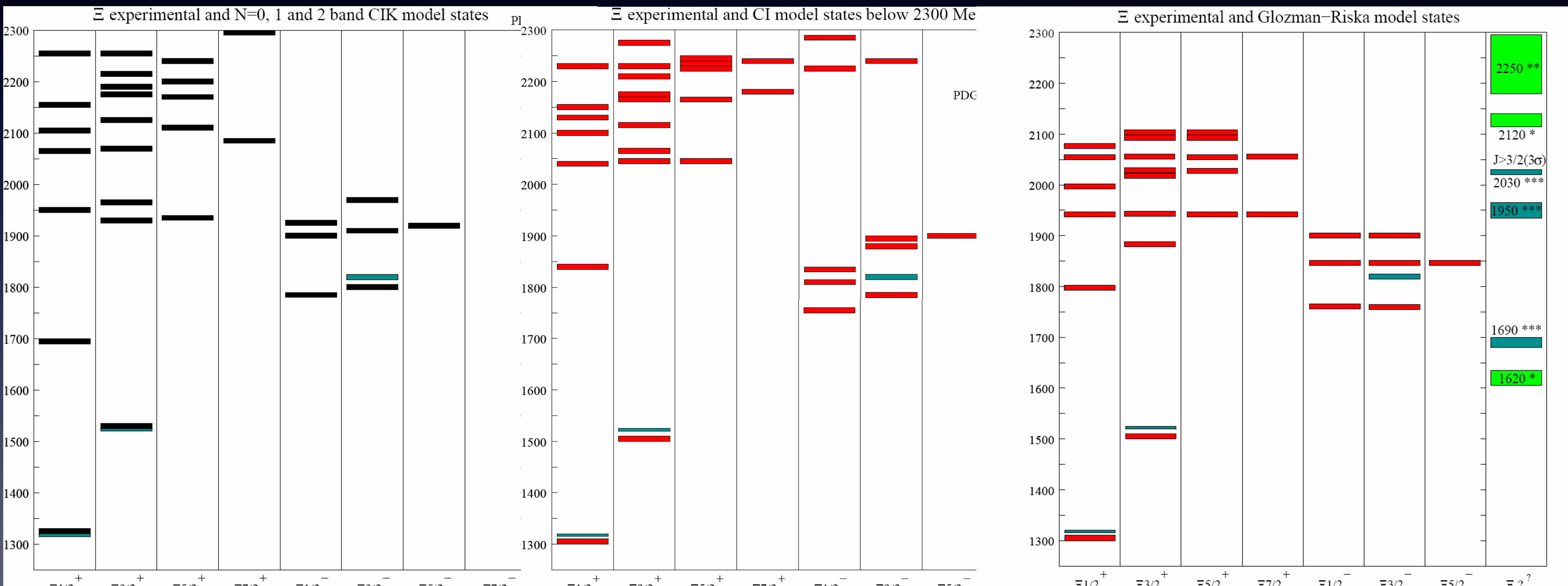


Particle Data Group

Missing resonances problem

Introduction & Motivation

Sensitivity of baryon spectrum on dynamics



NRQM

Chao, Isgur, Karl

RQM

Isgur, Karl

OBEM

Glozman, Riska

Introduction & Motivation

Highly model-dependent and sensitive to dynamics

Table 1. Low-lying Ξ and Ω baryon spectrum of spin 1/2 and 3/2 predicted by the non-relativistic quark model of Chao *et al.* (CIK), relativized quark model of Capstick and Isgur (CI), Glozman-Riska model (GR), large N_c analysis, algebraic model (BIL), and QCD sum rules (SR). The recent quark model prediction (QM) and the Skyrme model results (SK) are given as well. The mass is given in the unit of MeV.

State	CIK [4]	CI [5]	GR [6]	Large-N_c [7–11]	BIL [12]	SR [13, 14]	QM [15]	SK [1]
$\Xi(\frac{1}{2}^+)$	1325	1305	1320		1334	1320 (1320)	1325	1318
	1695	1840	1798	1825	1727		1891	1932
	1950	2040	1947	1839	1932		2014	
$\Xi(\frac{3}{2}^+)$	1530	1505	1516		1524		1520	1539
	1930	2045	1886	1854	1878		1934	2120
	1965	2065	1947	1859	1979		2020	
$\Xi(\frac{1}{2}^-)$	1785	1755	1758	1780	1869	1550 (1630)	1725	1614
	1890	1810	1849	1922	1932		1811	1660
	1925	1835	1889	1927	2076			
$\Xi(\frac{3}{2}^-)$	1800	1785	1758	1815	1828	1840	1759	1820
	1910	1880	1849	1973	1869		1826	
	1970	1895	1889	1980	1932			
$\Omega(\frac{1}{2}^+)$	2190	2220	2068	2408	2085		2175	2140
	2210	2255	2166		2219		2191	
$\Omega(\frac{3}{2}^+)$	1675	1635	1651		1670		1656	1694
	2065	2165	2020	1922	1998		2170	2282
	2215	2280	2068	2120	2219		2182	
$\Omega(\frac{1}{2}^-)$	2020	1950	1991	2061	1989		1923	1837
$\Omega(\frac{3}{2}^-)$	2020	2000	1991	2100	1989		1953	1978

Introduction & Motivation

Particle	$L_{2I \cdot 2J}$	Overall status
$N(939)$	P_{11}	****
$N(1440)$	P_{11}	****
$N(1520)$	D_{13}	****
$N(1535)$	S_{11}	****
$N(1650)$	S_{11}	****
$N(1675)$	D_{15}	****
$N(1680)$	F_{15}	****
$N(1700)$	D_{13}	***
$N(1710)$	P_{11}	***
$N(1720)$	P_{13}	****
$N(1900)$	P_{13}	**
$N(1990)$	F_{17}	**
$N(2000)$	F_{15}	**
$N(2080)$	D_{13}	**
$N(2090)$	S_{11}	*
$N(2100)$	P_{11}	*
$N(2190)$	G_{17}	****
$N(2200)$	D_{15}	**
$N(2220)$	H_{19}	****
$N(2250)$	G_{19}	****
$N(2600)$	$I_{1\,11}$	***
$N(2700)$	$K_{1\,13}$	**

$\Delta(1232)$	P_{33}	****
$\Delta(1600)$	P_{33}	***
$\Delta(1620)$	S_{31}	****
$\Delta(1700)$	D_{33}	****
$\Delta(1750)$	P_{31}	*
$\Delta(1900)$	S_{31}	**
$\Delta(1905)$	F_{35}	****
$\Delta(1910)$	P_{31}	****
$\Delta(1920)$	P_{33}	***
$\Delta(1930)$	D_{35}	***
$\Delta(1940)$	D_{33}	*
$\Delta(1950)$	F_{37}	****
$\Delta(2000)$	F_{35}	**
$\Delta(2150)$	S_{31}	*
$\Delta(2200)$	G_{37}	*
$\Delta(2300)$	H_{39}	**
$\Delta(2350)$	D_{35}	*
$\Delta(2390)$	F_{37}	*
$\Delta(2400)$	G_{39}	**
$\Delta(2420)$	$H_{3\,11}$	****
$\Delta(2750)$	$I_{3\,13}$	**
$\Delta(2950)$	$K_{3\,15}$	**

Λ states					Σ states				
State	J^P	Γ (MeV)	Rating	$\lvert g_{N\Lambda K}\rvert$	State	J^P	Γ (MeV)	Rating	$\lvert g_{N\Sigma K}\rvert$
$\Lambda(1116)$	$1/2^+$		****		$\Sigma(1193)$	$1/2^+$		****	
$\Lambda(1405)$	$1/2^-$	≈ 50	****		$\Sigma(1385)$	$3/2^+$	≈ 37	****	
$\Lambda(1520)$	$3/2^-$	≈ 16	****						
$\Lambda(1600)$	$1/2^+$	≈ 150	***	4.2	$\Sigma(1660)$	$1/2^+$	≈ 100	***	2.5
$\Lambda(1670)$	$1/2^-$	≈ 35	****	0.3	$\Sigma(1670)$	$3/2^-$	≈ 60	****	2.8
$\Lambda(1690)$	$3/2^-$	≈ 60	****	4.0	$\Sigma(1750)$	$1/2^-$	≈ 90	***	0.5
$\Lambda(1800)$	$1/2^-$	≈ 300	***	1.0	$\Sigma(1775)$	$5/2^-$	≈ 120	****	
$\Lambda(1810)$	$1/2^+$	≈ 150	***	2.8	$\Sigma(1915)$	$5/2^+$	≈ 120	****	
$\Lambda(1820)$	$5/2^+$	≈ 80	****		$\Sigma(1940)$	$3/2^-$	≈ 220	***	< 2.8
$\Lambda(1830)$	$5/2^-$	≈ 95	****		$\Sigma(2030)$	$7/2^+$	≈ 180	****	
$\Lambda(1890)$	$3/2^+$	≈ 100	****	0.8	$\Sigma(2250)$	$?^?$	≈ 100	***	
$\Lambda(2100)$	$7/2^-$	≈ 200	****						
$\Lambda(2110)$	$5/2^+$	≈ 200	***						
$\Lambda(2350)$	$9/2^+$	≈ 150	***						

At the mass region of > 1.8 GeV, many resonances are high spin states

$j \geq 5/2$

Introduction & Motivation

- Missing resonances – couple weakly to πN ?
- Search for resonances in the reactions other than the πN channel
- $\gamma N \to \omega N$
  with A.I. Titov and T.-S.H. Lee (2001) *
- $\gamma N \to \rho N$
  with T.-S.H. Lee (2004)
- $\gamma N \to \phi N$
  with A.I. Titov, S.N. Yang, T. Morii, H.-C. Bhang (1997,1999,2001)
- $\gamma N \to K^* \Lambda, K^* \Sigma$
  with Hungchong Kim (2006), with S.-H. Kim, S.-I. Nam, H.-Ch. Kim (2012) *
  B.-G. You and K.-J. Kong (2017)

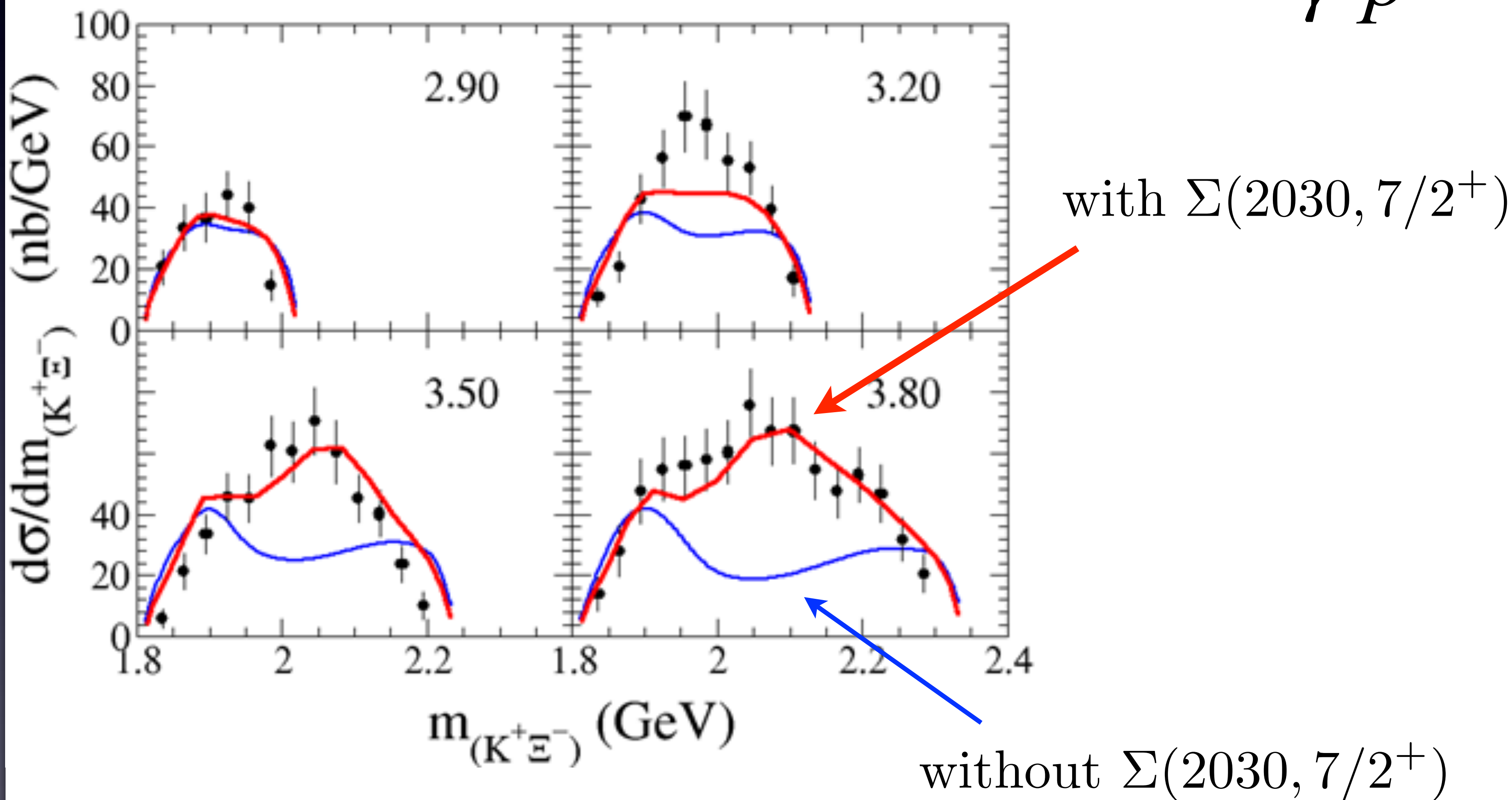
Introduction & Motivation

- $\gamma\, N \rightarrow K\, K\, \Xi$
  with K. Nakayama, H. Haberzettl (2006,2011) *
- $Kbar\, N \rightarrow K\, \Xi$
  with B. Jackson, K. Nakayama, H. Haberzettl (2012,2015) *
- $\gamma\, N \rightarrow K\, \Sigma^*(1385)$
  with K. Nakayama and C.M. Ko (2008)
- $\pi\, N \rightarrow \omega\, N$
  YO (2011) *
- What we need
  - vertices of $J^\pm \to 0^- + \frac{1}{2}^+, \quad J^\pm \to 1^- + \frac{1}{2}^+, J^\pm \to 0^- + \frac{3}{2}^+, \quad J^\pm \to 1^- + \frac{3}{2}^+$
    for an arbitrary value of J

Introduction & Motivation

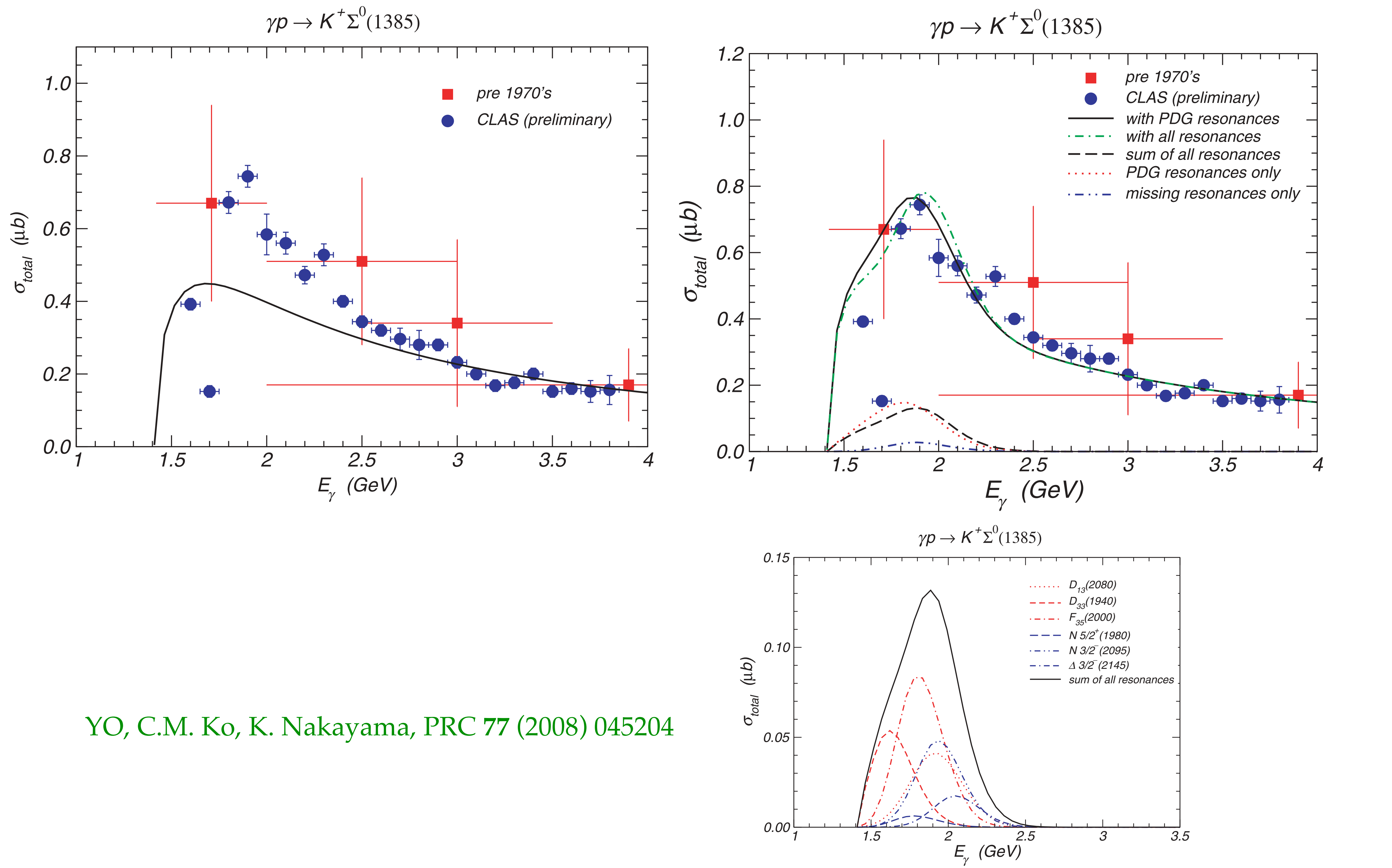
Importance of high-spin resonances

$\gamma\, p \longrightarrow K^+\, K^+\, \Xi^-$



Nakayama, YO, Haberzettl, PRC **74** (2006) 035205
Man, YO, Nakayama, PRC **83** (2011) 055201

Introduction & Motivation



YO, C.M. Ko, K. Nakayama, PRC **77** (2008) 045204

Introduction & Motivation

- Testing hadron models (such as quark models)
  - Data analyses: coupled-channels analyses
    - > extract coupling constants of effective interactions
    - > meson cloud effects (e.g. E2/M1 transition of $\Delta \to N$)
  - Quark models can give predictions on the decay amplitudes.
  - Decay width cannot determine the sign of the coupling constant (sign ambiguity)
    - need to work with decay amplitudes
    - need the relationship between coupling constants and the decay amplitudes predicted by baryon structure models
    - Tabel for $J^{\pm} \to 0^- + \frac{1}{2}^+, \quad J^{\pm} \to 1^- + \frac{1}{2}^+, J^{\pm} \to 0^- + \frac{3}{2}^+, \quad J^{\pm} \to 1^- + \frac{3}{2}^+$

Formalism

- Rarita-Schwinger fields
  - boson of spin-*j*: tensor of rank *n=j* $R_{\alpha_1\alpha_2\cdots\alpha_n}$

$$(\partial_\mu\partial^\mu + M^2)R_{\alpha_1\alpha_2\cdots\alpha_n} = 0 \quad \text{with} \quad R_{\alpha_1\cdots\alpha_i\cdots\alpha_j\cdots\alpha_n} = R_{\alpha_1\cdots\alpha_j\cdots\alpha_i\cdots\alpha_n}$$

subsidiary conditions

$$p^{\alpha_1}R_{\alpha_1\alpha_2\cdots\alpha_n} = 0, \quad g^{\alpha_1\alpha_2}R_{\alpha_1\alpha_2\cdots\alpha_n} = 0$$

  - fermion of spin-*j*: tensor of rank *n=j-1/2*

$$(i\partial\!\!\!/ - M)R_{\alpha_1\alpha_2\cdots\alpha_n} = 0. \quad \text{with} \quad R_{\alpha_1\cdots\alpha_i\cdots\alpha_j\cdots\alpha_n} = R_{\alpha_1\cdots\alpha_j\cdots\alpha_i\cdots\alpha_n}$$

subsidiary conditions

$$p^{\alpha_1}R_{\alpha_1\alpha_2\cdots\alpha_n} = 0, \quad g^{\alpha_1\alpha_2}R_{\alpha_1\alpha_2\cdots\alpha_n} = 0, \quad \gamma^{\alpha_1}R_{\alpha_1\alpha_2\cdots\alpha_n} = 0$$

Propagators

$$S(p) = \frac{1}{p^2 - M^2} \Delta^{\beta_1 \cdots \beta_n}_{\alpha_1 \cdots \alpha_n} \quad \text{for a boson}$$

$$S(p) = \frac{1}{p^2 - M^2} (\not{p} + M) \Delta^{\beta_1 \cdots \beta_n}_{\alpha_1 \cdots \alpha_n} \quad \text{for a fermion}$$

with the projection operator $\displaystyle\sum_{\text{spin}} R_{\alpha_1 \cdots \alpha_n} R^{\beta_1 \cdots \beta_n} = \Lambda_{\pm} \Delta^{\beta_1 \cdots \beta_n}_{\alpha_1 \cdots \alpha_n}$

where $\Lambda_{\pm} = \begin{cases} 1 & \text{for a boson} \\ (M \pm \not{p})/2M & \text{for a fermion} \end{cases}$

General form

Rushbrooke, PR 143 (66')
Behrends and Fronsdal, PR 106 (57')
Chang, PR 161 (67')

Boson $n = j$

$$\Delta^{\beta_1\cdots\beta_n}_{\alpha_1\cdots\alpha_n}(j,p) = \left(\frac{1}{n!}\right)^2 \sum_{P(\alpha),P(\beta)} \left[\prod_{i=1}^{n} \bar{g}^{\beta_i}_{\alpha_i} + a_1^{(n)} \bar{g}_{\alpha_1\alpha_2}\bar{g}^{\beta_1\beta_2}\prod_{i=3}^{n}\bar{g}^{\beta_i}_{\alpha_i} + \cdots + a^{(n)}_{n/2}\bar{g}_{\alpha_1\alpha_2}\bar{g}^{\beta_1\beta_2}\cdots\bar{g}_{\alpha_{n-1}\alpha_n}\bar{g}^{\beta_{n-1}\beta_n}\right]$$

for even *j*

$$\Delta^{\beta_1\cdots\beta_n}_{\alpha_1\cdots\alpha_n}(j,p) = -\left(\frac{1}{n!}\right)^2 \sum_{P(\alpha),P(\beta)} \left[\prod_{i=1}^{n} \bar{g}^{\beta_i}_{\alpha_i} + a_1^{(n)} \bar{g}_{\alpha_1\alpha_2}\bar{g}^{\beta_1\beta_2}\prod_{i=3}^{n}\bar{g}^{\beta_i}_{\alpha_i} + \cdots \right.$$
$$\left. + a^{(n)}_{(n-1)/2}\bar{g}_{\alpha_1\alpha_2}\bar{g}^{\beta_1\beta_2}\cdots\bar{g}_{\alpha_{n-2}\alpha_{n-1}}\bar{g}^{\beta_{n-2}\beta_{n-1}}\bar{g}^{\beta_n}_{\alpha_n}\right]$$

for odd *j*

Fermion $n = j - 1/2$

$$\Delta^{\beta_1\cdots\beta_{n-1}}_{\alpha_1\cdots\alpha_{n-1}}(j,p) = \frac{n}{2n+1}\gamma^\alpha\gamma_\beta\Delta^{\beta\beta_1\cdots\beta_{n-1}}_{\alpha\alpha_1\cdots\alpha_{n-1}}(j+\tfrac{1}{2},p)$$

$$\bar{g}_{\mu\nu} = g_{\mu\nu} - \frac{1}{M^2}p_\mu p_\nu.$$

$$a_r^{(n)} = \left(-\frac{1}{2}\right)^r \frac{n!}{r!(n-2r)!}\frac{1}{(2n-1)(2n-3)\cdots(2n-2r+1)}$$

Explicitly,

- **spin-1**

$$\Delta_\alpha^\beta(1,p) = -\bar{g}_\alpha^\beta = -\left(g_\alpha^\beta - \frac{1}{M^2}p_\alpha p^\beta\right).$$

- **spin-1/2**

$$\Delta(\tfrac{1}{2},p) = \tfrac{1}{3}\gamma^\alpha\gamma_\beta\Delta_\alpha^\beta(1,p) = 1.$$

- **spin-2**

$$\Delta_{\alpha_1\alpha_2}^{\beta_1\beta_2}(2,p) = \frac{1}{2}\left(\bar{g}_{\alpha_1}^{\beta_1}\bar{g}_{\alpha_1}^{\beta_2} + \bar{g}_{\alpha_1}^{\beta_1}\bar{g}_{\alpha_2}^{\beta_1} - \frac{2}{3}\bar{g}_{\alpha_1\alpha_2}\bar{g}^{\beta_1\beta_2}\right).$$

- **spin-3/2**

$$\begin{aligned}
\Delta_{\alpha_1}^{\beta_1}(\tfrac{3}{2},p) &= \frac{5}{2}\gamma^\alpha\gamma_\beta\Delta_{\alpha\alpha_1}^{\beta\beta_1}(2,p) \\
&= -\left(\bar{g}_{\alpha_1}^{\beta_1} - \frac{1}{3}\bar{\gamma}_{\alpha_1}\bar{\gamma}^{\beta_1}\right) \\
&= -g_{\alpha_1}^{\beta_1} + \frac{1}{3}\gamma_{\alpha_1}\gamma^{\beta_1} + \frac{1}{3M}\left(\gamma_{\alpha_1}p^{\beta_1} - p_{\alpha_1}\gamma^{\beta_1}\right) + \frac{2}{3M^2}p_{\alpha_1}p^{\beta_1}.
\end{aligned}$$

- **spin-3**

$$\Delta_{\alpha_1\alpha_2\alpha_3}^{\beta_1\beta_2\beta_3}(3,p) = -\frac{1}{36}\sum_{P(\alpha),P(\beta)}\left[\bar{g}_{\alpha_1}^{\beta_1}\bar{g}_{\alpha_2}^{\beta_2}\bar{g}_{\alpha_3}^{\beta_3} - \frac{3}{5}\bar{g}_{\alpha_1\alpha_2}\bar{g}^{\beta_1\beta_2}\bar{g}_{\alpha_3}^{\beta_3}\right].$$

Explicitly,

- **spin-5/2**

$$\Delta^{\beta_1\beta_2}_{\alpha_1\alpha_2}(\tfrac{5}{2},p) = \frac{3}{7}\gamma^\alpha\gamma_\beta\Delta^{\beta\beta_1\beta_2}_{\alpha\alpha_1\alpha_2}(3,p)$$
$$= \frac{1}{2}\left(\bar{g}^{\beta_1}_{\alpha_1}\bar{g}^{\beta_2}_{\alpha_2} + \bar{g}^{\beta_2}_{\alpha_1}\bar{g}^{\beta_1}_{\alpha_2}\right) - \frac{1}{5}\bar{g}_{\alpha_1\alpha_2}\bar{g}^{\beta_1\beta_2} - \frac{1}{10}\left(\bar{\gamma}_{\alpha_1}\bar{\gamma}^{\beta_1}\bar{g}^{\beta_2}_{\alpha_2} + \bar{\gamma}_{\alpha_1}\bar{\gamma}^{\beta_2}\bar{g}^{\beta_1}_{\alpha_2} + \bar{\gamma}_{\alpha_2}\bar{\gamma}^{\beta_1}\bar{g}^{\beta_2}_{\alpha_1} + \bar{\gamma}_{\alpha_2}\bar{\gamma}^{\beta_2}\bar{g}^{\beta_1}_{\alpha_1}\right). \tag{A10}$$

- **spin-4**

$$\Delta^{\beta_1\beta_2\beta_3\beta_4}_{\alpha_1\alpha_2\alpha_3\alpha_4}(4,p) = -\frac{1}{576}\sum_{P(\alpha),P(\beta)}\left[\bar{g}^{\beta_1}_{\alpha_1}\bar{g}^{\beta_2}_{\alpha_2}\bar{g}^{\beta_3}_{\alpha_3}\bar{g}^{\beta_4}_{\alpha_4} - \frac{6}{7}\bar{g}_{\alpha_1\alpha_2}\bar{g}^{\beta_1\beta_2}\bar{g}^{\beta_3}_{\alpha_3}\bar{g}^{\beta_4}_{\alpha_4} + \frac{3}{35}\bar{g}_{\alpha_1\alpha_2}\bar{g}_{\alpha_3\alpha_4}\bar{g}^{\beta_1\beta_2}\bar{g}^{\beta_3\beta_4}\right]. \tag{A11}$$

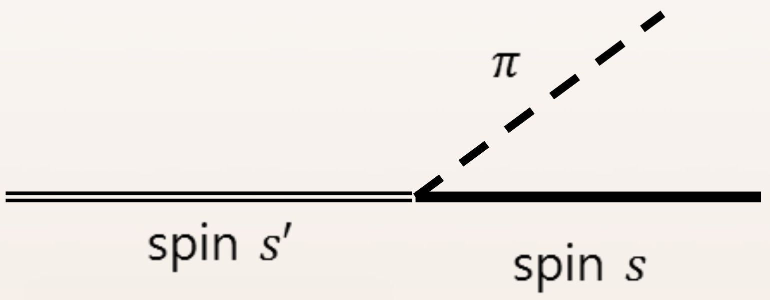
- **spin-7/2**

$$\Delta^{\beta_1\beta_2\beta_3}_{\alpha_1\alpha_2\alpha_3}(\tfrac{7}{2},p) = \frac{4}{9}\gamma^\alpha\gamma_\beta\Delta^{\beta\beta_1\beta_2\beta_3}_{\alpha\alpha_1\alpha_2\alpha_3}(4,p)$$
$$= -\frac{1}{36}\sum_{P(\alpha),P(\beta)}\left\{\bar{g}^{\beta_1}_{\alpha_1}\bar{g}^{\beta_2}_{\alpha_2}\bar{g}^{\beta_3}_{\alpha_3} - \frac{3}{7}\bar{g}^{\beta_1}_{\alpha_1}\bar{g}_{\alpha_2\alpha_3}\bar{g}^{\beta_2\beta_3} - \frac{3}{7}\bar{\gamma}_{\alpha_1}\bar{\gamma}^{\beta_1}\bar{g}^{\beta_2}_{\alpha_2}\bar{g}^{\beta_3}_{\alpha_3} + \frac{3}{35}\bar{\gamma}_{\alpha_1}\bar{\gamma}^{\beta_1}\bar{g}_{\alpha_2\alpha_3}\bar{g}^{\beta_2\beta_3}\right\}. \tag{A12}$$

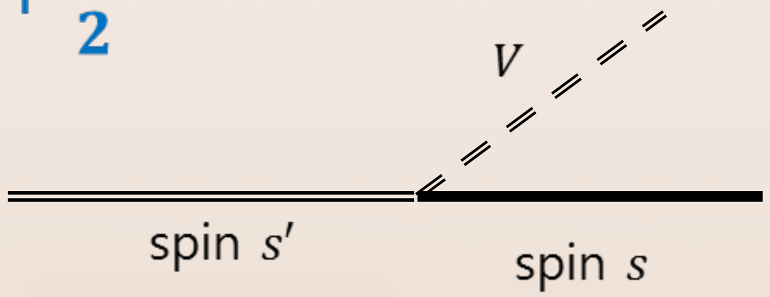
$$\bar{\gamma}^\mu = \gamma^\nu\bar{g}^\mu_\nu = \gamma^\mu - \frac{1}{M^2}\not{p}p^\mu.$$

Interactions

- Number of independent couplings
  - Angular momentum conservation
  - P and T invariance



- Pion ($J^P = 0^-$) vertex: $\boldsymbol{J^{\pm} \rightarrow 0^- + \frac{1}{2}^+}$
  - $RN\pi$ vertex where N: nucleon ($1/2^+$)
    R: nucleon resonance of J^P
    ⇨ only **one** interaction term



- Vector meson ($J^P = 1^-$) vertex: $\boldsymbol{J^{\pm} \rightarrow 1^- + \frac{1}{2}^+}$
  - RNV vertex
    ⇨ **two** interaction terms for R with $\frac{1}{2}^{\pm}$
    ⇨ **three** interaction terms for R with $J^{\pm}$ $(J \geq \frac{3}{2})$
  - If V is the photon, the numbers are reduced by one. $(\because q^2 = 0)$

$RN\pi$ Lagrangian

- $J^P = \frac{1}{2}^{\pm}$ case

$$\Gamma^{(\pm)} = \begin{pmatrix} \gamma_5 \\ 1 \end{pmatrix}, \quad \Gamma_\mu^{(\pm)} = \begin{pmatrix} \gamma_\mu \gamma_5 \\ \gamma_\mu \end{pmatrix}$$

$$\mathcal{L}_{1/2} = g_{\pi NR} \overline{N} \left[i\lambda \Gamma^{(\pm)} \pi \mp \frac{1-\lambda}{M_R \pm M_N} \Gamma_\mu^{(\pm)} \partial^\mu \pi \right] R + \text{H.c.}$$

- $J^P = \frac{3}{2}^{\pm}$ case

$$\mathcal{L}_{3/2} = \frac{g_{\pi NR}}{M_\pi} \overline{N} \Gamma^{(\mp)} \partial^\mu \pi R_\mu + \text{H.c.}$$

- $J^P = \frac{5}{2}^{\pm}$ case

$$\mathcal{L}_{RN\pi} = \frac{g_{RN\pi}}{M_\pi^{n-1}} \bar{N} \partial_{\mu_1} \cdots \partial_{\mu_{n-1}} \pi \left[i\Gamma^{(\pm)} \right] R^{\mu_1 \cdots \mu_{n-1}} + \text{H.c.},$$

$$\mathcal{L}_{5/2} = i \frac{g_{\pi NR}}{M_\pi^2} \overline{N} \Gamma^{(\pm)} \partial^\mu \partial^\nu \pi R_{\mu\nu} + \text{H.c.}$$

- $J^P = \frac{7}{2}^{\pm}$ case

$$\mathcal{L}_{7/2} = \frac{g_{\pi NR}}{M_\pi^3} \overline{N} \Gamma^{(\mp)} \partial^\mu \partial^\nu \partial^\alpha \pi R_{\mu\nu\alpha} + \text{H.c.}$$

Pion interaction Lagrangian

- The general expression for the decay widths

$$\Gamma(R \to N\pi) = \frac{3g_{\pi NR}^2}{4\pi}\frac{2^n(n!)^2}{n(2n)!}\frac{k_\pi^{2n-1}}{M_R M_\pi^{2(n-1)}}\left(E_N \pm M_N\right)$$

for $(-1)^n P_s = \pm 1$ with P_s being the parity of the spin - s resonance R

- Examples

$$\Gamma\left(\frac{1}{2}^{\pm} \to N\pi\right) = \frac{3g_{\pi NR}^2}{4\pi}\frac{k_\pi}{M_R}\left(E_N \mp M_N\right)$$

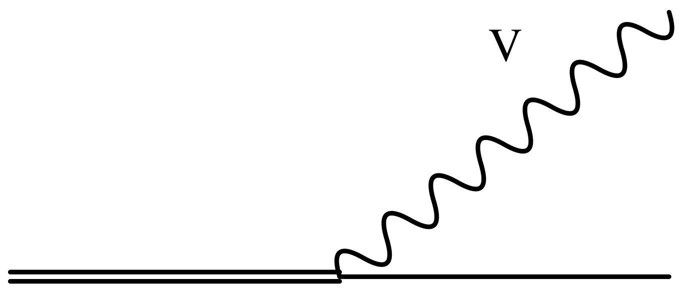
Isospin factor 3 is included.

$$\Gamma\left(\frac{3}{2}^{\pm} \to N\pi\right) = \frac{g_{\pi NR}^2}{4\pi}\frac{k_\pi^3}{M_R M_\pi^2}\left(E_N \pm M_N\right)$$

$$\Gamma\left(\frac{5}{2}^{\pm} \to N\pi\right) = \frac{g_{\pi NR}^2}{4\pi}\frac{2}{5}\frac{k_\pi^5}{M_R M_\pi^4}\left(E_N \mp M_N\right)$$

$$\Gamma\left(\frac{7}{2}^{\pm} \to N\pi\right) = \frac{g_{\pi NR}^2}{4\pi}\frac{6}{35}\frac{k_\pi^7}{M_R M_\pi^6}\left(E_N \pm M_N\right)$$

RNV interactions



	$(-1)^\gamma = +1$		$(-1)^\gamma = -1$	
	J_0	J_{+1}	J_0	J_{+1}
Nonidentical fermions $(s' = s)$	$j_{\min} + \frac{1}{2}$	$2j_{\min}$	$j_{\min} + \frac{1}{2}$	$2j_{\min}$
$(s' \neq s)$	$j_{\min} + \frac{1}{2}$	$2j_{\min} + 1$	$j_{\min} + \frac{1}{2}$	$2j_{\min} + 1$
Nonidentical bosons $(s' = s)$	$j_{\min} + 1$	$2j_{\min}$	$j_{\min}$	$2j_{\min}$
$(s' \neq s)$	$j_{\min} + 1$	$2j_{\min} + 1$	$j_{\min}$	$2j_{\min} + 1$
Identical fermions$(s' = s)$	$j_{\min} + \frac{1}{2}$	$j_{\min} + \frac{1}{2}$	—	—
Identical bosons $(s' = s)$	$j_{\min} + 1$	$j_{\min}$	—	—

TABLE II. Number of independent form factors for the vector current of a spin-s' particle into a spin-s particle transition [25]. Here $\gamma = s + s' + P$, where $(-1)^P$ is the relative parity of the initial and final states whose spins are s' and s, respectively, and $j_{\min} = \min(s, s')$. See Ref. [25] for the details.

Durand III, DeCelles, Marr, PR (1962)

RNV interactions

$$\Gamma(R \to NV) = \frac{q^2}{\pi} \frac{2M_N}{(2j+1)M_R} \times \left\{ \left|A_{1/2}\right|^2 + \left|A_{3/2}\right|^2 + \left|S_{1/2}\right|^2 \right\}$$

q: three-momentum of the vector meson in the rest frame of R

$$q = \frac{1}{2M_R}\sqrt{[M_R^2 - (M_N + M_V)^2][M_R^2 - (M_N - M_V)^2]}.$$

Helicity amplitude

$$A_\lambda(j) = \frac{1}{\sqrt{8M_N M_R q}} \frac{2j+1}{4\pi} \times \int d\cos\theta d\phi e^{-i(m-\lambda)\phi} d^j_{\lambda m}(\theta) \times \langle \mathbf{k}_\gamma, \lambda_\gamma, \lambda_N \mid -i\mathcal{M} \mid jm \rangle,$$

RNV Lagrangian

- $J^P = \frac{1}{2}^{\pm}$ case

$$\mathcal{L}_{1/2} = -\frac{1}{2M_N}\bar{N}\left[g_3\left(\pm\frac{\Gamma_\mu^{(\pm)}\partial^2}{M_R \mp M_N} - i\Gamma^{(\pm)}\partial_\mu\right)V^\mu - g_1\Gamma^{(\pm)}\sigma_{\mu\nu}\partial^\nu V^\mu\right]R + \mathrm{H.c.}$$

- $J^P = \frac{3}{2}^{\pm}$ case

$$\mathcal{L}_{3/2} = -i\frac{g_1}{2M_N}\bar{N}\Gamma_\nu^{(\pm)}V^{\mu\nu}R_\mu - \frac{g_2}{(2M_N)^2}\partial_\nu\bar{N}\Gamma^{(\pm)}V^{\mu\nu}R_\mu + \frac{g_3}{(2M_N)^2}\bar{N}\Gamma^{(\pm)}\partial_\nu V^{\mu\nu}R_\mu + \mathrm{H.c.}$$

- $J^P = \frac{5}{2}^{\pm}$ case

$$\mathcal{L}_{5/2} = \frac{g_1}{(2M_N)^2}\bar{N}\Gamma_\nu^{(\mp)}\partial^\alpha V^{\mu\nu}R_{\mu\alpha} - \frac{ig_2}{(2M_N)^3}\partial_\nu\bar{N}\Gamma^{(\mp)}\partial^\alpha V^{\mu\nu}R_{\mu\alpha} + \frac{ig_3}{(2M_N)^3}\bar{N}\Gamma^{(\mp)}\partial^\alpha\partial_\nu V^{\mu\nu}R_{\mu\alpha} + \mathrm{H.c.}$$

- $J^P = \frac{7}{2}^{\pm}$ case

$$\mathcal{L}_{7/2} = \frac{ig_1}{(2M_N)^3}\bar{N}\Gamma_\nu^{(\pm)}\partial^\alpha\partial^\beta V^{\mu\nu}R_{\mu\alpha\beta} + \frac{g_2}{(2M_N)^4}\partial_\nu\bar{N}\Gamma^{(\pm)}\partial^\alpha\partial^\beta V^{\mu\nu}R_{\mu\alpha\beta}$$

$$- \frac{g_3}{(2M_N)^4}\bar{N}\Gamma^{(\pm)}\partial^\alpha\partial^\beta\partial_\nu V^{\mu\nu}R_{\mu\alpha\beta} + \mathrm{H.c.}$$

RNV interactions

$$\Gamma(\tfrac{1}{2}^{\pm} \to NV) = \frac{1}{16\pi}\frac{q(E_N \mp M_N)}{M_R M_N^2}\left\{g_1^2\left[2(M_R \pm M_N)^2 + M_V^2\right] - 6g_1g_3\frac{M_V^2}{(M_R \mp M_N)^2}(M_R^2 - M_N^2)\right.$$

$$\left. + g_3^2\frac{M_V^2}{(M_R \mp M_N)^2}\left[(M_R \pm M_N)^2 + 2M_V^2\right]\right\},$$

$$A_{1/2}^{\pm} = \mp\frac{1}{2\sqrt{2}}\frac{\sqrt{E_N \mp M_N}}{M_N\sqrt{qM_N}} \times \left[(M_R \pm M_N)g_1 - \frac{M_V^2}{M_R \mp M_N}g_3\right],$$

$$S_{1/2}^{\pm} = \mp\frac{M_V}{4M_N}\sqrt{\frac{E_N \mp M_N}{qM_N}}\left[g_1 - \frac{M_R \pm M_N}{M_R \mp M_N}g_3\right].$$

$$\bar{g}_1 = \frac{g_1}{(2M_N)}, \quad \bar{g}_2 = \frac{g_3}{(2M_N)^2}, \quad \bar{g}_3 = \frac{g_3}{(2M_N)^2}.$$

$$\Gamma(\tfrac{3}{2}^{\pm} \to NV) = \frac{1}{12\pi}\frac{q}{M_R}(E_N \mp M_N) \times \left\{\bar{g}_1^2\left[2E_N(E_N \pm M_N) + (M_R \pm M_N)^2 + 2M_V^2\right] + \bar{g}_2^2\left[E_N^2(2M_R^2 + M_V^2) - 2M_N^2(M_R^2 - M_V^2)\right]\right.$$

$$+ \bar{g}_3^2M_V^2(E_N^2 - M_N^2 + 3M_V^2) \mp 2\bar{g}_1\bar{g}_2\left[\frac{E_N}{2}(3M_R^2 + M_N^2 \pm 2M_NM_R + 3M_V^2) - M_N^2(2M_R \pm M_N)\right]$$

$$\left. + 2\bar{g}_2\bar{g}_3M_V^2(E_N^2 + 2M_N^2 - 3E_NM_R) \pm 2\bar{g}_3\bar{g}_1M_V^2(3M_R - 2E_N \pm M_N)\right\}, \qquad (35)$$

$$A_{3/2}^{\pm} = \mp\frac{1}{4}\frac{\sqrt{E_N \mp M_N}}{M_N\sqrt{qM_N}}\left\{g_1(M_R \pm M_N) \mp \frac{g_2}{4M_N}(M_R^2 - M_N^2 - M_V^2) \pm \frac{g_3}{2M_N}M_V^2\right\},$$

$$A_{1/2}^{\pm} = \mp\frac{1}{4\sqrt{3}}\frac{\sqrt{E_N \mp M_N}}{M_N\sqrt{qM_N}}\left\{\frac{g_1}{M_R}\left[M_N(M_R \pm M_N) \mp M_V^2\right] + \frac{g_2}{4M_N}(M_R^2 - M_N^2 - M_V^2) - \frac{g_3}{2M_N}M_V^2\right\},$$

$$S_{1/2}^{\pm} = \mp\frac{1}{2\sqrt{6}}\frac{M_V\sqrt{E_N \mp M_N}}{M_N\sqrt{qM_N}}\left\{g_1 \mp \frac{g_2}{4M_NM_R}(M_R^2 + M_N^2 - M_V^2) \pm \frac{g_3}{4M_NM_R}(M_R^2 - M_N^2 + M_V^2)\right\}.$$

RNV interactions

$$\Gamma(\tfrac{5}{2}^{\pm} \to NV) = \frac{1}{60\pi}\frac{q^3}{M_R}(E_N \pm M_N)$$
$$\times \Big\{ \bar{g}_1^2 \left[4E_N(E_N \mp M_N) + (M_R \mp M_N)^2 + 4M_V^2\right]$$
$$+ \bar{g}_2^2 \left[E_N^2(3M_R^2 + 2M_V^2) - 3M_N^2(M_R^2 - M_V^2)\right] + \bar{g}_3^2 M_V^2(2E_N^2 - 2M_N^2 + 5M_V^2)$$
$$\pm 2\bar{g}_1\bar{g}_2 \left[E_N(2M_R^2 + M_N^2 \mp M_N M_R + 3M_V^2) - M_N^2(3M_R \mp M_N)\right]$$
$$+ 2\bar{g}_2\bar{g}_3 M_V^2(2E_N^2 + 3M_N^2 - 5E_N M_R) \mp 2\bar{g}_3\bar{g}_1 M_V^2(5M_R - 4E_N \mp M_N)\Big\},$$

$$\bar{g}_1 = \frac{g_1}{(2M_N)^2}, \quad \bar{g}_2 = \frac{g_3}{(2M_N)^3}, \quad \bar{g}_3 = \frac{g_3}{(2M_N)^3}.$$

$$A_{3/2}^{\pm} = \pm\frac{1}{4\sqrt{10}}\frac{\sqrt{E_N \pm M_N}}{M_N^2}\sqrt{\frac{q}{M_N}}\left\{g_1(M_R \mp M_N) \pm \frac{g_2}{4M_N}(M_R^2 - M_N^2 - M_V^2) \mp \frac{g_3}{2M_N}M_V^2\right\},$$

$$A_{1/2}^{\pm} = \pm\frac{1}{8\sqrt{5}}\frac{\sqrt{E_N \pm M_N}}{M_N^2}\sqrt{\frac{q}{M_N}}\Big\{\frac{g_1}{M_R}\left[M_N(M_R \mp M_N) \pm M_V^2\right]$$
$$+ \frac{g_2}{4M_N}(M_R^2 - M_N^2 - M_V^2) - \frac{g_3}{2M_N}M_V^2\Big\},$$

$$S_{1/2}^{\pm} = \pm\frac{1}{4\sqrt{10}}\frac{M_V\sqrt{E_N \pm M_N}}{M_N^2}\sqrt{\frac{q}{M_N}}\Big\{g_1 \pm \frac{g_2}{4M_R M_N}(M_R^2 + M_N^2 - M_V^2)$$
$$\mp \frac{g_3}{4M_R M_N}(M_R^2 - M_N^2 + M_V^2)\Big\}.$$

and so on …

$RN\gamma$ interaction

- Similar to RNV interaction
  - But with $g_3 = 0$

$$\mathcal{L}_{RN\gamma}\left(\tfrac{1}{2}^{\pm}\right) = \frac{ef_1}{2M_N}\bar{N}\Gamma^{(\mp)}\sigma_{\mu\nu}\partial^{\nu}A^{\mu}R + \text{H.c.},$$

$$\mathcal{L}_{RN\gamma}\left(\tfrac{3}{2}^{\pm}\right) = -\frac{ief_1}{2M_N}\overline{N}\Gamma_{\nu}^{(\pm)}F^{\mu\nu}R_{\mu} - \frac{ef_2}{(2M_N)^2}\partial_{\nu}\bar{N}\Gamma^{(\pm)}F^{\mu\nu}R_{\mu} + \text{H.c.},$$

$$\mathcal{L}_{RN\gamma}\left(\tfrac{5}{2}^{\pm}\right) = \frac{ef_1}{(2M_N)^2}\bar{N}\Gamma_{\nu}^{(\mp)}\partial^{\alpha}F^{\mu\nu}R_{\mu\alpha} - \frac{ief_2}{(2M_N)^3}\partial_{\nu}\bar{N}\Gamma^{(\mp)}\partial^{\alpha}F^{\mu\nu}R_{\mu\alpha} + \text{H.c.},$$

Helicity amplitudes

$$\Gamma(R \to N\gamma) = \frac{k_\gamma^2}{\pi}\frac{2M_N}{(2j+1)M_R}[|A_{1/2}|^2 + |A_{3/2}|^2],$$

$$A_{1/2}\left(\tfrac{1}{2}^{\pm}\right) = \mp\frac{ef_1}{2M_N}\sqrt{\frac{k_\gamma M_R}{M_N}},$$

$$A_{1/2}\left(\tfrac{3}{2}^{\pm}\right) = \mp\frac{e\sqrt{6}}{12}\sqrt{\frac{k_\gamma}{M_N M_R}}\left[f_1 + \frac{f_2}{4M_N^2}M_R(M_R \mp M_N)\right],$$

$$A_{3/2}\left(\tfrac{3}{2}^{\pm}\right) = \mp\frac{e\sqrt{2}}{4M_N}\sqrt{\frac{k_\gamma M_R}{M_N}}\left[f_1 \mp \frac{f_2}{4M_N}(M_R \mp M_N)\right],$$

$$A_{1/2}\left(\tfrac{5}{2}^{\pm}\right) = \pm\frac{e}{4\sqrt{10}}\frac{k_\gamma}{M_N}\sqrt{\frac{k_\gamma}{M_N M_R}} \times \left[f_1 + \frac{f_2}{4M_N^2}M_R(M_R \pm M_N)\right],$$

$$A_{3/2}\left(\tfrac{5}{2}^{\pm}\right) = \pm\frac{e}{4\sqrt{5}}\frac{k_\gamma}{M_N^2}\sqrt{\frac{k_\gamma M_R}{M_N}}\left[f_1 \pm \frac{f_2}{4M_N}(M_R \pm M_N)\right],$$

Interactions of R with spin-3/2 baryons

- $J^{\pm} \rightarrow 0^{-} + \frac{3}{2}^{+}$

Lagrangian

$$\mathcal{L}_{RK\Sigma^*}(\tfrac{1}{2}^{\pm}) = \frac{h_1}{M_K}\partial_\mu K\,\bar{\Sigma}^{*\mu}\Gamma^{(\mp)}R + \text{H.c.},$$

$$\mathcal{L}_{RK\Sigma^*}(\tfrac{3}{2}^{\pm}) = \frac{h_1}{M_K}\partial^\alpha K\,\bar{\Sigma}^{*\mu}\Gamma_\alpha^{(\pm)}R_\mu + \frac{ih_2}{M_K^2}\partial^\mu\partial^\alpha K\,\bar{\Sigma}_\alpha^{*}\Gamma^{(\pm)}R_\mu + \text{H.c.},$$

$$\mathcal{L}_{RK\Sigma^*}(\tfrac{5}{2}^{\pm}) = \frac{ih_1}{M_K^2}\partial^\mu\partial^\beta K\,\bar{\Sigma}^{*\alpha}\Gamma_\mu^{(\mp)}R_{\alpha\beta} - \frac{h_2}{M_K^3}\partial^\mu\partial^\alpha\partial^\beta K\,\bar{\Sigma}_\mu^{*}\Gamma^{(\mp)}R_{\alpha\beta} + \text{H.c.}.$$

By angular momentum and parity conservation,

1 coupling for the resonance with $j = 1/2$
2 couplings for the resonance with $j \geq 3/2$

Decay widths

$$\Gamma(\tfrac{1}{2}^{\pm} \to K\Sigma^*) = \frac{h_1^2}{2\pi}\frac{q^3 M_R}{M_K^2 M_{\Sigma^*}^2}(E_{\Sigma^*} \pm M_{\Sigma^*}),$$

$$\Gamma(\tfrac{3}{2}^{\pm} \to K\Sigma^*) = \frac{1}{24\pi}\frac{q}{M_R M_{\Sigma^*}^2}(E_{\Sigma^*} \mp M_{\Sigma^*}) \times \Bigg\{\frac{h_1^2}{M_K^2}(M_R \pm M_{\Sigma^*})^2 \times (2E_{\Sigma^*}^2 \mp 2E_{\Sigma^*}M_{\Sigma^*} + 5M_{\Sigma^*}^2) \mp 2\frac{h_1 h_2}{M_K^3}M_R q^2 (M_R \pm M_{\Sigma^*})(2E_{\Sigma^*} \mp M_{\Sigma^*}) + 2\frac{h_2^2}{M_K^4}M_R^2 q^4\Bigg\},$$

$$\Gamma(\tfrac{5}{2}^{\pm} \to K\Sigma^*) = \frac{1}{60\pi}\frac{q^3}{M_R M_{\Sigma^*}^2}(E_{\Sigma^*} \pm M_{\Sigma^*}) \times \Bigg\{\frac{h_1^2}{M_K^4}(M_R \mp M_{\Sigma^*})^2 \times (4E_{\Sigma^*}^2 \pm 4E_{\Sigma^*}M_{\Sigma^*} + 7M_{\Sigma^*}^2) \mp 4\frac{h_1 h_2}{M_K^5}M_R q^2 (M_R \mp M_{\Sigma^*})(2E_{\Sigma^*} \pm M_{\Sigma^*}) + 4\frac{h_2^2}{M_K^6}M_R^2 q^4\Bigg\},$$

Matching with quark model predictions

- Decay amplitude

$$\langle K(q)\Sigma^*(-q, m_f)| - i\mathcal{H}_{\rm int}|R(\mathbf{0}, m_j)\rangle = 2\pi M_R \sqrt{\frac{2}{q}} \sum_{\ell, m_\ell} \langle \ell m_\ell \tfrac{3}{2} m_f | j m_j\rangle Y_{\ell m_\ell}(\hat{q}) G(\ell),$$

$$\Gamma(R \to K\Sigma^*) = \sum_\ell |G(\ell)|^2,$$

Quark model predictions on G(l)

G's can be related to cc

For $\frac{1}{2}^+$ $$G(1) = -\frac{1}{\sqrt{2\pi}} \frac{q}{M_{\Sigma^*}} \sqrt{q M_R} \sqrt{E_{\Sigma^*} + M_{\Sigma^*}} \frac{h_1}{M_K},$$

For $\frac{1}{2}^-$ $$G(2) = -\frac{1}{\sqrt{2\pi}} \frac{q}{M_{\Sigma^*}} \sqrt{q M_R} \sqrt{E_{\Sigma^*} - M_{\Sigma^*}} \frac{h_1}{M_K},$$

For $\frac{3}{2}^+$ $$G(1) = G_{11}^{(3/2)} \frac{h_1}{M_K} + G_{12}^{(3/2)} \frac{h_2}{M_K^2}, \qquad G(3) = G_{31}^{(3/2)} \frac{h_1}{M_K} + G_{32}^{(3/2)} \frac{h_2}{M_K^2},$$

$$G_{11}^{(3/2)} = \frac{\sqrt{30}}{60\sqrt{\pi}} \frac{1}{M_{\Sigma^*}} \sqrt{\frac{q}{M_R}} \sqrt{E_{\Sigma^*} - M_{\Sigma^*}} (M_R + M_{\Sigma^*}) \times (E_{\Sigma^*} + 4M_{\Sigma^*}),$$

$$G_{12}^{(3/2)} = -\frac{\sqrt{30}}{60\sqrt{\pi}} \frac{q^2\sqrt{q M_R}}{M_{\Sigma^*}} \sqrt{E_{\Sigma^*} - M_{\Sigma^*}},$$

$$G_{31}^{(3/2)} = -\frac{\sqrt{30}}{20\sqrt{\pi}} \frac{1}{M_{\Sigma^*}} \sqrt{\frac{q}{M_R}} \sqrt{E_{\Sigma^*} - M_{\Sigma^*}} \times (M_R + M_{\Sigma^*})(E_{\Sigma^*} - M_{\Sigma^*}),$$

$$G_{32}^{(3/2)} = \frac{\sqrt{30}}{20\sqrt{\pi}} \frac{q^2\sqrt{q M_R}}{M_{\Sigma^*}} \sqrt{E_{\Sigma^*} - M_{\Sigma^*}},$$

and so on …

Application

TABLE I. Resonances listed in the review of PDG [29] and their decay amplitudes of $R \to K\Sigma(1385)$ and of $R \to N\gamma$ predicted in Refs. [10,30]. The coupling constants are calculated using the resonance masses of PDG.

Resonance	PDG [29]	Amplitudes of $R \to K\Sigma(1385)$[a]		h_1	h_2	Amplitudes of $R \to N\gamma$[b]		f_1	f_2
		$G(\ell_1)$	$G(\ell_2)$			$A^p_{1/2}$	$A^p_{3/2}$		
$N\frac{1}{2}^-(1945)$	$S^*_{11}(2090)$	$G(2) = +1.7$	–	-9.8	–	$+12$	–	-0.055	–
$N\frac{3}{2}^-(1960)$	$D^{**}_{13}(2080)$	$G(0) = +1.3$	$G(2) = +1.4$	0.24	-0.54	$+36$	-43	-1.25	1.21
$N\frac{5}{2}^-(2095)$	$D^{**}_{15}(2200)$	$G(2) = -2.0$	$G(4) = 0.0$	0.29	-0.08	-9	-14	0.37	-0.57
$\Delta\frac{3}{2}^-(2080)$	$D^*_{33}(1940)$	$G(0) = -4.1$	$G(3) = -0.5$	-0.68	1.00	-20	-6	0.39	-0.57
$\Delta\frac{5}{2}^+(1990)$	$F^{**}_{35}(2000)$	$G(1) = +4.0$	$G(3) = -0.1$	-0.87	0.11	-10	-28	-0.68	-0.062

[a]In $\sqrt{\text{GeV}}$.
[b]In $10^{-3}/\sqrt{\text{GeV}}$.

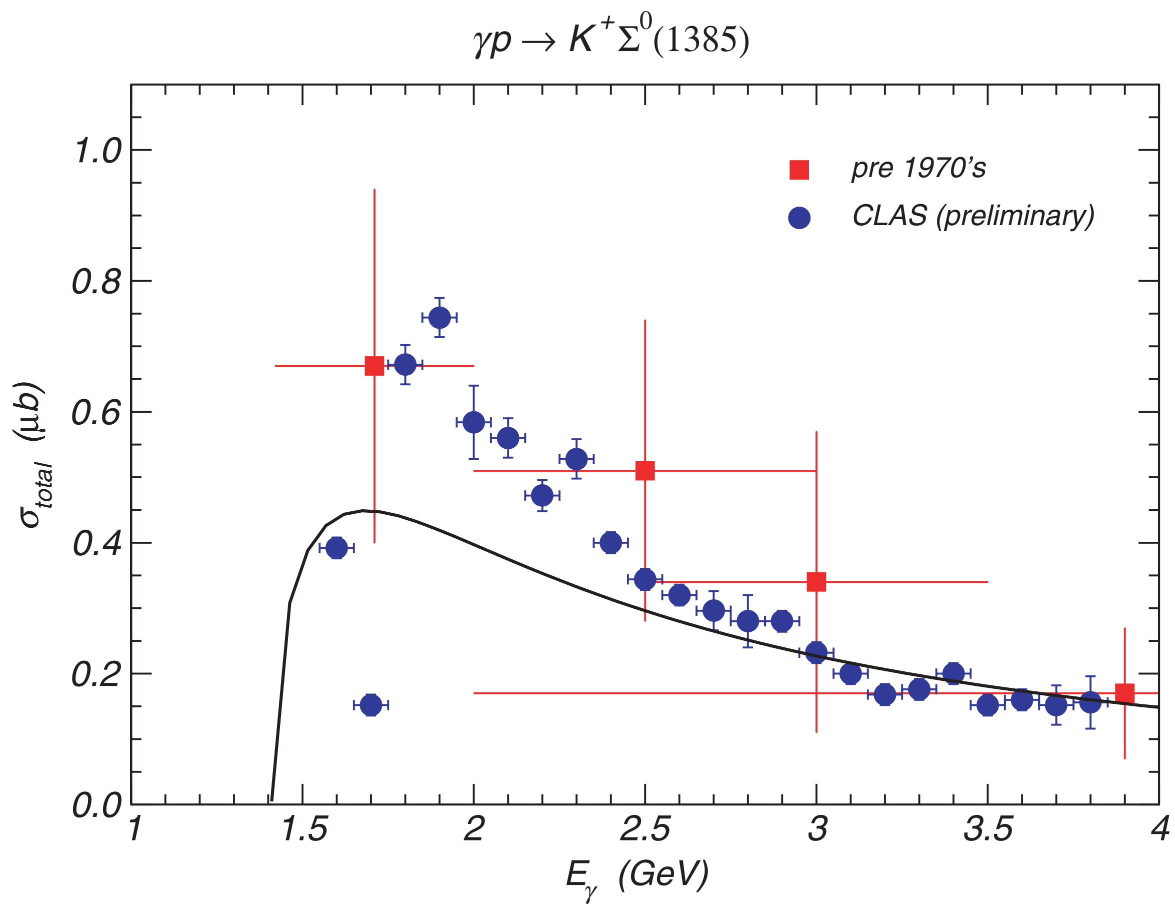
TABLE II. Missing resonances and their decay amplitudes predicted in Refs. [10,30].

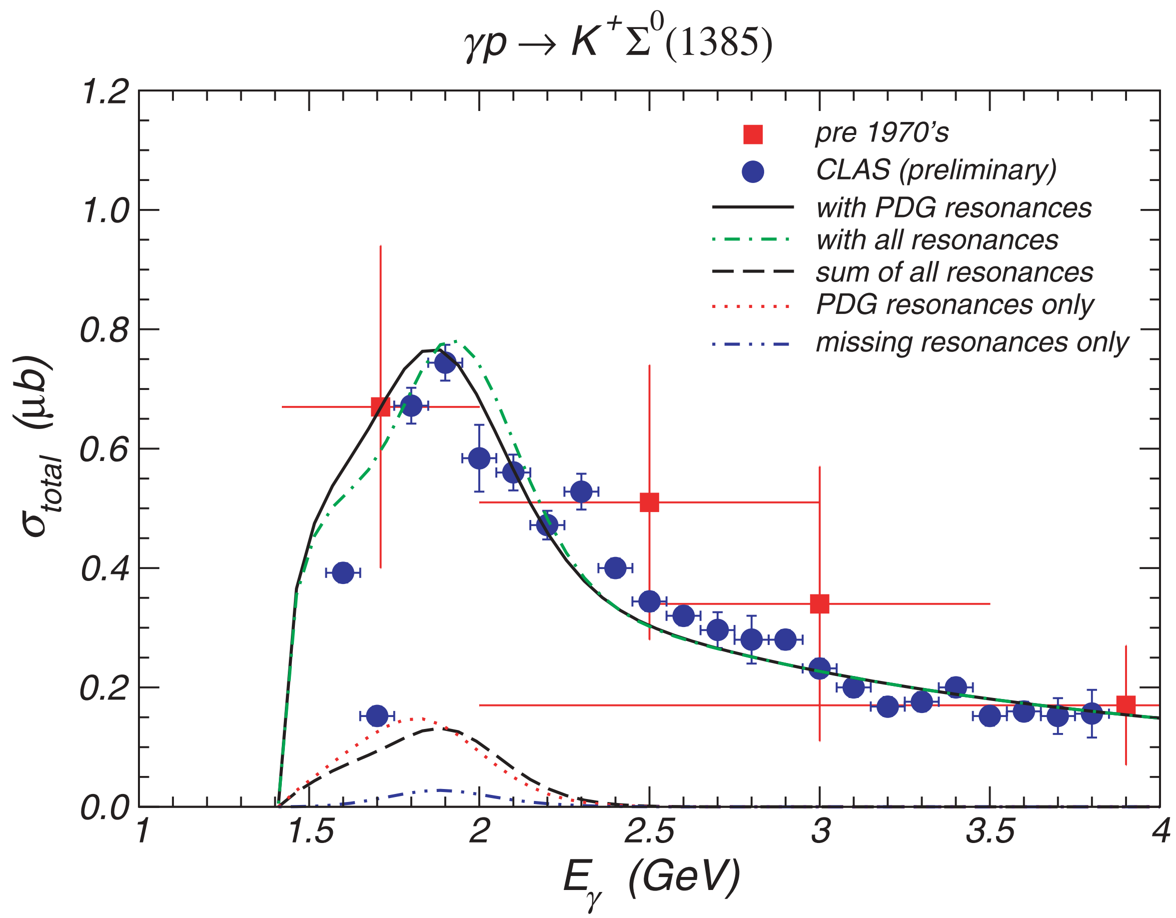
Resonance	Amplitudes of $R \to K\Sigma(1385)$[a]		h_1	h_2	Amplitudes of $R \to N\gamma$[b]		f_1	f_2
	$G(\ell_1)$	$G(\ell_2)$			$A^p_{1/2}$	$A^p_{3/2}$		
$N\frac{3}{2}^-(2095)$	$G(0) = +7.7$	$G(2) = -0.8$	0.99	0.27	-9	-14	0.49	-0.83
$N\frac{5}{2}^+(1980)$	$G(1) = -3.6$	$G(3) = -0.1$	0.59	0.24	-11	-6	0.019	-0.13
$\Delta\frac{3}{2}^-(2145)$	$G(0) = +5.2$	$G(2) = -1.9$	0.25	0.46	0	$+10$	0.11	-0.059

[a]In $\sqrt{\text{GeV}}$.
[b]In $10^{-3}/\sqrt{\text{GeV}}$.

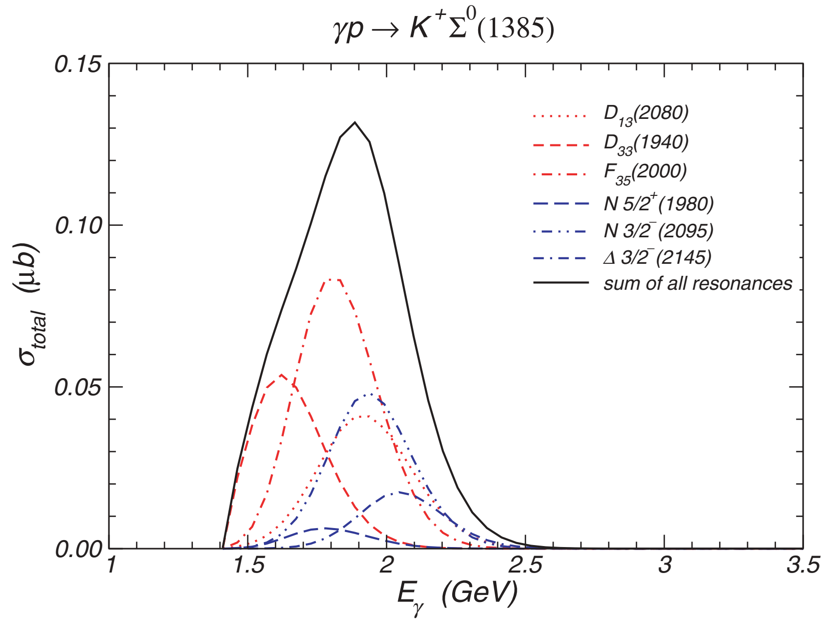
Based on the quark model of Capstick & Roberts

Application





YO, C.M. Ko, K. Nakayama, PRC **77** (2008) 045204



Summary and Outlook

- Needs for High-spin baryon resonances
  - to understand the production mechanisms of various reactions in the resonance region ~ 2 GeV
  - To search for the missing resonances
  - To test various models of baryon structure
- Further works
  - Extraction of coupling constants from various baryon models: A complete list for the coupling constants for various models
  - Issues on gauge invariance, off-shell parameters, etc
    (GWU group, V. Pascalutsa, T. Mart etc)
  - More results to come.

11th APCTP-BLTP JINR-PNPI NRC KI-SPbU Joint Workshop
"Modern problems in nuclear and elementary particle physics"
24-28 July, 2017, Petergof, St. Petersburg, Russia

YONGSEOK OH (KYUNGPOOK NATIONAL UNIV. / APCTP)

NUCLEAR ENERGY DENSITY FUNCTIONAL AND THE NUCLEAR ALPHA DECAY

KNU KYUNGPOOK NATIONAL UNIVERSITY
APCTP Asia Pacific Center for Theoretical Physics

CONTENTS

Ref.
E. Shin, Y. Lim, C.H. Hyun, Y. Oh, PRC 94, 024320 (2016)
Y. Lim, Y. Oh, PRC 95, 034311 (2017)

PHYSICAL REVIEW C **94**, 024320 (2016)

Nuclear isospin asymmetry in α decay of heavy nuclei

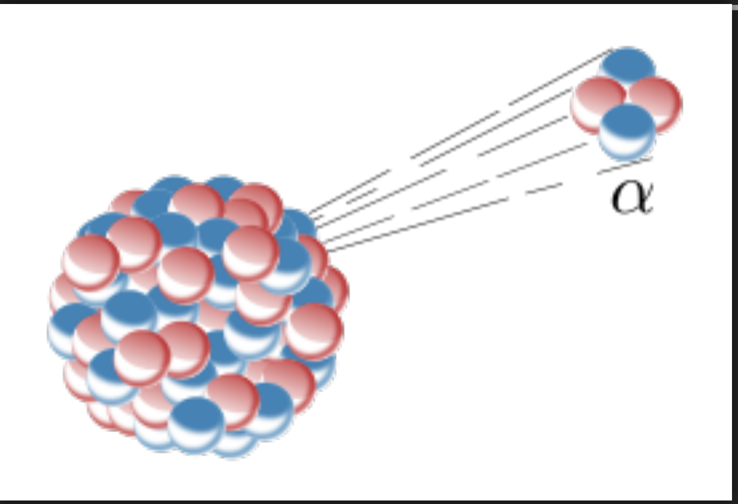
Eunkyoung Shin,[1,*] Yeunhwan Lim,[2,†] Chang Ho Hyun,[3,‡] and Yongseok Oh[1,4,§]
[1]*Department of Physics, Kyungpook National University, Daegu 41566, Korea*
[2]*Rare Isotope Science Project, Institute for Basic Science, Daejeon 34047, Korea*
[3]*Department of Physics Education, Daegu University, Gyeongsan, Gyeongbuk 38453, Korea*
[4]*Asia Pacific Center for Theoretical Physics, Pohang, Gyeongbuk 37673, Korea*
(Received 11 November 2015; revised manuscript received 20 June 2016; published 12 August 2016)

PHYSICAL REVIEW C **95**, 034311 (2017)

Nuclear energy density functional and the nuclear α decay

Yeunhwan Lim[1,*] and Yongseok Oh[2,3,†]
[1]*Cyclotron Institute and Department of Physics and Astronomy, Texas A&M University, College Station, Texas 77843, USA*
[2]*Department of Physics, Kyungpook National University, Daegu 41566, Korea*
[3]*Asia Pacific Center for Theoretical Physics, Pohang, Gyeongbuk 37673, Korea*
(Received 22 January 2017; revised manuscript received 8 February 2017; published 13 March 2017)

INTRODUCTION



- Nuclear α decay:
  the first observed nuclear reaction (1899, Rutherford)
- Why still α decay?
  - A tool to study structure of heavy nuclei
  - Identification of most heavy nucleus formation is made through decays such as α decay chain

Z=118: Oganesson (A=294)

  - E.g.: α decay of (unobserved nucleus) ^{296}Og is planned at JINR @ Dubna.
    (If confirmed, it would be the heaviest element observed so far.)
  - Theoretical understanding of the decay mechanism is needed
    - phenomenological vs fundamental approaches
    - towards more satisfactory theories on the nuclear α decay

THEORIES ON ALPHA DECAYS

- Quantum Tunneling Effects (1928)
  - G. Gamow
  - R.W. Gurney and E.U. Condon
    - one of the first applications of quantum mechanics

Wave Mechanics and Radioactive Disintegration.

AFTER the exponential law in radioactive decay had been discovered in 1902, it soon became clear that the time of disintegration of an atom was independent of the previous history of the atom and depended solely on chance. Since a nuclear particle must be held in the nucleus by an attractive field, we must, in order to explain its ejection, arrange for a spontaneous change from an attractive to a repulsive field. It has hitherto been necessary to postulate some special arbitrary ‘instability’ of the nucleus; but in the following note

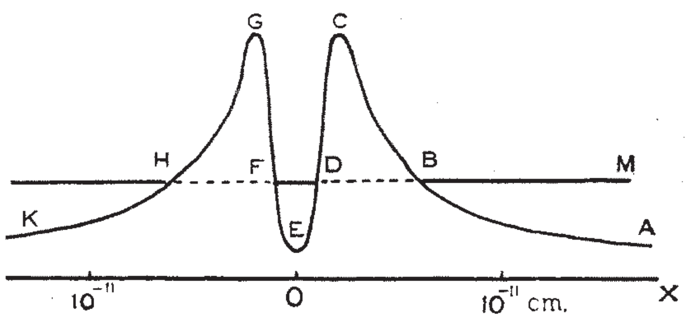


FIG. 1.

RONALD W. GURNEY.
EDW. U. CONDON.
Palmer Physical Laboratory,
Princeton University,
July 30.

Zur Quantentheorie des Atomkernes.

Von G. Gamow, z. Zt. in Göttingen.

Mit 5 Abbildungen. (Eingegangen am 2. August 1928.)

Es wird der Versuch gemacht, die Prozesse der α-Ausstrahlung auf Grund der Wellenmechanik näher zu untersuchen und den experimentell festgestellten Zusammenhang zwischen Zerfallskonstante und Energie der α-Partikel theoretisch zu erhalten.

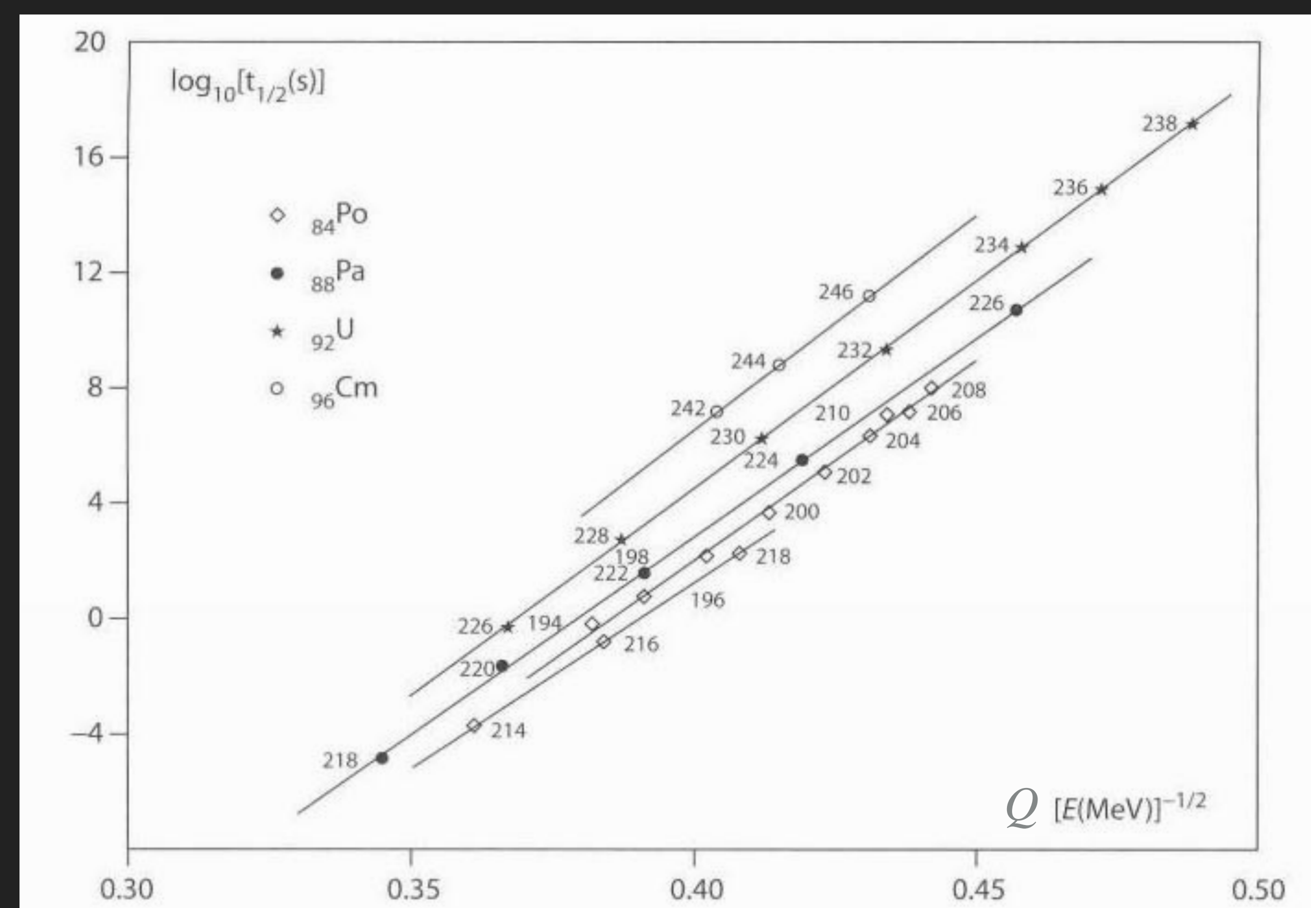
THEORIES ON ALPHA DECAYS

- Geiger-Nuttall law (1911)
  - Viola-Seaborg formula (1961)
  - phenomenological semi-empirical formula

$$\log_{10}(T_{1/2}) = \frac{aZ}{\sqrt{Q_\alpha}} + b$$

$$\log_{10}(T_{1/2}) = \frac{aZ + b}{\sqrt{Q_\alpha}} + cZ + d$$

C. Qi, A.N. Andreyev, M. Huyse, R.J. Liotta, P. Van Duppen, R. Wyss, PLB 734 (2014)



C.A. Bertulani, “Nuclear Physics in a Nutshell” (Princeton Univ. Press, 2007)

BASIS OF ALPHA DECAY THEORIES

- α cluster model
  - the α particle is preformed inside a nucleus.
- Models for effective α potential on nuclear interactions
  - square-well potential, cosh potential, double folding model, etc
- Calculation tool: WKB approximation
  - preformation factor ($\mathcal{P}$)
  - assaulting frequency ($\mathcal{F}$) of the α to the potential well - normalization
  - k: wave number of the α particle

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma} \qquad \Gamma = \mathcal{P}\mathcal{F}\frac{\hbar^2}{4m}\exp\left[-2\int dr k(r)\right] \qquad k(r) = \sqrt{\frac{2m}{\hbar^2}|Q_\alpha - V(r)|}$$

- Important factors which govern the α decay
  - Q values for the decay process (as can be seen in the GN formula)
    - For example, in $^{212}Po \rightarrow {}^{208}Pb + \alpha$
      - $\delta Q = 0.1$ MeV (where $Q_{exp} \approx 8.95$ MeV) causes a factor of 1.7 difference in lifetime
  - α potential: determined by the nucleon distributions - motivation of the present work
    - the interaction potential between the α particle and the rest of the nucleus (core or daughter nucleus)

α POTENTIALS

$$V = V_N + V_C + V_L$$

V_N : nuclear α potential

V_C : Coulomb potential

V_L : centrifugal potential

The Coulomb potential

$$V_C = 8\pi e^2 \left[\frac{1}{r} \int_0^r \rho_p(r') r'^2 dr' + \int_r^\infty \rho_p(r') r' dr' \right]$$

The centrifugal potential with the Langer modification

$$V_L = \frac{\hbar^2}{2mr^2} \left(\ell + \frac{1}{2} \right)^2$$

ISOSPIN EFFECTS IN NUCLEAR α POTENTIALS

E. Shin, Y. Lim, C.H. Hyun, YO, PRC 94 (2016)

- Effects of isospin asymmetry terms in nuclear α potentials
  - Define: I = (N−Z)/(N+Z),
    where N = neutron number and Z = proton number
  - Square-well potential

$$V_N = \begin{cases} V_0 + V_1 I + V_2 I^2, & r < R \\ 0, & r > R \end{cases}$$

  - Woods-Saxon potential

$$V_N = \frac{V_0 + V_1 I + V_2 I^2}{1 + \exp[(r - R)/a]}$$

  - Viola-Seaborg formula

$$\log_{10}(T_{1/2}) = \frac{aZ + b}{\sqrt{Q_\alpha}} + cZ + d + e_1 I + e_2 I^2$$

EFFECTS OF THE ISOSPIN TERMS 1

Square-well potential

TABLE I. Parameters of the SW potential fitted to the experimental data of Refs. [30,31]. The numbers in parentheses denote the fitted values without the V_1 and V_2 terms. The rms deviation σ is defined in Eq. (6).

Type	Number of events	V_0 (MeV)	V_1 (MeV)	V_2 (MeV)	σ
e-e	178	−140.035 (−132.415)	+57.567	−71.601	0.304 (0.319)
e-o	110	−175.980 (−140.416)	+524.995	−1737.533	0.596 (0.616)
o-e	137	−158.767 (−142.700)	+308.787	−1163.721	0.607 (0.630)
o-o	70	−152.100 (−144.250)	+56.482	−63.256	0.604 (0.609)

Woods-Saxon potential

TABLE IV. Fitted parameters of the WS potential. The notation is the same as in Table I.

Type	V_0 (MeV)	V_1 (MeV)	V_2 (MeV)	σ
e-e	−190.845 (−179.634)	+54.851	+56.370	0.302 (0.326)
e-o	−173.564 (−174.859)	+64.534	−38.600	0.211 (0.212)
o-e	−187.018 (−182.313)	+36.494	+127.714	0.248 (0.251)
o-o	−180.316 (−176.876)	−16.653	+86.544	0.254 (0.256)

EFFECTS OF THE ISOSPIN TERMS 2

Viola-Seaborg Forumla

TABLE VIII. Fitted coefficients of the modified VS formula. The values in parentheses are those of the unmodified VS formula, i.e., without the e_1 and e_2 terms.

Type	a	b	c	d	e_1	e_2	σ
e-e	1.53420 (1.48503)	4.20759 (5.26806)	−0.18124 (−0.18879)	−35.57934 (−33.89407)	5.28401	−38.17144	0.311 (0.359)
e-o	1.64322 (1.55427)	−2.33315 (1.23165)	−0.18749 (−0.18838)	−35.27841 (−34.29805)	1.19898	−31.24030	0.571 (0.608)
o-e	1.69868 (1.64654)	−5.67266 (−3.14939)	−0.22366 (−0.22053)	−32.02953 (−32.74153)	−12.96399	31.01813	0.542 (0.554)
o-o	1.37778 (1.34355)	13.63138 (13.92103)	−0.11009 (−0.12867)	−39.41075 (−37.19944)	5.98423	−52.56801	0.561 (0.617)

Conclusions

- Not a crucial effect on the alpha decay lifetime estimates
- But gives an improvement in RMSD (σ)

α POTENTIAL BASED ON SKYRME FORCE MODEL

- Following the standard Skyrme EDF, we write

$$\begin{aligned} v_{N\alpha}(\boldsymbol{k},\boldsymbol{k}') = {} & s_0\,(1+v_0P_\sigma)\,\delta(\boldsymbol{r}_{N\alpha}) \\ & + \frac{s_1}{2}\,(1+v_1P_\sigma)\,\left[\delta(\boldsymbol{r}_{N\alpha})\boldsymbol{k}^2 + \boldsymbol{k}'^2\delta(\boldsymbol{r}_{N\alpha})\right] \\ & + s_2\,\boldsymbol{k}'\cdot\delta(\boldsymbol{r}_{N\alpha})\boldsymbol{k} \\ & + iW_0^\alpha\boldsymbol{k}'\cdot(\boldsymbol{\sigma}\times\boldsymbol{k})\,\delta(\boldsymbol{r}_{N\alpha}) \\ & + \frac{s_3}{6}\,(1+v_3P_\sigma)\,\rho_N^\epsilon\delta(\boldsymbol{r}_{N\alpha}) \end{aligned}$$

- This leads to the form of the α potential in terms of nucleon densities as

$$V_N = \alpha\rho_N + \beta\left(\rho_n^{5/3} + \rho_p^{5/3}\right) + \gamma\rho_N^\epsilon\left(\rho_N^2 + 2\rho_n\rho_p\right) + \delta\frac{\rho_N'}{r} + \eta\rho_N''$$

where

$$\rho_N = \rho_p + \rho_n, \quad \rho_N' = d\rho_N/dr, \quad \rho_N'' = d^2\rho_N/dr^2$$

$$\rho_{p,n} = \frac{\rho_{p,n}^0}{1+\exp[(r-R_{p,n})/a_{p,n}]}$$

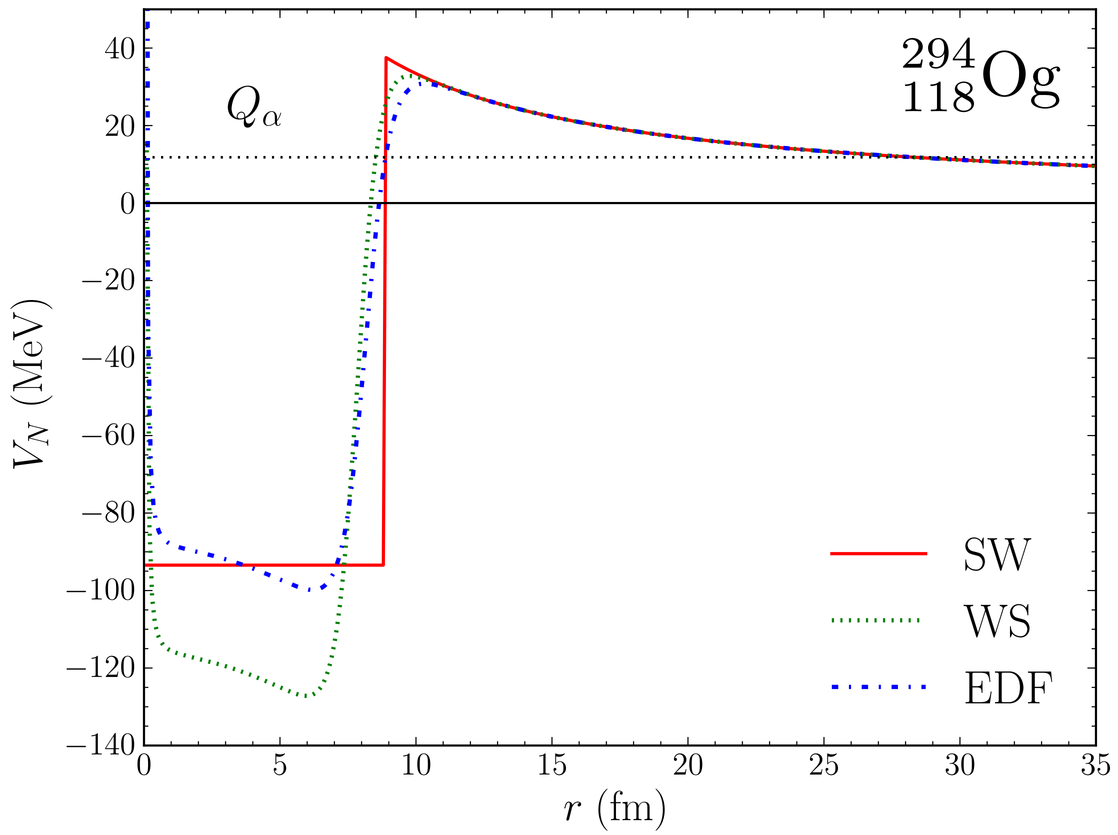
RESULTS

TABLE VII. Fitted parameters of the α-particle potential model based on the Skyrme EDF.

α (MeV fm^3)	β (MeV fm^5)	γ (MeV fm$^{6+3\epsilon}$)	δ (MeV fm^5)	η (MeV fm^5)
-1.6740×10^3	1.9208×10^3	1.7182×10^3	9.4166	-26.7616
σ(e-e)	σ(e-o)	σ(o-e)	σ(o-o)	σ(All)
0.319	0.276	0.283	0.301	0.296

$\epsilon = 1/6$

Better description than simple potential models



$$Q_\alpha^{\rm exp} = 11.81 \text{ MeV}$$

$$T_{1/2}^{\rm SW} = 1.46 \times 10^{-4} \text{ s}$$
$$T_{1/2}^{\rm WS} = 1.26 \times 10^{-4} \text{ s}$$
$$T_{1/2}^{\rm EDF} = 0.40 \times 10^{-4} \text{ s}$$
$$T_{1/2}^{\rm VS} = 0.31 \times 10^{-4} \text{ s}$$

exp: ~ 0.89 ms

α nuclear potential of the three potential models

RESULTS

TABLE XI. Results for α-decay half-lives of heavy nuclei. The upper and lower bounds of theoretical calculations are from the experimental errors of Q_α values.

(Z,A)	$Q_\alpha^{\text{Expt.}}$ (MeV)	$T_{1/2}^{\text{Expt.}}$	$T_{1/2}^{\text{SW}}$	$T_{1/2}^{\text{WS}}$	$T_{1/2}^{\text{EDF}}$	$T_{1/2}^{\text{VS}}$	References
(118,294)	11.81 ± 0.06	$0.89^{+1.07}_{-0.31}$ ms	$1.46^{+0.51}_{-0.38}$ ms	$1.26^{+0.45}_{-0.33}$ ms	$0.40^{+0.15}_{-0.11}$ ms	$0.31^{+0.12}_{-0.08}$ ms	[39]
(116,293)	10.67 ± 0.06	53^{+62}_{-19} ms	163^{+69}_{-48} ms	104^{+44}_{-31} ms	52^{+23}_{-16} ms	181^{+84}_{-57} ms	[40]
(116,292)	10.80 ± 0.07	18^{+16}_{-6} ms	78^{+39}_{-26} ms	69^{+35}_{-23} ms	25^{+13}_{-8} ms	20^{+10}_{-7} ms	[40]
(116,291)	10.89 ± 0.07	$6.3^{+11.6}_{-2.5}$ ms	47^{+23}_{-15} ms	31^{+15}_{-10} ms	16^{+8}_{-5} ms	46^{+25}_{-16} ms	[39]
(116,290)	11.00 ± 0.08	$7.1^{+3.2}_{-1.7}$ ms	$25.9^{+14.5}_{-9.2}$ ms	$23.2^{+13.2}_{-8.3}$ ms	$8.9^{+5.0}_{-3.3}$ ms	$7.2^{+4.2}_{-2.6}$ ms	[39]
(115,288)	10.61 ± 0.06	87^{+105}_{-30} ms	115^{+48}_{-34} ms	139^{+60}_{-41} ms	43^{+19}_{-13} ms	676^{+279}_{-196} ms	[41]
(115,287)	10.74 ± 0.09	32^{+155}_{-14} ms	55^{+37}_{-22} ms	50^{+34}_{-20} ms	21^{+15}_{-8} ms	131^{+97}_{-55} ms	[41]
(114,289)	9.96 ± 0.06	$2.7^{+1.4}_{-0.7}$ s	$2.8^{+1.3}_{-0.9}$ s	$3.1^{+1.5}_{-1.0}$ s	$1.1^{+0.5}_{-0.3}$ s	$4.8^{+2.5}_{-1.6}$ s	[40]
(114,288)	10.09 ± 0.07	$0.8^{+0.32}_{-0.18}$ s	$1.2^{+0.68}_{-0.43}$ s	$1.12^{+0.63}_{-0.40}$ s	$0.48^{+0.27}_{-0.17}$ s	$0.39^{+0.22}_{-0.14}$ s	[40]
(114,287)	10.16 ± 0.06	$0.48^{+0.16}_{-0.09}$ s	$0.80^{+0.36}_{-0.25}$ s	$0.53^{+0.24}_{-0.17}$ s	$0.32^{+0.15}_{-0.10}$ s	$1.23^{+0.61}_{-0.41}$ s	[39]
(114,286)	10.33 ± 0.06	$0.13^{+0.04}_{-0.02}$ s	$0.29^{+0.13}_{-0.09}$ s	$0.26^{+0.12}_{-0.08}$ s	$0.12^{+0.05}_{-0.04}$ s	$0.10^{+0.04}_{-0.03}$ s	[39]
(113,284)	10.15 ± 0.06	$0.48^{+0.58}_{-0.17}$ s	$0.40^{+0.18}_{-0.12}$ s	$0.50^{+0.23}_{-0.16}$ s	$0.28^{+0.13}_{-0.09}$ s	$2.12^{+0.93}_{-0.64}$ s	[41]
(113,283)	10.26 ± 0.09	100^{+490}_{-45} ms	209^{+152}_{-87} ms	62^{+45}_{-26} ms	91^{+69}_{-39} ms	563^{+445}_{-246} ms	[41]
(113,282)	10.83 ± 0.08	73^{+134}_{-29} ms	8^{+4}_{-3} ms	52^{+30}_{-19} ms	75^{+44}_{-28} ms	52^{+29}_{-18} ms	[42]
(112,285)	9.29 ± 0.06	34^{+17}_{-9} s	50^{+27}_{-17} s	34^{+18}_{-12} s	23^{+12}_{-8} s	133^{+76}_{-48} s	[40]
(112,283)	9.67 ± 0.06	$3.8^{+1.2}_{-0.7}$ s	$3.9^{+1.9}_{-1.3}$ s	$4.5^{+2.2}_{-1.5}$ s	$1.8^{+0.9}_{-0.6}$ s	$8.4^{+4.4}_{-2.9}$ s	[39]
(111,280)	9.87 ± 0.06	$3.6^{+4.3}_{-1.3}$ s	$0.50^{+0.23}_{-0.16}$ s	$3.7^{+1.7}_{-1.2}$ s	$6.0^{+2.9}_{-1.9}$ s	$2.4^{+1.1}_{-0.7}$ s	[41]
(111,279)	10.52 ± 0.16	170^{+810}_{-80} ms	10^{+16}_{-6} ms	62^{+96}_{-37} ms	110^{+177}_{-67} ms	23^{+39}_{-14} ms	[41]
(111,278)	10.89 ± 0.08	$4.2^{+7.5}_{-1.7}$ ms	$1.4^{+0.7}_{-0.5}$ ms	$2.7^{+1.5}_{-0.9}$ ms	$2.7^{+1.6}_{-1.0}$ ms	$8.2^{+4.4}_{-2.9}$ ms	[42]
(110,279)	9.84 ± 0.06	$0.20^{+0.05}_{-0.04}$ s	$0.28^{+0.13}_{-0.09}$ s	$0.18^{+0.08}_{-0.06}$ s	$0.13^{+0.06}_{-0.04}$ s	$0.59^{+0.30}_{-0.20}$ s	[39]
(109,276)	9.85 ± 0.06	$0.72^{+0.97}_{-0.25}$ s	$0.12^{+0.05}_{-0.04}$ s	$0.88^{+0.41}_{-0.28}$ s	$0.29^{+0.14}_{-0.09}$ s	$0.52^{+0.23}_{-0.16}$ s	[41]
(109,275)	10.48 ± 0.09	$9.7^{+46}_{-4.4}$ ms	$3.0^{+2.0}_{-1.2}$ ms	$18.6^{+12.5}_{-7.4}$ ms	$6.7^{+4.6}_{-2.7}$ ms	$6.3^{+4.5}_{-2.6}$ ms	[41]
(109,274)	9.95 ± 0.10	440^{+810}_{-170} ms	67^{+56}_{-30} ms	480^{+416}_{-220} ms	172^{+153}_{-80} ms	353^{+294}_{-159} ms	[42]
(108,275)	9.44 ± 0.06	$0.19^{+0.22}_{-0.07}$ s	$0.75^{+0.36}_{-0.24}$ s	$0.48^{+0.24}_{-0.16}$ s	$0.39^{+0.20}_{-0.13}$ s	$2.12^{+1.12}_{-0.73}$ s	[39]
(107,272)	9.15 ± 0.06	$9.8^{+11.7}_{-3.5}$ s	$2.3^{+1.2}_{-0.8}$ s	$5.3^{+2.8}_{-1.8}$ s	$7.0^{+3.7}_{-2.4}$ s	$8.7^{+4.3}_{-2.9}$ s	[41]
(107,270)	9.11 ± 0.08	61^{+292}_{-28} s	$3.1^{+2.3}_{-1.3}$ s	25^{+19}_{-11} s	60^{+46}_{-26} s	14^{+10}_{-6} s	[42]
(106,271)	8.67 ± 0.08	$1.9^{+2.4}_{-0.6}$ min	$0.51^{+0.41}_{-0.22}$ min	$2.06^{+1.71}_{-0.92}$ min	$1.67^{+1.41}_{-0.76}$ min	$2.28^{+2.01}_{-1.06}$ min	[39]
σ			0.616	0.290	0.238	0.513	

RESULTS

TABLE XII. Theoretical predictions on α-decay lifetimes of superheavy elements. The Q_α values are calculated with the WS4 mass table [44]. The modified and unmodified Viola-Seaborg formulas are represented by VS and VS0, respectively.

Nuclei (Z, A)	Q_α (MeV)	$T_{1/2}^{\rm SW}$ (s)	$T_{1/2}^{\rm WS}$ (s)	$T_{1/2}^{\rm EDF}$ (s)	$T_{1/2}^{\rm VS}$ (s)	$T_{1/2}^{\rm VS0}$ (s)
(122,307)	14.360	2.721×10^{-7}	1.417×10^{-7}	3.401×10^{-8}	3.315×10^{-8}	3.402×10^{-8}
(122,306)	13.775	2.641×10^{-6}	1.975×10^{-6}	3.777×10^{-7}	2.380×10^{-7}	2.026×10^{-7}
(122,305)	13.734	3.147×10^{-6}	1.749×10^{-6}	4.746×10^{-7}	5.266×10^{-7}	5.103×10^{-7}
(122,304)	13.710	3.503×10^{-6}	2.684×10^{-6}	5.544×10^{-7}	3.563×10^{-7}	2.669×10^{-7}
(122,303)	13.904	1.630×10^{-6}	9.198×10^{-7}	2.614×10^{-7}	2.468×10^{-7}	2.405×10^{-7}
(122,302)	14.208	5.069×10^{-7}	3.820×10^{-7}	8.078×10^{-8}	4.887×10^{-8}	3.438×10^{-8}
(121,306)	13.783	1.392×10^{-6}	1.396×10^{-6}	1.873×10^{-7}	6.268×10^{-6}	5.896×10^{-6}
(121,305)	13.242	1.296×10^{-5}	1.943×10^{-6}	1.999×10^{-6}	8.881×10^{-6}	8.478×10^{-6}
(121,304)	13.251	1.259×10^{-5}	1.302×10^{-5}	2.030×10^{-6}	6.994×10^{-5}	5.196×10^{-5}
(121,303)	13.283	1.109×10^{-5}	1.673×10^{-6}	1.864×10^{-6}	8.416×10^{-6}	7.039×10^{-6}
(121,302)	13.464	5.247×10^{-6}	5.273×10^{-6}	8.943×10^{-7}	3.391×10^{-5}	2.137×10^{-5}
(121,301)	13.795	1.391×10^{-6}	2.086×10^{-7}	2.344×10^{-7}	9.437×10^{-7}	7.494×10^{-7}

TABLE XIII. Half-lives of nuclides in the decay chain of the nucleus $^{294}117$. The experimental data are from Ref. [49].

(Z, A)	Q_α (MeV)	$T_{1/2}^{\rm Expt.}$	$T_{1/2}^{\rm SW}$	$T_{1/2}^{\rm WS}$	$T_{1/2}^{\rm EDF}$	$T_{1/2}^{\rm VS}$	$T_{1/2}^{\rm VS0}$
(117,294)	11.20 ± 0.04	51^{+94}_{-20} ms	17^{+4}_{-3} ms	34^{+9}_{-7} ms	22^{+6}_{-4} ms	96^{+23}_{-18} ms	75^{+18}_{-14} ms
(115,290)	10.45 ± 0.04	$1.3^{+2.3}_{-0.5}$ s	$0.29^{+0.08}_{-0.06}$ s	$2.0^{+0.56}_{-0.44}$ s	$2.3^{+0.64}_{-0.50}$ s	$1.40^{+0.37}_{-0.29}$ s	$1.28^{+0.33}_{-0.26}$ s
(113,286)	9.4 ± 0.3	$2.9^{+5.3}_{-1.1}$ s	53^{+398}_{-46} s	71^{+552}_{-62} s	24^{+191}_{-21} s	208^{+1452}_{-179} s	209^{+1390}_{-179} s
(111,282)	9.18 ± 0.03	$3.1^{+5.7}_{-1.2}$ min	$0.81^{+0.19}_{-0.16}$ min	$1.91^{+0.46}_{-0.37}$ min	$1.96^{+0.48}_{-0.38}$ min	$2.88^{+0.66}_{-0.54}$ min	$3.60^{+0.81}_{-0.66}$ min
(109,278)	9.59 ± 0.03	$3.6^{+6.5}_{-1.4}$ s	$0.61^{+0.13}_{-0.11}$ s	$4.70^{+1.03}_{-0.84}$ s	$1.44^{+0.32}_{-0.26}$ s	$2.13^{+0.45}_{-0.37}$ s	$3.63^{+0.75}_{-0.62}$ s
(107,274)	8.97 ± 0.03	30^{+54}_{-12} s	$8.0^{+1.9}_{-1.5}$ s	$18.8^{+4.5}_{-3.6}$ s	$22.9^{+5.6}_{-4.5}$ s	$23.6^{+5.5}_{-4.4}$ s	$48.0^{+10.8}_{-8.8}$ s
(105,270)	8.02 ± 0.03	$1.0^{+1.9}_{-0.4}$ h	$0.57^{+0.16}_{-0.12}$ h	$0.82^{+0.23}_{-0.18}$ h	$0.39^{+0.11}_{-0.09}$ h	$1.27^{+0.35}_{-0.27}$ h	$2.91^{+0.78}_{-0.61}$ h
σ			0.769	0.592	0.486	0.773	0.790
			0.625	0.185	0.340	0.173	0.241

Z=117: Tennessine

NEXT STEP – STRATEGY

Y. Lim, YO, PRC 95 (2017)

- Further development on the nuclear α potential based on the Skyrme EDF
- the form of the nuclear α potential

$$V_N = \alpha\rho_N + \beta\left(\rho_n^{5/3} + \rho_p^{5/3}\right) + \gamma\rho_N^{\epsilon}\left(\rho_N^2 + 2\rho_n\rho_p\right) + \delta\frac{\rho'_N}{r} + \eta\rho''_N$$

- Use more sophisticated EDF to obtain the nucleon density profiles in heavy nuclei
- Once the density profiles are obtained, fit the nuclear α potential parameters to the observed α decay data
- Then, apply the model to estimate the unobserved decays.

MODEL SETUP - FITTING PROCESS

- Q_α values: calculated from the observed masses

  E.L. Medeiros, M.M.N. Rodrigues, S.B. Duarte, O.A.P. Tavares, JPG 32, B23 (2006)

$$Q = \Delta M(Z,A) - \Delta M(Z-2,A-4) - \Delta M_\alpha + 10^{-6}\,k\left[Z^\beta - (Z-2)^\beta\right]$$

$$\Delta M_\alpha = 2.4249\ \mathrm{MeV}, \quad k = 8.7\ \mathrm{MeV}, \quad \beta = 2.517\ \text{for}\ Z \geq 60$$

- Nuclear density profiles - we consider 3 models
  - Skyrme SLy4 — E. Chabanat et al., NPA 635, 231 (1998)
  - Gogny D1S — J.F. Berger, M. Girod, D. Gogny, Com. Phys. Comm. 63, 365 (1991)
  - RMF DD-ME2 — G.A. Lalazissis, T. Niksic, D. Vretenar, P. Ring, PRC 71, 024312 (2005)

EDF MODELS

Skyrme SLy4

$$
\begin{aligned}
v_{ij} = {} & t_0\left(1+x_0 P_\sigma\right)\delta\left(\mathbf{r}_i-\mathbf{r}_j\right) \\
& + \frac{t_1}{2}\left(1+x_1 P_\sigma\right)\left[\delta\left(\mathbf{r}_i-\mathbf{r}_j\right)\mathbf{k}^2+\mathbf{k}'^2\delta\left(\mathbf{r}_i-\mathbf{r}_j\right)\right] \\
& + t_2(1+x_2 P_\sigma)\,\mathbf{k}'\cdot\delta(\mathbf{r}_i-\mathbf{r}_j)\mathbf{k} \\
& + \frac{t_3}{6}\left(1+x_3 P_\sigma\right)\rho^\epsilon\delta(\mathbf{r}_i-\mathbf{r}_j) \\
& + i\,W_0\,\mathbf{k}'\delta(\mathbf{r}_i-\mathbf{r}_j)\times\mathbf{k}\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)
\end{aligned}
$$

Gogny D1S

$$
\begin{aligned}
v_{12} = {} & \sum_{j=1,2}\exp\left\{-\frac{(\mathbf{r}_1-\mathbf{r}_2)^2}{\mu_j^2}\right\}(W_j+B_jP_\sigma-H_jP_\tau-M_jP_\sigma P_\tau) \\
& + t_0\left(1+x_0P_\sigma\right)\rho^\epsilon\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right)\delta\left(\mathbf{r}_1-\mathbf{r}_2\right) \\
& + iW_{LS}\,\mathbf{k}'\delta\left(\mathbf{r}_1-\mathbf{r}_2\right)\times\mathbf{k}\cdot\left(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2\right)
\end{aligned}
$$

RMF DD-ME2

$$
\begin{aligned}
\mathcal{L} = {} & \bar\psi\left(i\partial\!\!\!/-m\right)\psi+\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma-\frac{1}{2}m_\sigma\sigma^2-g_\sigma\bar\psi\sigma\psi \\
& -\frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu}+\frac{1}{2}m_\omega^2\omega^2-g_\omega\bar\psi\gamma^\mu\omega_\mu\psi \\
& -\frac{1}{4}\vec{R}^{\mu\nu}\cdot\vec{R}_{\mu\nu}+\frac{1}{2}m_\rho^2\vec\rho^{\,2}-g_\rho\bar\psi\gamma^\mu\vec\rho_\mu\cdot\vec\tau\psi \\
& -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-e\bar\psi\gamma^\mu A_\mu\frac{(1-\tau_3)}{2}\psi
\end{aligned}
$$

MODEL SETUP – NUCLEAR ALPHA POTENTIAL

TABLE III. Parameters for α particle potential in Eq. (13).

Parameter	SLy4	D1S	DD-ME2	Unit
α	-1484.58	-1499.04	-1524.24	MeV fm^3
β	1355.57	1248.80	1289.04	MeV fm^5
γ	1005.48	242.28	1137.21	MeV fm$^{6+\epsilon}$
δ	53.87	30.75	-41.84	MeV fm^5
η	-210.15	-178.12	-184.09	MeV fm^5
ϵ	$1/6$	$1/3$	$1/6$	

RESULTS – OBSERVED DECAYS

TABLE IV. Observed α decay half-lives of heavy nuclei and the results of the present paper. Unless specified, $\ell = 0$ is understood.

(Z, A)	Q_α^{Expt} (MeV)	$T_{1/2}^{\mathrm{Expt}}$	$T_{1/2}^{\mathrm{SLy4}}(\ell)$	$T_{1/2}^{\mathrm{D1S}}(\ell)$	$T_{1/2}^{\mathrm{DD\text{-}ME2}}(\ell)$	Reference
(118,294)	11.81 ± 0.06	$0.89^{+1.07}_{-0.31}$ ms	$0.50^{+0.18}_{-0.13}$ ms	$0.61^{+0.22}_{-0.16}$ ms	$0.43^{+0.15}_{-0.11}$ ms	[40]
(116,293)	10.67 ± 0.06	53^{+62}_{-19} ms	65^{+28}_{-20} ms	78^{+33}_{-23} ms	54^{+24}_{-16} ms	[41]
(116,292)	10.80 ± 0.07	18^{+16}_{-6} ms	31^{+16}_{-10} ms	38^{+19}_{-13} ms	26^{+13}_{-9} ms	[41]
(116,291)	10.89 ± 0.07	18^{+22}_{-6} ms	19^{+9}_{-6} ms	23^{+11}_{-7} ms	16^{+8}_{-5} ms	[40]
(116,290)	11.00 ± 0.08	$7.1^{+3.2}_{-1.7}$ ms	$10.6^{+6.1}_{-3.8}$ ms	$12.5^{+7.2}_{-4.5}$ ms	$8.6^{+5.0}_{-3.1}$ ms	[40]
(115,288)	10.61 ± 0.06	87^{+105}_{-30} ms	51^{+21}_{-15} ms	57^{+25}_{-17} ms	42^{+19}_{-13} ms	[42,43]
(115,287)	10.74 ± 0.09	32^{+155}_{-14} ms	25^{+17}_{-10} ms	28^{+20}_{-12} ms	21^{+15}_{-9} ms	[42,43]
(114,289)	9.96 ± 0.06	$2.7^{+1.4}_{-0.7}$ s	$1.3^{+0.6}_{-0.4}$ s	$1.5^{+0.7}_{-0.5}$ s	$1.0^{+0.5}_{-0.3}$ s	[41]
(114,288)	10.09 ± 0.07	$0.8^{+0.32}_{-0.18}$ s	$0.56^{+0.31}_{-0.20}$ s	$0.65^{+0.37}_{-0.23}$ s	$0.46^{+0.26}_{-0.16}$ s	[41]
(114,287)	10.16 ± 0.06	$0.48^{+0.16}_{-0.09}$ s	$0.37^{+0.17}_{-0.12}$ s	$0.42^{+0.20}_{-0.13}$ s	$0.31^{+0.15}_{-0.10}$ s	[40]
(114,286)	10.33 ± 0.06	$0.13^{+0.04}_{-0.02}$ s	$0.14^{+0.06}_{-0.04}$ s	$0.15^{+0.07}_{-0.05}$ s	$0.12^{+0.05}_{-0.04}$ s	[40]
(113,284)	10.15 ± 0.06	$0.48^{+0.58}_{-0.17}$ s	$0.20^{+0.09}_{-0.06}$ s	$0.23^{+0.10}_{-0.07}$ s	$0.28^{+0.13}_{-0.09}$ s ($\ell = 2$)	[42,43]
(113,283)	10.26 ± 0.09	100^{+490}_{-45} ms	106^{+77}_{-45} ms	120^{+89}_{-51} ms	94^{+70}_{-40} ms	[42,43]
(113,282)	10.83 ± 0.08	73^{+134}_{-29} ms	106^{+62}_{-38} ms ($\ell = 6$)	121^{+73}_{-45} ms ($\ell = 6$)	93^{+55}_{-34} ms ($\ell = 6$)	[44]
(112,285)	9.29 ± 0.06	34^{+17}_{-9} s	27^{+14}_{-10} ms	30^{+16}_{-10} s	22^{+13}_{-8} s	[41]
(112,283)	9.67 ± 0.06	$3.8^{+1.2}_{-0.7}$ s	$2.0^{+1.0}_{-0.7}$ s	$2.3^{+1.2}_{-0.8}$ s	$1.8^{+0.9}_{-0.6}$ s	[40]
(111,280)	9.87 ± 0.06	$3.6^{+4.3}_{-1.3}$ s	$1.4^{+0.7}_{-0.4}$ s ($\ell = 4$)	$1.6^{+0.8}_{-0.5}$ s ($\ell = 4$)	$7.2^{+3.4}_{-2.3}$ s ($\ell = 6$)	[42,43]
(111,279)	10.52 ± 0.16	170^{+810}_{-80} ms	157^{+251}_{-95} ms ($\ell = 6$)	176^{+276}_{-106} ms ($\ell = 6$)	138^{+219}_{-83} ms ($\ell = 6$)	[42,43]
(111,278)	10.89 ± 0.08	$4.2^{+7.5}_{-1.7}$ ms	$3.5^{+1.9}_{-1.3}$ ms ($\ell = 4$)	$3.9^{+2.2}_{-1.4}$ ms ($\ell = 4$)	$3.2^{+1.8}_{-1.1}$ ms ($\ell = 4$)	[44]
(110,279)	9.84 ± 0.06	$0.20^{+0.05}_{-0.04}$ s	$0.15^{+0.07}_{-0.05}$ s	$0.17^{+0.08}_{-0.05}$ s	$0.13^{+0.06}_{-0.04}$ s	[40]
(109,276)	9.85 ± 0.06	$0.72^{+0.97}_{-0.25}$ s	$0.37^{+0.17}_{-0.12}$ s ($\ell = 4$)	$0.41^{+0.19}_{-0.13}$ s ($\ell = 4$)	$0.33^{+0.16}_{-0.10}$ s ($\ell = 4$)	[42,43]
(109,275)	10.48 ± 0.09	$9.7^{+46}_{-4.4}$ ms	$8.7^{+5.9}_{-3.5}$ ms ($\ell = 4$)	$9.4^{+6.6}_{-3.8}$ ms ($\ell = 4$)	$7.9^{+5.4}_{-3.2}$ ms ($\ell = 4$)	[42,43]
(109,274)	9.95 ± 0.10	440^{+810}_{-170} ms	220^{+195}_{-99} ms ($\ell = 4$)	242^{+211}_{-112} ms ($\ell = 4$)	200^{+170}_{-94} ms ($\ell = 4$)	[44]
(108,275)	9.44 ± 0.06	$0.19^{+0.22}_{-0.07}$ s	$0.46^{+0.23}_{-0.15}$ s	$0.51^{+0.25}_{-0.17}$ s	$0.42^{+0.21}_{-0.14}$ s	[40]
(107,272)	9.15 ± 0.06	$9.8^{+11.7}_{-3.5}$ s	$9.0^{+4.7}_{-3.1}$ s ($\ell = 4$)	$9.7^{+5.1}_{-3.3}$ s ($\ell = 4$)	$7.9^{+4.1}_{-2.7}$ s ($\ell = 4$)	[42,43]
(107,270)	9.11 ± 0.08	61^{+292}_{-28} s	73^{+58}_{-30} s ($\ell = 6$)	84^{+64}_{-36} s ($\ell = 6$)	70^{+54}_{-30} s ($\ell = 6$)	[44]
(106,271)	8.67 ± 0.08	$1.9^{+2.4}_{-0.6}$ min	$2.10^{+1.77}_{-0.95}$ min ($\ell = 4$)	$2.27^{+1.99}_{-1.02}$ min ($\ell = 4$)	$1.83^{+1.54}_{-0.83}$ min ($\ell = 4$)	[40]
RMSD			0.209	0.198	0.218	

PREDICTIONS FOR UNOBSERVED DECAYS

- Q_α values: need a model for nuclear masses
  - modified Liquid Droplet Model (LDM)

W.D. Myers, W.J. Swiatecki, Ann. Phys. 55, 395 (1969)
A.W. Steiner, M. Prakash, J.M. Lattimer, P.J. Ellis, Phys. Rep. 411, 325 (2005)

$$E = f_B\,(A - N_s) + 4\pi R^2 \sigma(\mu_n) + \mu_n N_s + E_{\rm Coul} + E_{\rm pair} + E_{\rm shell},$$

f_B : binding energy per baryon in infinite nuclear matter

N_s : number of neutrons in the neutron skin

σ surface tension

$E_{\rm Coul}$: Coulomb energy

$E_{\rm paor}$: pairing energy

$E_{\rm shell}$: shell corrections

D.G. Ravenhall et al., NPA 407, 571 (1983)
J. Duflo, A.P. Zuker, PRC 52, R23 (1995)
A.E.L. Dieperink, P. Van Isacker, EPJA 42, 269 (2009)

Parameters are fitted by nuclear masses: Global fitting

PREDICTIONS FOR UNOBSERVED DECAYS

- Local formula for the Q_α values
  - Taylor expansion of the Q_α formula for heavy nuclei (large *N* and *Z*)

J. Dong, W. Zuo, J. Gu, Y. Wang, B. Peng, PRC 81, 064309 (2010)
T. Dong, Z. Rev, PRC 77, 064310 (2008)

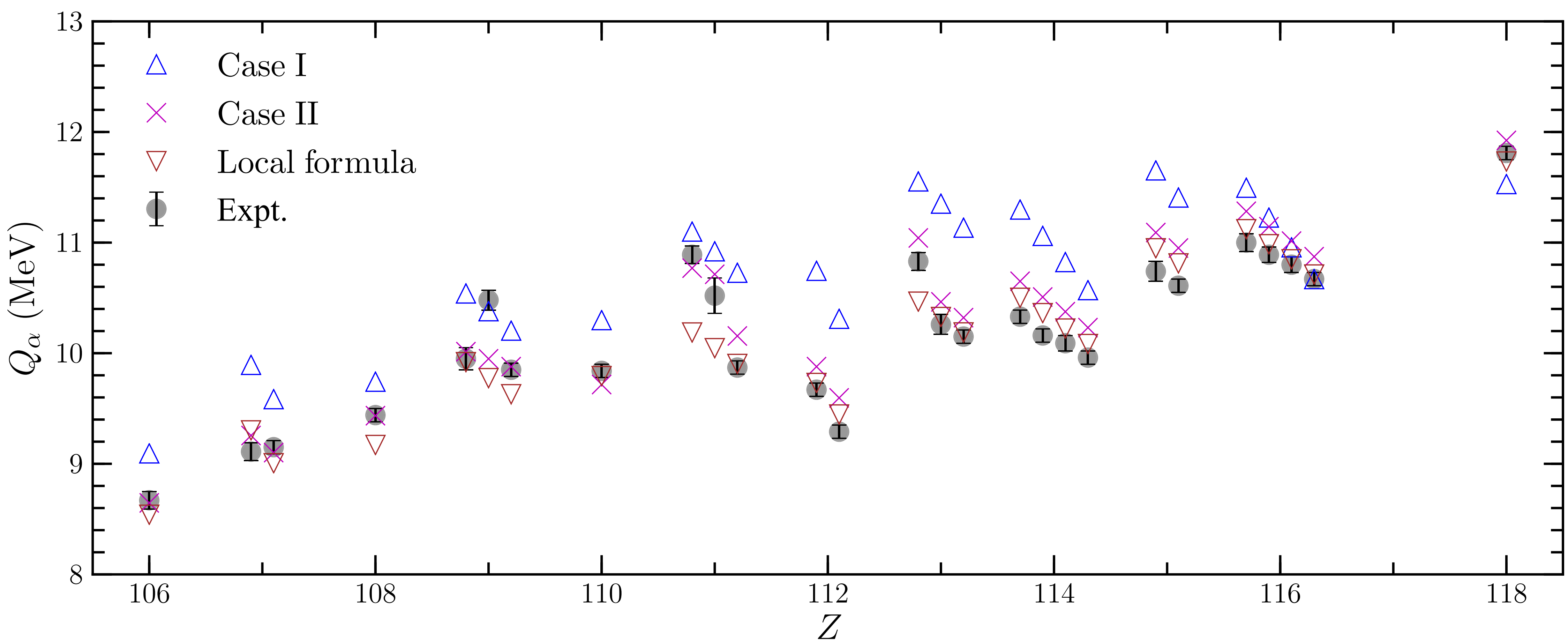
$$Q = a\frac{Z}{A^{4/3}}(3A - Z) + b\left(\frac{N-Z}{A}\right)^2 + c\left[\frac{|N-152|}{N} - \frac{|N-154|}{N-2}\right] + d\left[\frac{|Z-110|}{Z} - \frac{|Z-112|}{Z-2}\right] + e,$$

TABLE II. The best-fit parameters of Eq. (11). All parameters have a unit of MeV.

a	b	c	d	e	RMSD
0.90753	−97.84028	16.15924	−18.95722	−26.16600	0.255

$$Z \geq 90, \quad N \geq 140$$

COMPARISON – Q VALUES



Case I, Case II: two parameter sets for LDM

COMPARISON – ALPHA POTENTIAL

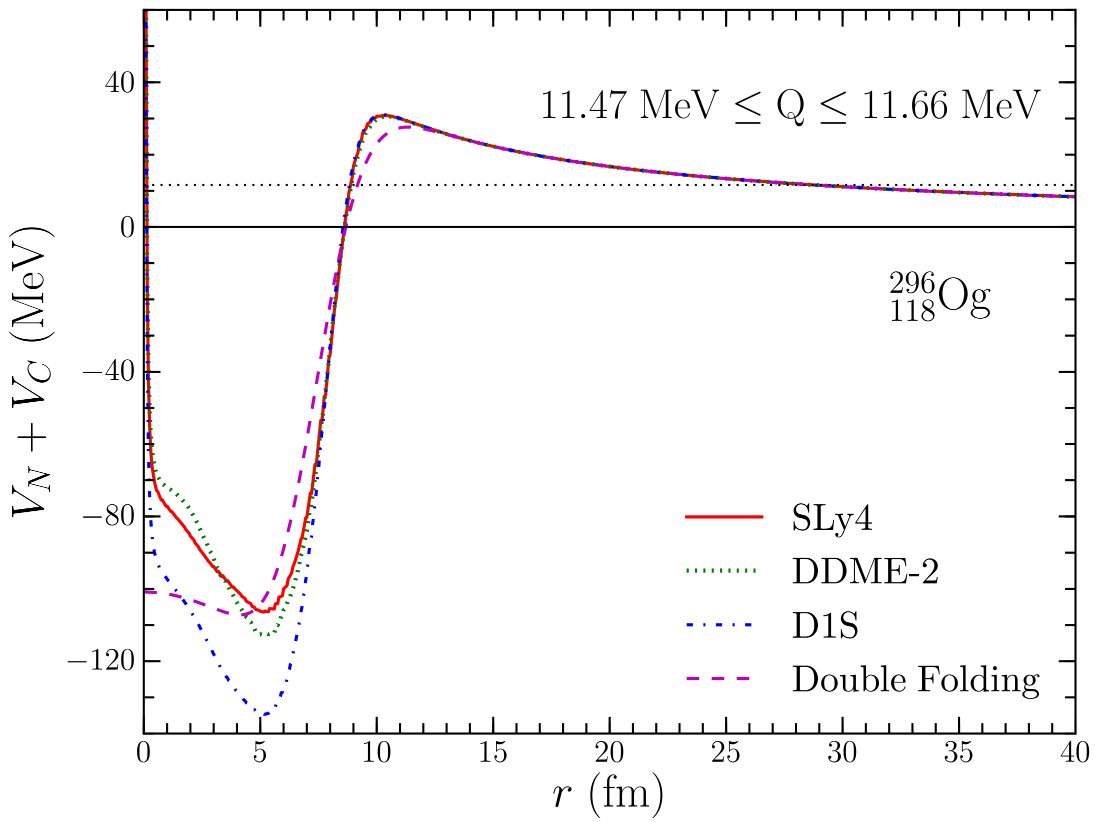


FIG. 3. The α nuclear and Coulomb potentials, $V_N + V_C$, for $^{296}_{118}$Og in the models of the present paper. The double folding potential for $^{296}_{118}$Og of Ref. [47] is also presented for comparison.

Double Folding Model

P. Mohr, PRC 95, 011302(R) (2017)

PREDICTIONS

TABLE V. Predictions on the α decay lifetimes for unobserved superheavy elements with Q values from the LDM (case II) and from the local formula.

Nuclei (Z, A)	Q (MeV) LDM	$T_{1/2}^{\text{SLy4}}$ (s)	$T_{1/2}^{\text{D1S}}$ (s)	$T_{1/2}^{\text{DD-ME2}}$ (s)	Q (MeV) Local formula	$T_{1/2}^{\text{SLy4}}$ (s)	$T_{1/2}^{\text{D1S}}$ (s)	$T_{1/2}^{\text{DD-ME2}}$ (s)
(122, 307)	12.594	9.467×10^{-5}	9.982×10^{-5}	6.999×10^{-5}	12.289	4.340×10^{-4}	4.514×10^{-4}	3.194×10^{-4}
(122, 306)	12.729	5.649×10^{-5}	5.836×10^{-5}	4.183×10^{-5}	12.420	2.517×10^{-4}	2.688×10^{-4}	1.891×10^{-4}
(122, 305)	12.853	3.334×10^{-5}	3.607×10^{-5}	2.525×10^{-5}	12.550	1.402×10^{-4}	1.539×10^{-4}	1.073×10^{-4}
(122, 304)	12.986	1.931×10^{-5}	2.100×10^{-5}	1.480×10^{-5}	12.679	7.919×10^{-5}	8.911×10^{-5}	6.193×10^{-5}
(122, 303)	13.108	1.145×10^{-5}	1.300×10^{-5}	9.047×10^{-6}	12.807	4.646×10^{-5}	5.237×10^{-5}	3.593×10^{-5}
(122, 302)	13.239	6.692×10^{-6}	7.539×10^{-6}	5.339×10^{-6}	12.935	2.646×10^{-5}	3.000×10^{-5}	2.099×10^{-5}
(121, 306)	12.114	5.360×10^{-4}	5.522×10^{-4}	3.846×10^{-4}	11.853	2.104×10^{-3}	2.175×10^{-3}	1.509×10^{-3}
(121, 305)	12.250	2.948×10^{-4}	3.093×10^{-4}	2.170×10^{-4}	11.985	1.143×10^{-3}	1.212×10^{-3}	8.467×10^{-4}
(121, 304)	12.367	1.664×10^{-4}	1.831×10^{-4}	1.274×10^{-4}	12.117	6.082×10^{-4}	6.787×10^{-4}	4.700×10^{-4}
(121, 303)	12.511	9.077×10^{-5}	1.030×10^{-4}	7.119×10^{-5}	12.248	3.317×10^{-4}	3.794×10^{-4}	2.593×10^{-4}
(121, 302)	12.636	5.323×10^{-5}	6.026×10^{-5}	4.191×10^{-5}	12.378	1.834×10^{-4}	2.093×10^{-4}	1.439×10^{-4}
(121, 301)	12.769	2.976×10^{-5}	3.401×10^{-5}	2.378×10^{-5}	12.508	1.027×10^{-4}	1.169×10^{-4}	8.201×10^{-5}
(120, 304)	11.790	1.567×10^{-3}	1.650×10^{-3}	1.167×10^{-3}	11.546	5.792×10^{-3}	6.146×10^{-3}	4.349×10^{-3}
(120, 303)	11.918	8.584×10^{-4}	9.358×10^{-4}	6.494×10^{-4}	11.679	2.987×10^{-3}	3.331×10^{-3}	2.289×10^{-3}
(120, 302)	12.055	4.456×10^{-4}	5.025×10^{-4}	3.459×10^{-4}	11.812	1.561×10^{-3}	1.761×10^{-3}	1.217×10^{-3}
(120, 301)	12.181	2.491×10^{-4}	2.816×10^{-4}	1.959×10^{-4}	11.944	8.288×10^{-4}	9.395×10^{-4}	6.575×10^{-4}
(120, 300)	12.317	1.342×10^{-4}	1.523×10^{-4}	1.068×10^{-4}	12.076	4.465×10^{-4}	5.053×10^{-4}	3.520×10^{-4}
(120, 299)	12.442	7.735×10^{-5}	8.978×10^{-5}	6.175×10^{-5}	12.207	2.436×10^{-4}	2.817×10^{-4}	1.957×10^{-4}
(119, 298)	11.973	4.022×10^{-4}	4.688×10^{-4}	3.243×10^{-4}	11.772	1.131×10^{-3}	1.322×10^{-3}	8.986×10^{-4}
(119, 297)	12.109	2.119×10^{-4}	2.415×10^{-4}	1.706×10^{-4}	11.904	5.932×10^{-4}	1.610×10^{-3}	4.795×10^{-4}
(119, 296)	12.234	1.181×10^{-4}	1.340×10^{-4}	9.719×10^{-5}	12.036	3.147×10^{-4}	3.587×10^{-4}	2.593×10^{-4}
(119, 295)	12.368	6.172×10^{-5}	7.814×10^{-5}	5.316×10^{-5}	12.167	1.643×10^{-4}	1.913×10^{-4}	1.405×10^{-4}
(119, 294)	12.492	3.425×10^{-5}	4.112×10^{-5}	2.983×10^{-5}	12.297	8.668×10^{-5}	1.044×10^{-4}	7.549×10^{-5}
(119, 293)	12.625	1.874×10^{-5}	2.264×10^{-5}	1.646×10^{-5}	12.427	4.775×10^{-5}	5.767×10^{-5}	4.168×10^{-5}
(118, 298)	11.393	4.077×10^{-3}	4.600×10^{-3}	3.215×10^{-3}	11.197	1.206×10^{-2}	1.373×10^{-2}	9.535×10^{-3}
(118, 297)	11.522	2.126×10^{-3}	2.488×10^{-3}	1.699×10^{-3}	11.332	5.977×10^{-3}	7.008×10^{-3}	4.774×10^{-3}
(118, 296)	11.660	1.068×10^{-3}	1.238×10^{-3}	8.599×10^{-4}	11.466	3.013×10^{-3}	3.481×10^{-3}	2.423×10^{-3}
(118, 295)	11.787	5.640×10^{-4}	6.577×10^{-4}	4.692×10^{-4}	11.600	1.500×10^{-3}	1.762×10^{-3}	1.244×10^{-3}
(118, 294)	11.924	2.824×10^{-4}	8.069×10^{-4}	2.412×10^{-4}	11.733	7.515×10^{-4}	9.050×10^{-4}	6.387×10^{-4}
(118, 293)	12.050	1.516×10^{-4}	1.835×10^{-4}	1.305×10^{-4}	11.865	3.832×10^{-4}	4.644×10^{-4}	3.289×10^{-4}
(117, 298)	10.779	6.202×10^{-2}	7.032×10^{-2}	4.795×10^{-2}	10.920	1.678×10^{-1}	1.916×10^{-1}	1.311×10^{-1}
(117, 297)	10.920	2.837×10^{-2}	3.274×10^{-2}	2.236×10^{-2}	10.749	7.769×10^{-2}	9.001×10^{-2}	6.129×10^{-2}
(117, 296)	11.051	1.409×10^{-2}	1.666×10^{-2}	1.126×10^{-2}	10.886	3.620×10^{-2}	4.330×10^{-2}	2.903×10^{-2}
(117, 295)	11.192	6.660×10^{-3}	7.806×10^{-3}	5.400×10^{-3}	11.023	1.735×10^{-2}	2.035×10^{-2}	1.396×10^{-2}
(117, 294)	11.321	3.310×10^{-3}	3.965×10^{-3}	6.634×10^{-3}	11.158	8.146×10^{-3}	9.736×10^{-3}	6.779×10^{-3}
(117, 293)	11.460	1.584×10^{-3}	1.941×10^{-3}	1.325×10^{-3}	11.293	3.885×10^{-3}	4.752×10^{-3}	3.244×10^{-3}

PREDICTIONS

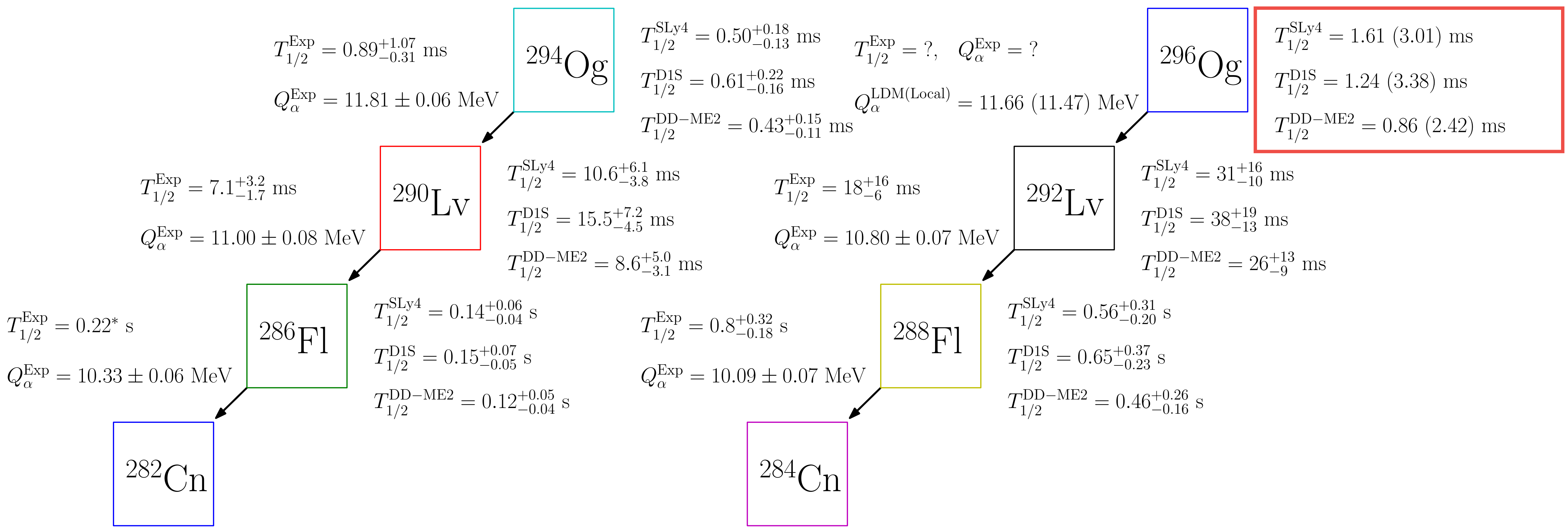


FIG. 2. Float charts for α decay chains for $^{294}_{118}$Og and $^{296}_{118}$Og. The measured half-life of $^{286}_{114}$Fl is about 0.13 s. Since the branching ratio of its α decay is about 60% [45,46], however, the half-life of its α decay is about 0.22 s.

For ^{296}Og

Our prediction: $0.86 \sim 3.48$ ms

0.5 ~ 4.8 ms: A. Sobiczewski, PRC 94, 051302(R) (2016)

0.825 ms: P. Mohr, PRC 95, 011302(R) (2017)

Planned to measure at JINR

CONCLUSIONS & OUTLOOK

- To develop more realistic theories on the nuclear α decay.
  - simple potential models
  - based on EDF
- Other elements
  - deformation
  - direct calculation using α cluster models
  - other theoretical framework

Hyperons in the mass-radius relations of neutron stars

Yongseok Oh (Kyungpook National Univ.)

in collaboration with

Yeunhwan Lim (Texas A&M) and Chang-Hwan Lee (Pusan National Univ.)

Effective interactions of hyperons and mass-radius relation of neutron stars

Yeunhwan Lim,[1,*] Chang-Hwan Lee,[2,†] and Yongseok Oh[3,4,‡]

[1]*Cyclotron Institute and Department of Physics and Astronomy, Texas A&M University, College Station, Texas 77843, USA*

[2]*Department of Physics, Pusan National University, Busan 46241, Korea*

[3]*Department of Physics, Kyungpook National University, Daegu 41566, Korea*

[4]*Asia Pacific Center for Theoretical Physics, Pohang, Gyeongbuk 37673, Korea*

(Dated: November 22, 2017)

We examine the role of hyperons in a neutron star based on the relativistic mean field approach. For nuclear matter below 1.5 times the normal nuclear density we constrain the model parameters by using the symmetric nuclear matter properties and theoretical investigations for neutron matter in the literature. We then extend the model to higher densities by including hyperons and isoscalar vector mesons that contain strangeness degree of freedom. We confirm that the ϕ meson induces a Λ repulsive force and hardens the equation of state. The hardening arising from the ϕ meson compensates the softening from the existence of hyperons. The flavor SU(3) and spin-flavor SU(6) relations are examined as well. We found that the coupling constants fitted by neutron matter properties could yield high enough maximum mass of a neutron star and the obtained results satisfy both the mass and radius constraints. The onset of the hyperon direct Urca process in neutron stars is also investigated using our parametrization.

PACS numbers: 26.60.-c, 21.65.-f, 26.60.Kp

arXiv:1708.01364

$$
\begin{aligned}
\mathcal{L}_{\sigma\omega\rho} = \sum_{B=n,p} \bar{\psi}_B \Big[& (i\partial\!\!\!/ - g_{\omega B}\gamma_\mu\omega^\mu) - g_{\rho B}\gamma_\mu\vec{\rho}^{\,\mu}\cdot\vec{\tau}_B \\
& - (M_B - g_{\sigma B}\sigma) - \frac{e}{2}\left(1+\tau_3\right)A_\mu\gamma^\mu \Big]\psi_B \\
& + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\,\sigma^2 - \frac{\kappa}{3!}\left(g_\sigma\sigma\right)^3 - \frac{\lambda}{4!}\left(g_\sigma\sigma\right)^4 \\
& + \frac{1}{2}m_\omega^2\,\omega_\mu\omega^\mu - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{\zeta}{4!}g_\omega^4\left(\omega_\mu\omega^\mu\right)^2 \\
& + \frac{1}{2}m_\rho^2\,\vec{\rho}_\mu\cdot\vec{\rho}^{\,\mu} - \frac{1}{4}\vec{\rho}_{\mu\nu}\cdot\vec{\rho}^{\,\mu\nu} + \frac{\xi}{4!}g_\rho^4\left(\vec{\rho}_\mu\cdot\vec{\rho}^{\,\mu}\right)^2 \\
& + f(\sigma,\omega_\mu\omega^\mu)g_\rho^2\left(\vec{\rho}_\mu\cdot\vec{\rho}^{\,\mu}\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
& + \sum_{l=e^-,\mu^-}\bar{\psi}_l\left(i\partial\!\!\!/ - m_l\right)\psi_l,
\end{aligned} \tag{1}
$$

TABLE I. The fitted parameter sets of SU(2) RMF models. RGCR represents RMF model with GCR5 parametrization and RDSS represents RMF models with DSS2 parametrization. For comparison, the fitted parameters in other works are also presented with references.

Parameter	RGCR	RDSS	IU-FSU [52]	SFHo [55]	GM1 [24]	NL3 [56]	Unit
m_σ	2.491	2.491	2.491	2.371	2.491	2.575	fm^{-1}
m_ω	3.966	3.966	3.966	3.864	3.966	3.966	fm^{-1}
m_ρ	3.929	3.929	3.867	3.902	3.867	3.867	fm^{-1}
$g_{\sigma N}$	8.005	7.985	9.971	7.536	8.553	10.217	
$g_{\omega N}$	9.235	9.235	13.032	8.782	10.603	12.868	
$g_{\rho N}$	11.108	11.033	13.590	9.384	8.121	8.922	
κ	6.603×10^{-2}	6.350×10^{-2}	1.713×10^{-2}	7.105×10^{-2}	2.805×10^{-2}	1.956×10^{-2}	fm^{-1}
λ	-2.900×10^{-2}	-2.474×10^{-2}	2.960×10^{-4}	-2.645×10^{-2}	-6.420×10^{-3}	-1.591×10^{-2}	
ζ	–	–	3.0×10^{-2}	-1.701×10^{-3}	–	–	
ξ	-3.807×10^{-5}	-1.088×10^{-7}	–	3.453×10^{-3}	–	–	
Λ_{s1}	-3.788×10^{-4}	4.467×10^{-4}	–	-3.054×10^{-2}	–	–	fm^{-1}
Λ_{s2}	1.810×10^{-2}	4.267×10^{-2}	–	1.021×10^{-2}	–	–	
Λ_{s3}	1.724×10^{-2}	-3.597×10^{-4}	–	8.048×10^{-4}	–	–	fm
Λ_{s4}	2.424×10^{-3}	2.550×10^{-4}	–	1.072×10^{-3}	–	–	fm^2
Λ_{s5}	-2.862×10^{-3}	2.588×10^{-3}	–	5.542×10^{-5}	–	–	fm^3
Λ_{s6}	-3.416×10^{-8}	9.217×10^{-8}	–	3.606×10^{-6}	–	–	fm^4
Λ_{v1}	1.131×10^{-4}	2.220×10^{-5}	4.60×10^{-2}	7.616×10^{-2}	–	–	
Λ_{v2}	-6.174×10^{-4}	-8.536×10^{-5}	–	-2.765×10^{-4}	–	–	fm^2
Λ_{v3}	1.563×10^{-5}	5.560×10^{-6}	–	6.861×10^{-4}	–	–	fm^4

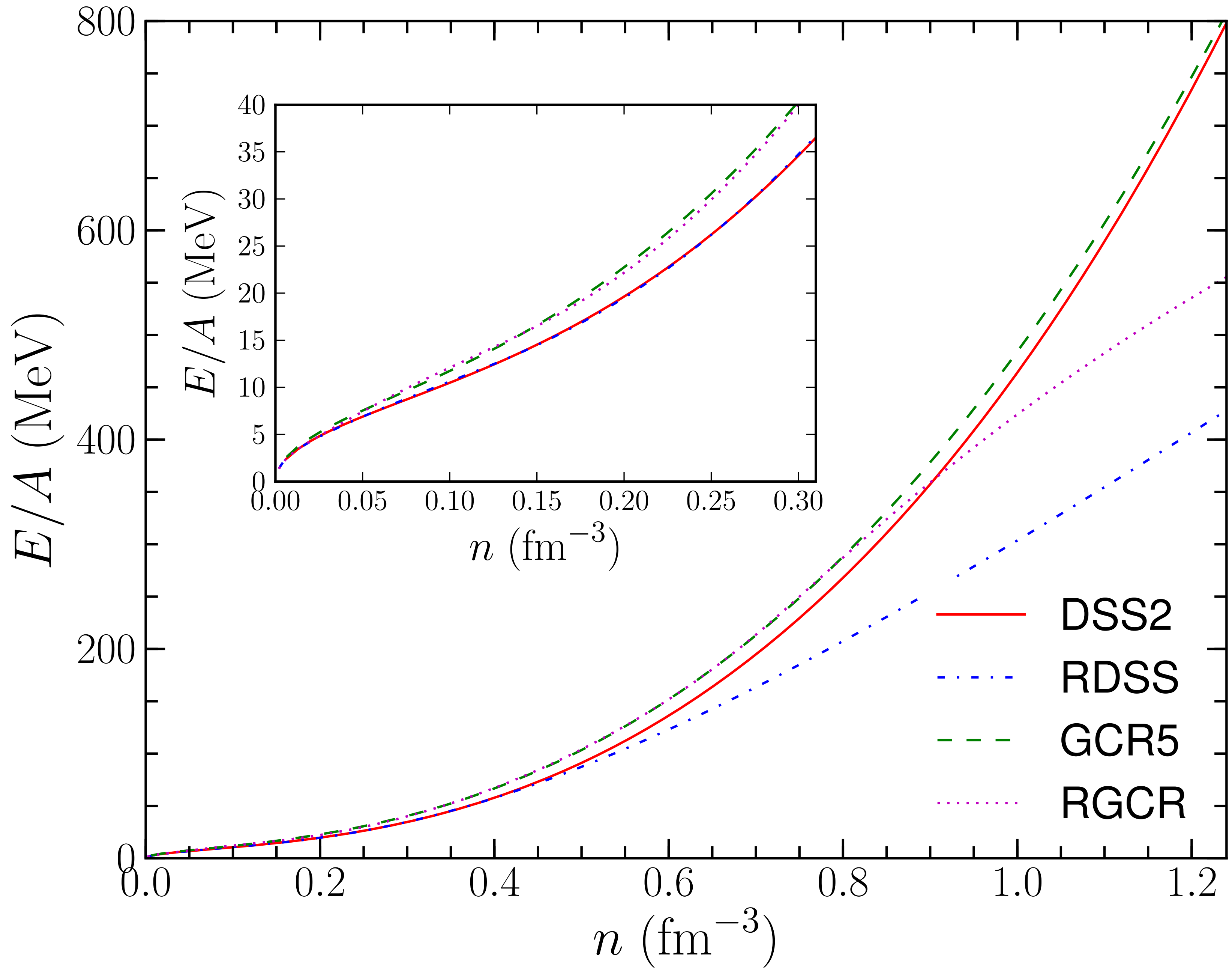


FIG. 1. Energy per baryon of pure neutron matter. RGCR and RDSS are obtained from the Lagrangian of Eq. (1) by fitting to the results of GCR5 [46] and DSS2 [47], respectively.

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & n \\ -\Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda^0 \end{pmatrix}. \tag{30}$$

Similarly, the vector meson octet can be written as

$$V_8 = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\sqrt{\frac{2}{3}}\omega_8 \end{pmatrix} \tag{31}$$

and the vector meson singlet takes the simple form of

$$V_1 = \frac{\omega_1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{32}$$

Then the flavor SU(3) invariant interactions between baryon octet and meson octet can be written as

$$\begin{aligned} \mathcal{L}_{V_8BB} = \sqrt{2}g_8\big\{&(d+f)\mathrm{Tr}\left(\bar{B}BV_8\right) \\ &+ (d-f)\mathrm{Tr}\left(\bar{B}V_8B\right)\big\}, \end{aligned} \tag{33}$$

while we have

$$\mathcal{L}_{V_1BB} = \sqrt{3}g_1\,\mathrm{Tr}\left(V_1\bar{B}B\right) \tag{34}$$

$$g_{\omega N} = \left\{ \sqrt{\frac{2}{3}} - \frac{1}{3} \left(1 - 4\alpha_V\right) z \right\} g_1,$$

$$g_{\phi N} = \left\{ -\frac{1}{\sqrt{3}} - \frac{\sqrt{2}}{3} \left(1 - 4\alpha_V\right) z \right\} g_1,$$

$$g_{\omega \Lambda} = \left\{ \sqrt{\frac{2}{3}} - \frac{2}{3} \left(1 - \alpha_V\right) z \right\} g_1,$$

$$g_{\phi \Lambda} = \left\{ -\frac{1}{\sqrt{3}} - \frac{2\sqrt{2}}{3} \left(1 - \alpha_V\right) z \right\} g_1,$$

$$g_{\omega \Sigma} = \left\{ \sqrt{\frac{2}{3}} + \frac{2}{3} \left(1 - \alpha_V\right) z \right\} g_1,$$

$$g_{\phi \Sigma} = \left\{ -\frac{1}{\sqrt{3}} + \frac{2\sqrt{2}}{3} \left(1 - \alpha_V\right) z \right\} g_1,$$

$$g_{\omega \Xi} = \left\{ \sqrt{\frac{2}{3}} - \frac{1}{3} \left(1 + 2\alpha_V\right) z \right\} g_1,$$

$$g_{\phi \Xi} = \left\{ -\frac{1}{\sqrt{3}} - \frac{\sqrt{2}}{3} \left(1 + 2\alpha_V\right) z \right\} g_1$$

$$g_{\rho N} = z g_1, \quad g_{\rho \Sigma} = 2\alpha_V z g_1,$$
$$g_{\rho \Xi} = -\left(1 - 2\alpha_V\right) z g_1.$$

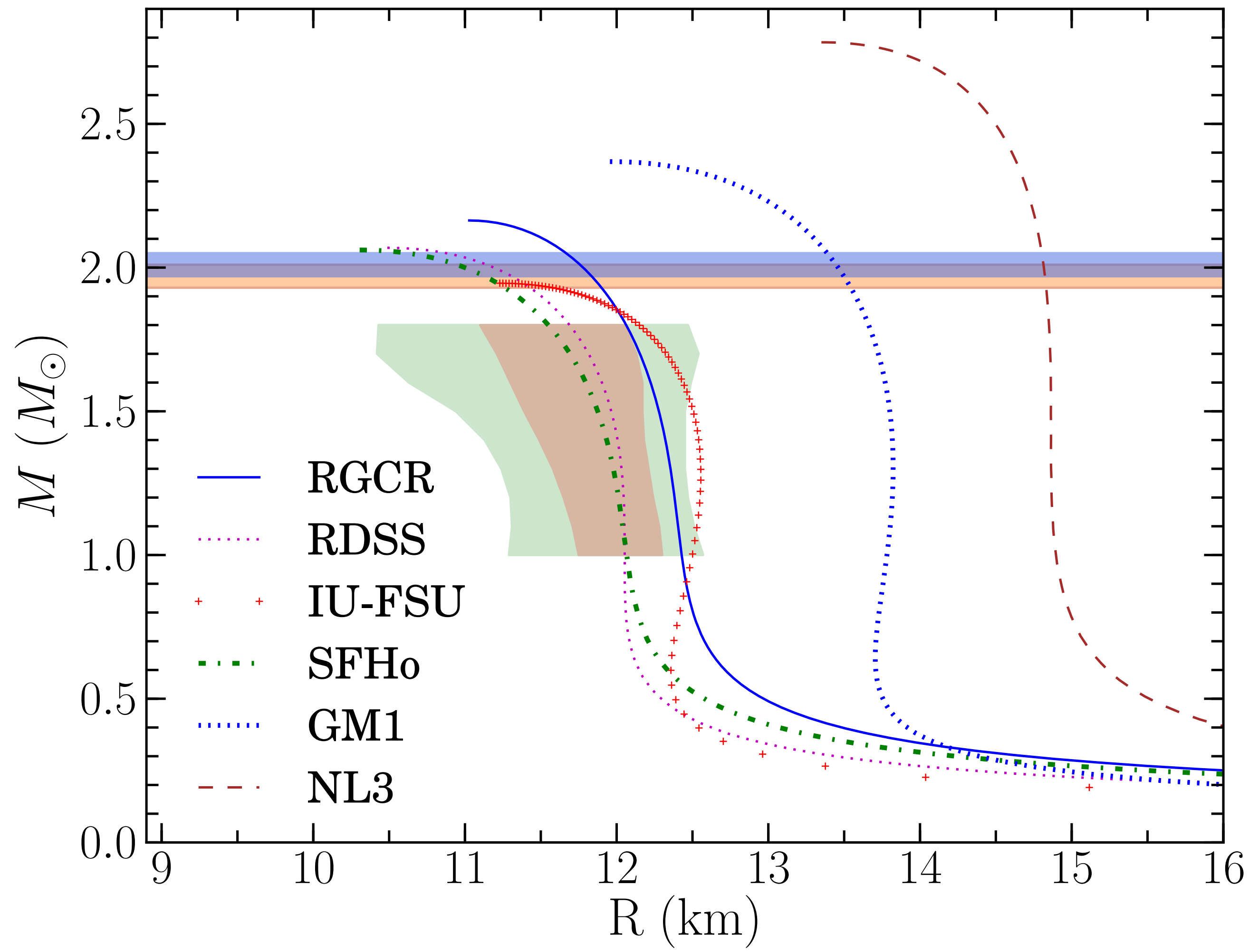


FIG. 4. Mass and radius of neutron stars using relativistic mean field models without hyperons. RGCR and RDSS models are the results of our calculation and the other models are explained in Table II. The horizontal lines indicate the observed neutron star masses of Ref. [3, 4]. The brown and green shaded areas show the allowed region of the mass-radius constraint of Ref. [38] at the 1σ and 2σ level, respectively.

TABLE III. The maximum mass of neutron stars (in units of $M_\odot$) in each model using $U_\Lambda^{(N)} = -30$ MeV, $U_\Sigma^{(N)} = +30$ MeV, $U_\Xi^{(N)} = -18$ MeV and $U_\Xi^{(\Xi)} = U_\Lambda^{(\Xi)} = 2U_\Xi^{(\Lambda)} = 2U_\Lambda^{(\Lambda)} = -10$ MeV. Note that SFHo and IU-FSU have non-vanishing ζ thus the maximum mass of neutron stars in case of II and III is not physical.

Model	z	α_V	RGCR	RDSS	IU-FSU	SFHo	GM1	NL3
SU(2)	—	—	2.22	2.07	1.94	2.06	2.36	2.78
Case I	$\frac{1}{\sqrt{6}}$	1	1.78	1.71	1.67	1.70	1.93	2.25
Case II	$\frac{1}{2\sqrt{6}}$	1	2.03	1.90	(1.93)	(1.88)	2.15	2.26
Case III	$\frac{1}{\sqrt{6}}$	$\frac{1}{2}$	1.98	1.91	(2.03)	(1.88)	2.14	2.51

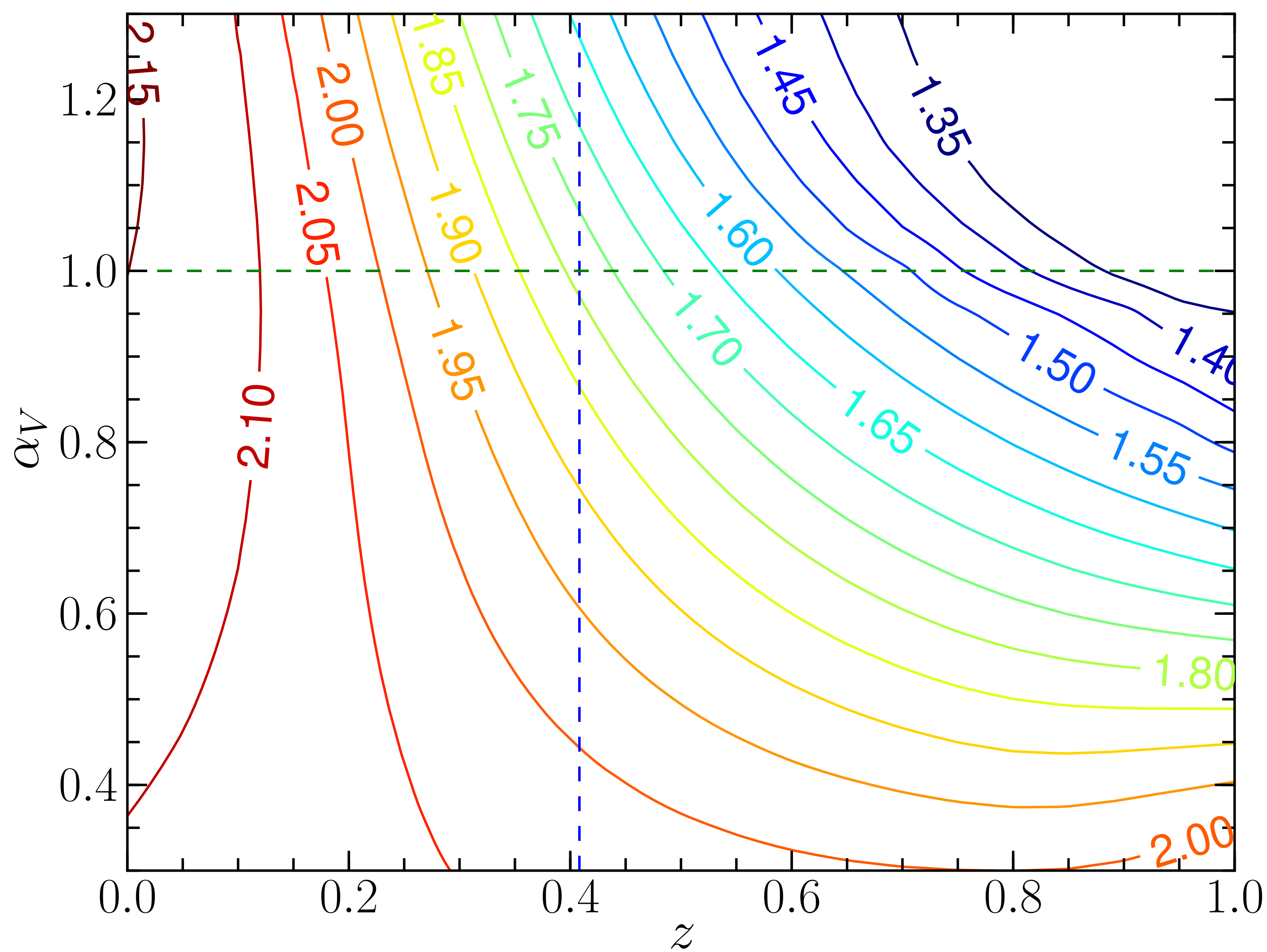


FIG. 7. Contour plot of the maximum mass of neutron stars as a function of z and α_V in the RGCR model with case IV. The horizontal and vertical dashed lines are $\alpha_V = 1.0$ and $z = 1/\sqrt{6}$, respectively.

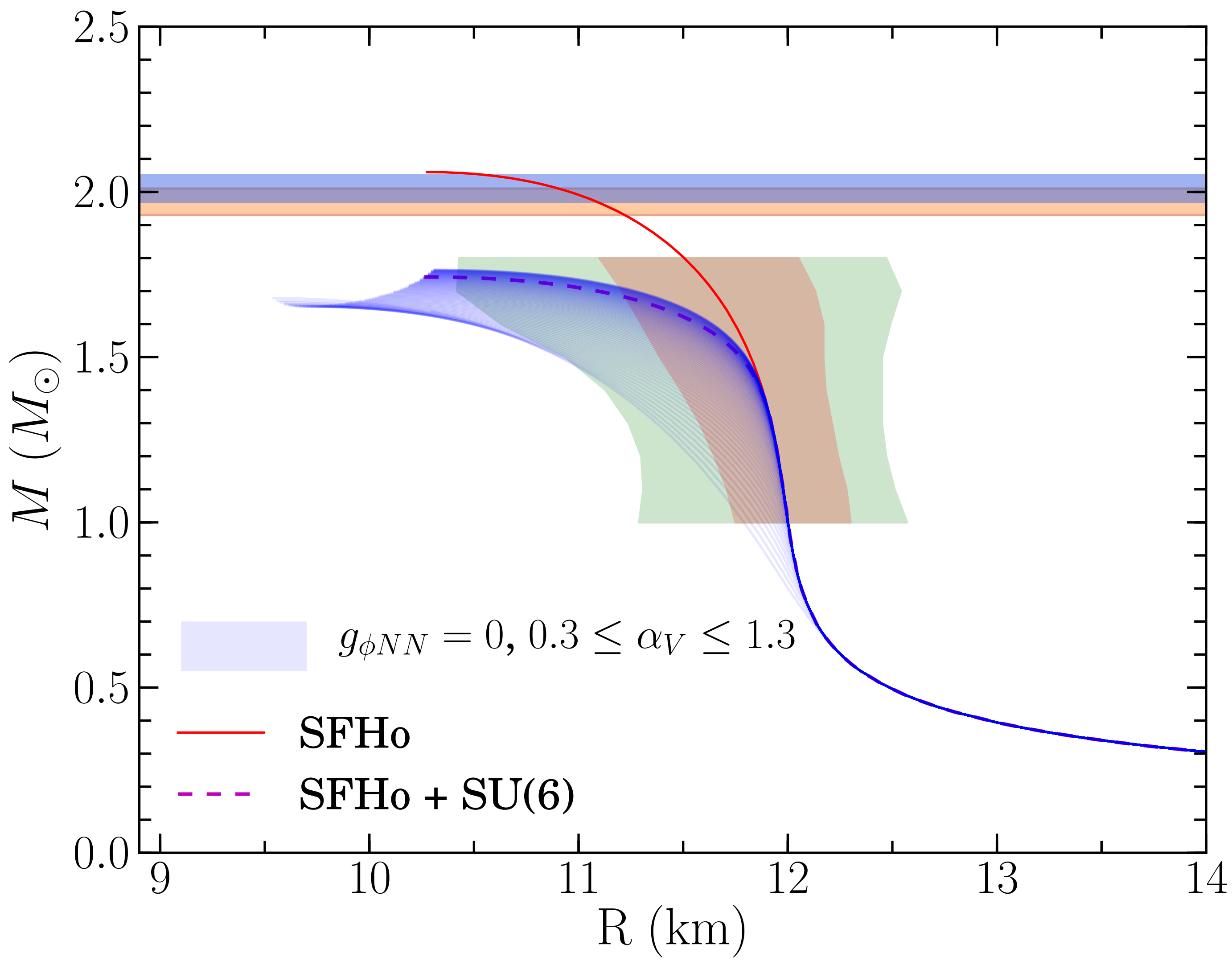


FIG. 9. Mass and radii curves with the variation of α_V constrained by $g_{\phi N} = 0$ in the SFHo model with case I. Smaller α_V gives smaller maximum mass of a neutron star for given EOS.

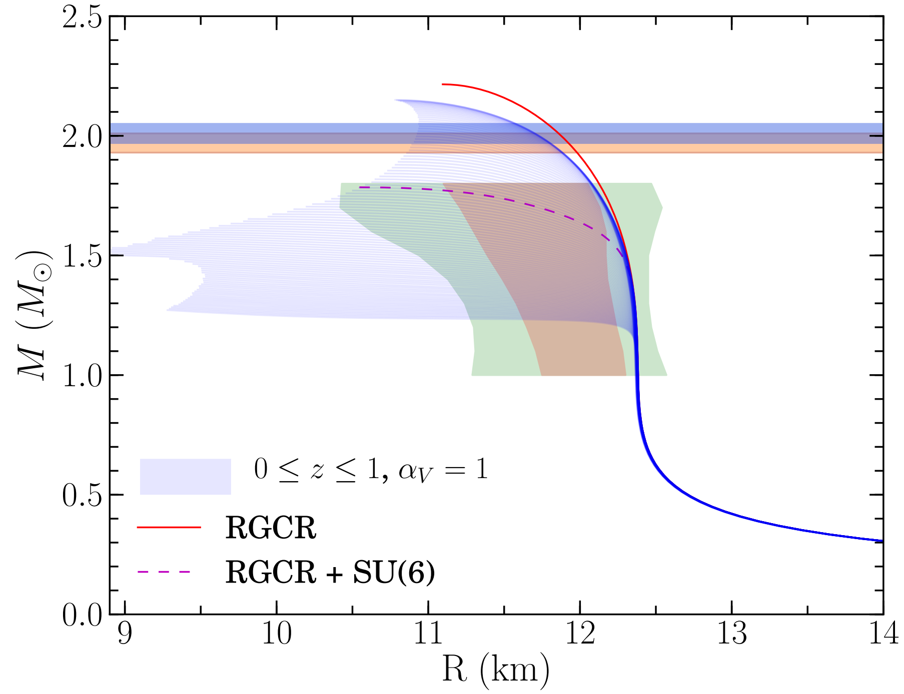
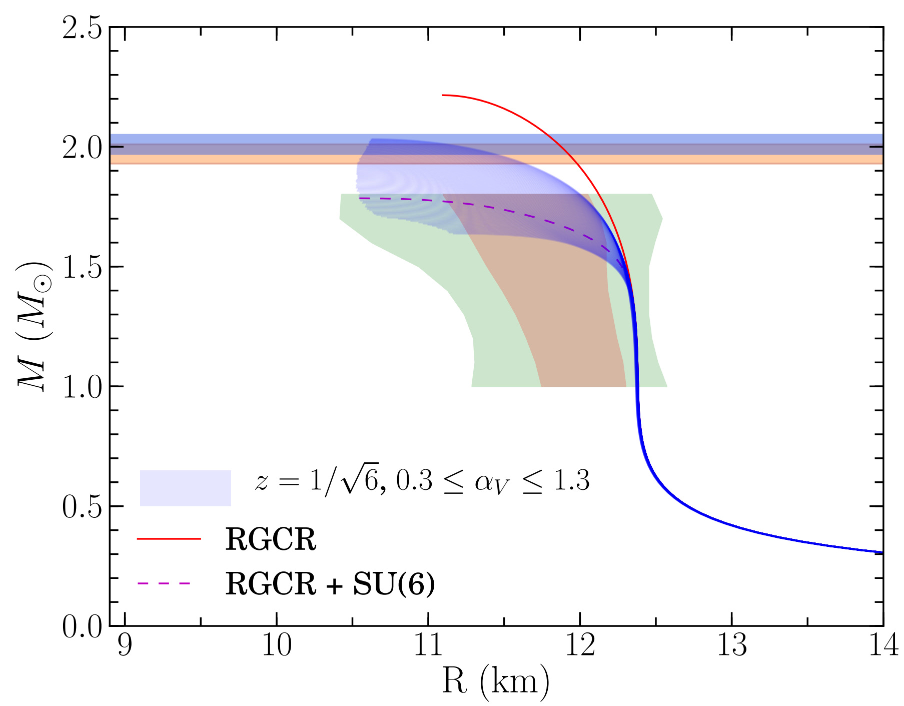


FIG. 10. Mass and radii curves with the variation of z with $\alpha_V = 1$ in the RGCR model, i.e., case II. The red solid line is the result of the model in the SU(2) case and the dashed line is that of case I.



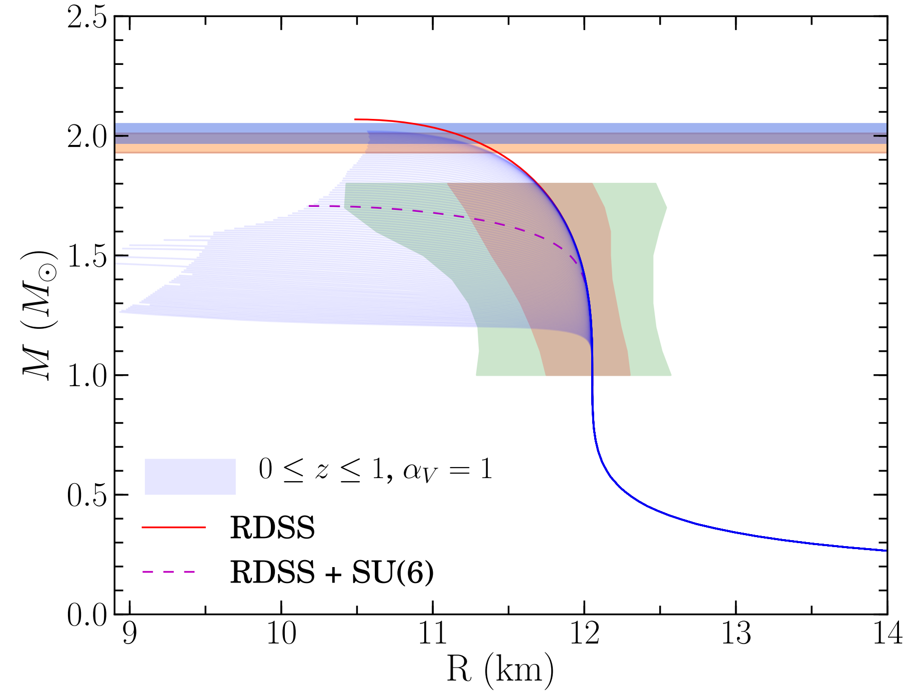


FIG. 12. Same as Fig. 10 but in the RDSS model.

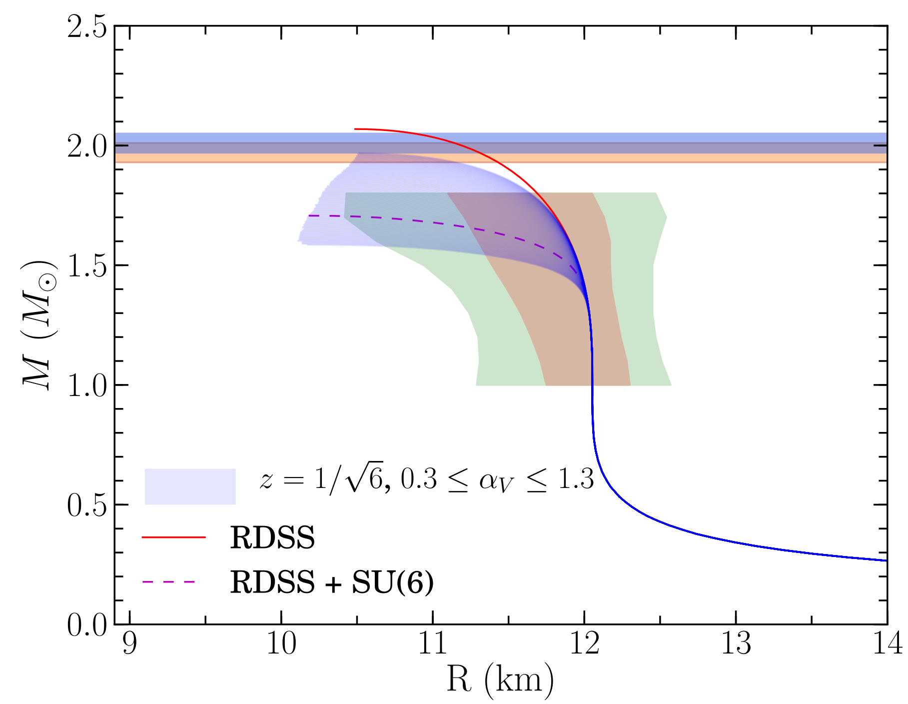


FIG. 13. Same as Fig. 11 but in the RDSS model.