Heavy Quarks in Thermal Environment from Lattice QCD

Seyong Kim

Sejong University

based on arXiv:1702.02297

Outline

Lattice Basics

- free parameters:
  quark masses (u,d,s), bare gauge coupling constant,
  number of quark flavors
- physical quark mass
- continuum limit (lattice spacing, $a \to 0$ limit)
- large enough spacetime
- computer doesn't know dimensional quantities
  everything expressed in terms of lattice spacing
  e.g., ρ meson mass is # $\times a^{-1}$

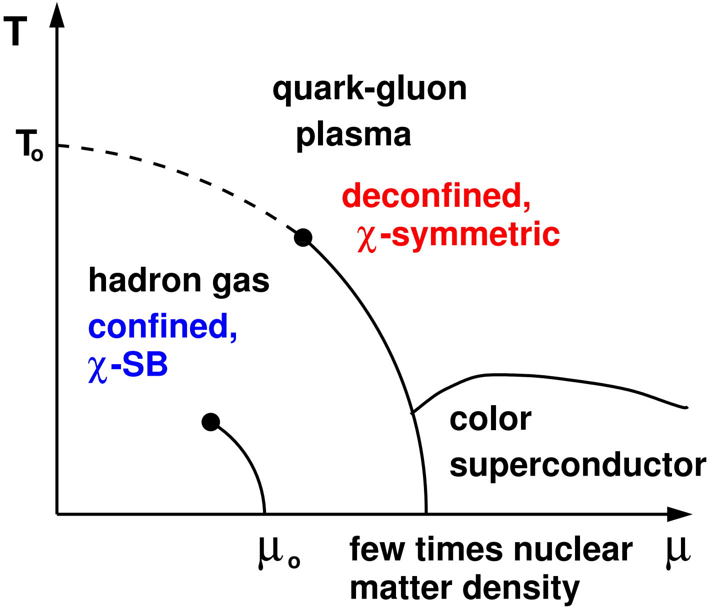
Lattice Basics

- simulation done with quark masses somewhat larger than physical u, d, s quark mass
  $\rightarrow$ need extrapolation

- simulation done with $a \neq 0$
  $\rightarrow$ scaling violation, need to check

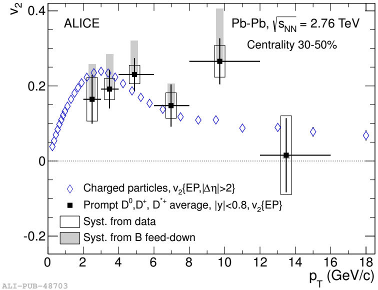
- computer power limit no. of degrees freedom
  $\rightarrow$ physical volume is limited, need to check

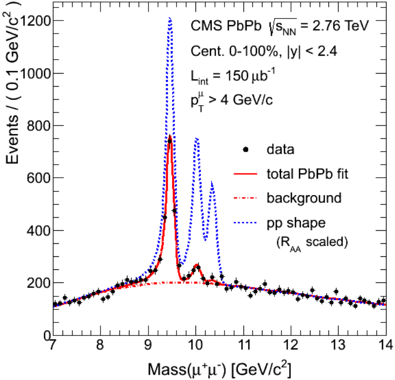
Lattice Basics



Opportunities

- elliptic flow (v_2) of $D-$meson: ALICE, PRC 90 (2014) 034904





- sequential suppression of $\Upsilon(1S, 2S, 3S)$: CMS, PRL109 (2012) 222301

Opportunities

• Investigation of QGP properties requires comparison between the baseline (p, p) and relativistic heavy ion collisions

• heavy quark system is one of better understood hadronic systems

• heavy quark mass scale(M) is large and the strong coupling at the mass scale is "small"

$\rightarrow$ separation of bound state dynamics from short distance perturbative dynamics

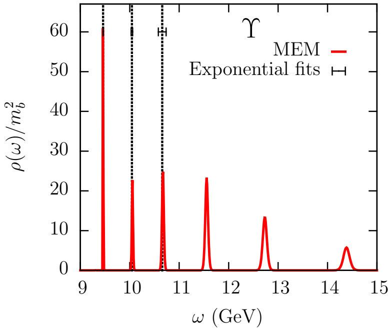
•effective field theory descriptions : NRQCD (pNRQCD), HQET (cf. G.T.Bodwin, E. Braaten, G.P. Legage, PRD51 (1995) 1125, N. Brambilla, A. Pineda, J. Soto, and A. Vairo, Rev. Mod. Phys 77 (2005) 1423, N. Isgur and M. Wise, PLB 237 (1990) 527)

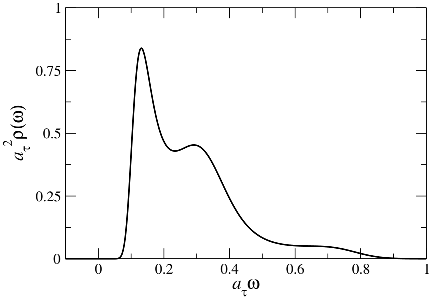
• with $T << M$, EFT still in operation (?)

Challenges

1. direct simulation of relativistic heavy quark system on lattice is difficult due to wide scale separations (confinement, heavy quark de Broglie wavelength, light quark de Broglie wavelength, lattice cutt-off)
2. calculating real time quantities using Euclidean lattices is diffcult
3. spectral behavior in non-zero temperature is more complex than the zero temperature case (thermal broadening, transport phenomena, disappearance of bound states, and etc)

Challenges





$$G(\tau) = A_0 e^{-E_0\tau} + A_1 e^{-E_1\tau} + \cdots \qquad \text{vs.} \qquad = \int \frac{d\omega}{2\pi} K(\tau,\omega)\,\rho(\omega)$$

Challenges

• obtaining spectral function from Euclidean correlators which is defined in terms of integral equation is numerically "ill-posed problem"

• the number of time-directional lattice sites (N_s) is usually smaller than that of space-directional lattice sites (N_τ) for $T \neq 0$

$\rightarrow$ in other words, we need to obtain more information from lesser amount of temporal correlator lattice data

need to overcome these difficulties somehow. In this conference,

• open charm on anisotropic lattice: J-I. Skullerud (Th. 14:00)

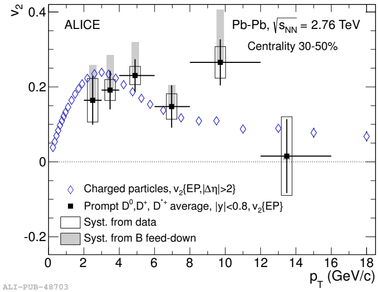
• new attempts to inverse integral transform problem: A. Ikeda (Th. 14:20), H. Shu (Th. 14:40), H. Ohno (Th. 15:00)

• charmonium diffusion constant: A. Ikeda (Th. 14:20) and H. Ohno (Th. 15:00)

• potential: A. Rothkopf (Th. 15:20) and screening mass: A. Steinberg (Th. 15:40))

Kinetic equilibriation of heavy quarks

- Can heavy quark achieves kinetic equilibrium ?



Kinetic equilibriation of heavy quarks

- diffusion constant(D)

$$D = \frac{1}{3\chi_{00}} \lim_{\omega \to 0} \sum_{i=1,..3} \frac{\rho_V^{ii}(\omega)}{\omega}$$

- momentum diffusion constant(κ) and diffusion constant (D) (cf. S. Caron-Huot, M. Laine, G.D. Moore, JHEP04 (2009) 053)

$$D = \frac{2T^2}{\kappa}$$

- drag coefficient and kinetic equilibriation time

$$\eta_D = \frac{\kappa}{2MT}, \quad \tau_{\rm kin} = \frac{1}{\eta_D}$$

Kinetic equilibriation of heavy quarks

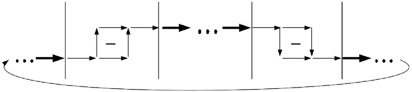
- momentum diffusion coefficient for heavy quark in QGP as the tail of correlation function: A. Francis, O. Kaczmarek, M. Laine, T. Neuhaus, and H. Ohno, PRD 92 (2015) 116003

- large quenched lattices ($64^3 \times 16 \sim 192^3 \times 48$) at $T = 1.5T_c$

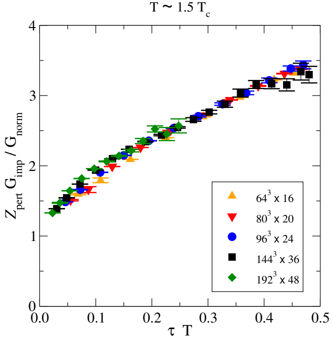
- use of HQET

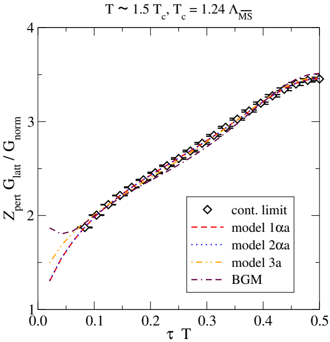
- modeling spectral functions in the continuum limit

Kinetic equilibribration of heavy quarks



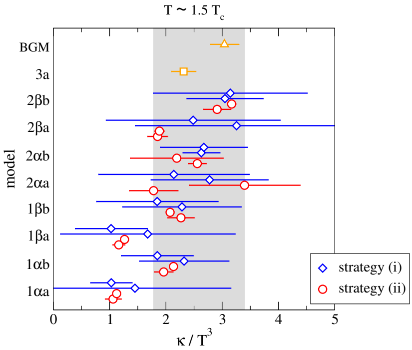
$$G_E(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3}\frac{\left\langle \mathrm{Re\,tr}\left[U(\beta;\tau)\,gE_i(\tau,\vec{0})\,U(\tau;0)\,gE_i(0,\vec{0})\right]\right\rangle}{\left\langle \mathrm{Re\,tr}[U(\beta;0)]\right\rangle}$$

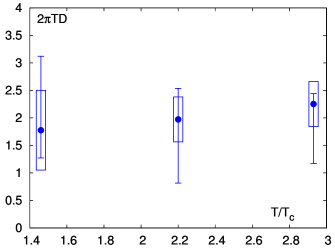




Kinetic equilibriation of heavy quarks

$$\kappa \equiv \lim_{\omega\to 0} \frac{2T\rho_E(\omega)}{\omega}, \quad \frac{\kappa}{T^3} \sim 1.8\cdots 3.4 \to \tau_{\rm kin} \sim 1{\rm fm}/c, DT = 0.59\cdots 1.1$$





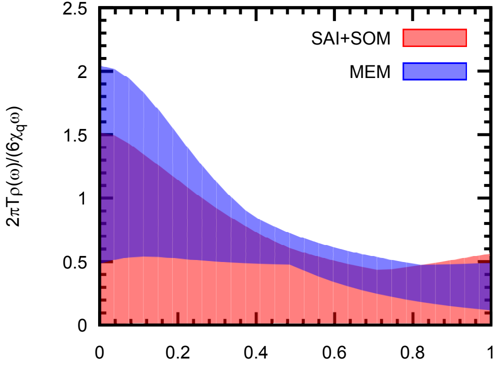
H.T. Ding et al, PRD86 (2012) 192302

Kinetic equilibriation of heavy quarks

- Talk by H. Ohno (Th. 15:00)

- single plaquette Wilson action, quenched lattices ($192^3 \times 48, 96(T = 0.75, 1.5T_c)$, MEM + new stochastic method (SAI and SOM: H-T. Shu, Th: 14:40)

- $TD = 0.24 \sim 0.32 \rightarrow \frac{\kappa}{T^3} = 6.3 \sim 8.3$

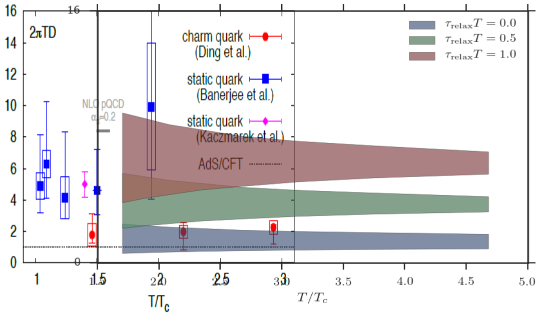


Kinetic equilibriation of heavy quarks

- Talk by A. Iketa (Th. 14:20)

- anisotropic quenched lattices ($\xi = 4$, upto $N_s = 128$), p-dependence of the mid-point of finite momentum temporal correlator assuming well-separated transport behavior from high frequency behavior of the spectral function

Ding et al. arXiv:1504.05274



Chemical equilibriation of heavy quarks

- chemical equilibriation rate as a transport coefficient of a density-density correlator: D. Bödeker and M. Laine, JHEP07 (2012) 130, JHEP01 (2013) 037)

- S.K. and M. Laine, arXiv:1602.08105

$$\Gamma_{\rm chem} \simeq \frac{8\pi\alpha_s^2}{3M^2}\left(\frac{MT}{2\pi}\right)^{3/2} e^{-M/T}\left[\frac{\bar{S}_1}{3}+\left(\frac{5}{6}+Nf\right)\bar{S}_8\right]$$

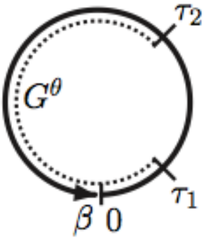
- Sommerfeld effect enhances the Born matrix elements

$$|\mathcal{M}_{\rm resummed}|^2 = \mathcal{S}|\mathcal{M}_{\rm tree}|^2$$

- first calculation for heavy quark chemical equilibriation rate in QGP using NRQCD

- uses FASTSUM 2nd Generation configurations: $24^3 \times N_\tau$, a-fixed ($a_s = 0.1227(8)$ fm), anistropic lattices ($a_s/a_\tau = 3.5$), 2+1 flavors with $m_\pi \simeq 400$MeV, $m_K \simeq 500$ MeV

Chemical equilibriation of heavy quarks



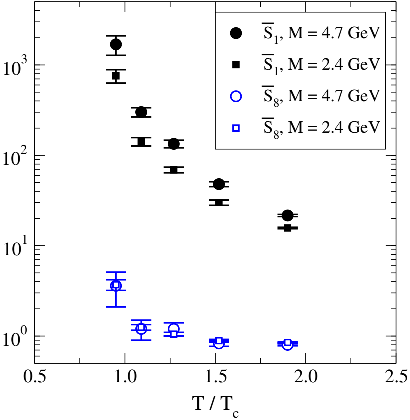
$$P_1 \equiv \frac{1}{2N_c} \mathrm{Re}\left\langle G^{\theta}_{\alpha\alpha;ii}(\beta,\vec{0};0,\vec{0})\right\rangle ,$$

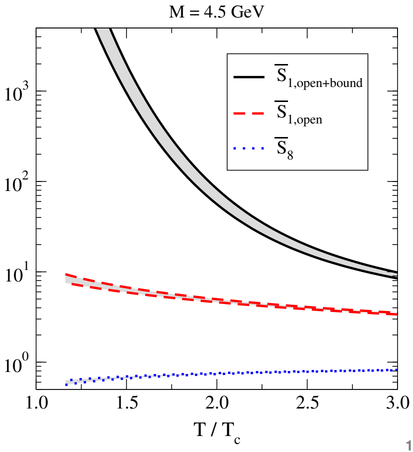
$$P_2 \equiv \frac{1}{2N_c} \left\langle G^{\theta}_{\alpha\gamma;ij}(\beta,\vec{0};0,\vec{0}) G^{\theta\dagger}_{\gamma\alpha;ji}(\beta,\vec{0};0,\vec{0})\right\rangle ,$$

$$P_3 \equiv \frac{1}{2N_c^2} \left\langle G^{\theta}_{\alpha\alpha;ij}(\beta,\vec{0};0,\vec{0}) G^{\theta\dagger}_{\gamma\gamma;ji}(\beta,\vec{0};0,\vec{0})\right\rangle .$$

Chemical equilibriation of heavy quarks

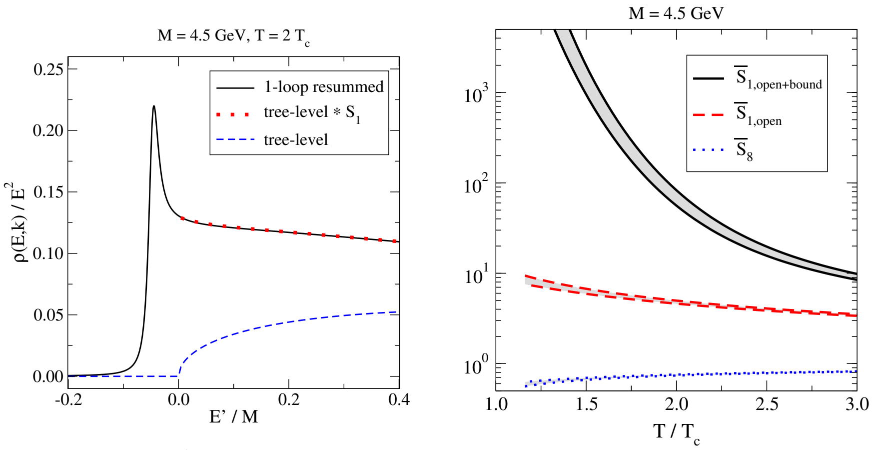
$$\bar{S}_1 = \frac{P_2}{P_1^2}, \quad \bar{S}_8 = \frac{N_c^2 P_3 - P_2}{(N_c^2 - 1) P_1^2}, \qquad \Gamma_{\rm chem}^{-1} \sim 150 {\rm fm}/c$$





Chemical equilibriation of heavy quarks

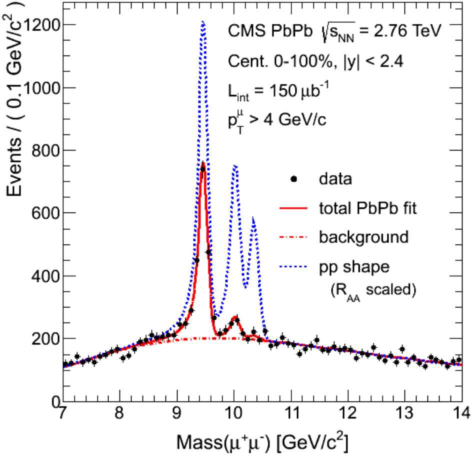
- understanding of lattice result by including bound state effect througth a spectral function



- implication for dark matter annihilation

Quarkonium melting at $T \neq 0$

- does quarkonium exist at $T \neq 0$? If so, at which temperature ?



Quarkonium melting at $T \neq 0$

- potential model consideration: T. Matsui and H. Satz, PLB178 (1986) 416

- first principle calculations ?

  - calculate $T \neq 0$ potential between heavy quarks, and solve Schrödinger equation (Y. Burnier, O. Kaczmarek, A. Rothkopf, arXiv:1606.06211 and a talk by A. Rothkopf (Th. 15:20))

  - calculate relativistic heavy quark correlator and quarkonium spectral functions (a talk by J.-I. Skullerud (Th.14:00))

  - calculate non-relativistic QCD correlator and quarkonium spectral functions (T. Harris et al, JHEP07 (2014) 097 ; S.K. A. Rothkopf, P. Petreczky, PRD91 (2015) 054511 and updates from both groups)

NRQCD for $T \neq 0$ quarkonium

- m_Q is “integrated out” and focus on the scale of “binding” $\rightarrow$ avoid large scale separation problem
- small statistical errors ($\sim \mathcal{O}(10^{-4})$)
- kernel $K(\tau,\omega)$ becomes $\sim e^{-\omega\tau}$ $\rightarrow$ temperature independent kernel
- intial value problem $\rightarrow$ larger τ range
- continuum limit can’t be taken ($m_Q a \sim \mathcal{O}(1)$)

Bayesian methods

- given $G(\tau)$ which is calculated on lattice, what is the spectral function, $\rho(\omega)$?

- Bayes theorm

$$P[X|Y] = P[Y|X]P[X]/P[Y]$$

- in other words

$$P[\rho|D,H] \propto P[D|\rho,H]P[\rho|H]$$

- systematic inclusion of prior knowledge (H)

$$P[D|\rho,H] = e^{-L}, \quad L = \frac{1}{2}\sum_i (D_i - D_i^{\rho})^2/\sigma_i^2$$

and

$$P[\rho|H] = e^{-S}, \quad S = S[\rho(\omega), m(\omega)]$$

where S is the prior and $m(\omega)$ is default model

Bayesian methods

- Shannon-Jaynes entropy for S (cf. Asakwa, Hatsuda, Nakahara, Prog. Part.Nucl.Phys. 45 (2001) 459)

$$S_{SJ} = \alpha \int d\omega \left(\rho - m - \rho \log(\frac{\rho}{m}) \right)$$

- new prior (cf. Y.Burnier, A. Rothkopf, PRL111 (2013) 182003)

$$S_{BR} = \alpha \int d\omega \left(1 - \frac{\rho}{m} + \log(\frac{\rho}{m}) \right)$$

Quarkonium – FASTSUM: NRQCD + MEM (previously)

• G. Aarts, C. Allton, T. Harris, S.K., M.P. Lombardo, M.B. Oktay, S.M. Ryan, D.K. Sinclair, J-I. Skullerud: NRQCD + MEM

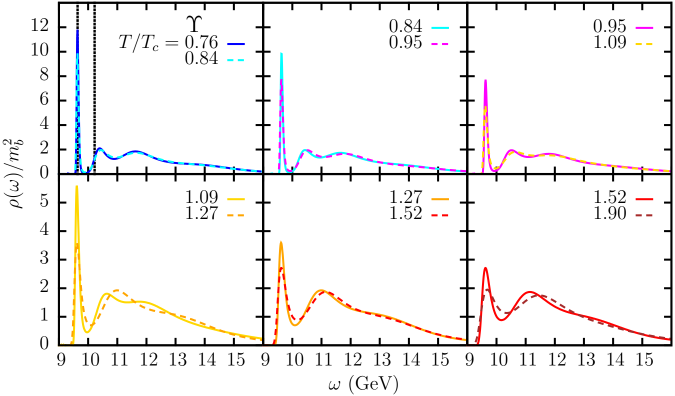
• anisotropic lattices with fixed scale, T change by N_τ: 1st Gen ($12^3 \times N_\tau, a_s/a_\tau = 6.0, N_f = 2$), 2nd Gen ($24^3 \times N_\tau, a_s/a_\tau = 3.5, N_f = 2+1$), 3rd Gen ($32^3 \times N_\tau, a_s/a_\tau \sim 7, N_f = 2+1$)

• S-wave bottomonium (PRL106 (2011) 061602, JHEP11 (2011) 013, JHEP03 (2013) 084), P-wave bottomonium (JHEP12 (2013) 064), and S- and P-wave (JHEP07 (2014) 097)

• detailed systematic errors study (default-model dependence, energy window, number of configurations, euclidean time window)

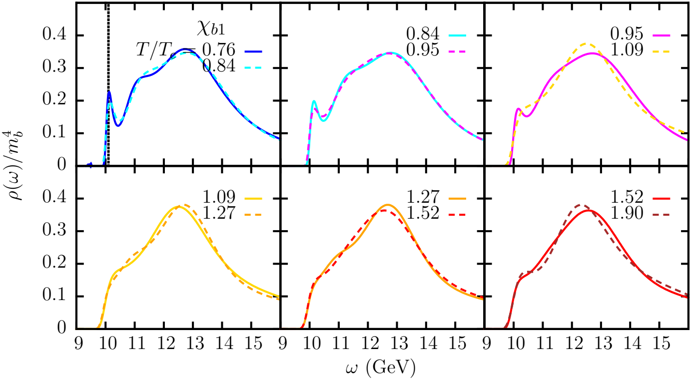
Quarkonium – FASTSUM: NRQCD + MEM (previously)

- S-wave (JHEP07 (2014) 097)

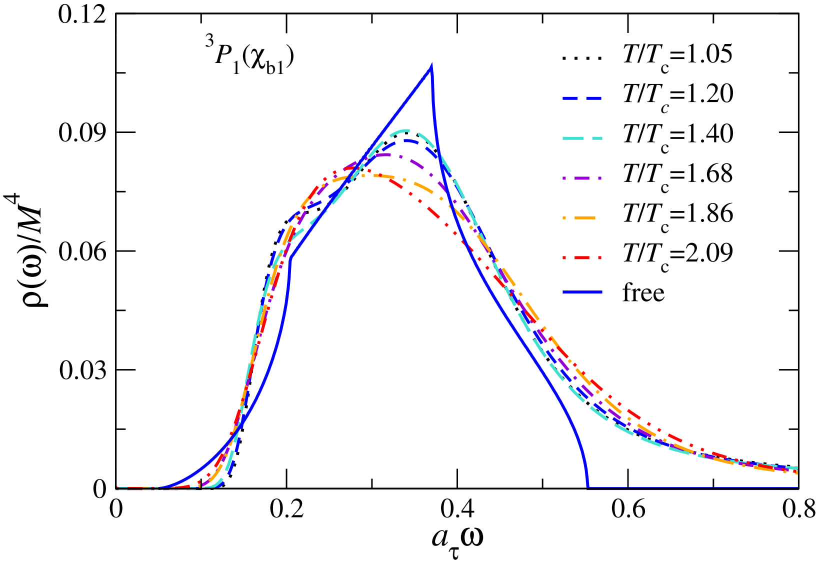


Quarkonium – FASTSUM: NRQCD + MEM (previously)

- P-wave (JHEP07 (2014) 097)



Quarkonium – FASTSUM: NRQCD + MEM (previously)



Quarkonium – FASTSUM: NRQCD + MEM (previously)

- sequential suppression of S-wave bottomonium
- survival of $\Upsilon(1S)$ upto $\sim 2.1\,T_c$ (1st Gen) and $\sim 1.9\,T_c$ (2nd Gen)
- immediate melting of P-wave bottomonium above T_c
- qualitatively similar for both $N_f = 2$ and $N_f = 2+1$ configurations

Quarkonium – KPR: NRQCD + MEM or New Bayesian approach (previously)

- S.K., A. Rothkopf, P. Petreczky: NRQCD + (MEM, new Bayesian)

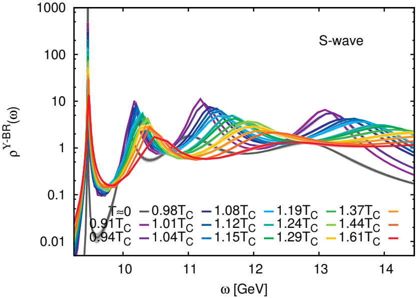
- isotropic lattices from HotQCD ($48^3 \times 12$, $N_f = 2+1$ light $m_\pi \sim 160$ MeV), T change by changing a (needs accompanying $T = 0$ calculation)

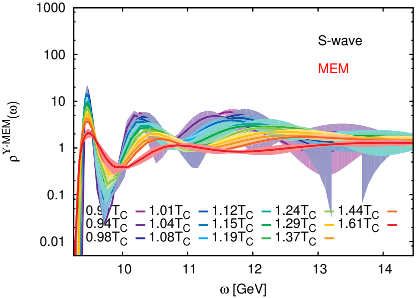
- S.K. A. Rothkopf, P. Petreczky, PRD 91 (2015) 054511

- detailed systematic errors study (default-model dependence, energy window, number of configurations, euclidean time window)

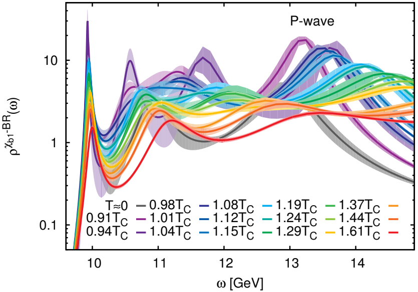
- investigation on the prior dependence (MEM vs new Bayesian)

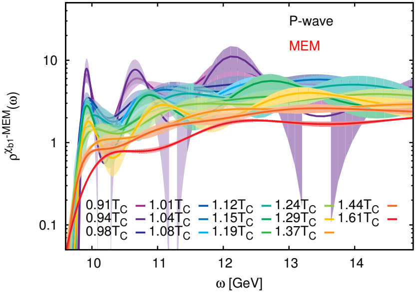
Quarkonium – KPR: NRQCD + MEM or New Bayesian approach (previously)





Quarkonium – KPR: NRQCD + MEM or New Bayesian approach (previously)



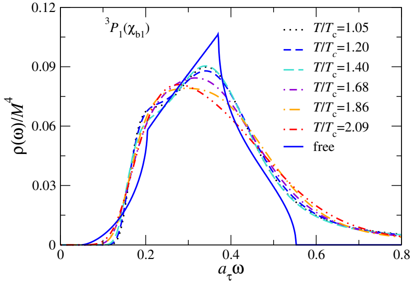


Quarkonium – KPR: NRQCD + MEM or New Bayesian approach (previously)

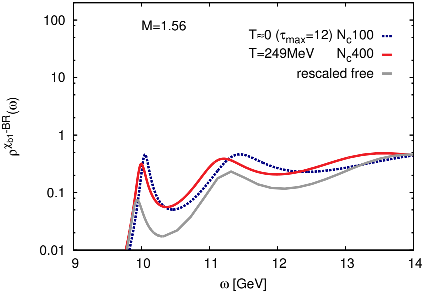
- sequential suppression of S-wave bottomonium
- survival of $\Upsilon(1S)$ upto $\sim 1.6\,T_c$
- survival of P-wave bottomonium upto $\sim 1.6\,T_c$

Quarkonium – KPR: NRQCD + MEM or New Bayesian approach (previously)

FASTSUM

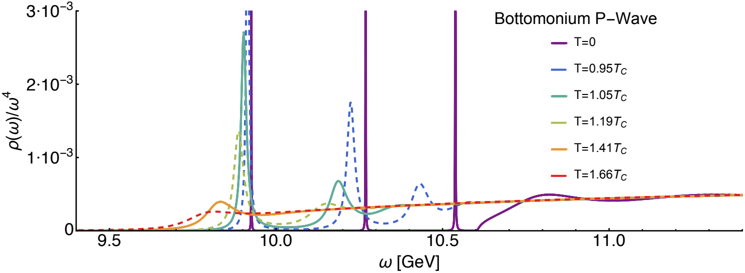


KPR

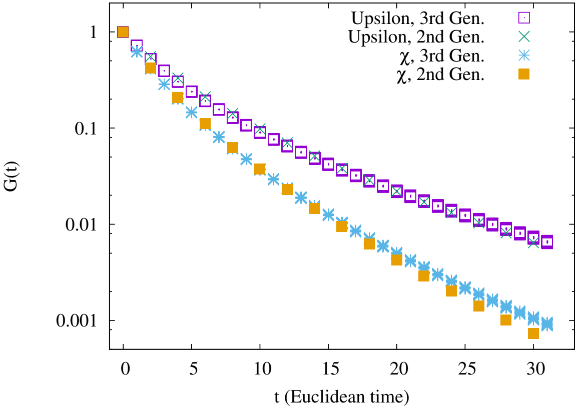


Some hint?

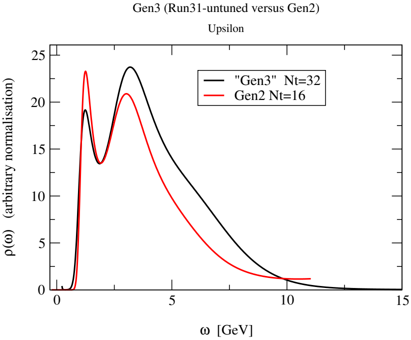
- Y. Burnier, O. Kaczmarek, A. Rothkopf, 1606.06211

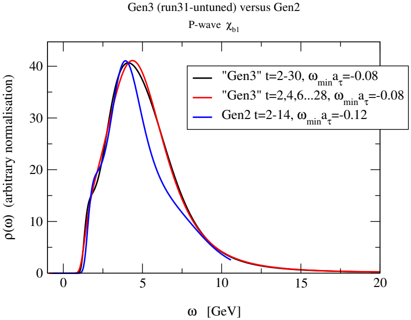


Quarkonium – FASTSUM (preliminary)



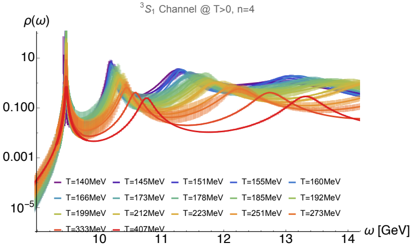
Quarkonium – FASTSUM (preliminary)



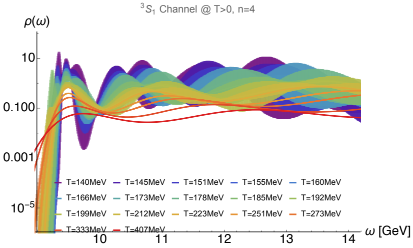


Quarkonium – KPR (update, preliminary)

- new data at $T = 273(1.84T_c), 333(2.25T_c), 407(2.75T_c)$ MeV with higher statistics ($\sim$ 4000 correlators)

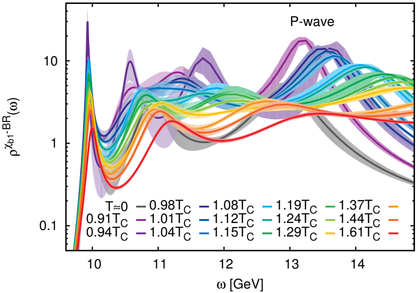


new Bayesian

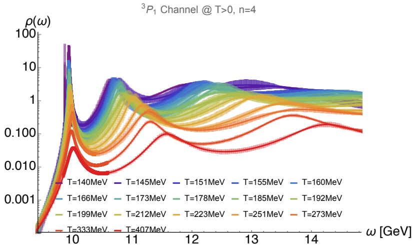


MEM

Quarkonium – KPR (update, preliminary)

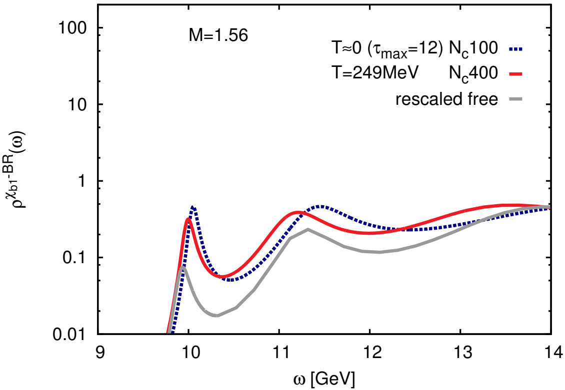


new Bayesian



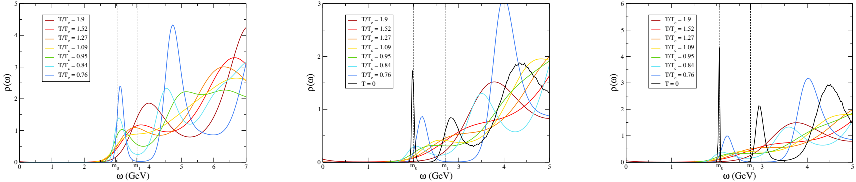
new Bayesian

Quarkonium – KPR (update, preliminary)



D Meson

- study on the spectral functions from relativistic charmonium correlators and (D, D_s) meson correlators by FASTSUM collaborations (cf. Talk by J-I. Skullerud (Th. 14:00))



Conclusion

• Heavy quarks and quarkonia is one of better understood hadronic system and $T \neq 0$ characteristics may be better understood by comparing experimental data and theoretical understanding

• Results on transport phenomena (kinetic equilibriation rate, chemical equilibriation rate) of heavy quarks are studied and further quantitative chacterization shall follow

• $T \neq 0$ behavior of S- and P-wave bottomonium spectral functions from NRQCD (with full QCD lattices)+ MEM shows qualitatively sequential melting (S-wave) and immediate melting above T_c (P-wave), which needs to be checked with other methods and with different prior functions

• We need to work harder