Multiple Level Spacing Distributions at the Mobility Edge of QCD Dirac Spectrum

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SMN-Giordano-Kovacs-Pittler, PoS Lattice 2013, 018 (plenary) 1312.3286 [hep-lat]

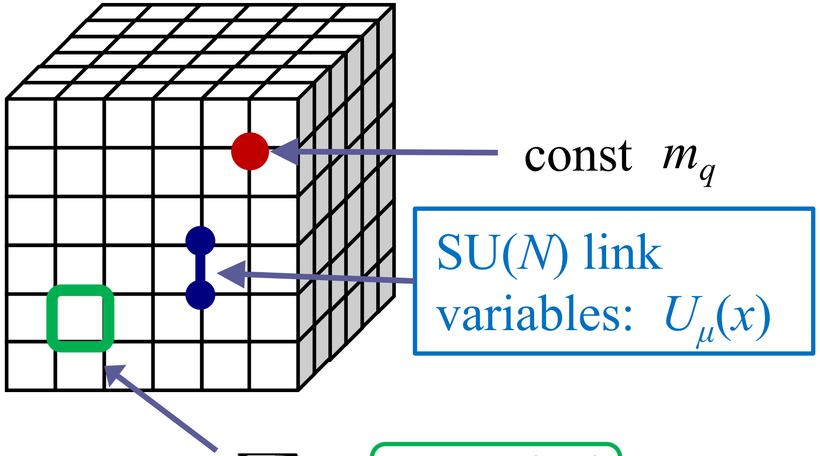
SMN, PoS Lattice 2015, 057 1606.00276 [hep-lat]

SMN-Yamamoto-et al., to be submitted to PTEP 17**.*****

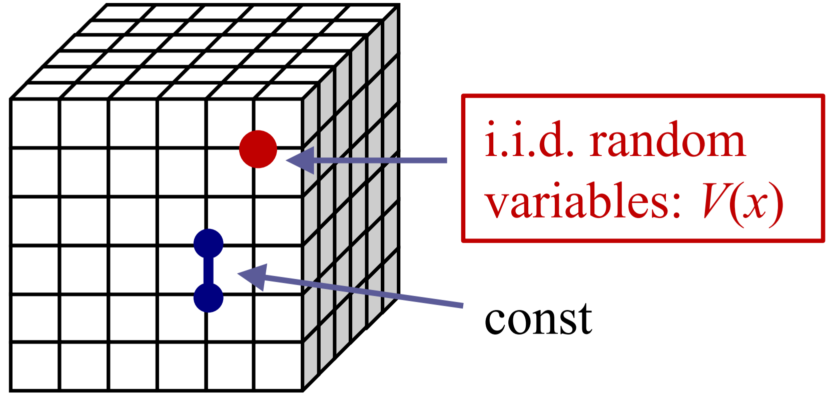
Workshop 2017 on "Discrete Approaches to the Dynamics of Fields and Space-Time"

2017.9.19, Asia Pacific Center for Theoretical Physics, Pohang

Wilson's Lattice Gauge Theory (4D) | Anderson's tight-binding H (3D)



$$\beta \sum_{\square} \mathrm{Tr}\, U_\bullet U_\bullet U_\bullet^\dagger U_\bullet^\dagger$$



Anderson Hamiltonian (AH):

$$H(x) = -\nabla^2 + V(x)$$

Anderson Localization in LGT? [Halasz-Verbaarschot 95,…]

- if AL occurred above T_{pc} ,

extended Dirac modes only

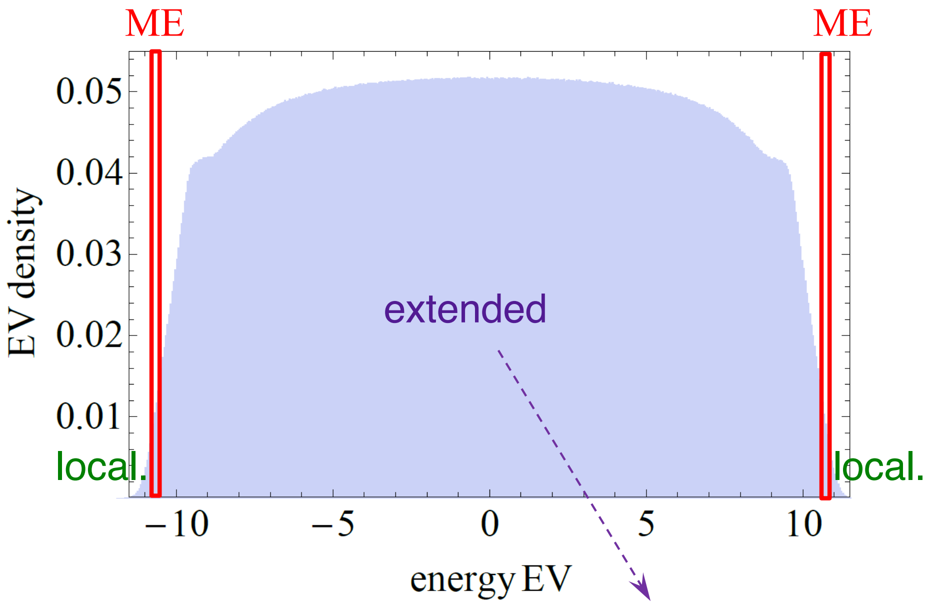
$$\langle x|(D+m)^{-1}|y\rangle = \sum_k \frac{\phi_k^*(x)\phi_k(y)}{i\lambda_k + m} \approx \sum_{|\lambda_k| \geq \bar{\lambda}} \frac{\phi_k^*(x)\phi_k(y)}{i\lambda_k + m}$$

$\bar{\lambda}$: Mobility Edge

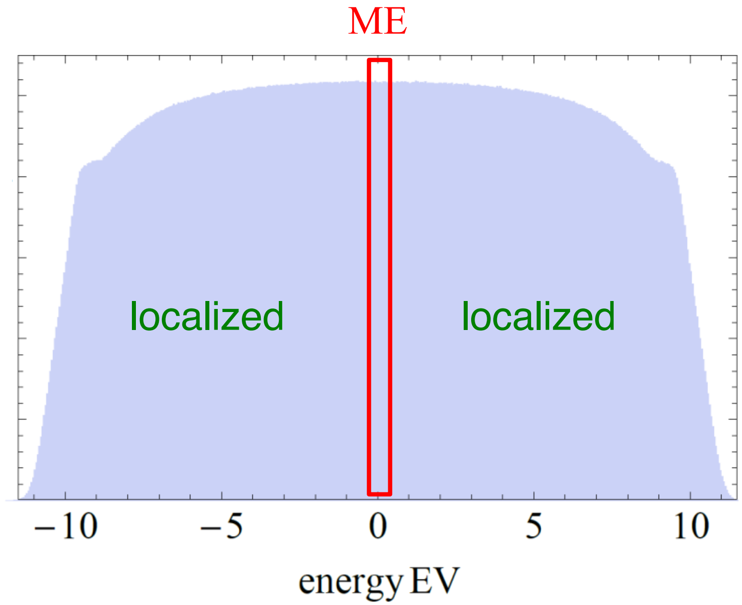
quark propagator in QGP would be drastically affected…

- or AL can even be a mechanism connecting Confinement & χSB

Anderson H : Level Statistics

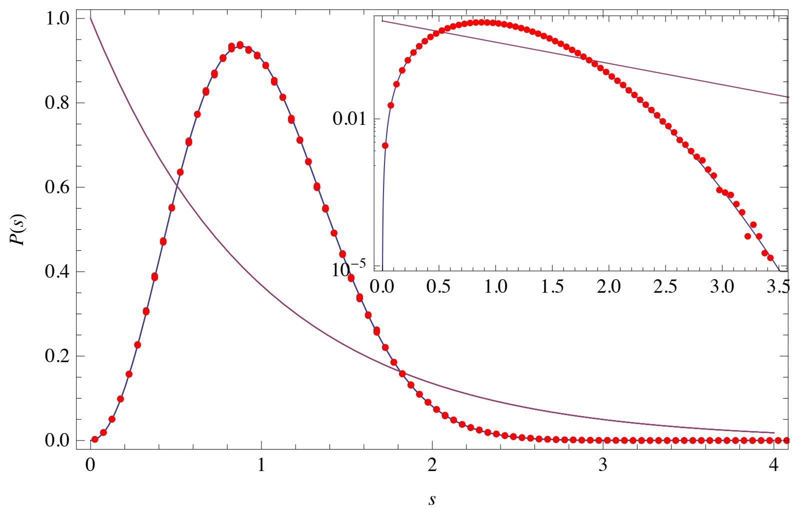


random ness



Level Spacing Distribution

$$s \equiv \frac{\lambda_{k+1} - \lambda_k}{\text{mean spacing } \Delta}$$



3D AH + ***B***
on 20^3, $N_{conf}=10^4$

vs

Gauβ Unitary Ensemble

extended modes : level statistics ⊂ RM universality

Random Matrices

$$H = \begin{pmatrix} \# & & & \# & & & \cdots \\ & \# & & & & \# & \\ & & \# & & & & \\ \# & & & \# & \# & & \# \\ & & & \# & \# & & \\ & \# & & & & \# & \\ \vdots & & & \# & & & \ddots \end{pmatrix} \quad \rightarrow \quad H = \begin{pmatrix} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} & H_{16} & \cdots \\ H_{21} & H_{22} & H_{23} & H_{24} & H_{25} & H_{26} & \cdots \\ H_{31} & H_{32} & H_{33} & H_{34} & H_{35} & H_{36} & \cdots \\ H_{41} & H_{42} & H_{43} & H_{44} & H_{45} & H_{46} & \cdots \\ H_{51} & H_{52} & H_{53} & H_{54} & H_{55} & H_{56} & \cdots \\ H_{61} & H_{62} & H_{63} & H_{64} & H_{65} & H_{66} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

sparse, dimensionful

dense, indep. Gauβ random

sharing involutive symmetries

$$d\mu(H) = dH\, \mathrm{e}^{-\mathrm{Tr}\, H^2} \propto \prod_i d\lambda_i\, \underbrace{\mathrm{e}^{-\lambda_i^2} \prod_{i>j} |\lambda_i - \lambda_j|^{\beta}}_{} \quad (=1,2,4)$$

harm. osc. WF

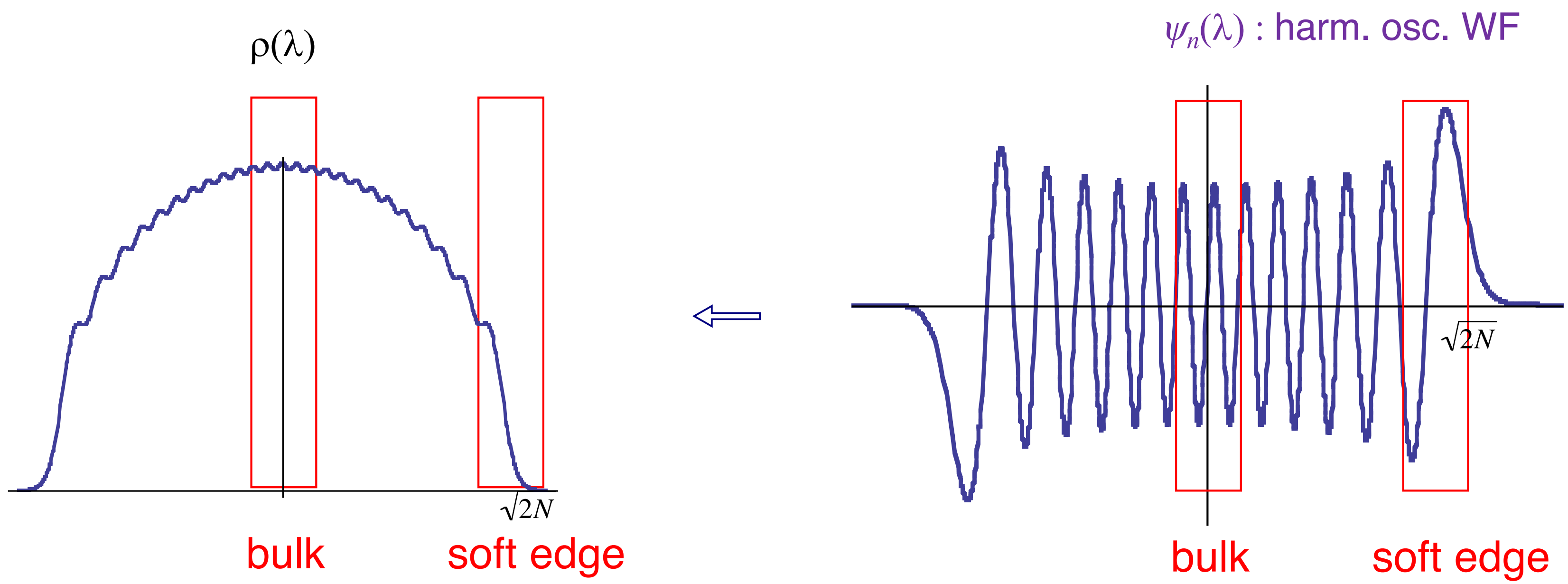
$\beta = 2$ case

$$\det\left[\sum_{n=0}^{N-1} \psi_n(\lambda_i)\psi_n(\lambda_j)\right]_{i,j=1}^{N} = \det\left[K(\lambda_i,\lambda_j)\right]_{i,j=1}^{N}$$

EV density correlator $\langle \rho(\lambda_1) \ldots \rho(\lambda_p) \rangle = \det\left[K(\lambda_i, \lambda_j)\right]_{i,j=1}^{p}$: Det process

⇨ $\mathrm{Prob}(k \text{ EVs} \in I) = \frac{1}{k!} \partial_z^k \mathrm{Det}\,(1 + z\, K|_I)\Big|_{z=-1}$: Fredholm Det

RM : local EV correlation



mean EV spacing Δ

bulk unfolding $\lambda = \frac{1}{N^{1/2}} x$

$$\Rightarrow \quad \psi_n(x) \sim \begin{cases} \cos x & (n \text{ even}) \\ \sin x & (n \text{ odd}) \end{cases}$$

$$K_{\sin}(x,y) = \frac{\sin(x-y)}{x-y}$$

soft edge unfolding $\lambda = \sqrt{2N} + \frac{1}{N^{1/6}} x$

$$\Rightarrow \quad \psi_n(x) \sim \mathrm{Ai}(x)$$

$$K_{\mathrm{Ai}}(x,y) = \frac{\mathrm{Ai}(x)\mathrm{Ai}'(y) - \mathrm{Ai}'(x)\mathrm{Ai}(x)}{x-y}$$

NLσM from AH

[Wegner '80s]

$$H = -\nabla^2 + V(x)$$

Gaussian av. over $V(x)$

$$Z(\varepsilon) = \left\langle \det(\varepsilon_1 - H)^{n_1} \det(\varepsilon_2 - H)^{n_2} \right\rangle \quad \text{(replica)}$$

$$\text{or } = \left\langle \frac{\det(\varepsilon_1 - H)}{\det(\varepsilon_1' - H)} \frac{\det(\varepsilon_2 - H)}{\det(\varepsilon_2' - H)} \right\rangle \quad \text{(SUSY)}$$

⇓ H-S transf

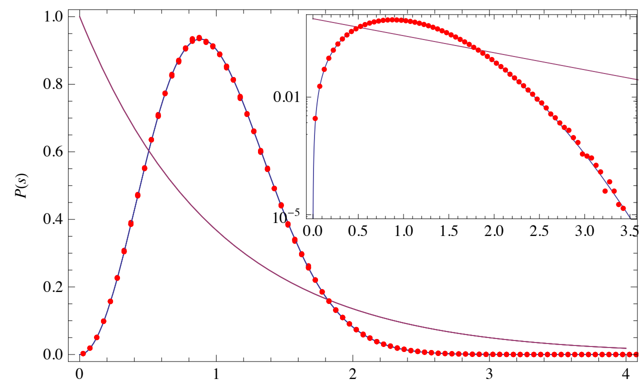
diffusion cst

$$Z(\varepsilon) = \int_{\text{NG mfd}} DU \exp\left(\frac{1}{V\Delta} \int d^d x \left\{ D \operatorname{tr}(\nabla U)^2 + (\varepsilon_1 - \varepsilon_2) \operatorname{tr} \Gamma U \right\} \right)$$

⇓ $E_{\text{Th}} \equiv D/L^2 >> \delta\varepsilon = O(\Delta)$: ε-regime = 0 mode dominance

$$Z(\varepsilon) = \int dU \, e^{\frac{\delta\varepsilon}{\Delta} \operatorname{tr} \Gamma U}$$

: 0d NLσM ⇔ RM



NLσM from AH

$$Z(\varepsilon) = \int DU \, \mathrm{e}^{-S[U]}$$

$$S[U] = \frac{1}{V\Delta} \int d^d x \left\{ D \, \mathrm{tr}(\nabla U)^2 + \delta\varepsilon \, \mathrm{tr} \, \Gamma U \right\}$$

$\downarrow$ $E_{\mathrm{Th}} \equiv D/L^2 >> \delta\varepsilon = O(\Delta)$: ε regime, 0 mode dominance

$$Z(\varepsilon) = \int dU \, \mathrm{e}^{\frac{\delta\varepsilon}{\Delta} \mathrm{tr} \, \Gamma U}$$

: 0D NLσM ⇔ RM

ergodic regime $E_{\mathrm{Th}} >> \Delta$: RMT √

diffusive regime $E_{\mathrm{Th}} > \Delta$: perturbation √

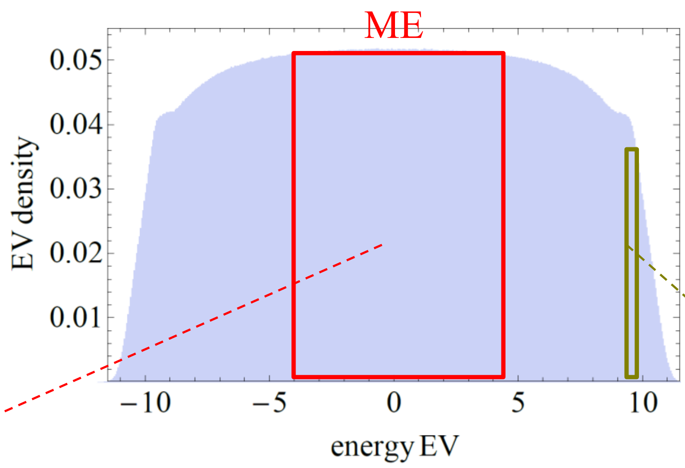
“mobility edge” $E_{\mathrm{Th}} \sim \Delta$: perturbation ×, fixed point

→ phenomenological model desirable

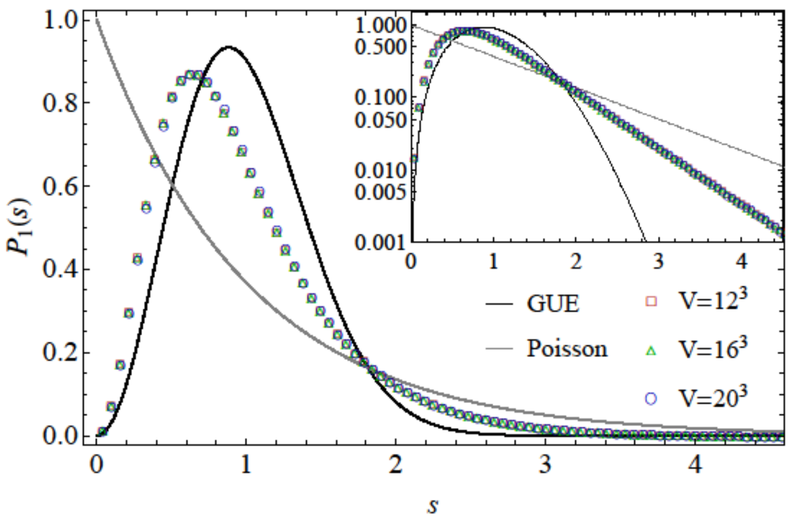
II. Critical Statistics & Deformed RM

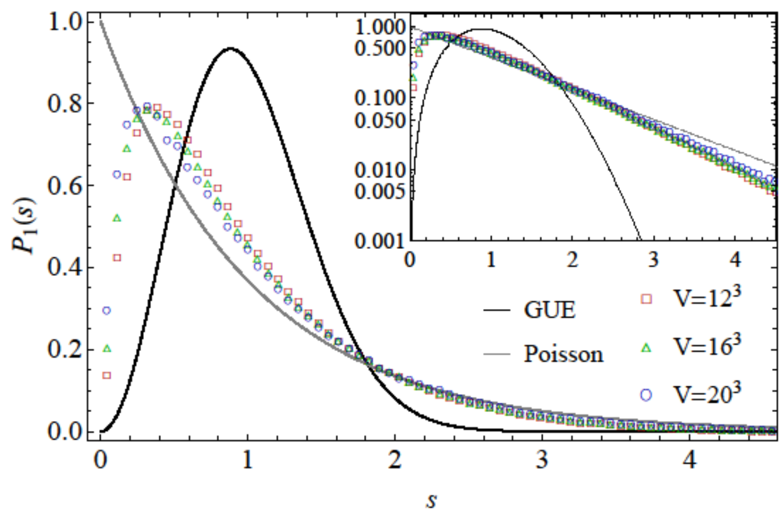
Critical Statistics

[Shklovskii et al ’93]



on 12^3, 16^3, 20^3, $N_{\text{conf}}=10^4$
randomness W=18.1
mag. flux $\Phi=0.4\pi$





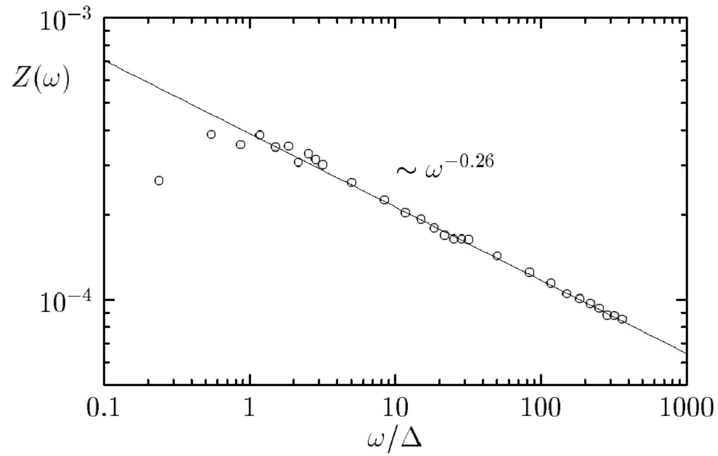
multifractal WF $\xi \sim L$

localized WF $\xi \ll L$

Scale Invariant Critical Statistics

no repulsion → Poisson

Critical Statistics



Anomalous inverse part. ratio

$$\sum_x \left\langle |\psi_i(x)|^{2p} \right\rangle \propto L^{-D_p(p-1)}$$

Sparse overlap

$$\sum_x \left\langle |\psi_i(x)|^2 \, |\psi_j(x)|^2 \right\rangle \propto |E_i - E_j|^{-(1-D_2)/d}$$

[Chalker '90]
[Zharekeshev-Kramer '97]

distant levels become less repulsive

		s small	s large
Level Spacing	$P(s)$	$\propto s^{\beta}$	$\propto e^{-\kappa s}$
Level # Variance	$\Sigma^2(S)$	$\propto \log s$	$\propto \chi S$

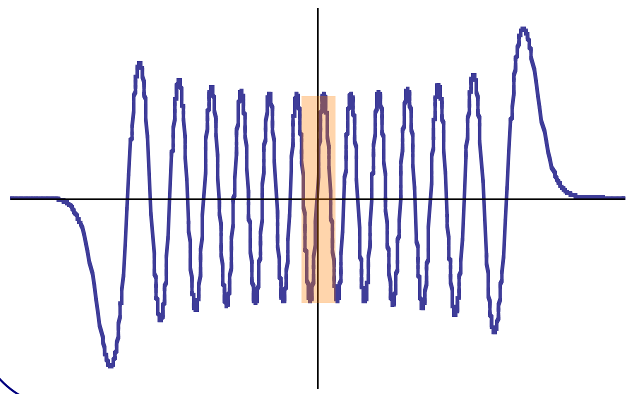
Poisson-like

"Level Repulsion without Rigidity"

Universality of 10 RM classes is very robust

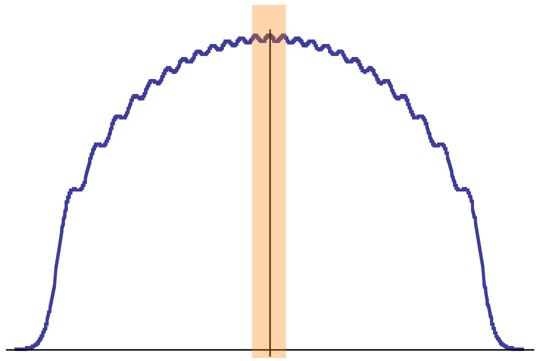
(s)OP method

$\psi_n(\lambda)$: WF



EV distribution, correlation

$\rho(\lambda)$



NLσM method

$$Z(\varepsilon) = \int dU \, e^{\frac{\delta\varepsilon}{\Delta} \mathrm{tr}\, \Gamma U}$$

NG mfd : 10 Riemann SS

$U(n+m)/U(n)\times U(m)$ for GUE

proven to be stable against deformations

like $e^{-\mathrm{tr}\, H^2} \rightarrow e^{-\mathrm{tr}\, H^4}, e^{-\mathrm{tr}\, (H+A)^2}, \ldots$

$$\lambda H_n(\lambda) = H_{n+1}(\lambda) + n H_{n-1}(\lambda)$$

$$\downarrow$$

$$\lambda H_n(\lambda; q) = H_{n+1}(\lambda; q) + [n]_q H_{n-1}(\lambda; q)$$

proposal: *q*-analog of OP

*disclaimer: individual impressions may vary

q^{-1}-Hermite Ensemble

[Muttailb-Chen-Ismail-Nicopoulos '93]

$$h_n(\lambda\,;q)=\sum_{k=0}^{n}\begin{bmatrix} n \\ k \end{bmatrix}_q q^{-k(n-k)}(-)^k \mathrm{e}^{(n-2k)u},\quad \lambda\equiv\sinh u \qquad (0<q<1)$$

$$\int_{-\infty}^{\infty} h_n(\sinh u\,;q)\, h_m(\sinh u\,;q)\frac{du}{(-qe^{-2u},q)_\infty(-qe^{2u},q)_\infty}=0 \quad (n\neq m)$$

$$V(\lambda)\approx\frac{1}{2a}(\sinh^{-1}\lambda)^2 \quad (x>>1)$$: weakly confining potential

$$q:=\mathrm{e}^{-2a},\quad p:=\mathrm{e}^{-\pi^2/a}$$

almost* unique deformation of GUE universality

$$K_N^{(a)}(\lambda,\lambda')\sqrt{d\lambda d\lambda'}\xrightarrow{N\to\infty}K_a(x,x)\sqrt{dxdx'}$$

$$\lambda\mapsto x=\frac{1}{2a}\sinh^{-1}\lambda$$: exponential unfolding

$$K_a(x,x')=cst.\underbrace{\frac{\sqrt{\cosh 2ax\cosh 2ax'}}{\cosh a(x+x')}\frac{\vartheta_4(\pi(x+x');p)}{\sqrt{\vartheta_4(\pi x;p)\vartheta_4(\pi x';p)}}}_{\text{transl. non-inv.}\ \to 1}\frac{\vartheta_1(\pi(x-x');p)}{\sinh a(x-x')},\qquad K_a(x,x)=1$$

$$\xrightarrow[\text{bulk limit}]{x,x\to\infty,\quad x-x=O(1)}$$

$$\boldsymbol{K}_a(x-x')=cst.\frac{\vartheta_1(\pi(x-x');\mathrm{e}^{-\pi^2/a})}{\sinh a(x-x')}$$: Elliptic Kernel

q^{-1}-Hermite Ensemble

the essence is simple…

$$K_a(x) = cst.\frac{\vartheta_1(\pi x; e^{-\pi^2/a})}{\sinh ax}$$

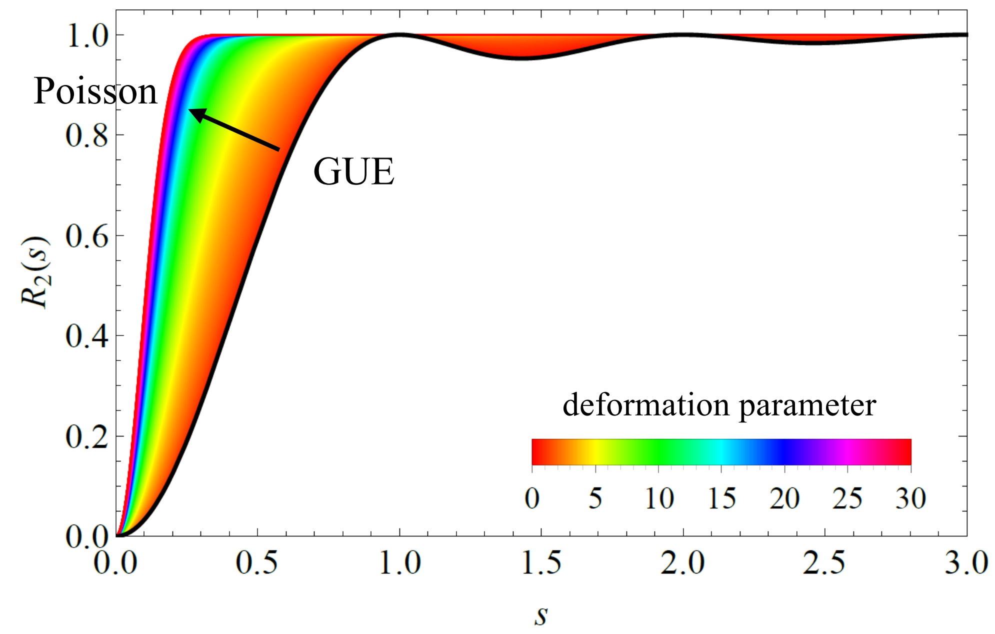
$$V(\lambda) \approx \frac{1}{2a}(\log\lambda)^2 \quad \Rightarrow \quad \rho(\lambda) \approx \frac{1}{2a\lambda} \quad \Rightarrow \quad \upsilon = \int^{\lambda} \rho(\lambda)\, d\lambda \approx \frac{1}{2a}\log\lambda$$

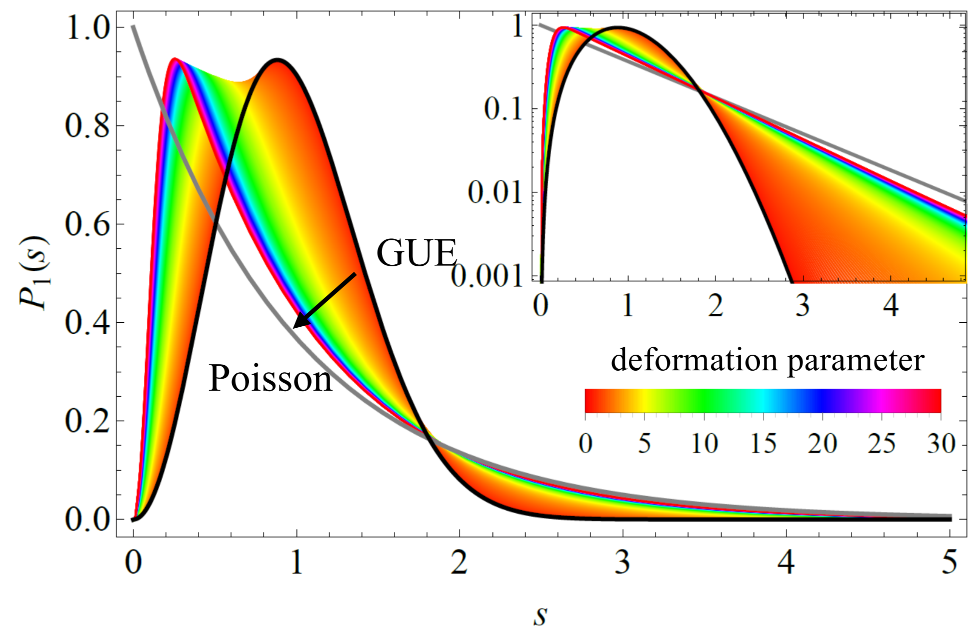
⇨ C-S MM

denom. of Ch-D formula : $\lambda - \lambda' \approx e^{2ax} - e^{2ax'} = e^{a(x+x')}\sinh a(x-x')$

2-level correlation function $R_2(s) = 1 - K_a(s)^2$

LSD $P_1(s) = \mathrm{Det}\,(1 - K_a|_{[0,s]})$ [SMN '98, '99]





III. Ordered EV Statistics

Nystrom approximation to Fredholm Det

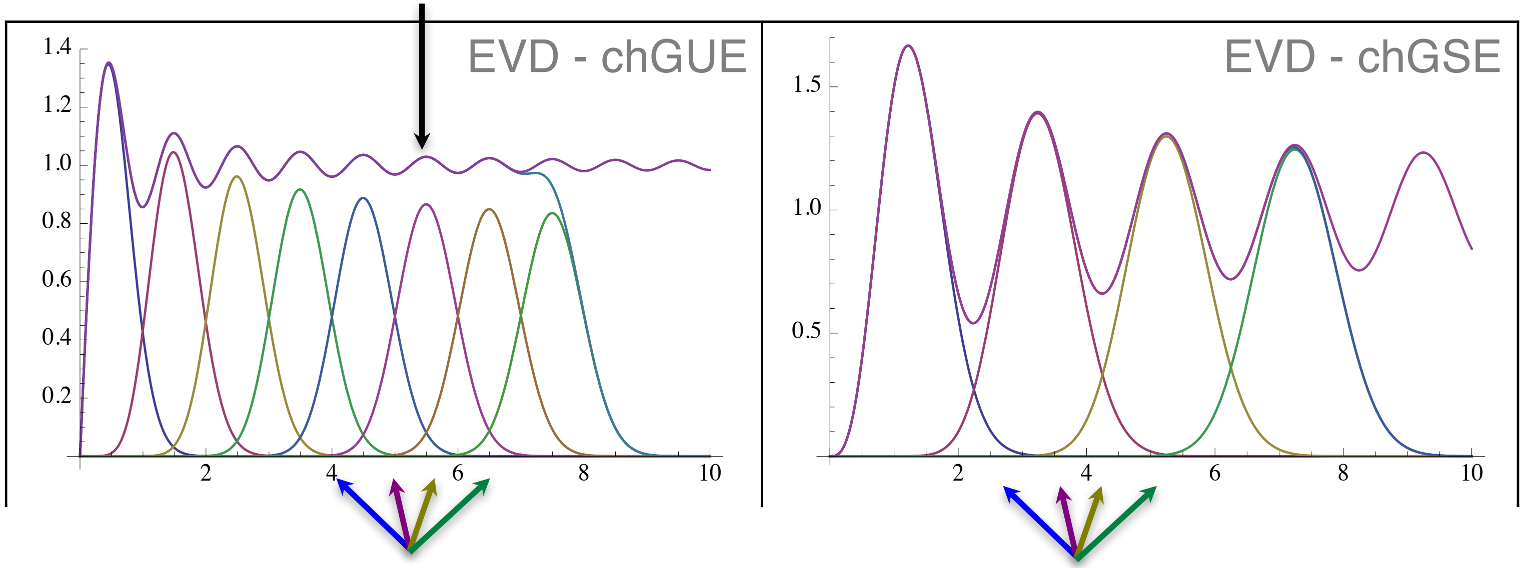
[Bornemann '10]

Gauss-Legendre Quadrature : $\{x_1,\ldots,x_M\}\in I,\ \ \{\Delta x_1,\ldots,\Delta x_M\}>0$

$$\int_I f(x)\,dx \cong \sum_{i=1}^{M} f(x_i)\Delta x_i \text{ , exact for } f(x)=x^M+\text{lower}$$

$$\mathrm{Det}\left(1-K_I\right) \cong \det\left[\delta_{ij}-K(x_i,x_j)\sqrt{\Delta x_i\Delta x_j}\right]_{i,j=1}^{M} + \text{relative error } O(\mathrm{e}^{-\mathrm{const}.M})$$

flattening oscillation ⇒ larger error for fitting



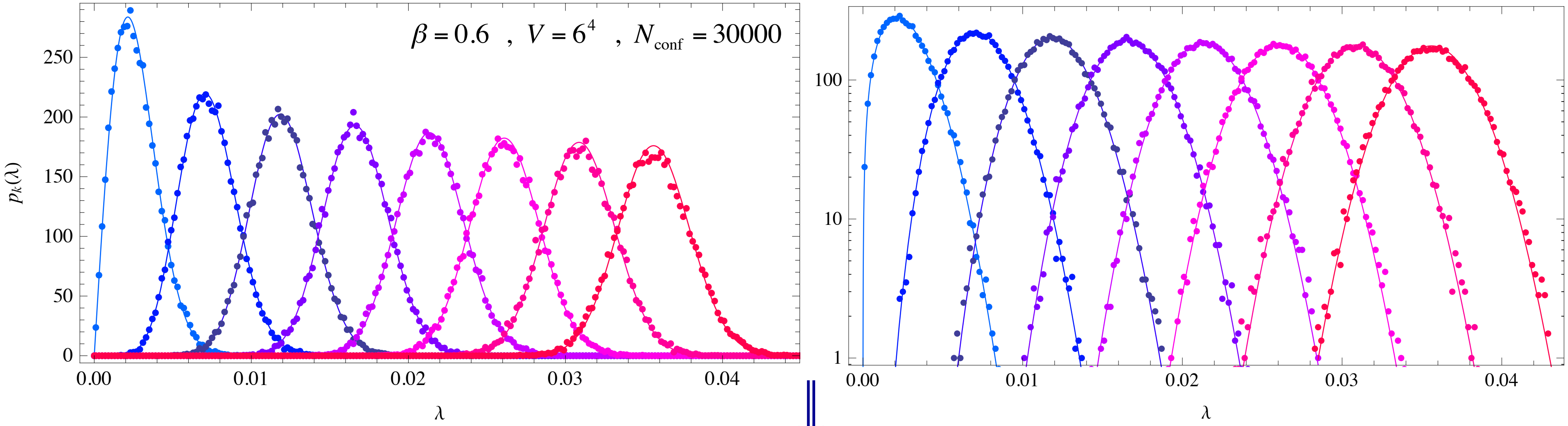
Gauβ-like ⇒ precise fitting possible

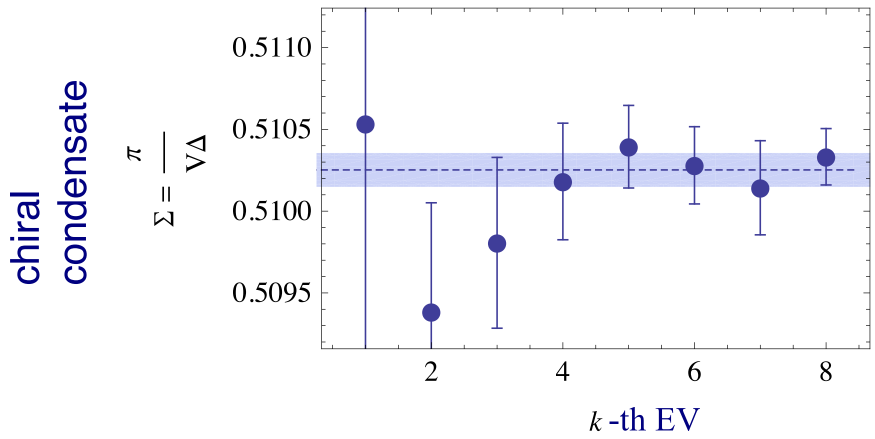
[Damgaard-SMN 01]

Chiral condensate from ordered EVDs

[SMN '16]

exercise 1 : quenched compact U(1) Dirac spectrum vs chGUE



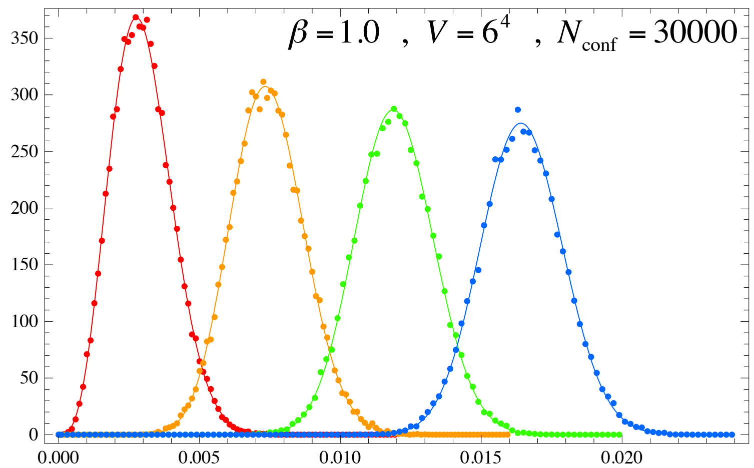


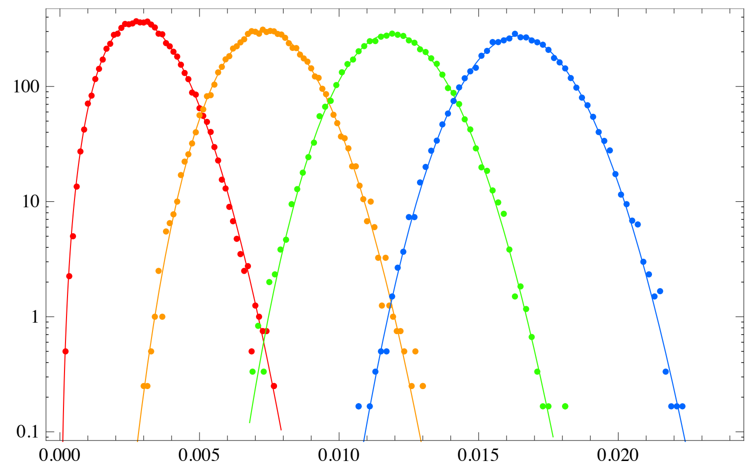
$$\Sigma a^3 = 0.51025(10)$$

Chiral condensate from ordered EVDs

[SMN '16]

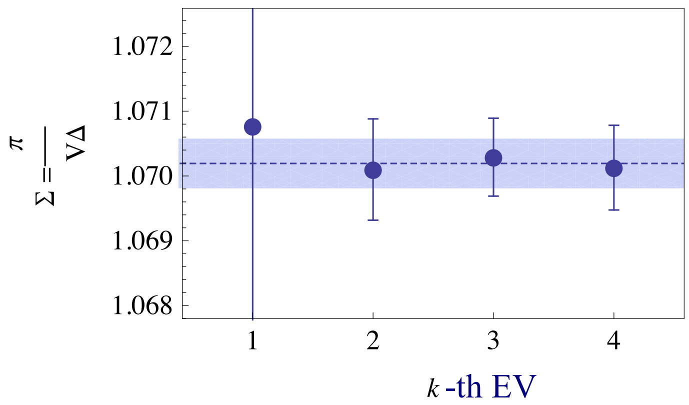
exercise 2 : quenched SU(2) Dirac spectrum vs chGSE





⇓

chiral condensate

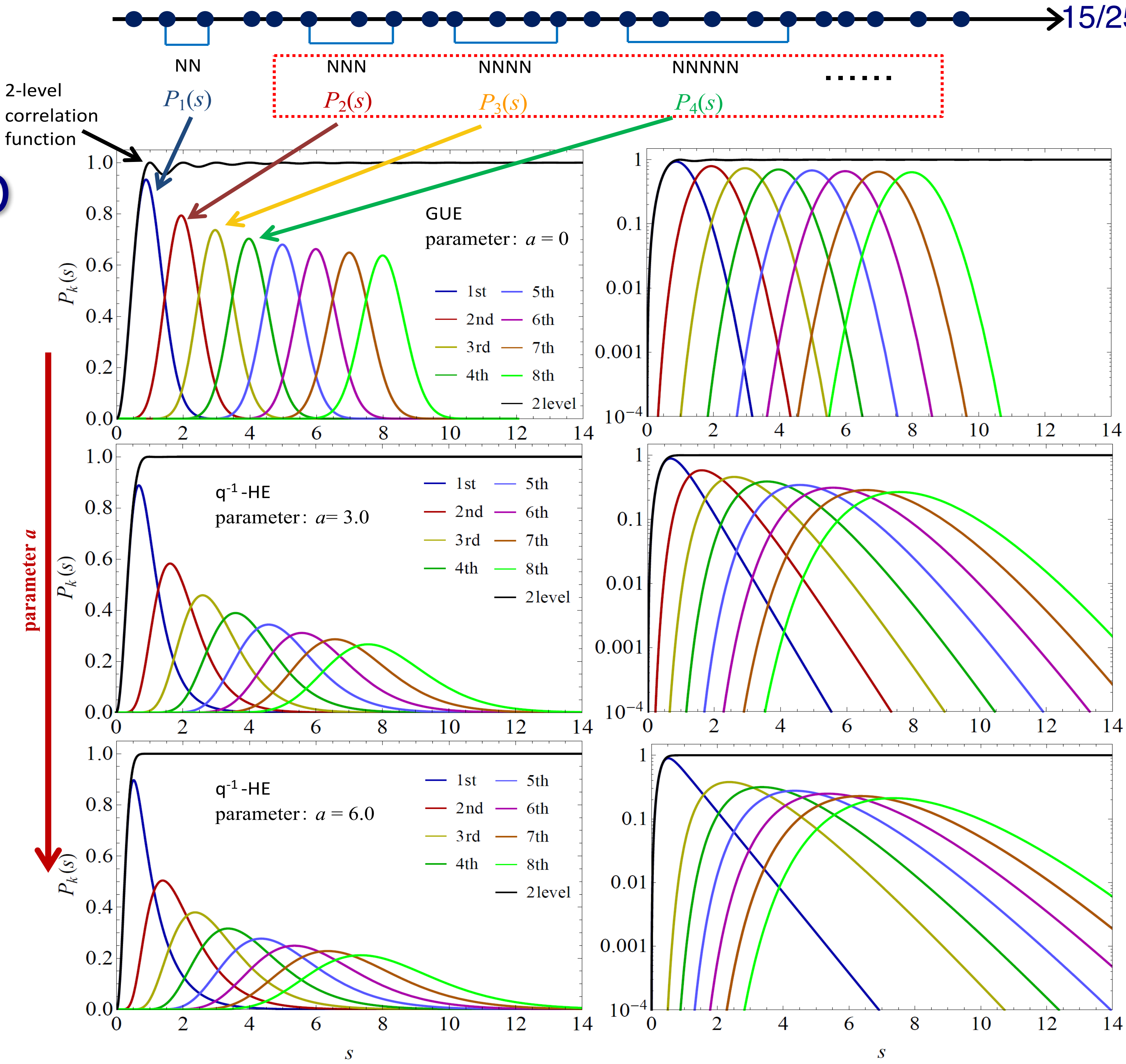


$$\Sigma a^3 = 1.07019(38)$$

Σ within $O(10^{-4})$ error!

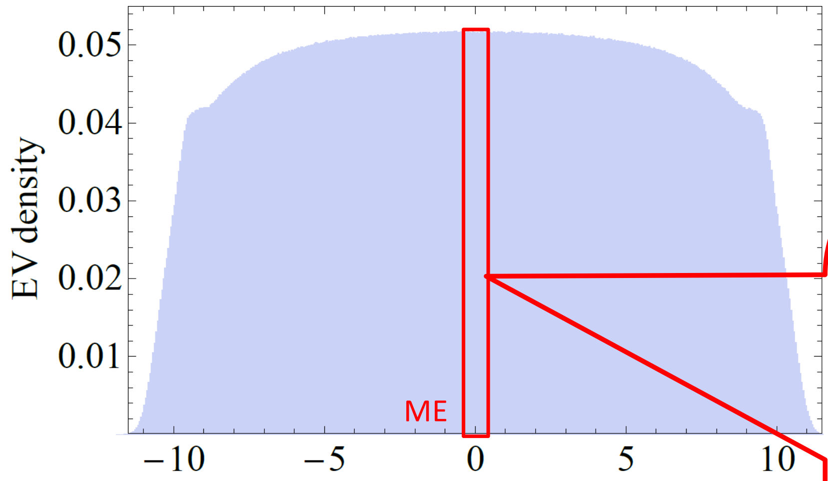
q^{-1}-HE: multi-LSD

[SMN '16]

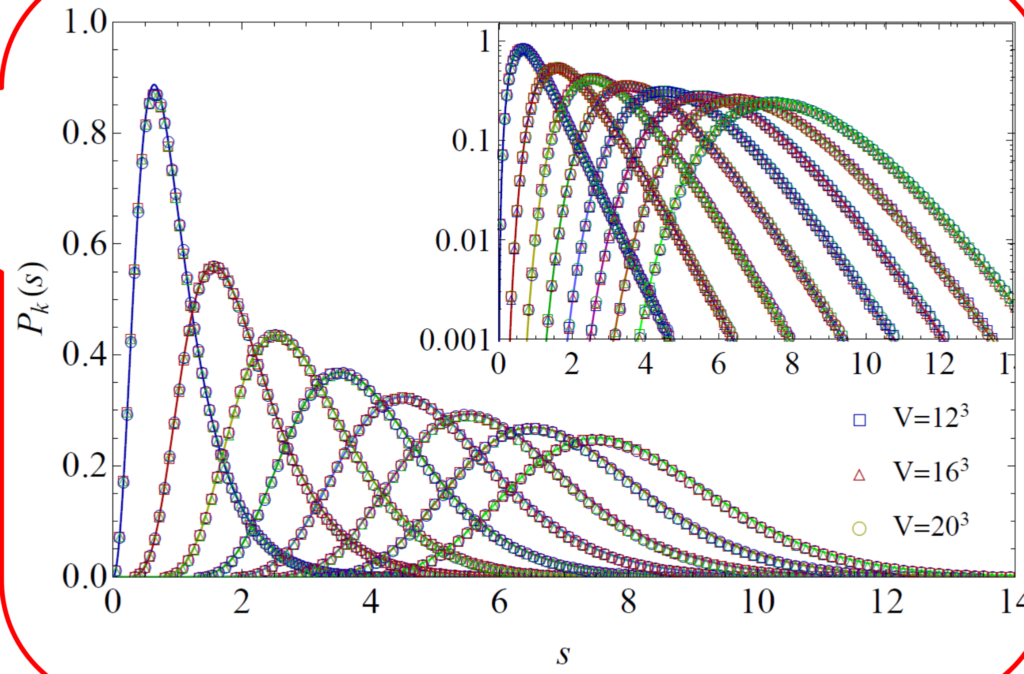


q^{-1}-HE vs AH@ME

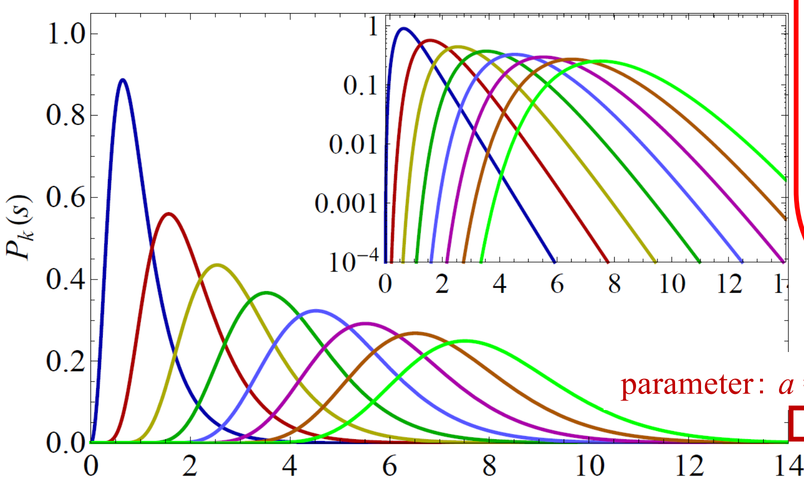
[SMN ’99, ’16]



AH on $12^3, 16^3, 20^3$, $N_{\rm conf}=10^4$
randomness W=18.1
mag. flux $\Phi=0.4\pi$



MLSDs: q^{-1}-HE



fit MLSDs of AH at ME with MLSDs of q^{-1}-HE

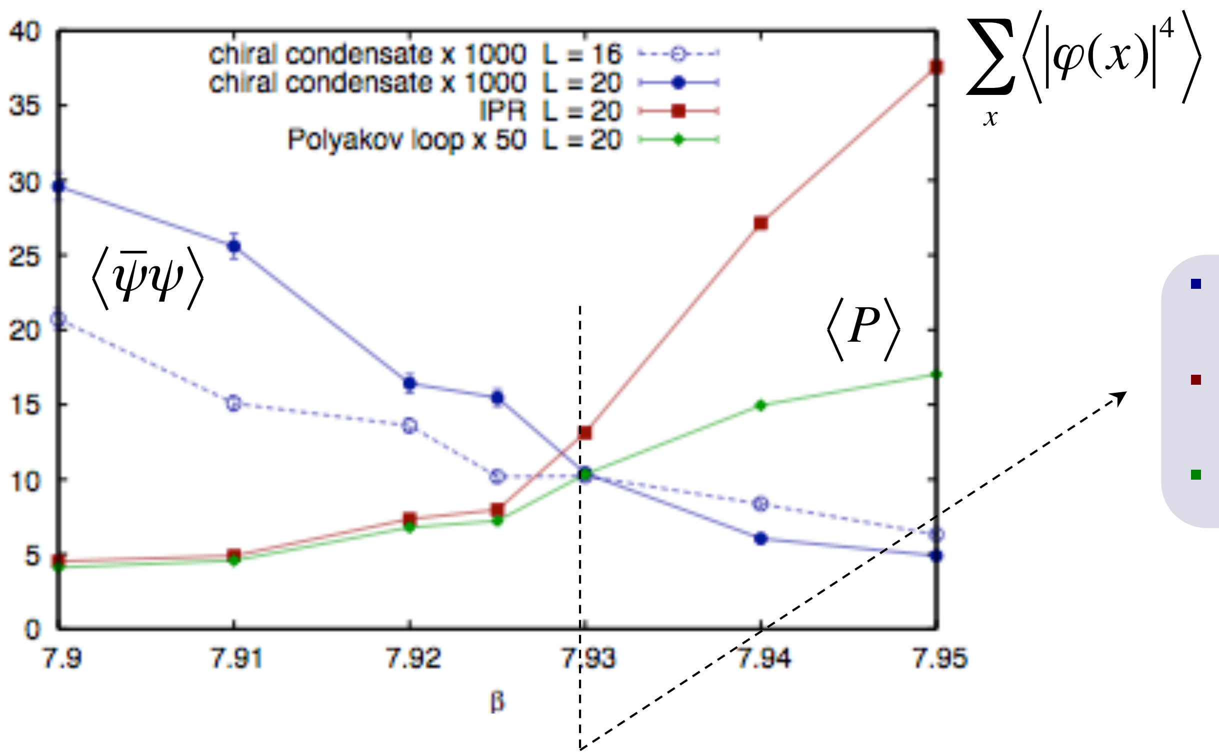
move on to QCD Dirac spectra at high T

IV. QCD Dirac Spectrum at high T

Localization and QCD transition

SU(3) quenched LGT
on $16^3 \sim 20^3 \times 4$, KS Dirac op.

first attempt by [Garcia-Osborn '07]



- chi symm restoration
- localization
- deconfinement

simultaneous?

Lattice setup

◆ QCD at physical point [Aoki, *et. al.*, '06]

gauge: Symanzik improved action
quark: 2-level stout-smeared staggered Dirac op.

N_C	N_F	β	m_{ud}	m_s	a [fm]
3	2+1	3.75	0.001786	0.05030	0.125

physical point determined by BMW Coll.

◆ Dirac EVs at high T [Kovacs-Pittler '12, '13]

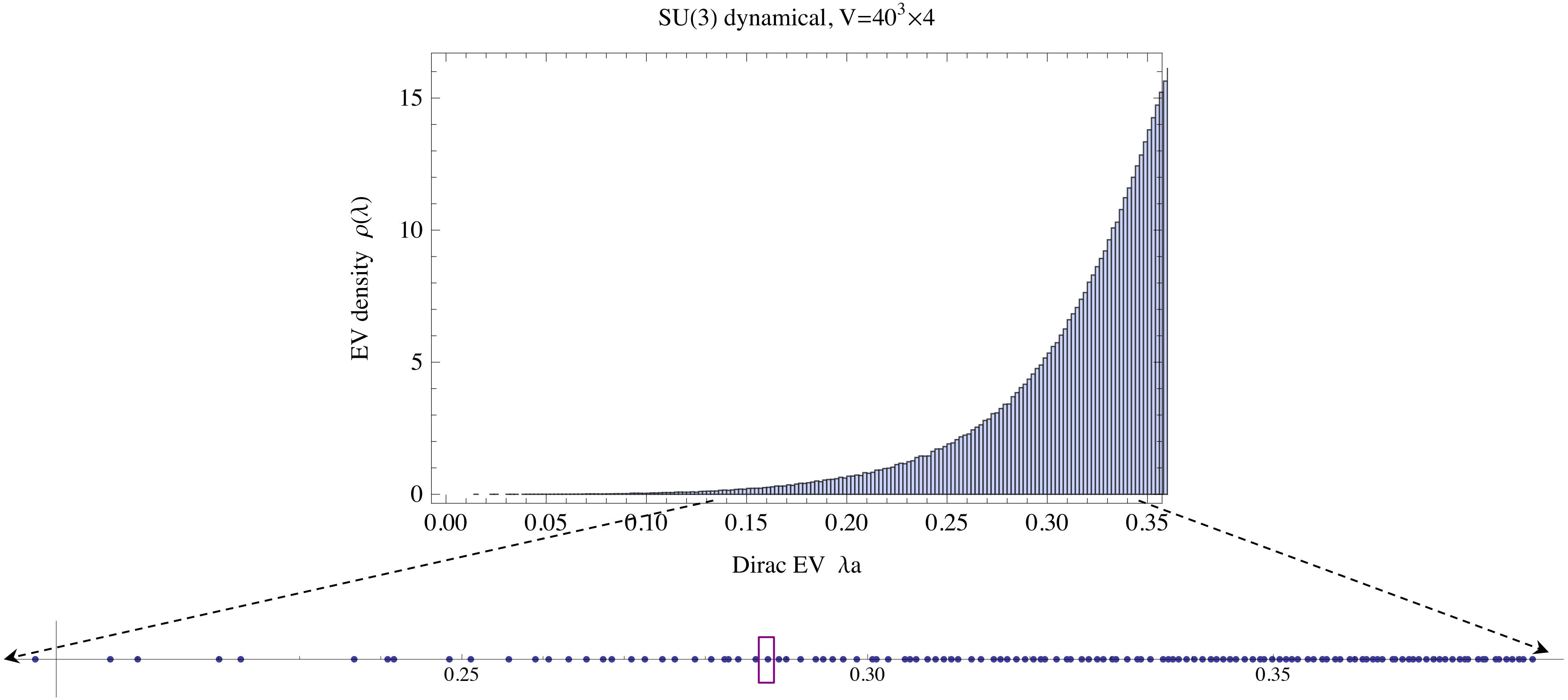
L_s	L_t	T	$N_{\rm conf}$	$N_{\rm EV}$
24	4	394[MeV]	45122	256
28	4		24794	400
32	4		19523	512
36	4		13994	550
40	4		8815	615
44	4		6507	820
48	4		6989	1000
56	4		3091	1590

$T = 2.6\ T_C$

#samples increased (rows L_s = 40, 44, 48, $N_{\rm conf}$)

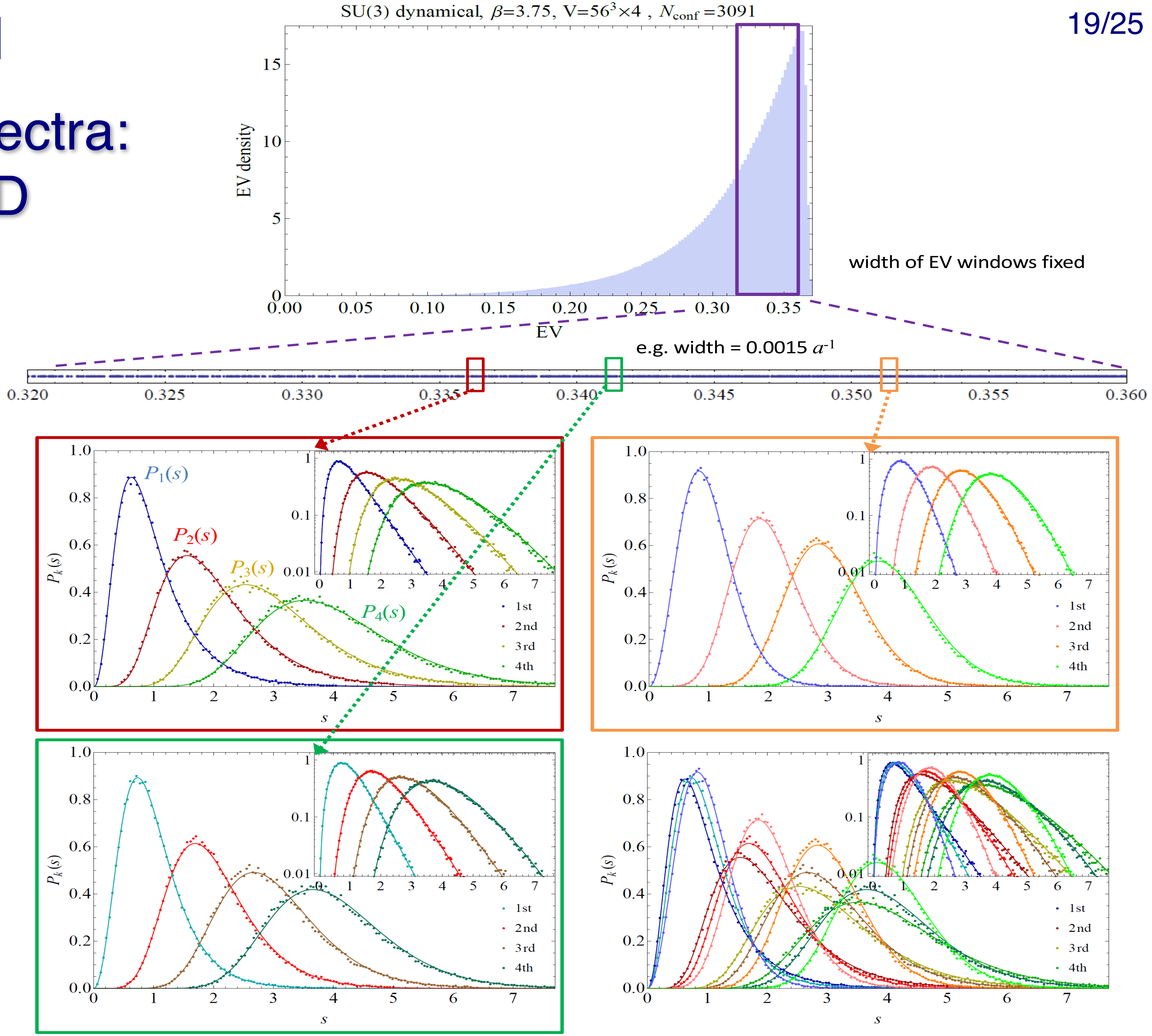
new (row L_s = 56)

Dirac spectra



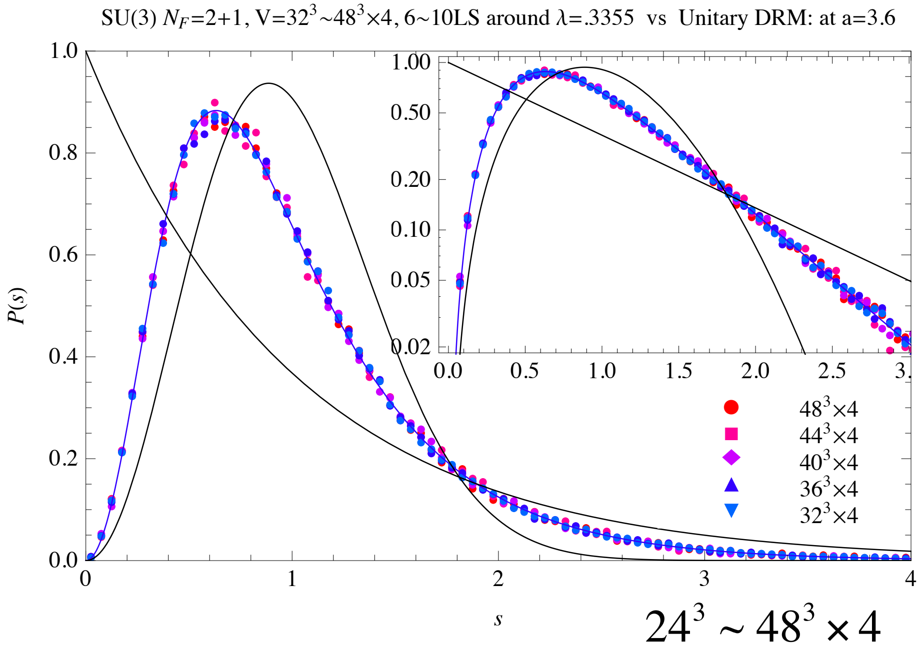
local EV window → MLSD

Dirac spectra: Multi LSD

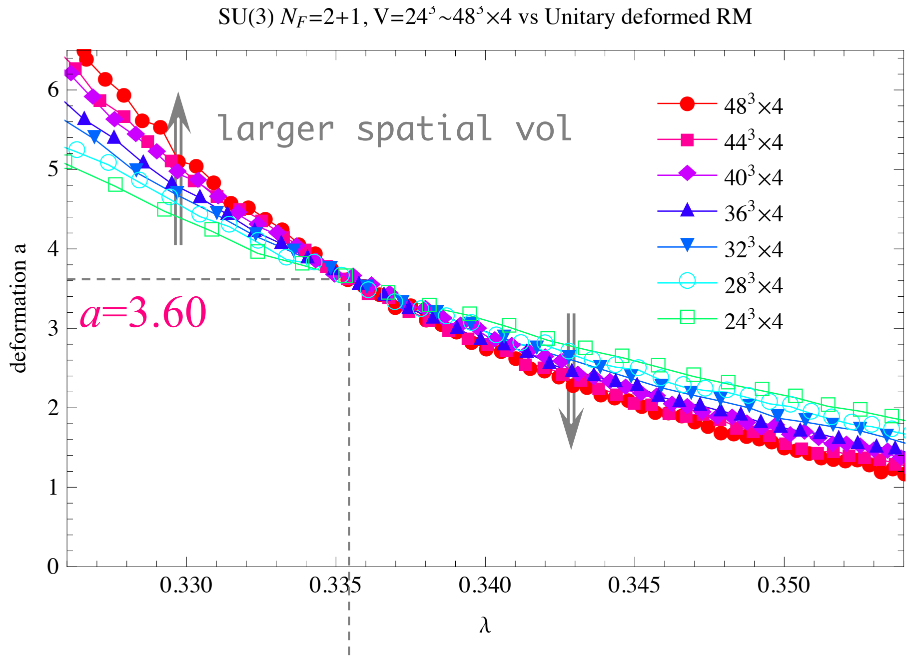


Single LSD

@ME



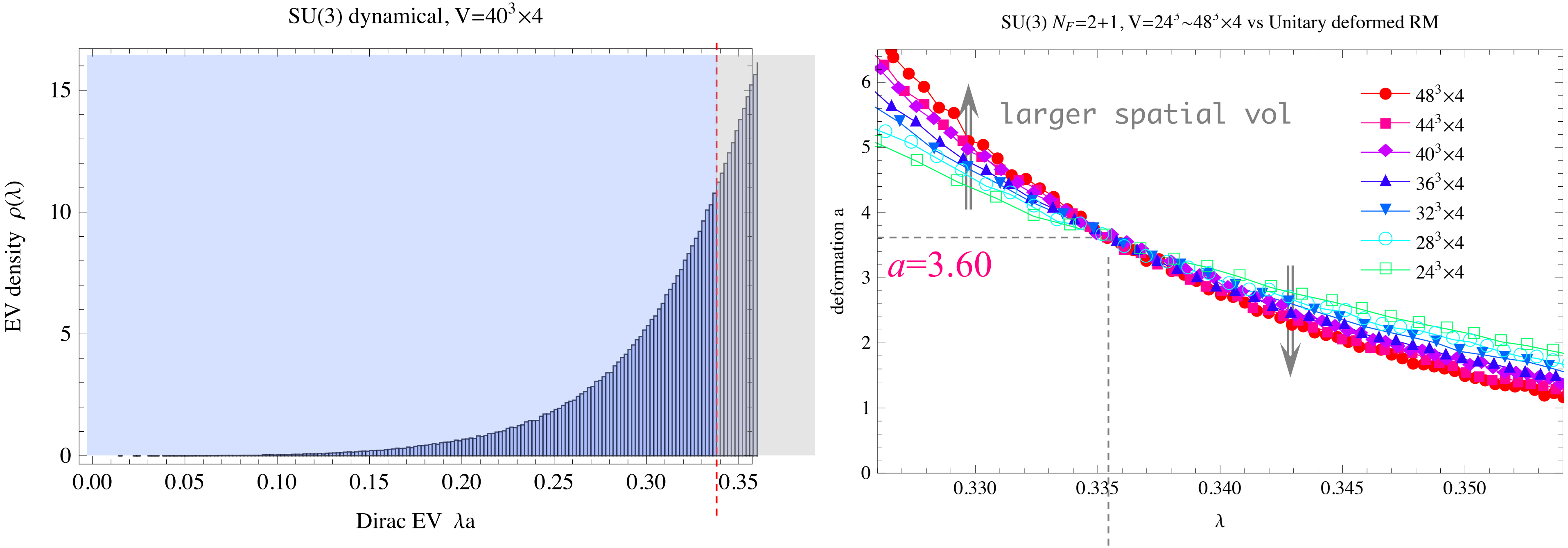
χ^2/d.o.f = 0.92 ~ 1.03



scale inv ME

finite fraction of small EVs Anderson-localizes
even in presence of very light quarks

Single LSD

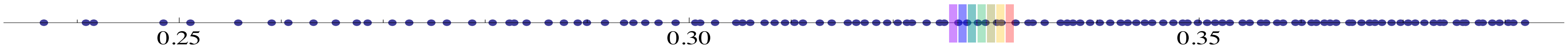


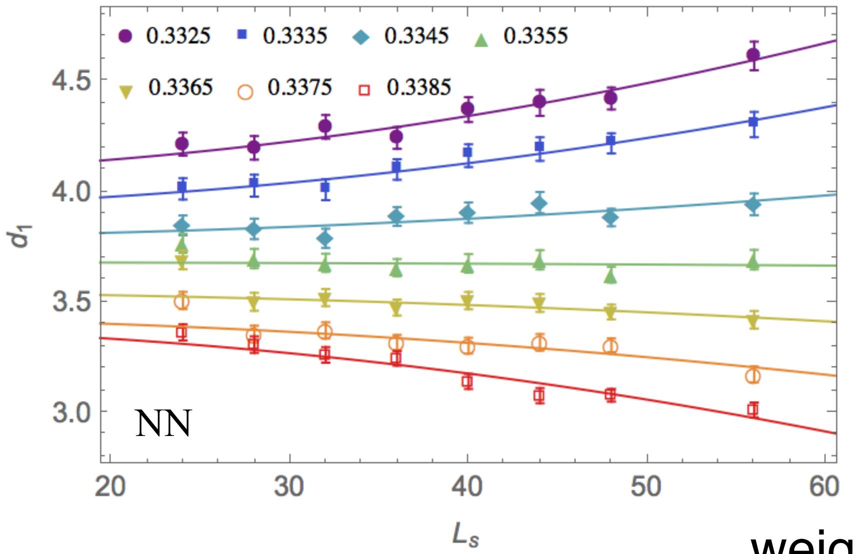
TDL : localized←ME→extended   scale inv ME

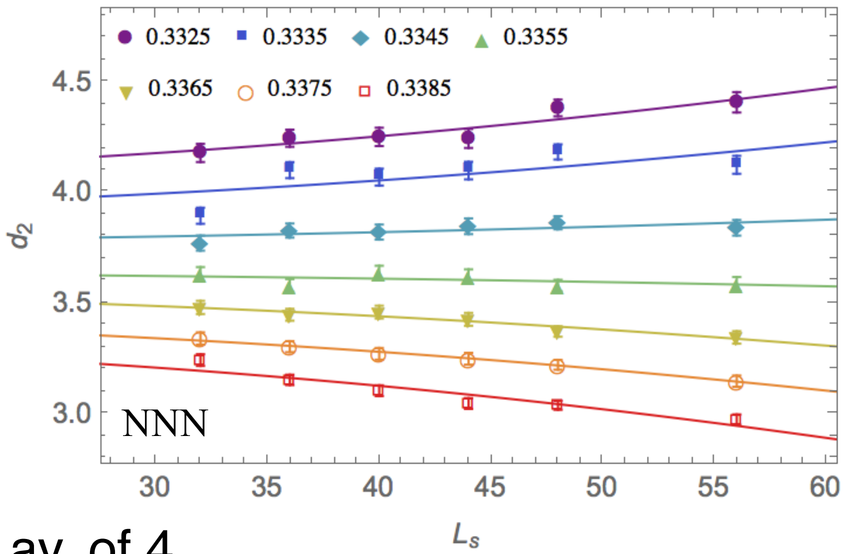
finite fraction of small EVs Anderson-localizes
even <u>in presence of very light quarks</u>

Multi LSD

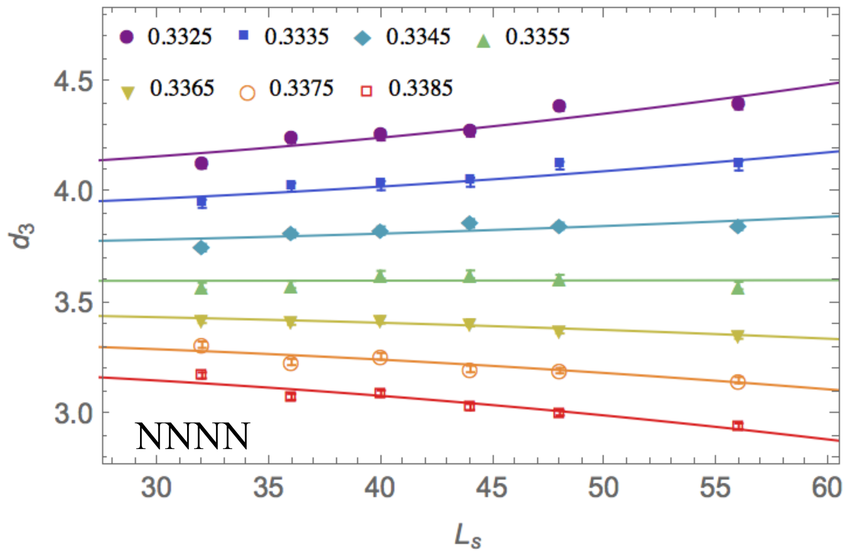
spatial size vs deform parameter

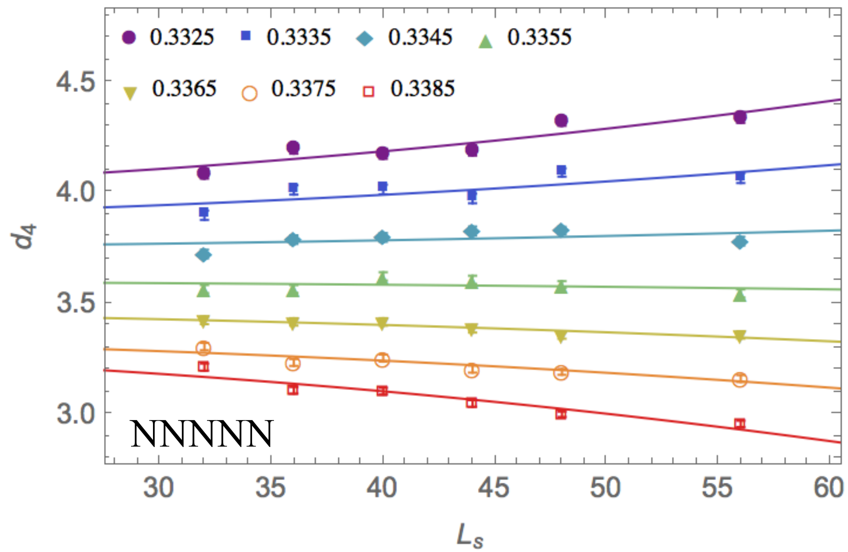




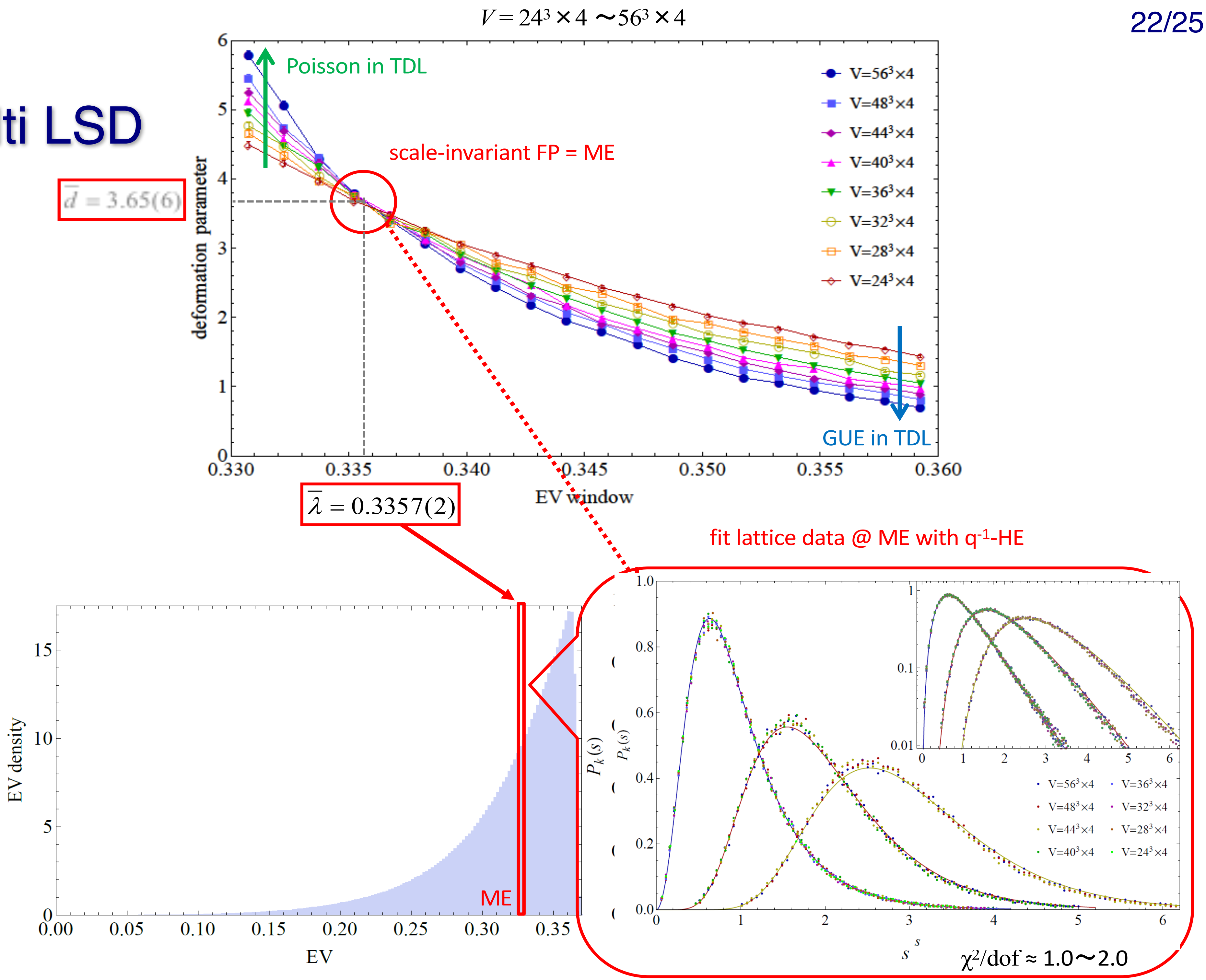


weighted av. of 4



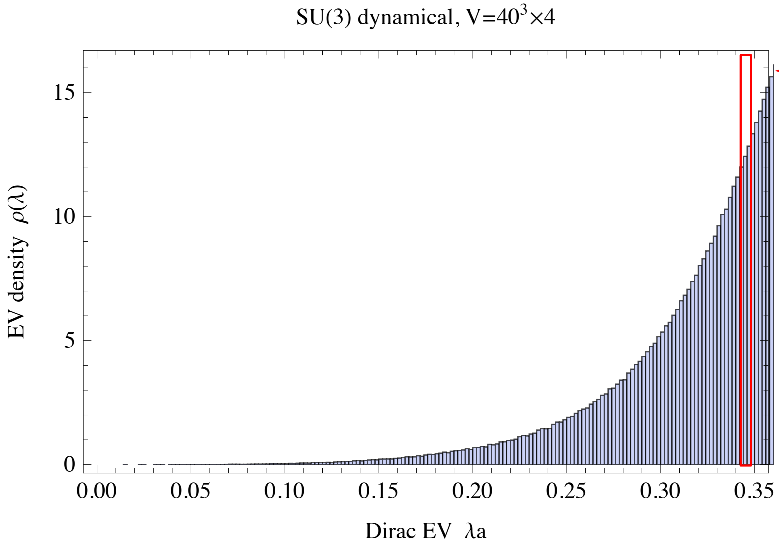


Multi LSD

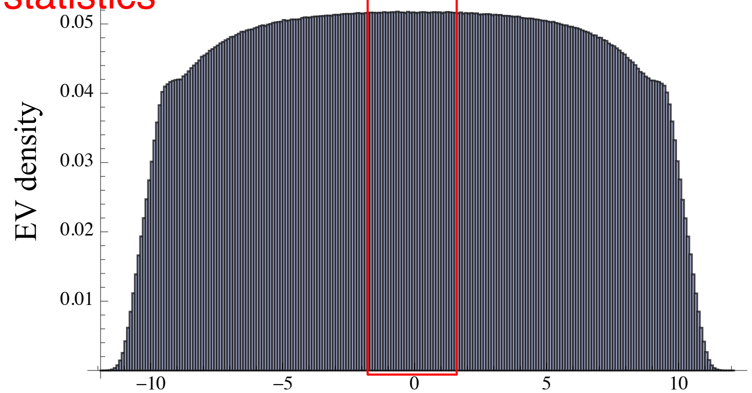

$V = 24^3 \times 4 \sim 56^3 \times 4$
$\overline{d} = 3.65(6)$
deformation parameter
Poisson in TDL
scale-invariant FP = ME
GUE in TDL
EV window
V=56³×4
V=48³×4
V=44³×4
V=40³×4
V=36³×4
V=32³×4
V=28³×4
V=24³×4
$\overline{\lambda} = 0.3357(2)$
fit lattice data @ ME with q⁻¹-HE
EV density
EV
ME
$P_k(s)$
s
χ^2/dof ≈ 1.0〜2.0

V. Conclusion

QCD $\not{D}$ on $L^3 \times 1/T\,(<1/T_c)$



Anderson H on L^3

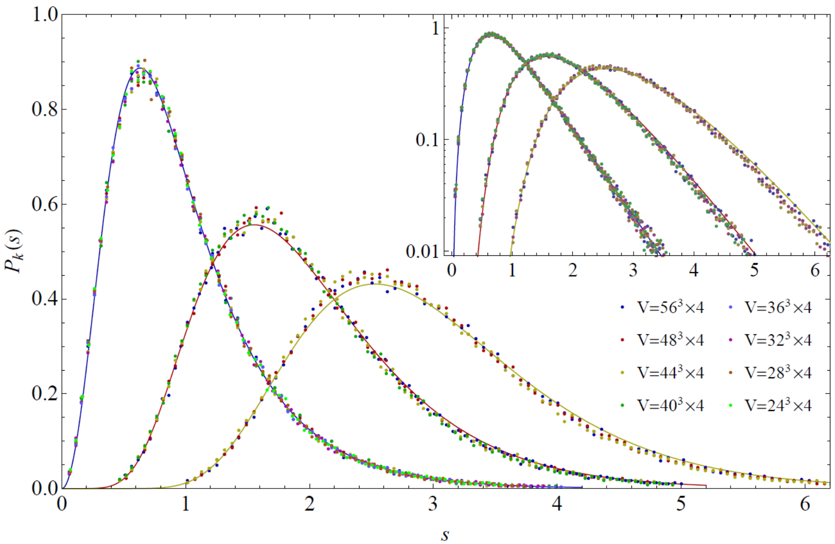


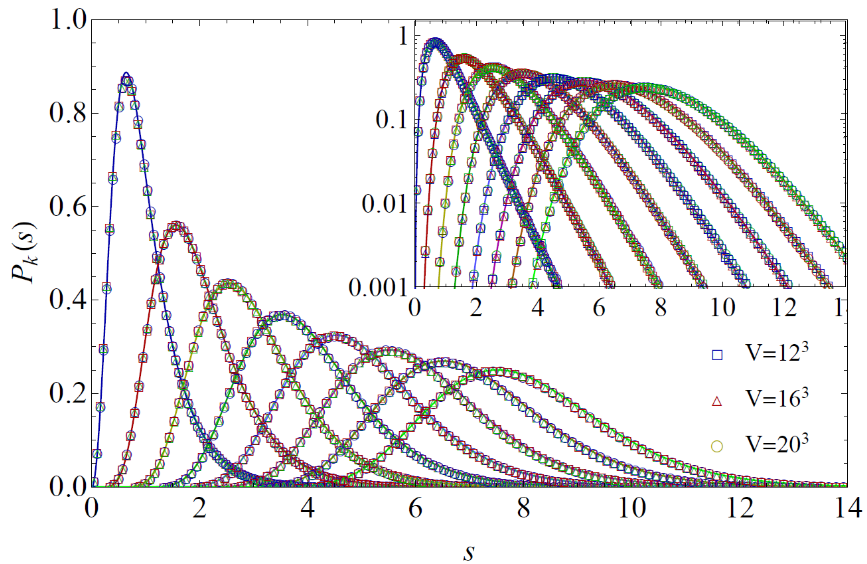
ME : "identical" critical statistics

q-deformed RM

a=3.65(6)

a=3.57(2)



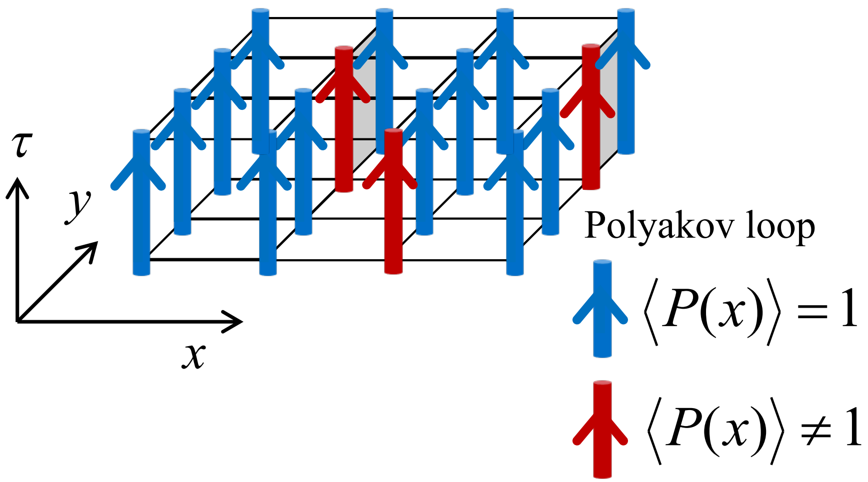


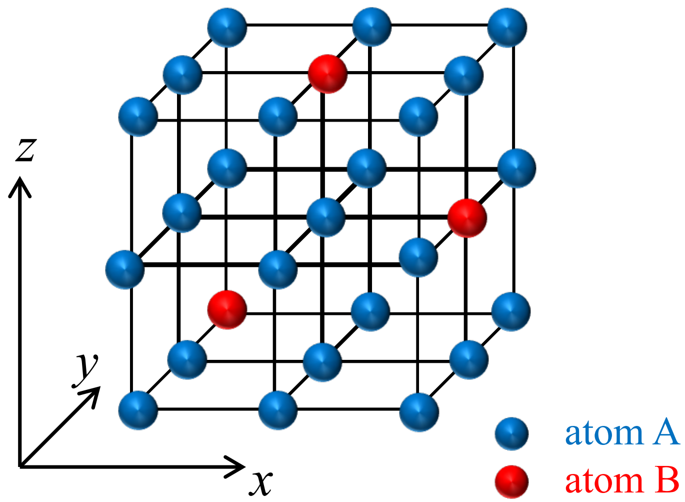
Polyakov loop as Impurity?

CS at ME: (3+1) D QCD at high T. ⟷ 3D unitary Anderson model

[Bruckmann-Kovacs-Schierenberg, 2011] $T > T_C$ (deconfinement phase)

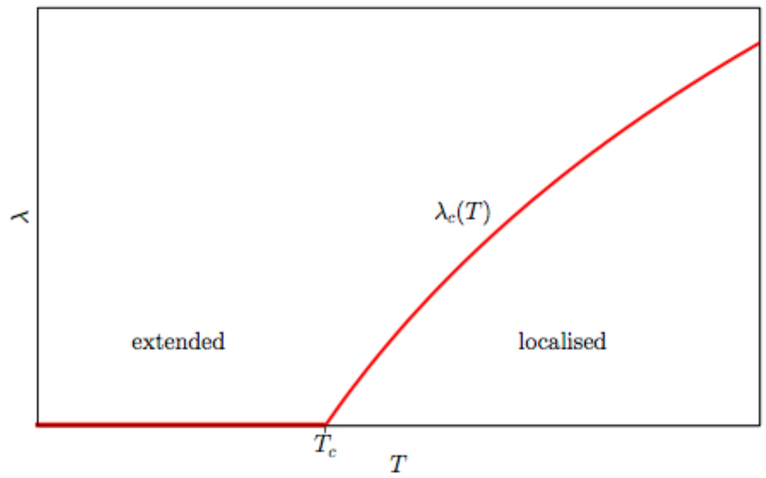
islands of Polyakov loops as dislocations ⟷ impurities of Anderson model

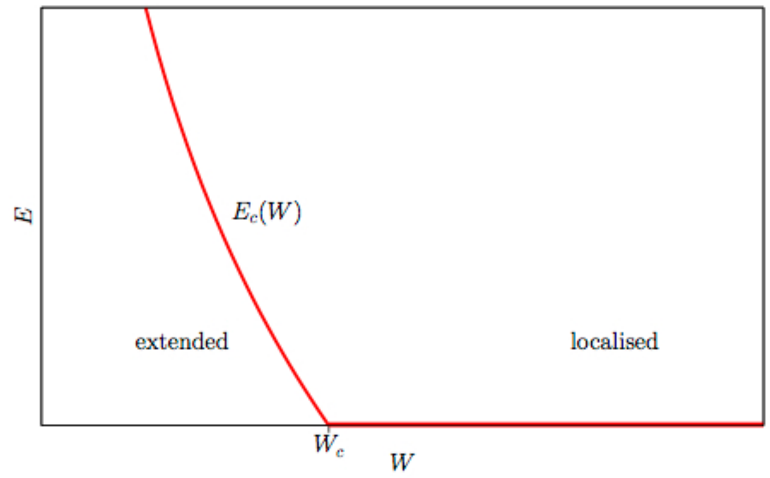




QCD $\not{D}$ on $L^3 \times 1/T\,(<1/T_c)$	Anderson H on L^3

media:	vacuum	electrons in crystal
scale:	$O(10^8 eV)$	$O(1 eV)$
disorder:	Polyakov loop	impurity





deformation par. : $a=3.65(6)$ ⟷ $a=3.57(2)$

critical exponent: $\nu=1.43(6)$ $\nu=1.43(4)$

conclusion from multi-LSD improvement : DIFFERENT critical statistics