Spontaneous symmetry breaking

What is important?

Independence of details of theory

Low-energy theorem

Ex.) Goldberger-Treiman relation

$$g_{\pi NN} = 2m_N g_A / f_\pi$$

Relation between different vertices.

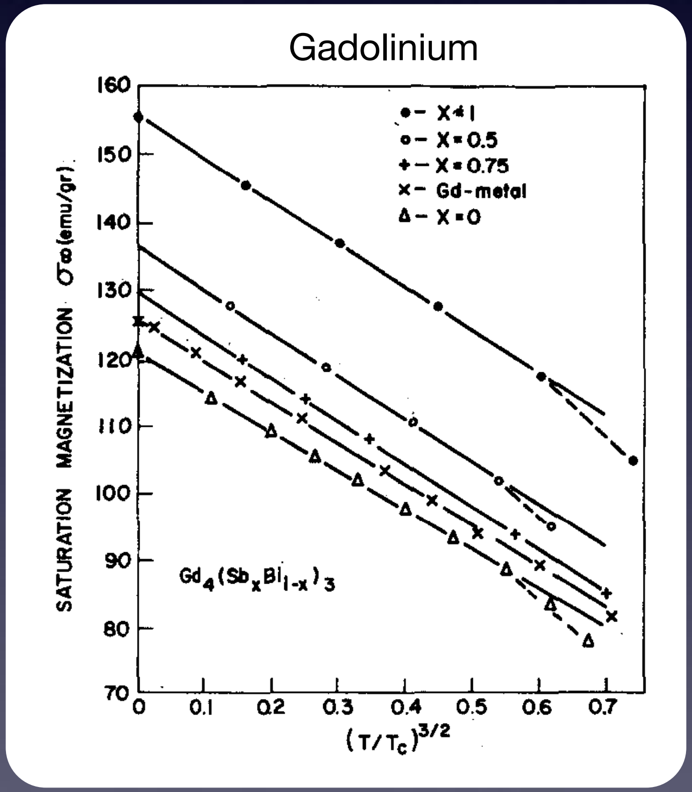
Spontaneous symmetry breaking

Why important?

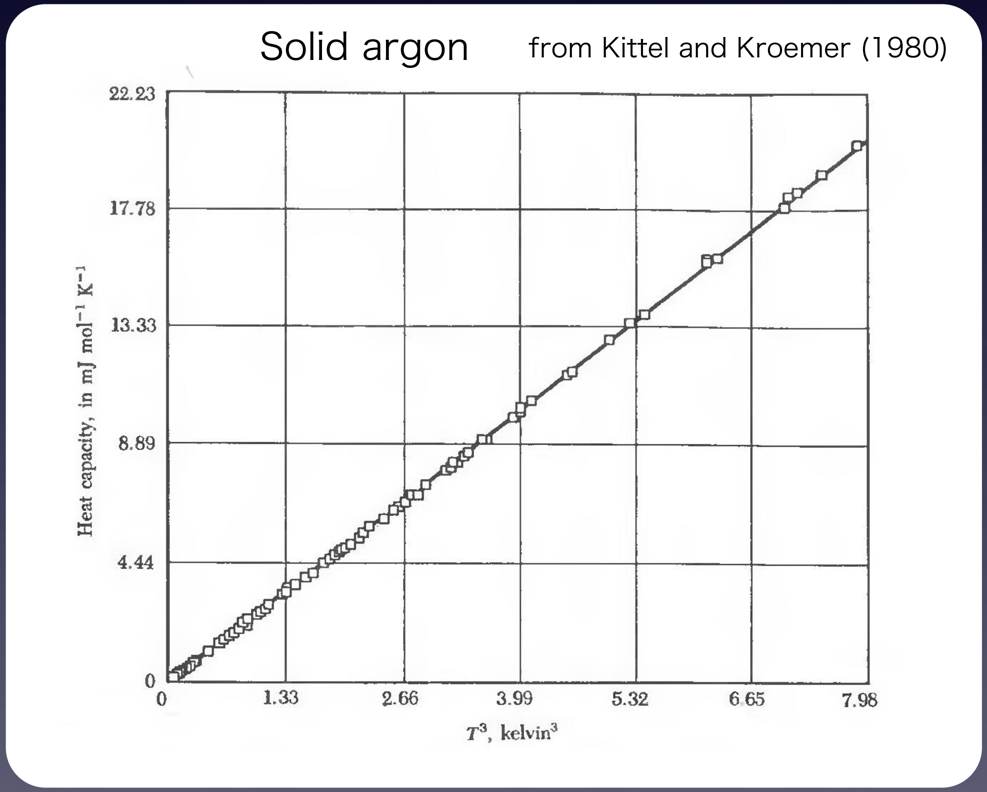
Without detail of systems, one can predict many things:

dispersion relations, low-energy theorem,...

Bloch $T^{3/2}$ law, Debye T^3 law, ...



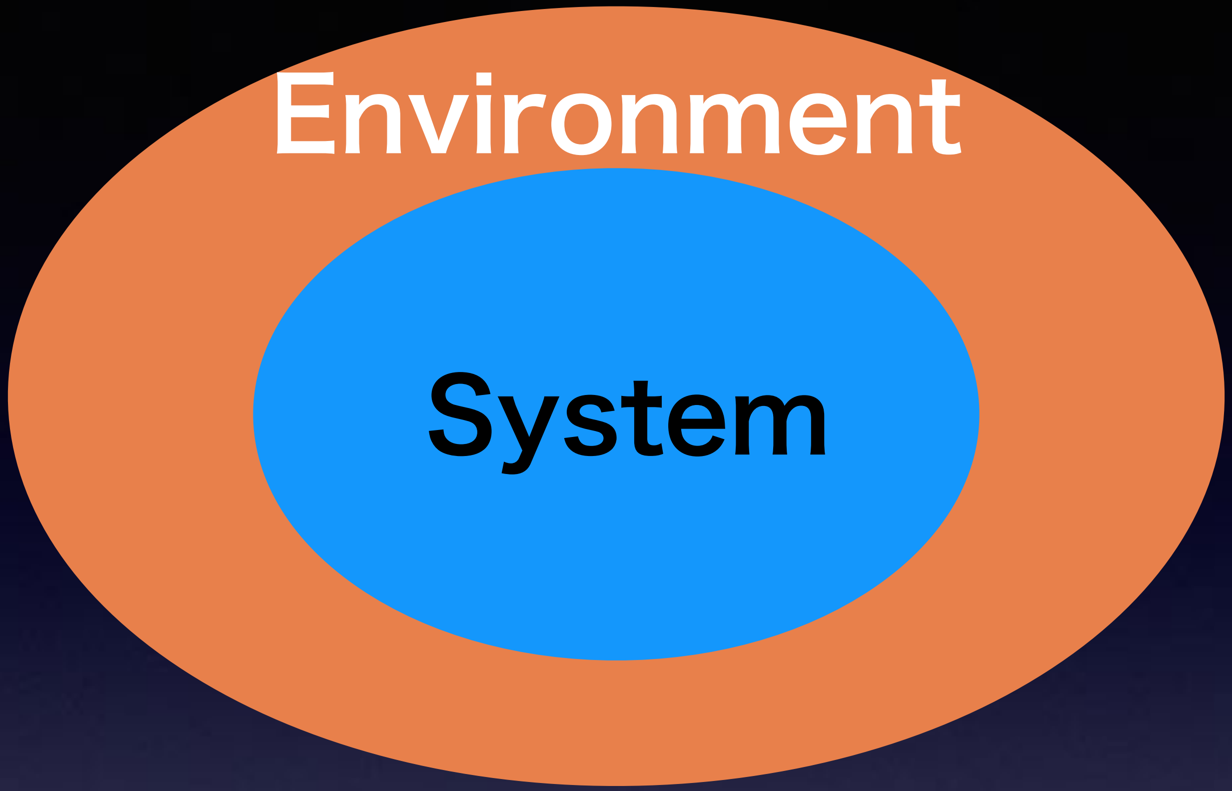
Holtzberg, McGuire, M'ethfessel, Suits, J. Appl. Phys. 35,1033 (1964)



Magnon: $\omega \sim k^2$

Phonon: $\omega \sim k$

Open systems



Example) Active matter

Questions

Hamiltonian systems

Continuum symmetry $\Leftrightarrow$ $\partial_\mu J^\mu = 0$

Open systems

$\partial_\mu J^\mu \neq 0$ because of friction

What is the symmetry?

Is there any symmetry breaking?

Does a NG mode appear?

Classification of Nambu-Goldstone modes in Hamiltonian system

Exception of NG theorem

NG modes with $N_{\rm BS} \neq N_{NG}$ and $\omega \neq k$ exist

NG modes in Kaon condensed CFL phase

Miransky, Shovkovy ('02) Schafer, Son, Stephanov, Toublan, and Verbaarschot ('01)

$$SU(2)_I \times U(1)_Y \to U(1)_{\rm em}$$

$$N_{\rm BS} = 3, \quad N_{\rm NG} = 2$$

Dispersion: $\omega \propto k$ and $\omega \propto k^2$

Magnon

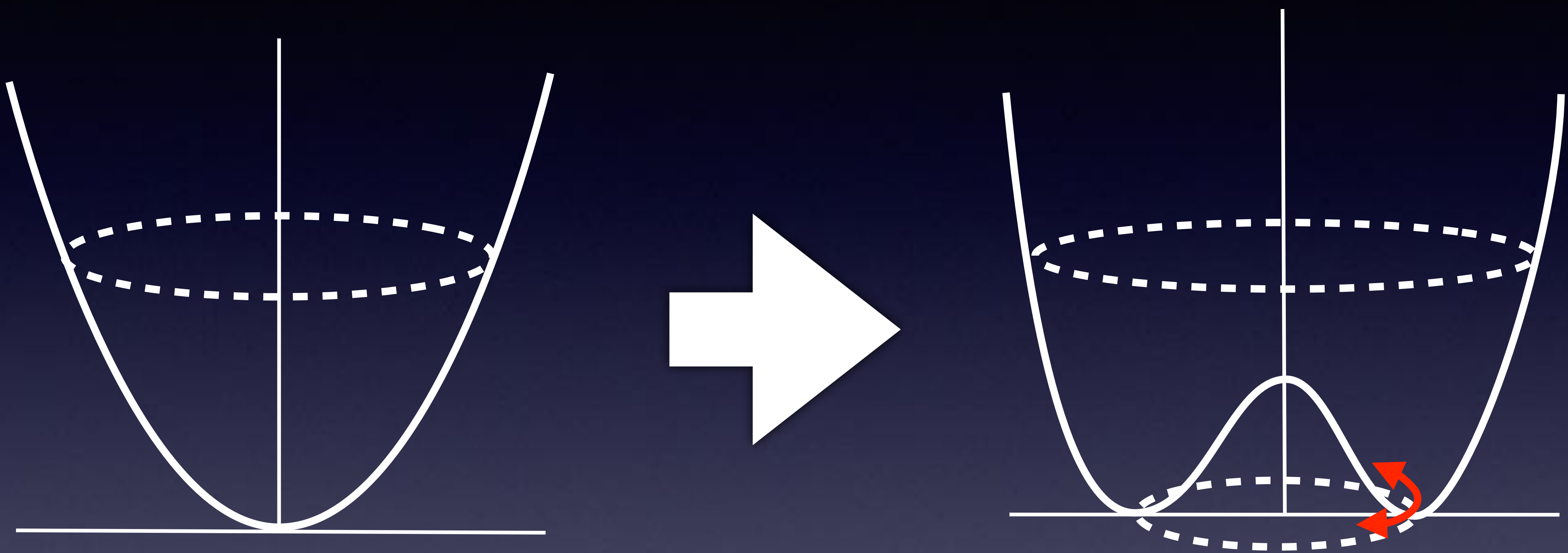
spin rotation $SO(3) \to SO(2)$

$$N_{\rm BS} = \dim(G/H) = 2 \quad N_{\rm NG} = 1$$

Dispersion: $\omega \propto k^2$

Internal symmetry breaking

Symmetry group $G \Rightarrow H$



# of flat direction

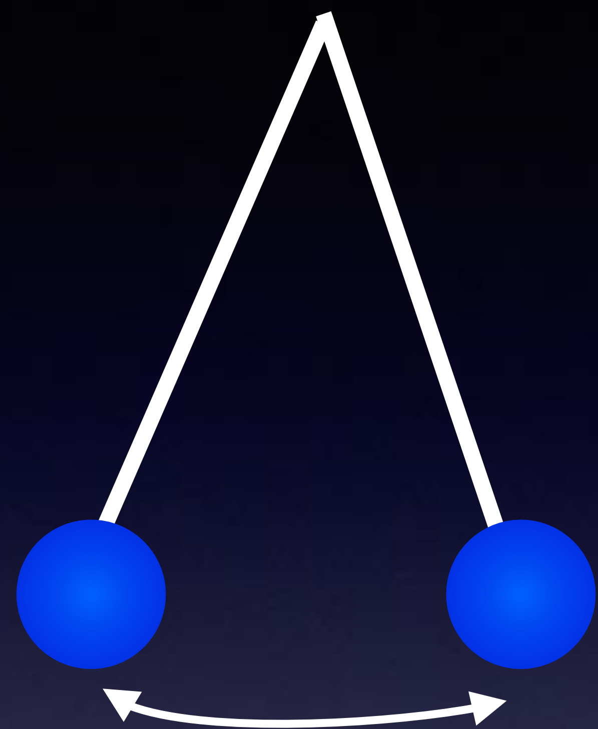
$$N_{\rm BS} = \dim(G/H)$$

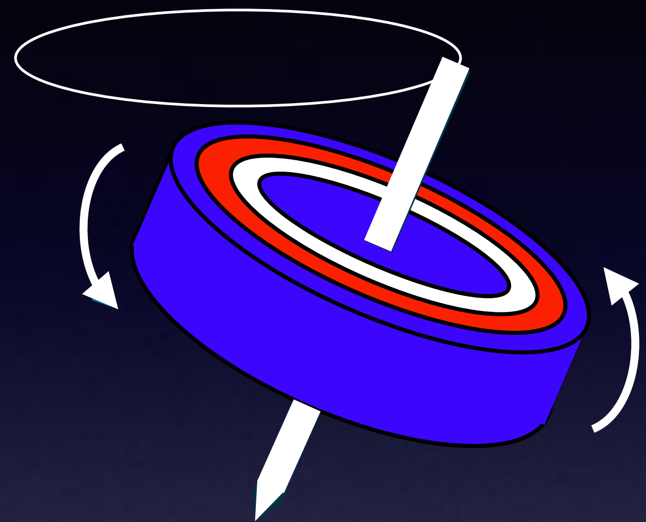
This does work in nonrelativistic system at zero and finite temperature

Classification of NG modes

Watanabe, Murayama ('12), YH ('12)

cf. Takahashi, Nitta ('14), Beekman ('14)





Type-A

Harmonic oscillation

$$N_A = N_{\rm BS} - {\rm rank}\langle [iQ_a, Q_b]\rangle$$

Ex.) superfluid phonon

Type-B

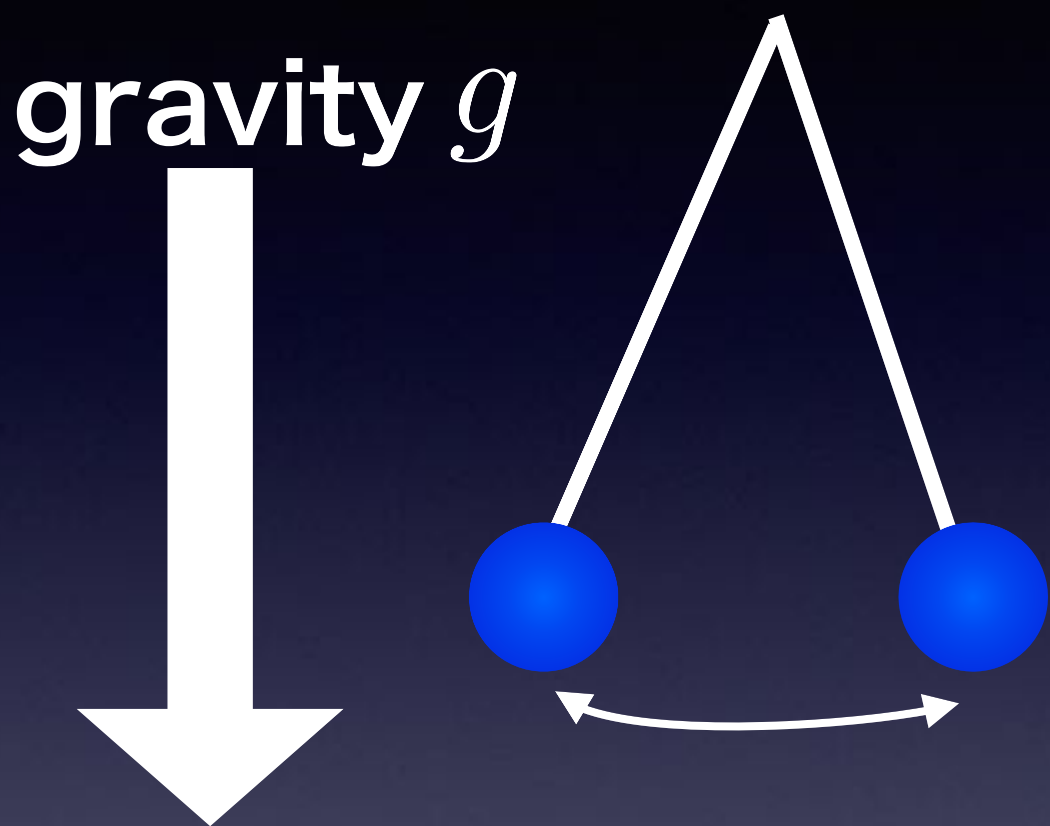
Precession

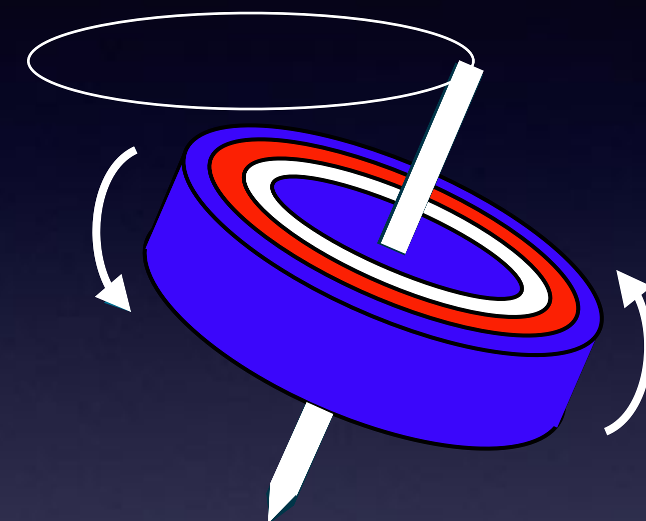
$$N_B = \frac{1}{2}{\rm rank}\langle [iQ_a, Q_b]\rangle$$

Ex.) magnon

$$\Rightarrow \quad N_{\rm NG} = N_{\rm BS} - \frac{1}{2}\langle i[Q_a, Q_b]\rangle$$

Dispersion relation





Type-A

$$\omega \sim \sqrt{g} \sim \sqrt{k^2}$$

Type-B

$$\omega \sim g \sim k^2$$

Examples of Type-B NG modes

	$N_{\rm BS}$	$N_{\text{type-A}}$	$N_{\text{type-B}}$	$\frac{1}{2}\text{rank}\langle[Q_a, Q_b]\rangle$	$N_{\text{type-A}} + 2N_{\text{type-B}}$
Spin wave in ferromanget SO(3)→SO(2)	2	0	1	1	2
NG modes in Kaon condensed CFL $SU(2)\times SU(1)_Y \to U(1)_{em}$	3	1	1	1	3
Spinor BEC SO(3)xU(1)→U(1)	3	1	1	1	3
nonrelativistic massive CP^1 model $U(1)\times\mathbf{R}^3\to\mathbf{R}^2$	2	0	1	1	2

$$N_{\text{type-A}} + 2N_{\text{type-B}} = N_{\rm BS} \qquad N_{\rm BS} - N_{\rm NG} = \frac{1}{2}\text{rank}\langle[Q_a, Q_b]\rangle$$

At finite temperature

Hayata, YH ('14)

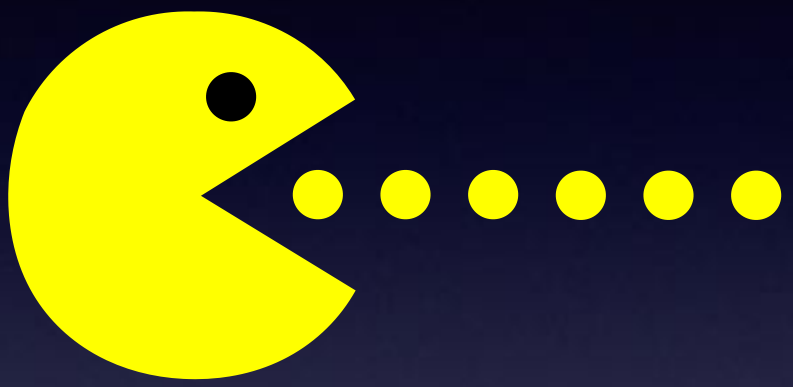
The interaction with thermal particles modifies the dispersion relation

Type-A: $\omega = ak - ibk^2$

Type-B: $\omega = a'k^2 - ib'k^4$

Open system

Model of active particles
Ex)Active Brownian model



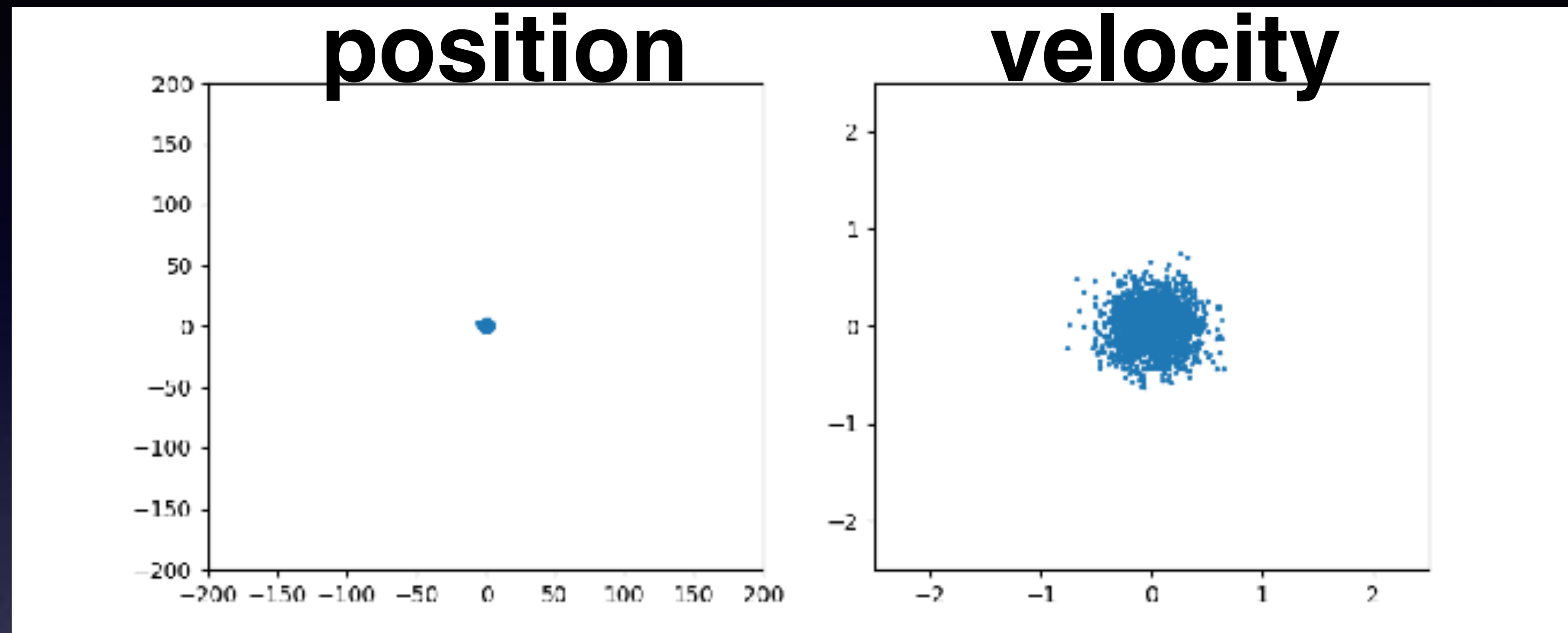
$$\partial_t \epsilon = q - c\epsilon - \kappa \boldsymbol{v}^2 \epsilon$$

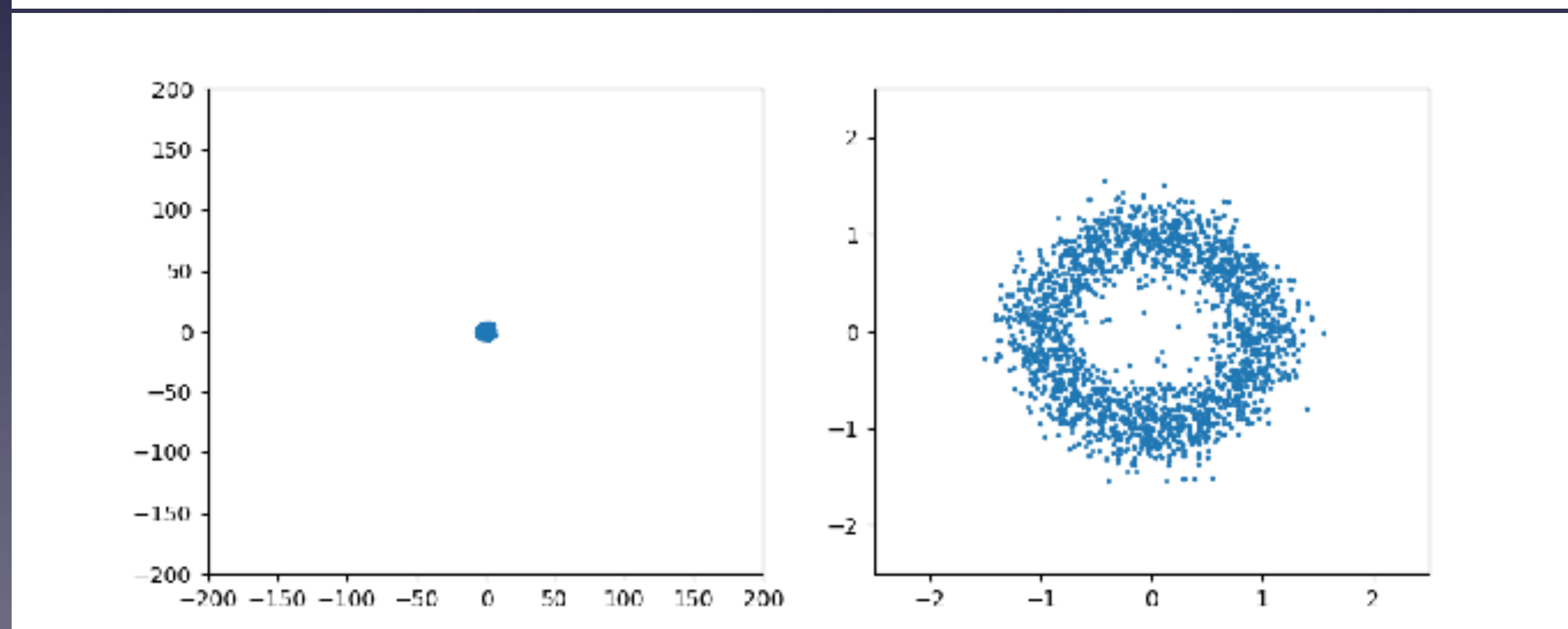
feed **metabolic** **kinematic energy loss**

$$m\partial_t \boldsymbol{v} = -m\gamma \boldsymbol{v} + \epsilon\kappa \boldsymbol{v} + \boldsymbol{\xi}$$

friction **propelling force** **noise**

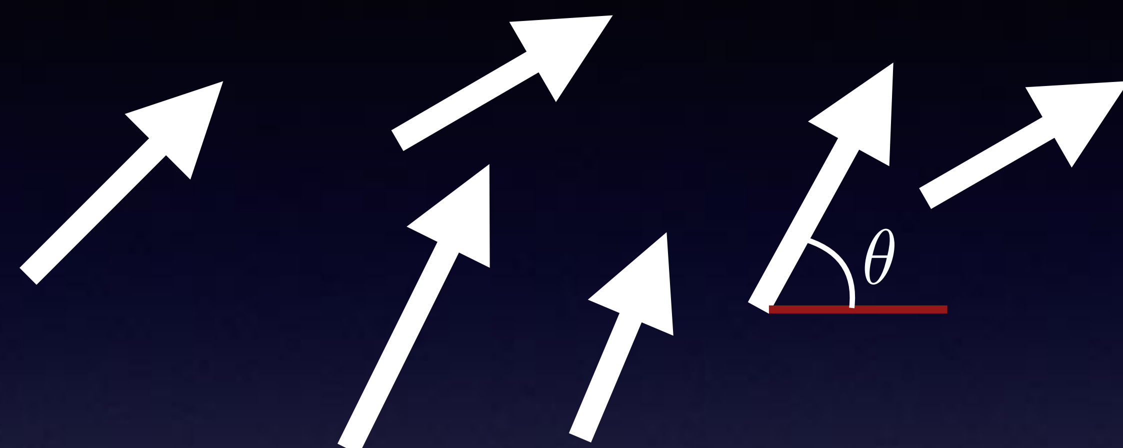
Ex1)Active Brownian model





Ex2) Vicsek model

T. Vicsek, et al., PRL (1995).



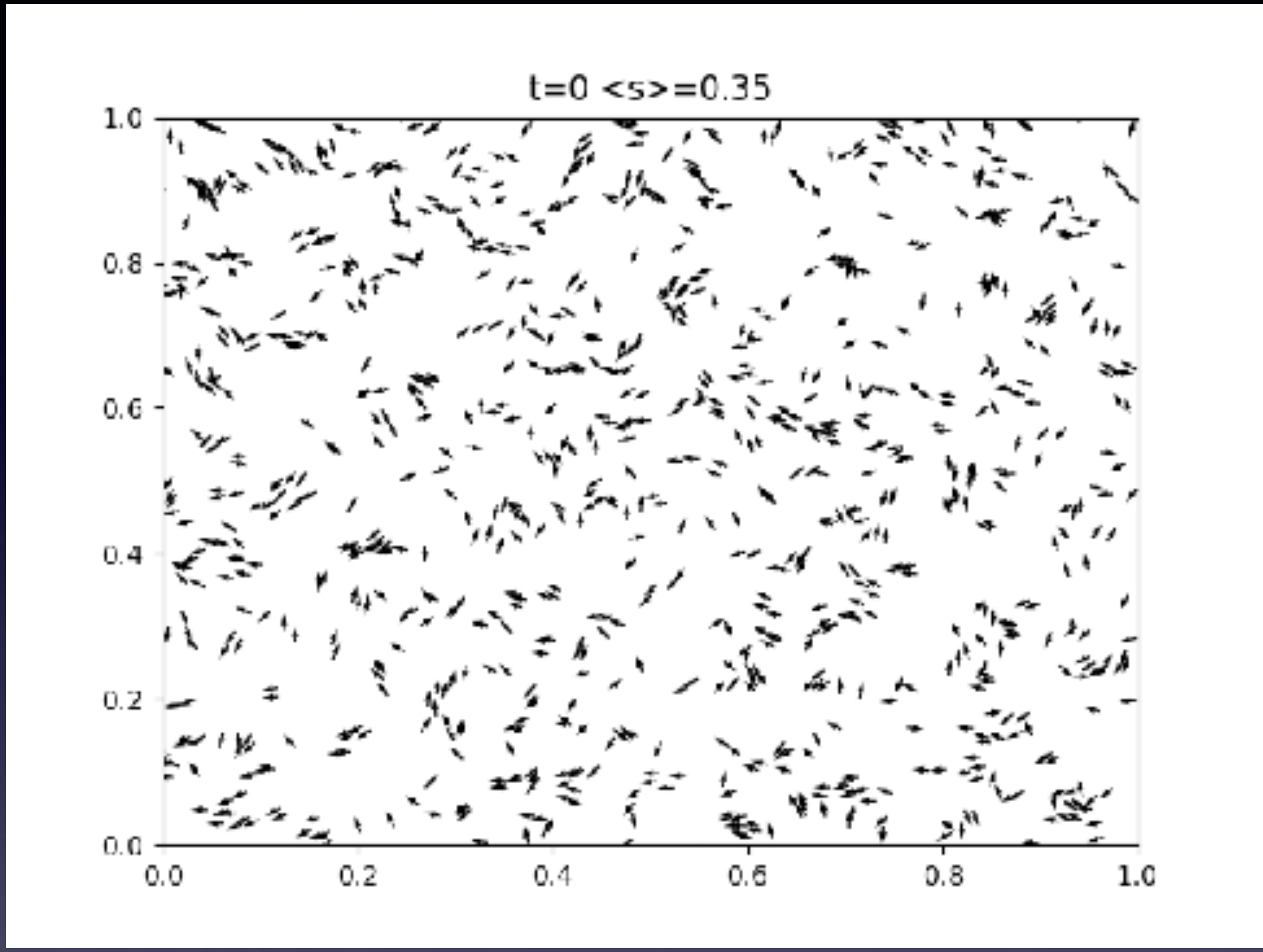
$$\boldsymbol{x}_i(t+\Delta t) = \boldsymbol{x}_i(t) + \boldsymbol{v}_i \Delta t$$

velocity

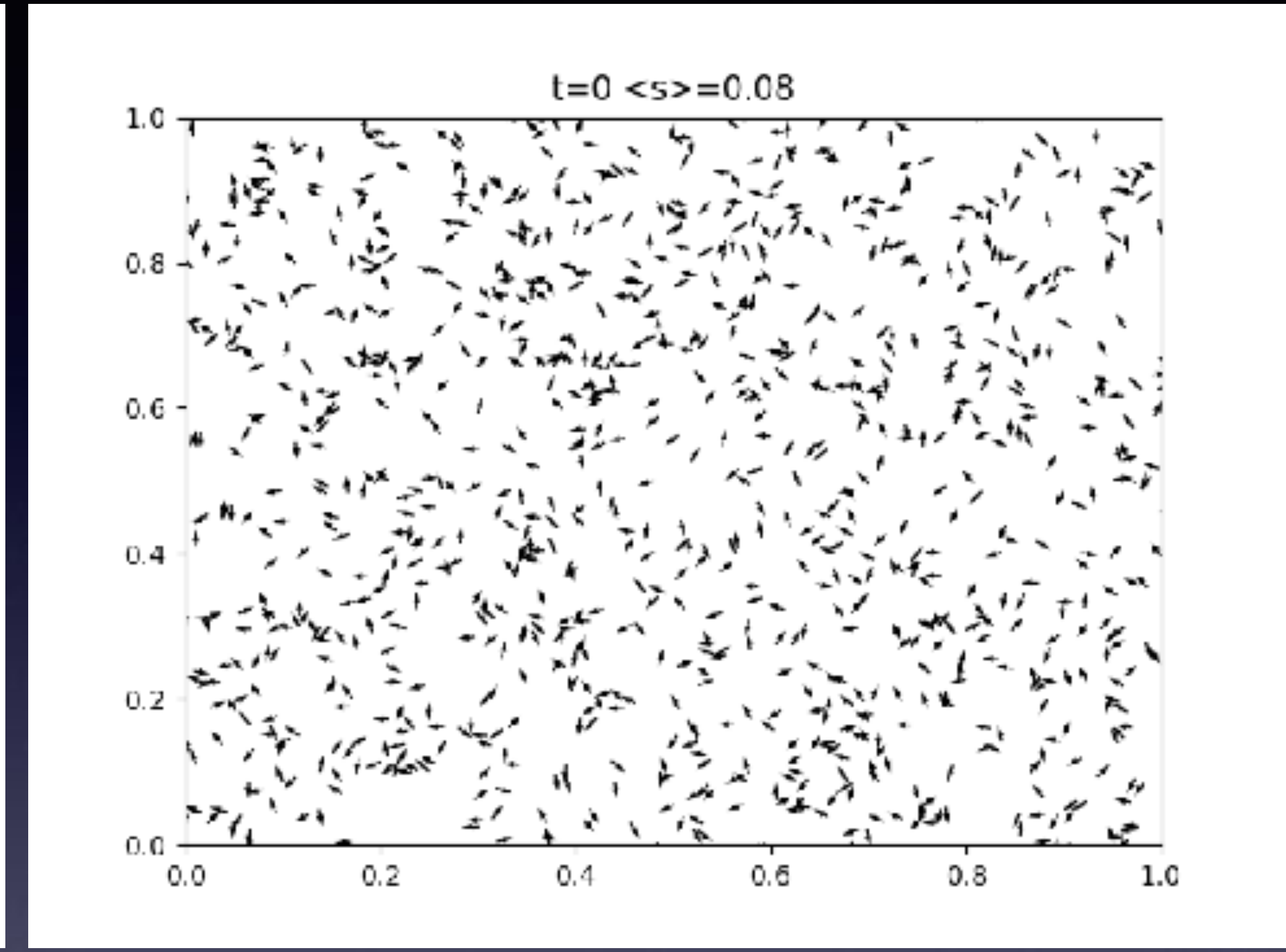
$$\boldsymbol{v}_i = v_0(\cos\theta_i, \sin\theta_i)$$

$$\theta_i(t+\Delta t) = \langle \theta_i(t) \rangle_r + \xi_i$$

angle of velocity **average angle** **noise**



t=0 <s>=0.35
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

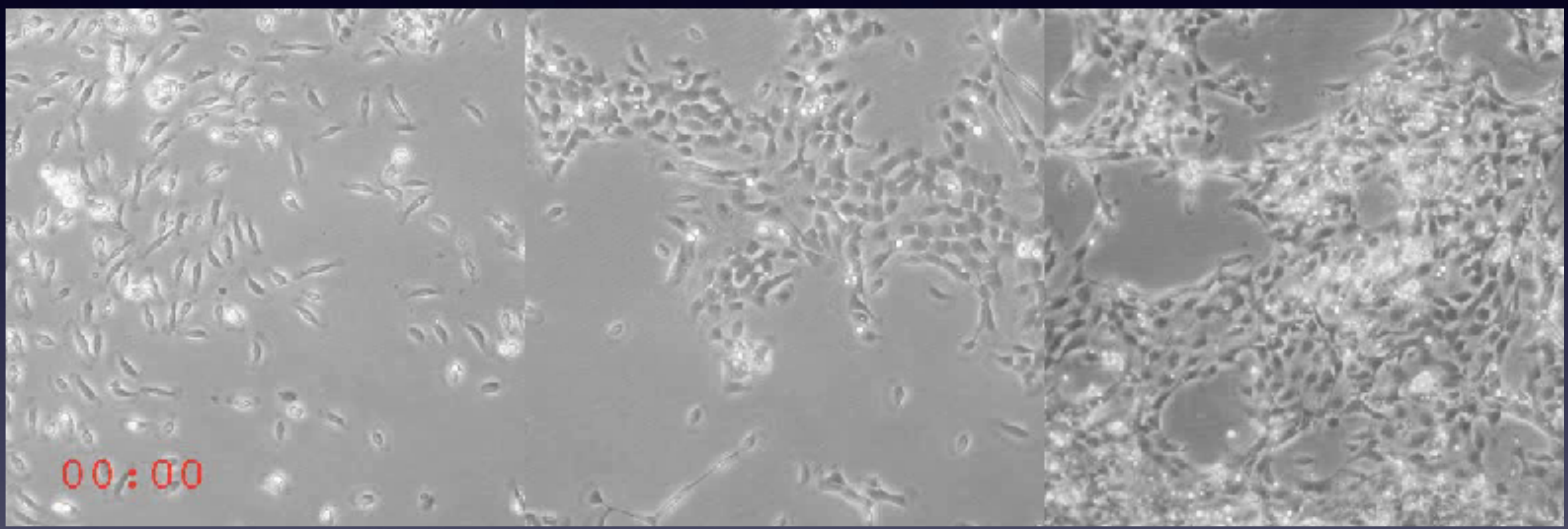


t=0 <s>=0.08
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

An example of SSB in open systems

Some cells on fish skin

low density **high density**



B. Szabo, et al., Phys. Rev. E 74, 061908 (2006)

Model of active matter: Vicseck model, Active hydrodynamics, ….

T. Vicsek, et al., PRL (1995). J. Toner, and Y. Tu, PRL (1995).

Field theoretical model

Ex.) NG mode in Active hydrodynamics

J. Toner, and Y. Tu, PRE (1998)

$$\partial\rho + \nabla\cdot(\rho\boldsymbol{v}) = 0$$

$$\partial_t\boldsymbol{v} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = \alpha\boldsymbol{v} - \beta\boldsymbol{v}^2\boldsymbol{v} - \nabla P + D_L\nabla(\nabla\cdot\boldsymbol{v}) + D_l(\boldsymbol{v}\cdot\nabla)^2\boldsymbol{v} + \boldsymbol{f}$$

nonconserved term

noise

Steady state solution: $\boldsymbol{v}^2 = \alpha/\beta \equiv v_0^2$

Symmetry breaking: $O(3) \to O(2)$

Fluctuation: $\boldsymbol{v} = (v_0 + \delta v_x, \delta v_y, \delta v_z)$

⇒ $\omega = ck$ $\omega = i\Gamma k^2$ **NG modes**

propagating **diffusive**

Can we discuss symmetry breaking without ordinary conservation law?

Ex) Symmetry of Brownian motion

Langevin equation

$$\frac{d}{dt}\boldsymbol{x}(t) = \boldsymbol{u}(t)$$

$$\frac{d}{dt}\boldsymbol{u}(t) = -\gamma \boldsymbol{u}(t) + \boldsymbol{\xi}(t)$$

$$\langle \xi_i(t)\xi_j(t')\rangle = 2\delta_{ij}\gamma T \delta(t-t')$$

Angular momentum $\boldsymbol{L} = \boldsymbol{x} \times \boldsymbol{u}$

$$\frac{d}{dt}\langle \boldsymbol{L}(t)\rangle = -\gamma\langle \boldsymbol{x} \times \boldsymbol{u}(t)\rangle \neq 0$$

not conserved

Langevin equation

$$\frac{d}{dt}\boldsymbol{u}(t) = -\gamma \boldsymbol{u}(t) + \boldsymbol{\xi}(t)$$

⇕

Fokker-Planck equation

$$\partial_t P(t,u) = \frac{\partial}{\partial u_i}\Big(\gamma T \frac{\partial}{\partial u_i} + \gamma u_i\Big) P(t,u)$$

⇕

Path integral

Martin-Siggia-Rose formalism

$$Z = \int \mathcal{D}\chi \mathcal{D}u e^{iS[\chi,u]}$$

Dynamic action: $iS = \int dt \Big[i\chi_i \Big(\frac{d}{dt} u_i + \gamma u_i \Big) - T\gamma \chi_i^2 \Big]$

Symmetry of Dynamic action

$$iS = \int dt \Big[i\chi_i \Big(\frac{d}{dt} u_i + \gamma u_i \Big) - T\gamma\chi_i^2 \Big]$$

O(3) symmetry

$\chi_i \to R_{ij}\chi_j \quad u_i \to R_{ij}u_j$ with $R_{ik}R_{kj} = \delta_{jk}$

Noether charge

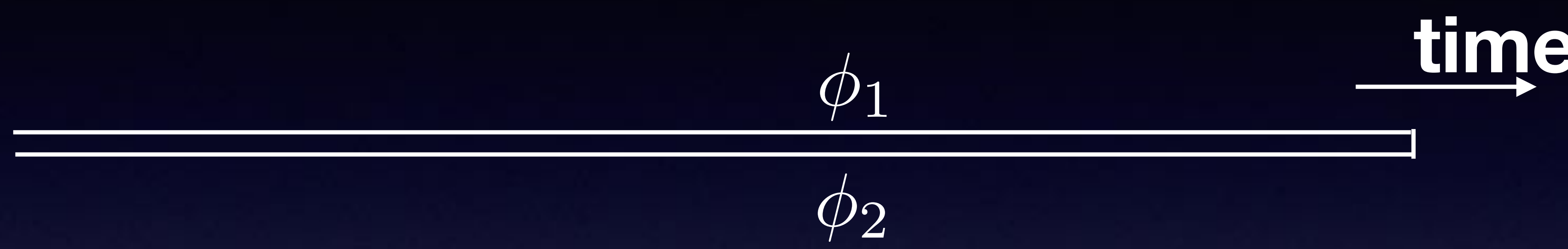
$$\boldsymbol{L}_{\mathrm{MSR}} = \boldsymbol{\chi} \times \boldsymbol{u} \qquad \boldsymbol{L} = \boldsymbol{x} \times \boldsymbol{u}$$

$$L_{\mathrm{MSR}} \neq L$$

Open quantum system

cf. for review, Sieberer, Buchhold, Diehl, 1512.00637

Schwinger-Keldysh Path integral



$$Z = \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 \exp\left[iS[\phi_1] - iS[\phi_2] + iS_{12}[\phi_1, \phi_2]\right]$$

complex

Q_1, Q_2 :Symmetry generators:

$S[\phi_1], S[\phi_2]$ are invariant.

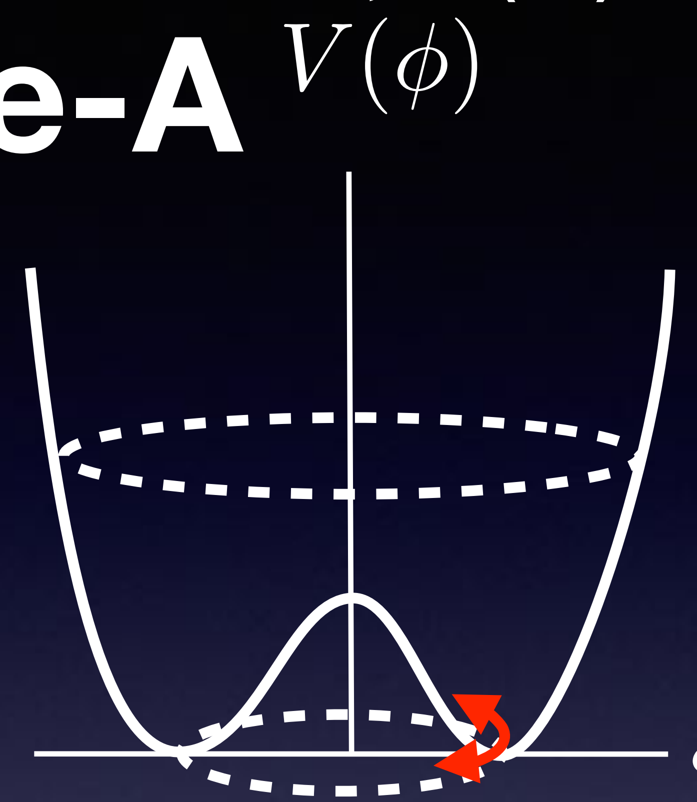
Suppose $S_{12}[\phi_1, \phi_2]$ is invariant under $Q_A = \frac{Q_1 - Q_2}{2}$

We also define $Q_R = \frac{Q_1 + Q_2}{2}$

Spontaneous symmetry breaking

Minami, YH ('15)

Ex1) SU(2)xU(1) model Type-A



Langevin equation

$$(\partial_0^2 + \gamma\partial_0 - \nabla^2)\phi_a = -\frac{\partial V}{\partial \phi_a} + \xi_a$$

Linear analysis

$$(\partial_0^2 + \gamma\partial_0 - \nabla^2)\pi_a = 0$$ **NG type-A mode**

$$\Rightarrow -\omega^2 - i\gamma\omega + k^2 = 0$$

$$\Rightarrow \omega = \frac{-i\gamma}{2} \pm \frac{i}{2}\sqrt{\gamma^2 - 4k^2} \sim -\frac{i}{\gamma}k^2, -i\gamma + \frac{i}{\gamma}k^2$$

diffusion mode

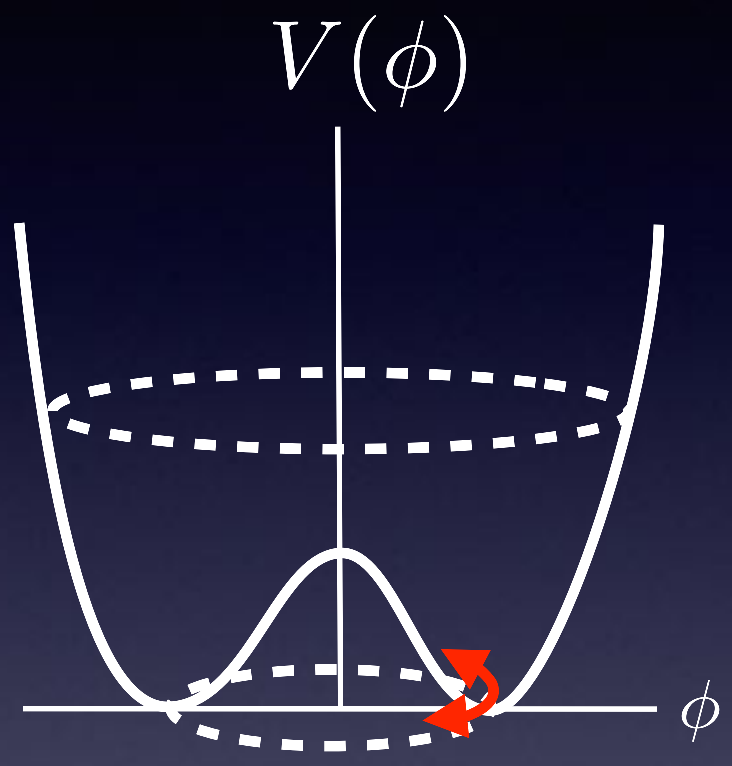
Spontaneous symmetry breaking

Minami, YH ('15)

Ex2) SU(2)xU(1)model Type-B

with chemical potential

$$\begin{pmatrix} -\partial_0^2 - \gamma\partial_0 + \nabla^2 & 2\mu\partial_0 \\ -2\mu\partial_0 & -\partial_0^2 - \gamma\partial_0 + \nabla^2 \end{pmatrix} \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} = 0,$$



$$\Rightarrow \quad \omega = \frac{k^2}{4\mu^2 + \gamma^2}(\pm 2\mu - i\gamma)$$

quadratic dispersion

Inverse propagator and dispersion

Minami, YH ('17)

$$[G_\pi^{-1}(k)]^{\beta\alpha} = iC^{\mu;\beta\alpha}k_\mu + C^{\mu\nu;\beta\alpha}k_\mu k_\nu + \cdots$$

Hamiltonian system

Open system

$$C^{\mu;\beta\alpha} = -\langle [iQ_R^\alpha, j_A^{\beta\mu}(0)] \rangle$$

$$-i\int d^D x \langle [iQ_R^\alpha, \mathcal{L}_{12}(x)] j_A^{\beta\mu}(0) \rangle_{\pi\mathrm{c}}$$

$$C^{\mu\nu;\beta\alpha} = i\int d^D x \langle j_R^{\alpha\mu}(x) j_A^{\beta\nu}(0) \rangle_{\pi\mathrm{c}}$$

$$- \lim_{k\to 0} \frac{\partial}{\partial k_\nu} i \int d^D x e^{ik_\rho x^\rho} \langle [iQ_R^\alpha, \mathcal{L}_{12}(x)] j_A^{\beta\mu}(0) \rangle_{\pi\mathrm{c}}$$

Our result is too general

Need to impose symmetry of S_{12}

Ex)'Standard' Fokker-Plank eq.

Type-A mode	Type-B mode
Diffusive $\omega = -ik^2\Gamma$	$\omega = ak^2 - ik^2\Gamma'$
$N_A = N_{\mathrm{BS}} - \mathrm{rank}\langle[iQ_R^\alpha, Q_A^\beta]\rangle$	$N_B = \dfrac{1}{2}\mathrm{rank}\langle[iQ_R^\alpha, Q_A^\beta]\rangle$

Next step: classification

Spontaneous breaking of symmetry of Dynamic action

Two-type of diffusive NG modes

Type-A mode

Diffusive $\omega = -ik^2\Gamma$

Type-B mode

$\omega = ak^2 - ik^2\Gamma'$

Next step: classification

Type	Dispersion		Conserved charge	Examples
	Re	Im		
A	k	k^2	Q_A, Q_R	Superfluid, etc.
	0	k^2	Q_A	Flock of birds
B $\langle [Q_A, Q_R] \rangle \neq 0$	k^2	k^4	Q_A, Q_R	Ferromagnet
	k^2	k^2	Q_A	Magnetotactic bacteria?

What is the condition satisfying this table?