Supergeometry and unified picture of fluxes

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based on the collaboration with

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§1. Introduction

- The keyword "Duality" is conceptually hard to incorporate with the geometry. To understand the stringy geometry, we need to accommodate them.

- The symmetry is, as usual, the guiding principle to find a proper way.

But we need a kind of generalization of the standard symmetry in standard field theory, namely,

Lie algebra and Diffeomorphism

There are several directions, super, higher, Lie algebroid, ...

For these generalizations, we also need to generalize the geometry. This gives a natural way to a stringy geometry.

There are several generalizations of the geometry.

One direction is (geometry and symmetry)

1. Generalized geometry [Hitchin]: we consider the geometry of $E = TM \oplus T^*M$ Exact Courant algebroid

   NSNS sector of closed string/SUGRA

2. B_{2n} generalized geometry [Baraglia]

   $E = TM \oplus \mathbf{1} \oplus T^*M$ Courant algebroid

   Type I and Heterotic string/SUGRA

3. Exceptional generalized geometry [Hull, Pacheco-Waldram, Baraglia]

   $E = TM \oplus \wedge^2 T^*M \oplus \wedge^5 T^*M$ Leibniz algebroid

   M-theory

4. Double field theory [Hull, Zwiebach]

   base manifold $M \times \tilde{M}$ manifest $O(D, D)$

5. Exceptional field theories [Hull, Pacheco-Waldram]

Note: all can be combined with NCG.

Introduction

All fit into the supergeometry formulation. Let me talk about the generalized geometry as a basic example to introduce terminologies

Generalized geometry is manifestation of the $O(D,D)$ symmetry of T-duality. In the generalized geometry, we consider

1. generalized tangent bundle $E = TM \oplus T^*M$
2. its section : generalized vector $\Gamma(E) \ni u = X^i\partial_i + \alpha_i dx^i$

There is a generalization of Lie bracket called Dorfman bracket

$$[X+\alpha, Y+\beta]_D = L_X(Y+\beta) - i_Y d\alpha$$

Together with $O(D,D)$ invariant product $\langle u, v\rangle$

$$\langle X+\alpha, Y+\beta\rangle = \iota_X\beta + \iota_Y\alpha$$

and the anchor map $\rho \ : \ \Gamma(E) \to \Gamma(TM)$ $\rho(u) = X$

$\Gamma(E)$ becomes a Courant algebroid $(\Gamma(E), [-,-]_D, \rho, \langle -,-\rangle)$

Generalized geometry

In this context, the 3-form H-flux is introduced as a kind of twist of the bracket:

$$[x+\alpha, Y+\beta]_H = [X+\alpha, Y+\beta]_D - i_X i_Y H$$

The generalized Lie derivative $L_u v = [u,v]_D$ gives the symmetry of the Courant algebroid. The parameter is the generalized vector.

$$u = X^i \partial_i + \alpha_i dx^i$$

It is just the gauge symmetry of $g+B$. $Diffeo. \ltimes \Omega^2_{closed}$

One can construct the connection and curvature using the Lie algebroid structure in CA. And then the generalized scalar curvature gives the bosonic part of SUGRA action.

In this talk, I want to discuss these algebra/algebroid structure from the supergeometric point view.

Fluxes

In this talk, I also want to discuss the structure of the fluxes appearing in the T-duality chain of NS-fluxes.[Shelton Taylor Wecht]

$$H_{abc} \overset{T_a}{\longleftrightarrow} F^a_{bc} \overset{T_b}{\longleftrightarrow} Q^{ab}_c \overset{T_c}{\longleftrightarrow} R^{abc}$$

This is obtained by naive application of the Buscher rule.
These fluxes appear also in the compactified theory as structure constants of the gauge algebra in gauged supergravity.[Kaloper,Myers]

$$[e_a, e_b] = F^c_{ab} e_c + H_{abc} e^c$$
$$[e_a, e^b] = Q^{bc}_a e_c - F^b_{ac} e^c$$
$$[e^a, e^b] = R^{abc} e_c + Q^{ab}_c e^c$$

Fluxes

The true origin of these fluxes is still to be clarified. There are already the some formulations, in such as β gravity, DFT, ….
We want to derive those fluxes from the supergeometric construction of the algebra/algebroid.

Plan of talk

1. Introduction
2. Supergeometry
3. Examples
4. Applications (review of our works)
5. Fluxes in Supergeometry
6. Examples
7. Double Field Theory in Supergeometry
8. Discussion

§2. Supergeometry (Lie algebra) [AKSZ, Strobl, Ikeda,

Before giving the general definition, we look at how Lie algebra relates to supergeometry.
We notice soon that it is very familiar formulation, since it is simply the ghost part of BRST-BV formulation

1. For a generator $T_a \in \mathfrak{g}$ we introduce coordinates,

   q^a and its dual p_a

2. We assign a degree (ghost number) $|q^a| = |p_a| = 1$

   This superspace is called $T^*[2]\mathfrak{g}[1]$

$$(p_a, q^a) \sim (1, 1)$$

Supergeometry (Lie algebra)

In the superspace $T^*[2]\mathfrak{g}[1]$ $(p_a, q^a) \sim (1,1)$

3. We define an graded Poisson bracket $\{q^a, p_b\} = \delta^a_b$

4. On this "phase space" we may define a "Hamiltonian".

   For the Lie algebra $\Theta = \frac{1}{2} f_{bc}{}^a p_a q^b q^c$

5. Then the algebra structure is encoded in the "classical master equation" $\{\Theta, \Theta\} = 0 \quad \Leftrightarrow \quad f_{[ab}{}^c f_{cd]}{}^e = 0$

6. "Derived bracket" defines the Lie bracket
$$[p_a, p_b] = -\{\{p_a, \Theta\}, p_b\} = f_{ab}{}^c p_c$$

So we can identify the vector space spanned by p_a with $\mathfrak{g}$

Supergeometry as Diff. Graded Symplectic Manifold

In general, we require the following structures

1. Take a local coordinate and its dual with degree

$$(q^a, p_a, \cdots) \sim (1, n-1, \cdots)$$

2. Poisson structure or equivalently graded symplectic structure

$$\{q^a, p_b\} = (-1)^n \{p_b, q^a\} = \delta^a_b \qquad \omega = (-1)^n dq^a \wedge dp_a + \cdots$$

3. Hamiltonian Θ master equation $\{\Theta, \Theta\} = 0$

This is also called QP-manifold of degree n $|\omega| = n$ $|\Theta| = n+1$

We automatically have the following operation.

1. Nilpotent differential of degree 1 $Q = \{\Theta, -\}$ Q-structure

2. derived bracket $[-,-] = -\{\{-, \Theta\}, -\}$

§3. Examples Lie bialgebra

In the previous example of Lie algebra, the nilpotent differential gives the cobracket in $\mathfrak{g}^*$

$$Qq^a = \frac{1}{2} f_{bc}{}^a q^b q^c$$

Since the role of the coordinates and dual coordinates are symmetric, we can define the bracket on $\mathfrak{g}^*$ by the Hamiltonian

$$\mathcal{S} = \frac{1}{2} c^{ab}{}_c p_a p_b q^c$$

This defines a Lie bialgebra structure. The Hamiltonian $\Theta_B = \Theta + \mathcal{S}$ defines a Lie algebra structure of the Drinfeld double. Master equation requires generalized Jacobi identities and derived bracket gives

$$[e_a, e_b] = f_{ab}{}^c e_c$$
$$[e_a, e^b] = c^{bc}{}_a e_c - f_{ac}{}^b e^c$$
$$[e^a, e^b] = c^{ab}{}_c e^c$$

Examples Generalized geometry

For the generalized geometry, we take the following QP manifold

$$T^*[2]T[1]M \qquad (\xi_i, p_i, q^i, x^i) \sim (2,1,1,0)$$

$$\{x^i, \xi_j\} = \delta^i_j \qquad \{q^i, p_j\} = \delta^i_j$$

1. The Hamiltonian function is $\Theta = \xi_i q^i$

2. Embedding $j : (dx^i, \partial_i) \rightarrow (q^i, p_i)$

3. generalized vectors correspond to the degree 1 subspace:

$$j^*(X^i p_i + \alpha_i q^i) = X^i \partial_i + \alpha_i dx^i \in \Gamma(TM + T^*M)$$

4. Nilpotent operator Q gives de Rahm differential

for example, on the function $f(x)$ $\quad Qf = \{\Theta, f\} = q^i \partial_i f$

This encodes the Courant algebroid structure completely

Examples Generalized geometry

Courant algebroid structure from supergeometry:

1. Dorfman bracket $[X+\alpha, Y+\beta] \equiv -\{\{X+\alpha, \Theta\}, Y+\beta\}$

   by applying j^* $[X+\alpha, Y+\beta] = L_X(Y+\beta) - i_Y d\alpha$

2. anchor $\rho(X+\alpha) = X^i\partial_i$

   For this we consider the function $f(x)$ then the derived bracket $\rho(X+\alpha)(f) \equiv -\{\{X+\alpha, \Theta\}, f\} = X^i\partial_i f$

3. $O(D,D)$ invariant product $\langle X+\alpha, Y+\beta\rangle \equiv \{X+\alpha, Y+\beta\}$

The derived bracket can be defined on any polynomial of the graded coordinate. On the other hand j^* sends them to the p-forms and k-vectors. For example $B = \frac{1}{2}B_{ij}q^iq^j$ can be identified with 2-form B field. A derived bracket with the generalized vector defines the generalized Lie derivative w.r.t. the vector X and 1-form α, as expected.

§4. Applications of supergeometry

We have formulated some of generalized gauge theories using the supergeometry. They are specified by

1. Superspace $\mathcal{M}$ 2. Hamiltonian Θ

1. Algebroid gauge theory: (generalized gauged sigma model)

$$\mathcal{M} = T^*[n]E[1] \qquad (x^i, q^a)(\xi_i, p_a) \sim (0,1), (n, n-1)$$

where E is vector bundle $V \to E \to M$

$$\Theta = \rho^i{}_a(x)\xi_i q^a + \frac{1}{2} f^c{}_{ab}(x) q^a q^b p_c$$

gauge symmetry is $[p_a, p_b] = f^c_{ab}(x) p_c$

Embedding $x^i \sim \phi^i(\sigma) : \Sigma \to M$

gauge field $q^a \sim A^a(\sigma)_\mu d\sigma^\mu$

[U.C-Watamura,M.Heller,N.Ikeda,T.Kaneko,S.W, PTEP 2017,083B01;arXiv:1612.02612]

Applications of supergeometry

2. Higher gauge theory: Non-abelian p-form theory Multi-M5

$$\mathcal{M} = T^*[n](\mathfrak{g}[1] \oplus \mathfrak{h}[2]) \qquad (q^a, Q^A), (p_a, P_A)$$

$$(1, 2), (n-1, n-2)$$

$$\Theta = t^a_A Q^A p_a - (-1)^n \frac{1}{2} f^c_{ab} q^a q^b p_c + \alpha^B_{aA} q^a Q^A P_B$$

gauge symmetry $\quad \mathfrak{g} \ltimes \mathfrak{h} \quad$ Lie 2-algebra

$$[g_a, g_b] = f^c_{ab} g_c \qquad [h_A, h_B] = \tilde{f}^C_{AB} h_C \qquad \tilde{f}^C_{AB} = t^a_A \alpha^C_{aB}.$$

two maps $\quad \underline{\alpha} : \mathfrak{g} \times \mathfrak{h} \to \mathfrak{h} \qquad \underline{\alpha}(g_a) h_A = \alpha^B_{aA} h_B$

$\quad \underline{t} : \mathfrak{h} \to \mathfrak{g} \qquad \underline{t}(h_A) = t^a_A g_a \qquad$ crossed module

$$F^a = dA^a - \frac{1}{2} f^a_{bc} A^b \wedge A^c - t^a_A B^A \; D_A,$$

$$H^A = dB^A + \alpha^A_{aB} A^a \wedge B^B - \frac{1}{3!} T^A_{abc} A^a \wedge A^b \wedge A^c + s^{bA} C_b + n_a^{AB} A^a \wedge D_B,$$

[U.Carow-Watamura,M.Heller,N.Ikeda,Y.Kaneko,S.W., JHEP(2016)125]

Applications of supergeometry

3. Poisson Courant algebroid: Variant of generalized geometry

$$\mathcal{M} = T^*[2]T[1]M$$

$$\Theta = \pi^{ij}\xi_i p_j - \frac{1}{2} Q_k^{ij} q^k p_i p_j + \frac{1}{6} R^{ijk} p_i p_j p_k$$

[T. Asakawa,H.Muraki,S.Sasa,S.W.,IJMP, A30 (2015) 30,1550182]

Gravity on Poisson manifold and non geometric flux:

[T. Asakawa,H.Muraki,S.W.,Fortsch.Phys. 63 (2015) 683-704]

Contravariant gravity; Muraki's Talk

[H.Muraki,Y.Kaneko,S.W.,Class.Quant.Grav. 34 (2017) no.11, 11500]

Semiclassical approximation of noncommutative gravity, Kaneko's talk

§5. Unified Picture of Fluxes

Canonical transformation and fluxes

nQP manifold is a graded phase space and thus we can consider the canonical transformation. Let the exponential adjoint action

$$e^{\delta_A} B \equiv B + \{B, A\} + \frac{1}{2}\{\{B, A\}, A\} + \cdots$$

where A,B are any function.

If $|A| = n$, then this action defines the canonical transformation.

$$e^{\delta_A}\{B, C\} = \{e^{\delta_A} B, C\} + \{B, e^{\delta_A} C\}$$

For Courant algebroid, n=2 and thus the parameters of the canonical transformation are for example $B = \frac{1}{2} B_{ij} q^i q^j$ $\beta = \frac{1}{2}\beta^{ij} p_i p_j$
They generate so-called B-transformation and β-transformation which are elements of O(D,D) transformation.

Fluxes in Supergeometry

Now we want to formulate the fluxes using the above supergeometric construction

The first example is H-flux. As I explained in the introduction, H-flux can be introduced by a twist of the Courant algebroid. This twist is known as B-transformation and naturally we apply the canonical transformation by B field:

$$B = \frac{1}{2} B_{ij} q^i q^j \qquad \Theta_0 = \xi_i q^i \qquad \begin{aligned} \Theta_H &= e^{-\delta_B} \Theta_0 \\ &= \Theta_0 + dB \end{aligned}$$

The derived bracket gives

$$[u, v] = -\{\{u, \Theta_H\}, v\} = [u, v]_D + i_X i_Y dB$$

From this, we can identify the local expression of H-flux as $H = dB$

With this identification, the classical master equation for $\Theta_H = \Theta_0 + H$ gives the Bianchi identity $dH = 0$

Geometric Fluxes

Here we introduce the geometric flux. For this, we need to introduce the local Lorentz frame. Thus we consider the frame bundle and the coordinate (q^a, p_a)

The vielbein is now assigned as $e = e_a^i q^a p_i \quad , \quad e^{-1} = e_i^a q^i p_a$

The canonical transformation is given by $\mathcal{D}_e \equiv e^{-\delta_e} e^{\delta_{e^{-1}}} e^{-\delta_e}$

$$\Theta_f = \mathcal{D}_e \xi_i q^i = \xi_i {e_a}^i q^a + \frac{1}{2} f_{ab}^c q^a q^b p_c + ({e_a}^i \partial_i e_b^j) e_k^b q^a p_j q^k$$

$$f_{bc}^a = -e_b^j e_c^i \partial_{[j} e_{i]}^a$$

This is the local expression of the geometric flux appearing in the commutator $[e_b^i \partial_i, e_c^j \partial_j] = f_{bc}^a e_a^i \partial_i$

The classical master equation gives $(e_{[d}^i \partial_i f_{bc]}^a) + f_{[da'}^a f_{bc]}^{a'} = 0$

General Fluxes

Now we introduce the full twist. The corresponding Hamiltonian is

$$\Theta_{e\beta B} = \mathcal{D}_e e^{-\delta_\beta} e^{-\delta_B} \Theta_0$$

$$\Theta_{B\beta e} = e_b{}^i q^b \xi_i + e^b{}_l \beta^{lm} p_b \xi_m - e^b{}_l \beta^{lm} \partial_m e_a{}^j e^a{}_i q^i p_j p_b + e_b{}^m \partial_m e_a{}^j e^a{}_i q^i q^b p_j$$
$$+ \frac{1}{3!} H_{abc} q^a q^b q^c + \frac{1}{2} F^a_{bc} p_a q^b q^c + \frac{1}{2} Q^{bc}_a q^a p_b p_c + \frac{1}{3!} R^{abc} p_a p_b p_c,$$

The local expression of the fluxes are

$$H_{abc} = 3\nabla_{[a} B_{bc]},$$
$$F^a_{bc} = f^a_{bc} - H_{mns} \beta^{si} e^a{}_i e_b{}^m e_c{}^n,$$
$$f^a_{bc} = 2e_{[b}{}^m \partial_m e_{c]}{}^j e^a{}_j,$$
$$H_{mns} = 3\partial_{[m} B_{ns]},$$
$$Q^{bc}_a = \partial_a \beta^{bc} + f^b_{ad} \beta^{dc} - f^c_{ad} \beta^{db} + H_{isr} \beta^{sh} \beta^{rk} e_a{}^i e^b{}_h e^c{}_k,$$
$$R^{abc} = 3(\beta^{[a|m|} \partial_m \beta^{bc]} + f^{[a}_{mn} \beta^{b|m|} \beta^{c]n}) - H_{mns} \beta^{mi} \beta^{nh} \beta^{sk} e^a{}_i e^b{}_h e^c{}_k.$$

General Fluxes

The classical master equation with these identification leads to the Bianchi identity:

$$e_a = e^i_a \partial_i$$

$$e_{[a}{}^m \partial_{|m|} H_{bcd]} - \frac{3}{2} F^e_{[ab} H_{|e|cd]} = 0,$$

$$e^{[a}{}_l \beta^{|lm|} \partial_m R^{bcd]} - \frac{3}{2} Q_e^{[ab} R^{|e|cd]} = 0,$$

$$e^d{}_l \beta^{ln} \partial_n H_{[abc]} - 3 e_{[a}{}^n \partial_n F^d_{bc]} - 3 H_{e[ab} Q^{ed}_{c]} + 3 F^d_{e[a} F^e_{bc]} = 0,$$

$$-2 e^{[c}{}_l \beta^{|ln|} \partial_n F^{d]}_{[ab]} - 2 e_{[a}{}^n \partial_n Q^{[cd]}_{b]} + H_{e[ab]} R^{e[cd]} + Q^{[cd]}_e F^e_{[ab]} + F^{[c}_{e[a} Q^{|e|d]}_{b]} = 0,$$

$$3 e^{[b}{}_l \beta^{|ln|} \partial_n Q^{cd]}_a - e_a{}^n \partial_n R^{[bcd]} + 3 F^{[b}_{ea} R^{|e|cd]} - 3 Q^{[bc}_e Q^{|e|d]}_a = 0.$$

This is the same as the closure condition for the

$$[e_a, e_b] = F^c_{ab} e_c + H_{abc} e^c_\sharp,$$

$$[e_a, e^b_\sharp] = Q^{bc}_a e_c - F^b_{ac} e^c_\sharp,$$

$$[e^a_\sharp, e^b_\sharp] = R^{abc} e_c + Q^{ab}_c e^c_\sharp,$$

where $e_a = e^i_a \partial_i$ $\quad e^a_\sharp = \beta^{ab} e_a$ [Blumenhagen, Deser, Plauschinn, Rennecke]

§6. DFT and supergeometry

Here we discuss about the supergeometric formulation of the DFT and flux [Deser-Stasheff, Deser-Saemann, Ikeda-Heller-S.W.]

Since the base manifold is also doubled in DFT, we generalize the supermanifold correspondingly

$$T^*[2]T[1](M \times \tilde{M}) \quad (x^M, q^M, p_M, \xi_M) \sim (0,1,1,2)$$

where all coordinates are doubled as $x^M = (\tilde{x}_m, x^m)$, $q^M = (\tilde{q}_m, q^m)$

The P-structure is given by $\{x^M, \xi_N\} = \delta^M_N$, $\{q^M, p_M\} = \delta^M_N$

The Hamiltonian is

$$\Theta_{DFT} = \xi_M(q^M + \eta^{MN} p_M) = \xi_i(q^i + \tilde{p}^i) + \tilde{\xi}^i(p_i + \tilde{q}_i)$$

This Hamiltonian does not satisfy the classical master equation.
This supermanifold is called pre-QP-manifold.

DFT and supergeometry

The classical master equation gives now

$$\{\Theta_{DFT}, \Theta_{DFT}\} = 4\eta^{MN}\xi_M\xi_N$$

Instead of requiring the CME, we restrict the algebra in subspace

$$\{\{A, \{\Theta_{DFT}, \Theta_{DFT}\}\}, B\} = \partial^M A \partial_M B = 0$$

This is the strong constraint in DFT.

The derived bracket is equivalent to the D-bracket in DFT.

$$-\{\{\Theta, X_N Q^N\}, Y_L Q^L\} = (X^M(\partial_M Y_N) - Y^M \partial_M X_N + Y^M \partial_N X_M) Q^N = j_*[X, Y]_D$$

where $Q^M = \frac{1}{\sqrt{2}}(q^M + \eta^{MN} p_M)$ and $\Theta = \xi_M Q^M$

DFT and supergeometry

Fully twisted Hamiltonian

$$
\begin{aligned}
\tilde{\Theta}_{B\beta e} = {} & e_d{}^i \xi_i q^d - e_d{}^i B_{mi} \tilde{\xi}^m q^d + e^c{}_l \tilde{\xi}^l p_c - \beta^{ml} e^c{}_l \xi_m p_c + e^c{}_l B_{nm} \beta^{ml} \tilde{\xi}^n p_c \\
& + e_d{}^i (\partial_i + B_{im} \tilde{\partial}^m) e_a{}^j e^a{}_k p_j q^k q^d + e^c{}_l (\tilde{\partial}^l + \beta^{lm} \partial_m + \beta^{lm} B_{mn} \tilde{\partial}^n) e_a{}^j e^a{}_k p_j q^k p_c \\
& \quad + (\xi_i + \partial_i e_a{}^j e^a{}_k p_j q^k + \partial_i e_a{}^j e^b{}_j q^a p_b) \tilde{p}^i + (\tilde{\xi}^i + \tilde{\partial}^i e_a{}^j e^a{}_k p_j q^k + \tilde{\partial}^i e_a{}^j e^b{}_j q^a p_b) \tilde{q}_i \\
& + \frac{1}{2} (\partial_i B_{jk} \tilde{p}^i + \tilde{\partial}^i B_{jk} \tilde{q}_i) e_a{}^j e_b{}^k q^a q^b + \frac{1}{2} (\partial_i \beta^{jk} \tilde{p}^i + \tilde{\partial}^i \beta^{jk} \tilde{q}_i) e^b{}_j e^c{}_k p_b p_c \\
& - \partial_i B_{jk} \beta^{km} e_b{}^j e^c{}_m \tilde{p}^i q^b p_c - \tilde{\partial}^i B_{jk} \beta^{km} e_b{}^j e^c{}_m \tilde{q}_i q^b p_c + \frac{1}{2} \partial_i B_{jk} \beta^{jm} \beta^{kn} e^b{}_m e^c{}_n \tilde{p}^i p_b p_c \\
& + \frac{1}{2} \tilde{\partial}^i B_{jk} \beta^{jm} \beta^{kn} e^b{}_m e^c{}_n \tilde{q}_i p_b p_c \\
& + \frac{1}{3!} H_{abc} q^a q^b q^c + \frac{1}{2} F^a_{bc} p_a q^b q^c + \frac{1}{2} Q^{bc}_a q^a p_b p_c + \frac{1}{3!} R^{abc} p_a p_b p_c,
\end{aligned}
$$

DFT and supergeometry

Local expression of the fluxes

$$
\begin{aligned}
H_{abc} &= 3(\nabla_{[a}B_{bc]} + B_{[a|m|}\tilde{\partial}^m B_{bc]} + \tilde{f}^{mn}_{[a}B_{b|m|}B_{c]n}),\\
F^a_{bc} &= f^a_{bc} - H_{mns}\beta^{si}e^a{}_i e_b{}^m e_c{}^n + \tilde{\partial}^a B_{bc} + \tilde{f}^{ad}_b B_{dc} - \tilde{f}^{ad}_c B_{db},\\
Q^{bc}_a &= \tilde{f}^{bc}_a + \partial_a\beta^{bc} + f^b_{ad}\beta^{dc} - f^c_{ad}\beta^{db} + H_{isr}\beta^{sh}\beta^{rk}e_a{}^i e^b{}_h e^c{}_k\\
&\quad + B_{am}\tilde{\partial}^m\beta^{bc} + \tilde{\partial}^{[b}B_{ae}\beta^{e|c]} + 2B_{[a|e}\tilde{f}^{be}_{d]}\beta^{dc} - 2B_{[a|e}\tilde{f}^{ce}_{d]}\beta^{db},\\
R^{abc} &= 3(\beta^{[a|m|}\partial_m\beta^{bc]} + f^{[a}_{mn}\beta^{b|m|}\beta^{c]n} + \tilde{\partial}^{[a}\beta^{bc]} - \tilde{f}^{[ab}_d\beta^{|d|c]}\\
&\quad + B_{ln}\tilde{\partial}^l\beta^{[ab}\beta^{|n|c]} + \tilde{\partial}^{[a}B_{ed}\beta^{|e|b}\beta^{|d|c]} + \tilde{f}^{[a|e|}_n B_{ed}\beta^{|n|b|}\beta^{|d|c]})\\
&\quad - H_{mns}\beta^{mi}\beta^{nh}\beta^{sk}e^a{}_i e^b{}_h e^c{}_k,\\
H_{mns} &= 3(\partial_{[m}B_{ns]} + B_{[m|l|}\tilde{\partial}^l B_{ns]}),\\
\tilde{f}^{ab}_c &= 2e^{[a}{}_m\tilde{\partial}^m e^{b]}{}_j e_c{}^j
\end{aligned}
$$

DFT and supergeometry

Bianchi identities:

$$e_{[a}{}^{i} B_{in} \tilde{\partial}^{n} H_{bcd]} - \frac{3}{2} F^{e}_{[ab} H_{|e|cd]} = 0,$$

$$(e^{[a}{}_{n} + e^{[a}{}_{l} \beta^{lm} B_{mn}) \tilde{\partial}^{n} R^{bcd]} - \frac{3}{2} Q_{e}^{[ab} R^{|e|cd]} = 0,$$

$$(e^{d}{}_{n} + e^{d}{}_{l} \beta^{lm} B_{mn}) \tilde{\partial}^{n} H_{[abc]} - 3 e_{[a}{}^{i} B_{in} \tilde{\partial}^{n} F^{d}_{bc]} - 3 H_{e[ab} Q_{c]}^{ed} + 3 F^{d}_{e[a} F^{e}_{bc]} = 0,$$

$$- 2(e^{[c}{}_{n} + e^{[c}{}_{l} \beta^{lm} B_{mn}) \tilde{\partial}^{n} F^{d]}_{[ab]} - 2 e_{[a}{}^{i} B_{in} \tilde{\partial}^{n} Q_{b]}^{[cd]} + H_{e[ab]} R^{e[cd]} + Q_{e}^{[cd]} F^{e}_{[ab]} + F^{[c}_{e[a} Q_{b]}^{|e|d]} =$$

$$3(e^{[b}{}_{n} + e^{[b}{}_{l} \beta^{lm} B_{mn}) \tilde{\partial}^{n} Q_{a}^{cd]} - e_{a}{}^{i} B_{in} \tilde{\partial}^{n} R^{[bcd]} + 3 F^{[b}_{ea} R^{|e|cd]} - 3 Q_{e}^{[bc} Q_{a}^{|e|d]} = 0.$$

After imposing the strong constraints, these local expressions of the fluxes and Bianchi identities reduce to the known examples.

§7. Discussions

1. The supergeometric method is very powerful to investigate the generalization of geometry and gauge symmetries.
2. Apllying to the generalized geometry and DFT, the local expression of the fluxes, and Bianchi identities can be derived by canonical transformations.
3. Application to DFT is still under investigation. Especially, the Riemannian structure and the characterization of the invariant action are to be done. However, for this it seems we need to generalize the supergeometric method.
4. This formulation fits also to the Leibniz algebroid, and thus also to the formulation of EFT.

Discussions

Physically, the geometries based on Lie algebroid, Courant algebroid and Leibniz algebroid in general, are very interesting.

1. Based on the Poisson-Lie algebroid, we can construct the contra variant gravity, gravity on the poisson manifold. [H.Muraki,Y.Kaneko,SW]
2. This may be considered as the semiclassical limits of the gravity on the noncommutative geometry.

Finally, these class of symmetries are subalgebra of so-called L_∞ algebra which appears as the symmetry of closed string field theo