A model for emergence of space and time

(因果力学的単体分割による宇宙創成)

Talk @ Discrete Approaches to the Dynamics of Fields and Space-Time held at APTCP on 22/9/2017

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(in collaboration with Jan Ambjorn)

MENU 1

1. **Review of DT**
   a. What is DT **(Dynamical Triangulation)** ?
   b. DT expressed by string field theory
2. **Review of CDT** (I only treat 2dim CDT)
   a. What is CDT **(Causal Dynamical Triangulation)** ?
   b. CDT expressed by string field theory
3. **DT and CDT with $\mathcal{W}$-algebra**

   [Ambjørn, Watabiki: arXiv:1505.04353]

   a. The mode expansion of DT and reduced $\mathcal{W}$-algebra
   b. The mode expansion of CDT and $\mathcal{W}$-algebra
   c. The emergence of spacetime

MENU 2

4. **Higher-dimensional CDT with $\mathcal{W}$-algebra**

   [Ambjørn, Watabiki: arXiv:1703.04402]

   a. Higher-dimensional CDT with $\mathcal{W}$-algebra
   b. New $\mathcal{W}$-algebra with Jordan algebra
   c. Tangent and Hypaboric Tangent expansions
   d. Dimension enhancement and Vanishing cosmo. const.

5. **Modified Friedmann Equation**

   [Ambjørn, Watabiki: arXiv:1709.06497]

   a. The modified Friedmann equation
   b. Accelerating Universe
   c. Predictions

6. **Summary and Discussions**

1. Review of DT

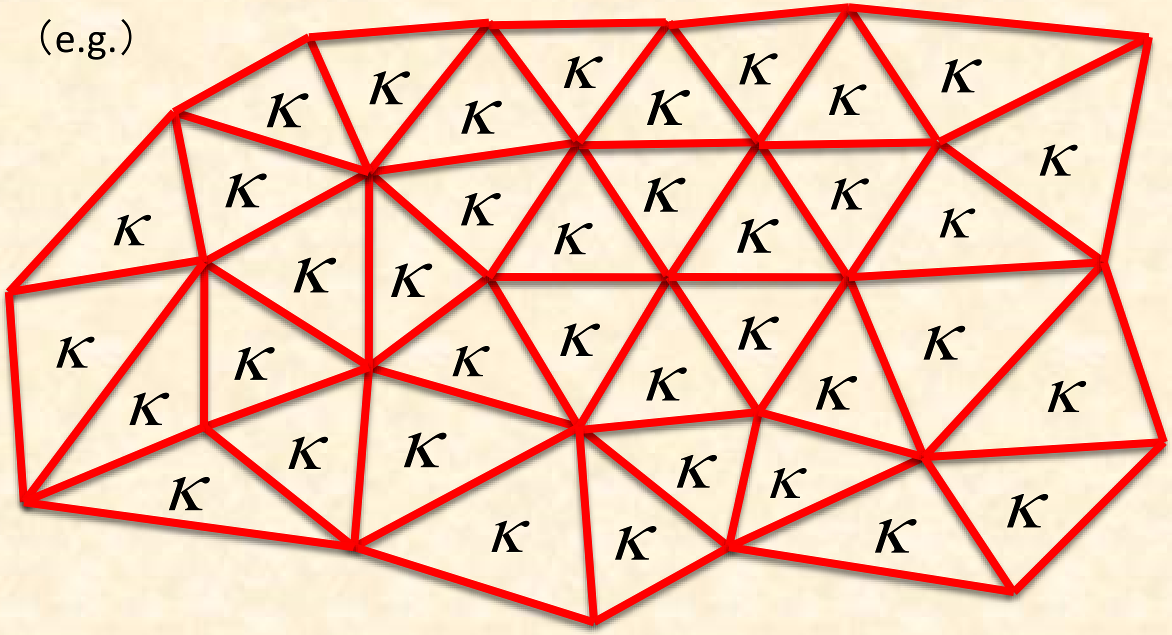
a. What is DT (Dynamical Triangulation) ?

- **Definition of DT**

Construction of lattice by "equilateral triangles"

(e.g.)

All triangles are the same equilateral triangle



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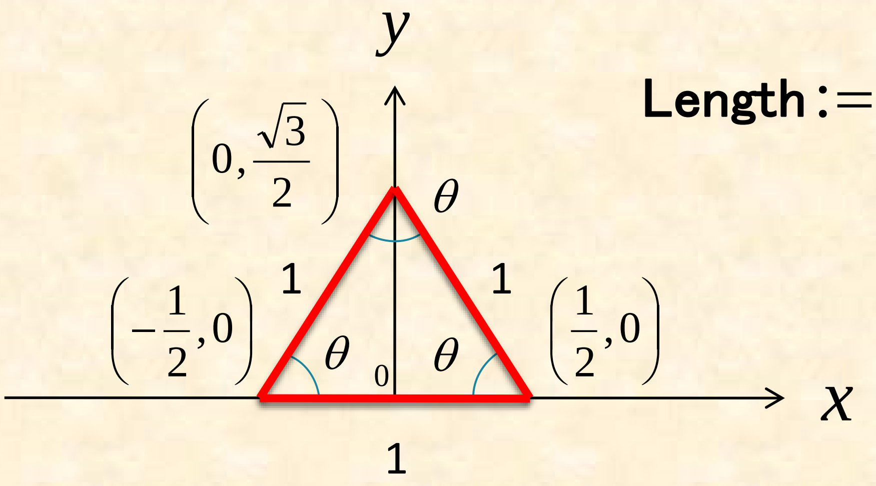
Each triangle is the same size and equilateral.



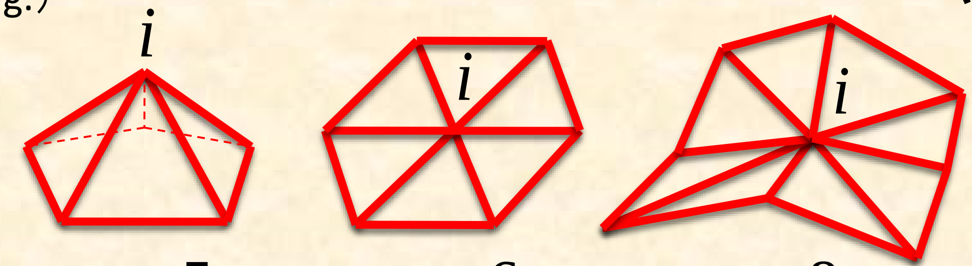
Length $:= \sqrt{x^2 + y^2}$

Curvature of site $\boldsymbol{i}$, $\sqrt{g}R_i$ is

(curvature exists only on sites)

$$\sqrt{g}R_i = (6 - q_i)\theta$$

(e.g.)



$q_i = 5$ Curv. > 0

$q_i = 6$ Curv.=0

$q_i = 8$ Curv.<0

q_i is the nr. of triangles together to the site $\boldsymbol{i}$

1- a

- **Partition function of DT**

Quantum gravity is the path integral of metric $g_{\mu\nu}$
(μ is the cosmological constant)

$$Z = \int Dg_{\mu\nu}\, \mathrm{e}^{\int \mathrm{d}^2x\sqrt{g}\,(-\frac{R}{4\pi G}+\mu)}$$

The metric $g_{\mu\nu}$ expresses various curved spaces, so the path integral is the summation of all kinds of triangulated spaces.

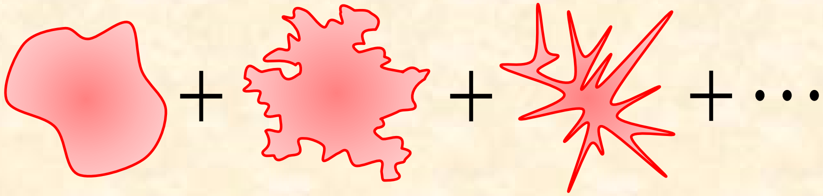
ε^2 is the area of one triangle.

(κ is cosmological constant at lattice level, N_2 is nr. of triangles)

One triangle corresponds to one κ

$$Z = \sum_{\text{summation of triangulated lattices}} N^{\chi}\kappa^{N_2} \quad \left\{ \begin{array}{l} N = \mathrm{e}^{1/G} \qquad \kappa = \mathrm{e}^{\varepsilon^2\mu} \\ N_2 = \dfrac{1}{\varepsilon^2}\int \mathrm{d}^2x\sqrt{g} \end{array} \right.$$

The summation is performed by all possible triangulated lattices by



- **DT and Amplitudes** (Discrete Laplace transf.)

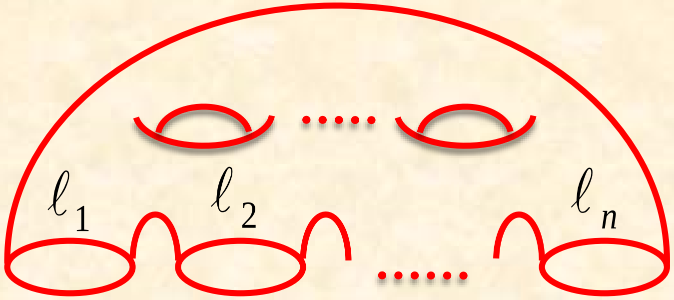
Definition of Amplitudes

The partition fun with general topology is obtained by summing up the lattice with the following topology

$$W(x_1,\ldots,x_n) \ := \ N^{n-2}\sum_{\ell_1=0}^{\infty}\cdots\sum_{\ell_n=0}^{\infty} x^{-\ell_1-1}\cdots x^{-\ell_n-1} W(\ell_1,\ldots,\ell_n)$$

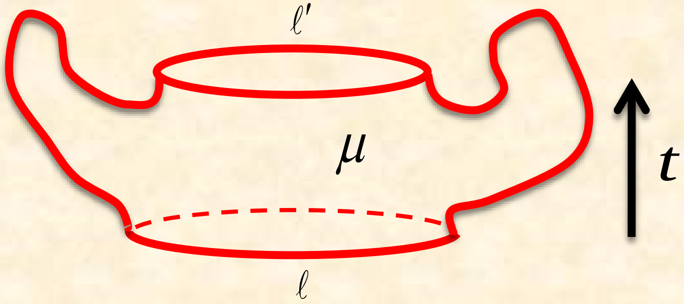
W is the partition fun. fixing the topology

$\left(\begin{array}{l}\text{Surface with } n \text{ holes} \\ \text{and several handles}\end{array}\right)$



Green function is

$$G(x,y;t) \ = \ \sum_{\ell=1}^{\infty}\sum_{\ell'=1}^{\infty} x^{\ell} y^{\ell'} G(\ell,\ell';t)$$



Continuum limit of Amplitudes

The continuum limit is obtained by

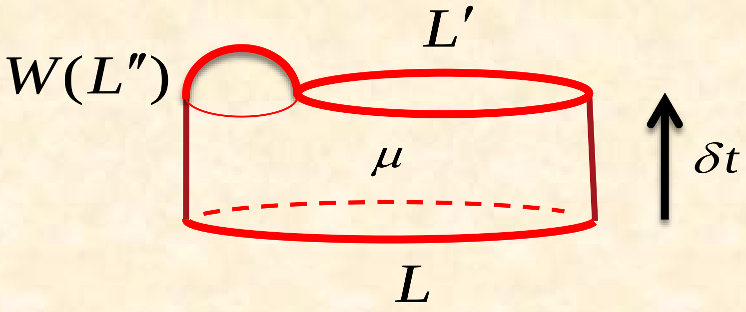
$$t \to \frac{t}{\varepsilon^{1/2}} \qquad x \to x_{\mathrm{c}} \mathrm{e}^{-\varepsilon\xi} \qquad \kappa \to \kappa_{\mathrm{c}} \mathrm{e}^{\varepsilon^2 \mu}$$

The disk amplitude is

$$W(\xi) = \left(\xi - \frac{\sqrt{\mu}}{2}\right)\sqrt{\xi + \sqrt{\mu}}$$

The differential equation of Green fun is

$$\frac{\partial}{\partial t} G(\xi,\eta;t) = -2g \frac{\partial}{\partial \xi}\big(W(\xi)G(\xi,\eta;t)\big)$$



b. DT expressed by string field theory

- Creation op. and annihilation op. (L : length)

$$[\Psi(L),\Psi^{\dagger}(L')] = L\,\delta(L-L')$$

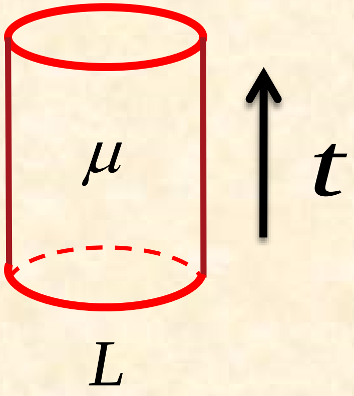
(others are zero)

- Free Hamiltonian and Green function

$$H_0 = 0$$

$$G(L,L',t) = \langle 0|\Psi(L')\mathrm{e}^{-tH_0}\Psi^{\dagger}(L)|0\rangle$$

$$= \delta(L-L')$$



- Hamiltonian with interactions

$$H_{\mathrm{DT}} = H_0 - g\int \mathrm{d}L_1 \int \mathrm{d}L_2 \Psi^{\dagger}(L_1)\Psi^{\dagger}(L_2)\Psi(L_1+L_2)$$

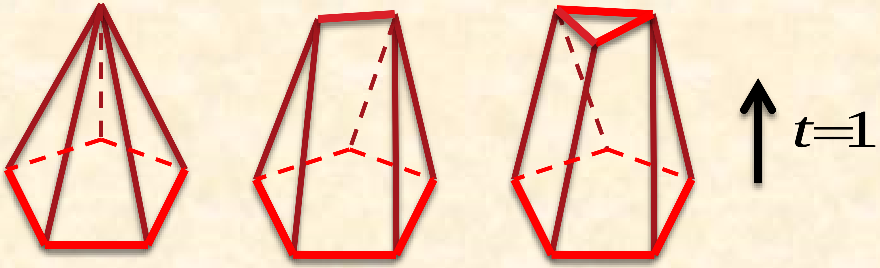
$$- Gg\int \mathrm{d}L_1 \int \mathrm{d}L_2 \Psi^{\dagger}(L_1+L_2)\Psi(L_1)\Psi(L_2)$$

$$- \int \frac{\mathrm{d}L}{L}\rho(L)\Psi(L)$$

$$\rho(L) = 3\delta''(L) - \frac{3\mu}{4}\delta(L)$$

g is a coupling constant of string theory

G is the constant which counts nr. of handles



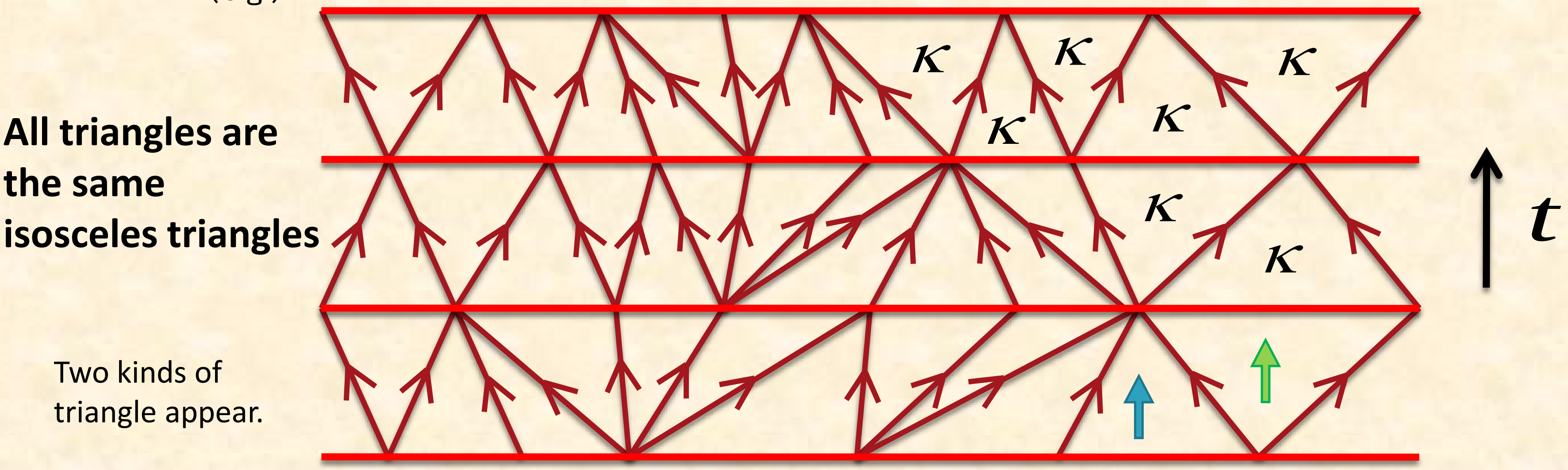
2. Review of CDT

a. What is CDT (Causal Dynamical Triangulation) ?

- Definition of CDT

Construction of lattice by “time (isosceles) triangles

（e.g.）



The direction of time is unique and causal.

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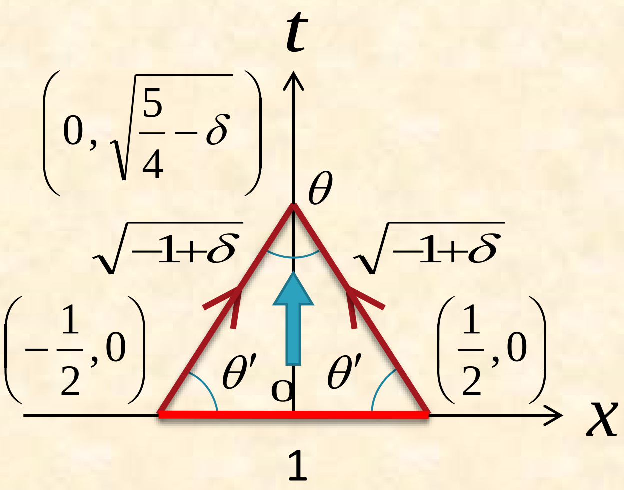
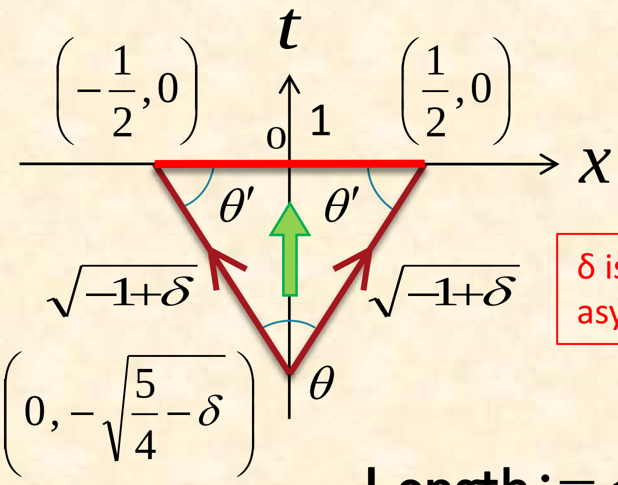
Each triangle is the same size and isosceles.



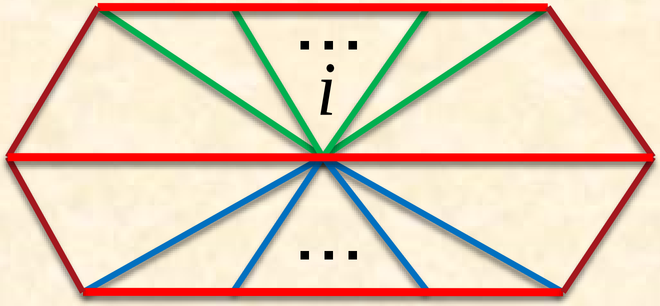


$(\delta < 1)$

δ is related with the asymmetry of *x* and *t*

Length $:= \sqrt{x^2 - t^2}$

The curvature of a site i, $\sqrt{g}R_i$ **is expressed by**



$$\sqrt{g}R_i = (4 - k_i - j_i)\theta$$

k_i Is nr. of green links

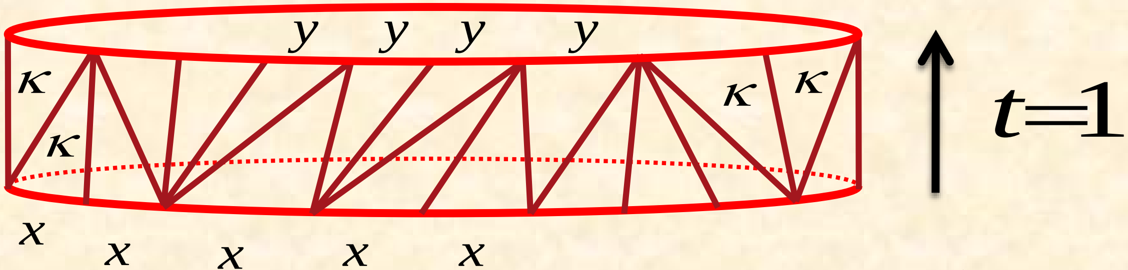
j_i Is nr. of blue links

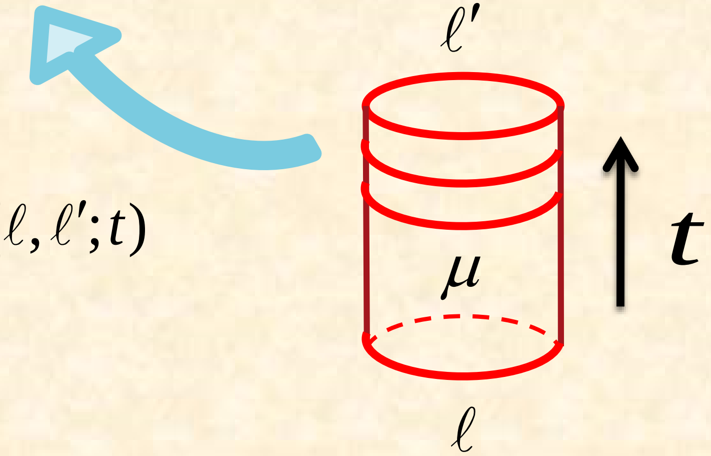
- CDT and Green fun (Discrete Laplace transf.)

Definition of Green fun

The partition fun with cylinder topology is obtained by piling the following lattice

(e.g.)





Green function is

$$G(x,y;t) = \sum_{\ell=1}^{\infty}\sum_{\ell'=1}^{\infty} x^{\ell} y^{\ell'} G(\ell,\ell';t)$$

Continuum limit of Amplitudes

The continuum limit is obtained by

$$t \to \frac{t}{\varepsilon} \qquad x \to x_{\mathrm{c}} \mathrm{e}^{-\varepsilon \xi} \qquad \kappa \to \kappa_{\mathrm{c}} \mathrm{e}^{\varepsilon^2 \mu}$$

The disk amplitude is

$$W(\xi) = \int_0^\infty \mathrm{d}t\, G(\xi, \ell' = 0; t) = \frac{1}{\xi + \sqrt{\mu}}$$

The differential equation of Green fun is

$$\frac{\partial}{\partial t} G(\xi, \eta; t) = -\frac{\partial}{\partial \xi}\Big((\xi^2 - \mu) G(\xi, \eta; t)\Big)$$

$$\frac{\partial}{\partial t} G(L, L', t) = L\left(-\frac{\partial^2}{\partial L^2} + \mu\right) G(L, L', t)$$

2- b

b. CDT expressed by string field theory

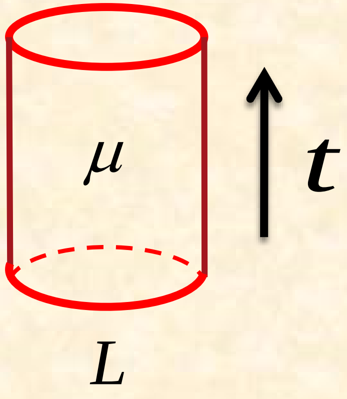
- Creation op. and annihilation op. (L : length)

$$[\Psi(L)\,,\Psi^{\dagger}(L')] = L\,\delta(L-L')$$

(others are zero)

- Free Hamiltonian and Green function

$$H_0 = \int_0^{\infty}\frac{\mathrm{d}L}{L}\Psi^{\dagger}(L)\,L\left(-\frac{\partial^2}{\partial L^2}+\mu\right)\Psi(L)$$



(Time reversal symmetry is not broken.)

$$G(L,L',t) = \langle 0|\Psi(L')\mathrm{e}^{-tH_0}\Psi^{\dagger}(L)|0\rangle$$

$$= \frac{\sqrt{\mu LL'}}{\sinh\sqrt{\mu}\,T}\mathrm{e}^{-\sqrt{\mu}(L+L')\coth\sqrt{\mu}\,T}I_1\left(\frac{2\sqrt{\mu LL'}}{\sinh\sqrt{\mu}\,T}\right)$$

- Hamiltonian with interactions

$$H_{\mathrm{CDT}} = H_0 - g\int \mathrm{d}L_1 \int \mathrm{d}L_2 \Psi^\dagger(L_1)\Psi^\dagger(L_2)\Psi(L_1+L_2)$$

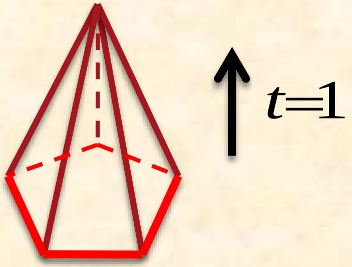
$$-Gg\int \mathrm{d}L_1 \int \mathrm{d}L_2 \Psi^\dagger(L_1+L_2)\Psi(L_1)\Psi(L_2)$$

$$-\int \frac{\mathrm{d}L}{L}\rho(L)\Psi(L)$$

$$\rho(\mathrm{L}) = \delta(L)$$

g is a coupling constant of string theory

G is the constant which counts nr. of handles



3. DT and CDT with $\mathcal{W}$-algebra

a. The mode expansion of DT and reduced $\mathcal{W}$-algebra

- Laplace transformation of the string field of DT

$$\Psi^\dagger(\zeta) = \int_0^\infty dL\, e^{-\varsigma L}\, \Psi^\dagger(L) \qquad \Psi(\zeta) = \int_0^\infty dL\, e^{-\varsigma L}\, \Psi(L)$$

- Mode expansion of the string field of DT

$$\Psi^\dagger(\zeta) = (\text{polynomial of } \zeta) + \zeta^{3/2} - \frac{3}{8}\mu\zeta^{-1/2} + \sum_{l=1}^{\infty} \zeta^{-l/2-1}\phi_l^\dagger$$

$$\Psi(\zeta) = \sum_{l=1}^{\infty} \zeta^{l/2}\phi_l \qquad [\phi_m^\dagger, \phi_n] = m\delta_{m+n,0}$$

μ is the cosmological constant.

3- a

- **Hamiltonian and 2-reduced $\mathcal{W}$ operator**

$$H_{\mathrm{DT}} = -2\sqrt{G}\,\overline{W}^{(3)}_{-2} + \frac{1}{2\sqrt{G}}\phi_6^{\dagger} - \frac{3\mu}{8\sqrt{G}}\phi_2^{\dagger}$$

$$\overline{W}^{(3)}_{n} = \frac{1}{4}\left(\frac{1}{3}\sum_{k+l+m=2n} :\alpha_k\alpha_l\alpha_m: + \frac{1}{4}\alpha_{2n}\right)$$

$$\alpha_n = \begin{cases} n\,(\lambda_n - \sqrt{G}\phi_{-n}) & (n<0) \\ 0 & (n=0) \\ \dfrac{1}{\sqrt{G}}\phi_n^{\dagger} & (n>0) \end{cases}$$

$$[\alpha_m, \alpha_n] = m\delta_{m+n,0}$$

$$\lambda_5 = \frac{1}{\sqrt{G}} \qquad \lambda_1 = -\frac{3\mu}{8\sqrt{G}}$$

- The absolute vacuum $|0\rangle$ and the coherent states $|\lambda_5, \lambda_1\rangle$

Only $\lambda_5 \neq 0 \quad \lambda_1 \neq 0$

$$\alpha_{-5} | \lambda_5, \lambda_1\rangle = \lambda_5 | \lambda_5, \lambda_1\rangle \qquad \alpha_{-1} | \lambda_5, \lambda_1\rangle = \lambda_1 | \lambda_5, \lambda_1\rangle$$

then

$$H_{\mathrm{DT}} | \lambda_5, \lambda_1\rangle = 0 \qquad \phi_n | \lambda_5, \lambda_1\rangle = 0 \quad (n = 1, 2, \dots)$$

The physical vacuum is expressed by

$$| \lambda_5, \lambda_1\rangle = e^{\lambda_5 \alpha_5 + \lambda_1 \alpha_1} | 0\rangle$$

$$\alpha_{-n} | 0\rangle = 0 \quad (n = 0, 1, 2, \dots)$$

b. The mode expansion of CDT and *W*-algebra

- Laplace transformation of the string field of DT

$$\Psi^{\dagger}(\zeta)=\int_0^{\infty} dL\, e^{-\varsigma L}\,\Psi^{\dagger}(L) \qquad \Psi(\zeta)=\int_0^{\infty} dL\, e^{-\varsigma L}\,\Psi(L)$$

- Mode expansion of the string field of DT

$$\Psi^{\dagger}(\zeta)=(\text{polynomial of }\zeta)+\zeta^{-1}+\sum_{l=1}^{\infty}\zeta^{-l-1}\phi_l^{\dagger}$$

$$\Psi(\zeta)=\sum_{l=1}^{\infty}\zeta^{l}\phi_l$$

- Hamiltonian and $\mathcal{W}$ operator

$$H_{\mathrm{CDT}} = -g\sqrt{G}\,W_{-2}^{(3)} + \frac{1}{G}\left(\frac{\mu^2}{4g} + \frac{1}{4g}\phi_4^\dagger - \frac{\mu}{2g}\phi_2^\dagger + \phi_1^\dagger \right)$$

$$W_n^{(3)} = \frac{1}{3}\sum_{k+l+m=n} :\alpha_k \alpha_l \alpha_m:$$

$$[\alpha_m, \alpha_n] = m\delta_{m+n,0}$$

$$\alpha_n = \begin{cases} n(\lambda_n - \sqrt{G}\phi_{-n}) & (n<0) \\ p & (n=0) \\ \dfrac{1}{\sqrt{G}}\phi_n^\dagger & (n>0) \end{cases}$$

$$\lambda_3 = \frac{1}{6g\sqrt{G}} \qquad \lambda_1 = -\frac{\mu}{2g\sqrt{G}}$$

$$\nu = \frac{1}{\sqrt{G}}$$

- The absolute vacuum $|0\rangle$ and the coherent states $|\lambda_3,\lambda_1,\nu\rangle$

Only $\lambda_3 \neq 0 \quad \lambda_1 \neq 0 \quad \nu \neq 0$

$$\alpha_{-3}|\lambda_3,\lambda_1,\nu\rangle = \lambda_3|\lambda_3,\lambda_1,\nu\rangle \quad \alpha_{-1}|\lambda_3,\lambda_1,\nu\rangle = \lambda_1|\lambda_3,\lambda_1,\nu\rangle$$

$$p|\lambda_3,\lambda_1,\nu\rangle = \nu|\lambda_3,\lambda_1,\nu\rangle$$

then

$$H_{\mathrm{CDT}}|\lambda_3,\lambda_1,\nu\rangle = 0 \qquad \phi_n|\lambda_3,\lambda_1,\nu\rangle = 0 \ \ (n = 1, 2, \ldots)$$

The physical vacuum is expressed by

$$|\lambda_3,\lambda_1,\nu\rangle = e^{\lambda_3\alpha_3+\lambda_1\alpha_1+i\nu q}|0\rangle$$

$$\alpha_{-n}|0\rangle = 0 \quad (n = 0, 1, 2, \ldots)$$

C. The emergence of spacetime

- **DT case**

$$H_{\mathrm{DT}} \,|\, \lambda_5, \lambda_1 \rangle = 0$$

$$\overline{H}_W \,|\, \lambda_5, \lambda_1 \rangle = -\frac{1}{\sqrt{G}} \left(\frac{1}{2} \phi_6^\dagger - \frac{3\mu}{8} \phi_2^\dagger \right) |\, \lambda_5, \lambda_1 \rangle \neq 0$$

$$\overline{H}_W := -2\sqrt{G}\, \overline{W}_{-2}^{(3)}$$

$$\lim_{T \to \infty} \langle 0 | \, e^{-TH} \phi_{2n}^\dagger \, | \lambda_5, \lambda_1 \rangle = 0$$ (Disk amplitude with boundary $2n$)

→ There is no essential difference between H_{DT} and $\overline{H}_W$

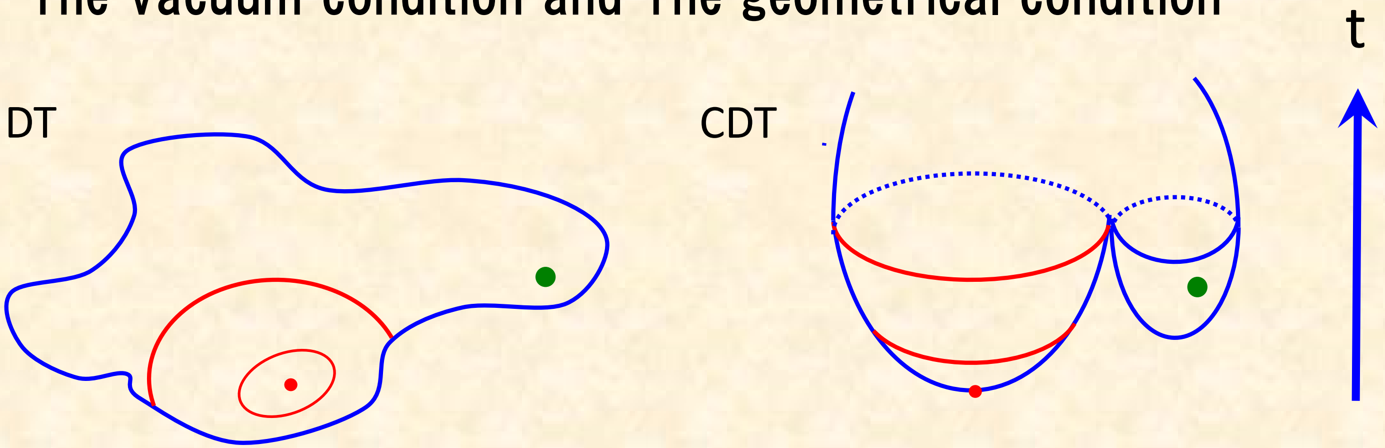
- **CDT case**

$$H_{\mathrm{CDT}}\,|\,\lambda_3,\lambda_1,\nu\rangle = 0$$

$$H_W\,|\,\lambda_3,\lambda_1,\nu\rangle = -\frac{1}{G}\left(\frac{\mu^2}{4g}+\frac{1}{4g}\phi_4^\dagger-\frac{\mu}{2g}\phi_2^\dagger+\phi_1^\dagger\right)|\,\lambda_3,\lambda_1,\nu\rangle \neq 0$$

$$H_W := -g\sqrt{G}\,W_{-2}^{(3)}$$

- **The vacuum condition and The geometrical condition**



You can (not) reach ● from ●

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- **From now on, we assume the following Hamiltonian**

$$
\begin{aligned}
H_W &= \mu\phi_1 - 2g\phi_2 - gG\phi_1\phi_1 \boxed{-\frac{1}{G}\left(\frac{\mu^2}{4g}+\frac{1}{4g}\phi_4^\dagger-\frac{\mu}{2g}\phi_2^\dagger+\phi_1^\dagger\right)} \\
&\quad -\sum_{l=1}^{\infty}\phi_{l+1}^\dagger l\phi_l+\mu\sum_{l=2}^{\infty}\phi_{l-1}^\dagger l\phi_l-2g\sum_{l=3}^{\infty}\phi_{l-2}^\dagger l\phi_l \\
&\quad -g\sum_{l=4}^{\infty}\sum_{n=1}^{l-3}\phi_n^\dagger\phi_{l-n-2}^\dagger l\phi_l-gG\sum_{l=1}^{\infty}\sum_{m=\max(3-l,1)}^{\infty}\phi_{m+l-2}^\dagger m\phi_m l\phi_l \\
&= -g\sqrt{G}\,W_{-2}^{(3)}
\end{aligned}
$$

$\boxed{}$ Is added from the viewpoint of $\mathcal{W}$-symmetry.

Emergence of time

$H_W = -g\sqrt{G}\,W_{-2}^{(3)}$ is not Hermitian.

So, Energy conservation law doesn’t exist.

The time and physical vacuum were born by the interaction between different Hilbert spaces of $\mathcal{W}$−algebra.

(ex)

$$|\ldots,\lambda_3'',\lambda_2'',\lambda_1'',\nu''\rangle \Leftrightarrow |\lambda_3,\lambda_1,\nu\rangle\otimes|\ldots,\lambda_3',\lambda_2',\lambda_1',\nu'\rangle$$

The $\mathcal{W}$−algebra world (WAW) is described

by the static picture or the picture using the fictitious time.

The above process is not spontaneous, but we cannot deny the possibility of spontaneous symmetry breaking.

4. Higher-dim. CDT with $\mathcal{W}$-algebra

a. Higher−dimensional CDT with $\mathcal{W}$−algebra

- **The Hamiltonian**

$$H_W = -g\sqrt{G}\,W_{-2}^{(3)}$$

$$W_n^{(3)} = \frac{1}{3}\sum_{a,b,c} d_{abc} \sum_{k+l+m=n} :\alpha_k^{(a)}\alpha_l^{(b)}\alpha_m^{(c)}:$$

$$[\lambda_a, \lambda_b] = \sum_c c_{abc}\lambda_c \qquad \{\lambda_a, \lambda_b\} = \frac{4}{3}\delta_{ab} + \sum_c d_{abc}\lambda_c$$

Let N be 1, 2, 4, 8 for real, complex, quaternion, octonion.

- $N=2$ case (complex version)

λ_a $[a=1,2,\ldots,8]$ are Gell-mann matrices.

$$d_{118}=d_{228}=d_{338}=\frac{1}{\sqrt{3}} \qquad d_{888}=-\frac{1}{\sqrt{3}}$$

$$d_{448}=d_{558}=d_{668}=d_{778}=-\frac{1}{2\sqrt{3}}$$

$$d_{344}=d_{355}=\frac{1}{2} \qquad d_{366}=d_{377}=-\frac{1}{2}$$

$$d_{146}=d_{157}=d_{256}=\frac{1}{2} \qquad d_{247}=-\frac{1}{2}$$

8th and 3rd spaces play a special role if c.c. is positive!

b. New $\mathcal{W}$-algebra with Jordan algebra

- **Operators**

$$J^{(a|n)}(z) := \partial^{n+1}\phi^{(a)}(z)$$

$$T^{(m,n)}(z) := \frac{1}{2}\sum_{a,b}\delta_{ab} : J^{(a|m)}(z) J^{(b|n)}(z) :$$

$$W^{(l,m,n)}(z) := \frac{1}{3}\sum_{a,b,c} d_{abc} : J^{(a|l)}(z) J^{(b|m)}(z) J^{(c|n)}(z) :$$

$$\Lambda^{(k.l,m,n)}(z) := \frac{1}{4}\sum_{a,b,c,d,e} d_{abe} d_{cde} : J^{(a|k)}(z) J^{(b|l)}(z) J^{(c|m)}(z) J^{(d|n)}(z) :$$

$$\vdots$$

$$W^{(0,0,0)}(z) = \sum_{n=-\infty}^{\infty} W_n^{(3)} z^{-n-3}$$

- **New $\mathcal{W}$-algebra**

$$W^{(0,0,0)}(z)W^{(0,0,0)}(w) \sim \frac{8\Lambda^{(1,0,0,0)}(w)}{z-w}+\frac{4\Lambda^{(0,0,0,0)}(w)}{(z-w)^2}$$

$$+\frac{N+2}{6}\left(\frac{2T^{(3,0)}(w)}{3(z-w)}+\frac{2T^{(2,0)}(w)}{(z-w)^2}\right.$$

$$\left.+\frac{4T^{(1,0)}(w)}{(z-w)^3}+\frac{4T^{(0,0)}(w)}{(z-w)^4}\right)$$

$$+\frac{N(N+2)}{9(z-w)^6}$$

$$\vdots$$

C. Tangent and Hyperbolic Tangent expansion (THT-expansion)

- **Physical vacuum** (we choose the following physical vacuum)

$$\alpha_{-3}^{(8)}|\text{phys}\rangle = \lambda_3^{(8)}|\text{phys}\rangle \qquad \alpha_{-1}^{(8)}|\text{phys}\rangle = \lambda_1^{(8)}|\text{phys}\rangle$$

$$\alpha_{-3}^{(3)}|\text{phys}\rangle = \lambda_3^{(3)}|\text{phys}\rangle \qquad \alpha_{-1}^{(3)}|\text{phys}\rangle = \lambda_1^{(3)}|\text{phys}\rangle$$

- **Hamiltonian**

$$\begin{aligned} H_W = & - d_{aa8}\sum_{l=1}^{\infty}\phi_{l+1}^{(a)\dagger} l \phi_l^{(a)} + \mu^{(8)} d_{aa8}\sum_{l=2}^{\infty}\phi_{l-1}^{(a)\dagger} l \phi_l^{(a)} \\ & - d_{aa3}\sum_{l=1}^{\infty}\phi_{l+1}^{(a)\dagger} l \phi_l^{(a)} + \mu^{(3)} d_{aa3}\sum_{l=2}^{\infty}\phi_{l-1}^{(a)\dagger} l \phi_l^{(a)} \\ & - g\, d_{aa8}\sum_{l=4}^{\infty}\sum_{n=1}^{l-3}\phi_n^{(8)\dagger}\phi_{l-n-2}^{(a)\dagger} l \phi_l^{(a)} + \ldots \end{aligned}$$

- Hamiltonian which gives 3, 4, 6, 10-dim model

$$\lambda_1^{(8)} = -\frac{\sqrt{3}\mu}{2g\sqrt{G}} \qquad \lambda_1^{(3)} = -\frac{\mu}{2g\sqrt{G}}$$

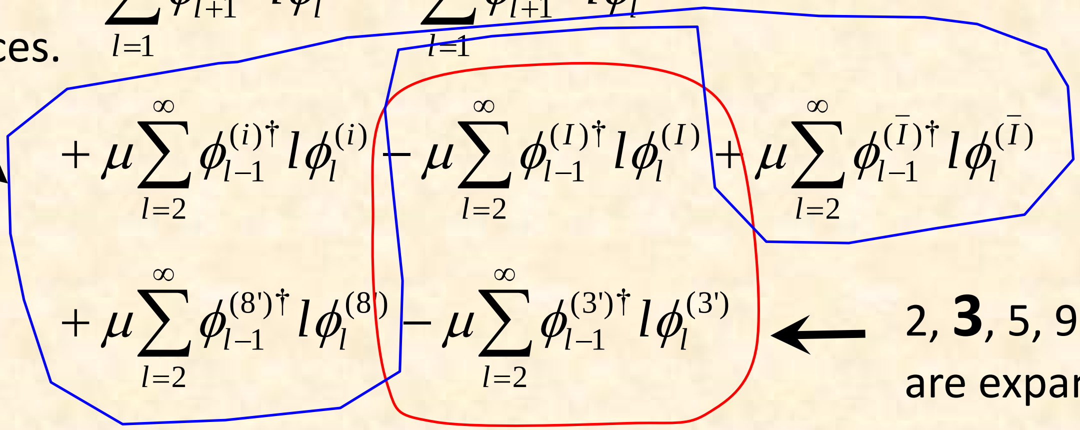
$$H_W = -\sum_{l=1}^{\infty}\phi_{l+1}^{(i)\dagger} l \phi_l^{(i)} - \sum_{l=1}^{\infty}\phi_{l+1}^{(I)\dagger} l \phi_l^{(I)} - \sum_{l=1}^{\infty}\phi_{l+1}^{(\bar{I})\dagger} l \phi_l^{(\bar{I})}$$

$$-\sum_{l=1}^{\infty}\phi_{l+1}^{(3')\dagger} l \phi_l^{(3')} - \sum_{l=1}^{\infty}\phi_{l+1}^{(8')\dagger} l \phi_l^{(8')}$$

$i = 1, 2$

$I = 4, 5$

$\bar{I} = 6, 7$



$$+\mu\sum_{l=2}^{\infty}\phi_{l-1}^{(i)\dagger} l \phi_l^{(i)} - \mu\sum_{l=2}^{\infty}\phi_{l-1}^{(I)\dagger} l \phi_l^{(I)} + \mu\sum_{l=2}^{\infty}\phi_{l-1}^{(\bar{I})\dagger} l \phi_l^{(\bar{I})}$$

$$+\mu\sum_{l=2}^{\infty}\phi_{l-1}^{(8')\dagger} l \phi_l^{(8')} - \mu\sum_{l=2}^{\infty}\phi_{l-1}^{(3')\dagger} l \phi_l^{(3')}$$

(2, **4**, 8, 16) +1 spaces are expanding and form a compact spaces.

2, **3**, 5, 9 spaces are expanding.

$\phi_l^{(3')\dagger}$ and $\phi_l^{(8')\dagger}$ are linear combination of $\phi_l^{(3)\dagger}$ and $\phi_l^{(8)\dagger}$

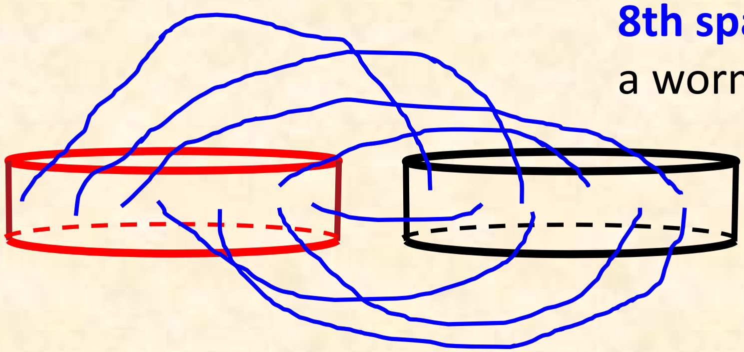
- Tangent and Hyperbolic Tangent expansion（THT-expansion)

The length of space is growing as

$$L(t)=\frac{\int_0^\infty \frac{dL}{L}\,L\,G(0,L;t)}{\int_0^\infty \frac{dL}{L}\,G(0,L;t)}=\begin{cases}\frac{1}{\sqrt{\mu}}\tanh\sqrt{\mu}\,t & (\mu>0)\\ \frac{1}{\sqrt{-\mu}}\tan\sqrt{-\mu}\,t & (\mu<0)\end{cases}$$

d. Dimension Enhancement and Vanishing Cosmo. Const.

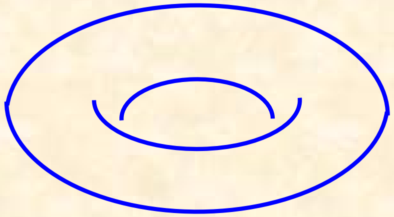
- **Dimension Enhancement** (Knitting of spaces)



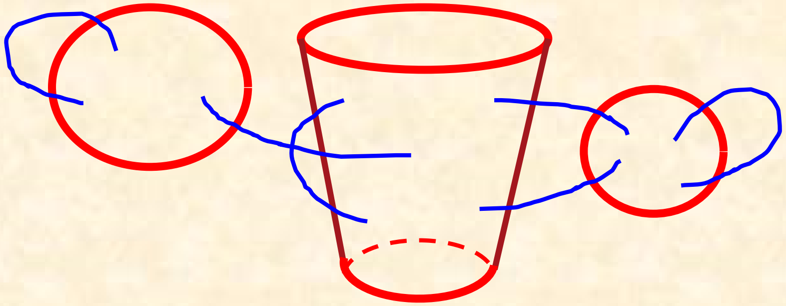
8th space plays a wormhole.

If the lengths of wormholes are zero,

The set of wormholes gives toroidal space.



- **Vanishing cosmological constant** (Coleman mechanism)



Summing up all possible wormholes,

cosmological term disappear.

$$\mu \phi_{l-1}^{(a)\dagger} l \phi_l^{(a)} \to 0$$

5. Modified Friedmann equation

a. The modified Friedmann equation

- **The Hamiltonian from** $-\phi^{\dagger}_{l+1} l \phi_l + \mu \phi^{\dagger}_{l-1} l \phi_l - 2g \phi^{\dagger}_{l-2} l \phi_l$

$$\mathcal{H} = -NL\left(\Pi^2 - \mu + \frac{2g}{\Pi}\right) \qquad \{L, \Pi\} = 1$$

$N(t)$ is introduced to realize the reparametrization invariance of time.

Then, we obtain

$$\left(\frac{\dot{L}}{2NL}\right)^2 = \mu - \frac{2gNL}{\dot{L}} \frac{1+3F(x)}{(F(x))^2} \qquad x := -\frac{8gL^3}{\dot{L}^3}$$

$$(F(x))^3 - (F(x))^2 + x = 0$$

- **Assumption after the Big Bang**

Both μ and g comes from the baby universe production. But, μ disappears because of the Coleman mechanism. On the other hand, g should survives because it plays the coupling constant of wormholes.

In CDT, matter fields are considered to be integrated out as the same as in DT. So, we assume matter fields appear effectively after the Big Bang.

Assumption: $\mu \to \frac{\kappa\rho}{12}$ $g \to -\frac{B}{8}$

ρ is the energy density of matter and if space dimension is 3, we have $\kappa\rho = \frac{(\text{const.})}{L^3}$

5- a

The modified Friedmann equation

Setting the gauge fixing $N(t)=1$
we obtain the modified Friedmann equation

$$\left(\frac{\dot{L}}{L}\right)^2 = \frac{\kappa\rho}{3} + \frac{BL}{\dot{L}}\frac{1+3F(x)}{\left(F(x)\right)^2} \qquad x := \frac{BL^3}{\dot{L}^3}$$

The modified Friedmann equation is also written as

$$\Omega_{\mathrm{m}} + \Omega_B + \Omega_K + \Omega_\Lambda = 1$$

$$\Omega_{\mathrm{m}} = \frac{\kappa\rho}{3H^2} \qquad \Omega_B = \frac{x\left(1+3F(x)\right)}{\left(F(x)\right)^2}$$

$$\Omega_K = \Omega_\Lambda = 0 \qquad H := \frac{\dot{L}}{L}$$

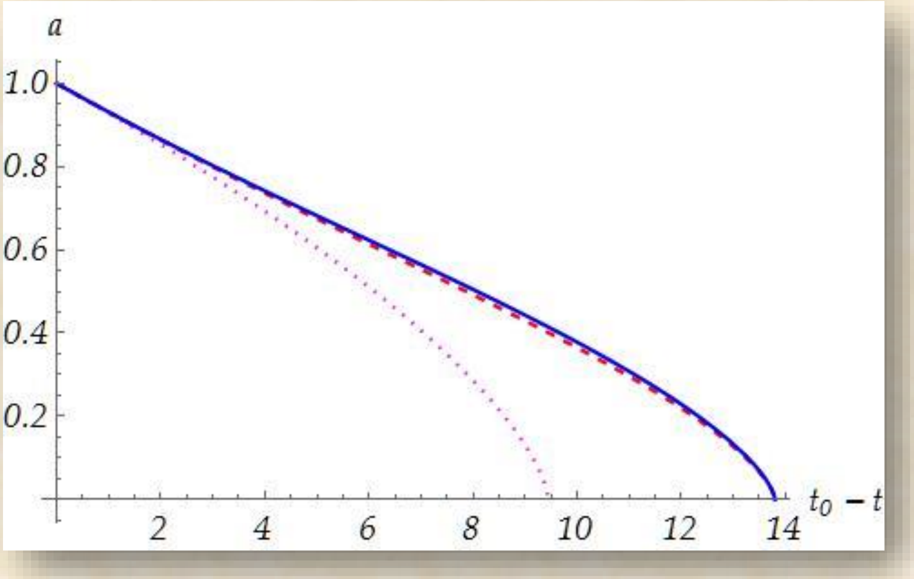
b. Accelerating Universe

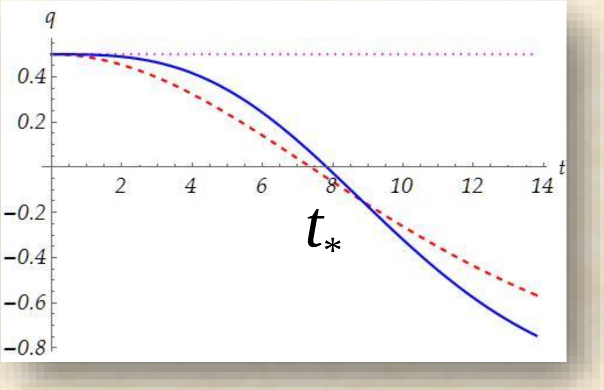
- **The expansion of Universe**

Solid line is our model

Dashed line is Λ-CDM

Dotted line is CDM



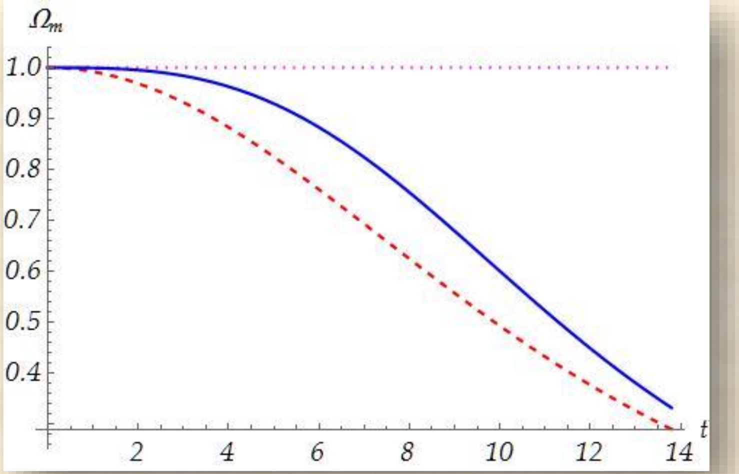


$$a(t) := \frac{L(t)}{L(t_0)} \qquad \left(z(t) + 1 = \frac{1}{a(t)} \right)$$

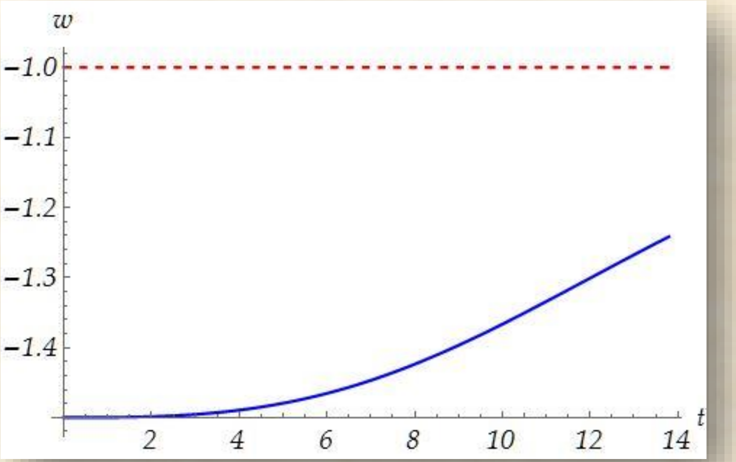
$$q(t) := -\frac{\ddot{L}(t)}{\left(H(t)\right)^2 L(t)}$$

- Other graphs

Solid line is our model
Dashed line is Λ-CDM
Dotted line is CDM



$$\Omega_{\mathrm{m}}(t) := \frac{\kappa\rho(t)}{3\left(H(t)\right)^2}$$



$$w(t) := \frac{2q(t)-1}{3\left(1-\Omega_{\mathrm{m}}(t)\right)}$$

Cf. $$w(t) := \frac{p(t)}{\rho(t)}$$

C. Predictions

- We here assume

$$H(t_0) = 69 \ [\mathrm{km\ s^{-1} Mpc^{-1}}] \qquad t_0 = 13.8 \ [\mathrm{Gyr}]$$

- We predict

$$\Omega_{\mathrm{m}}(t_0) \approx 0.33 \qquad w(t_0) \approx -1.2 \qquad q(t_0) \approx -0.74$$

$$t_* \approx 7.8 \ [\mathrm{Gyr}] \qquad z(t_*) \approx 0.60$$

- Observed value

$$w(t_0) \approx -1.16 \ \pm 0.19$$ (Planck + WMAP + BAO + SNIa)

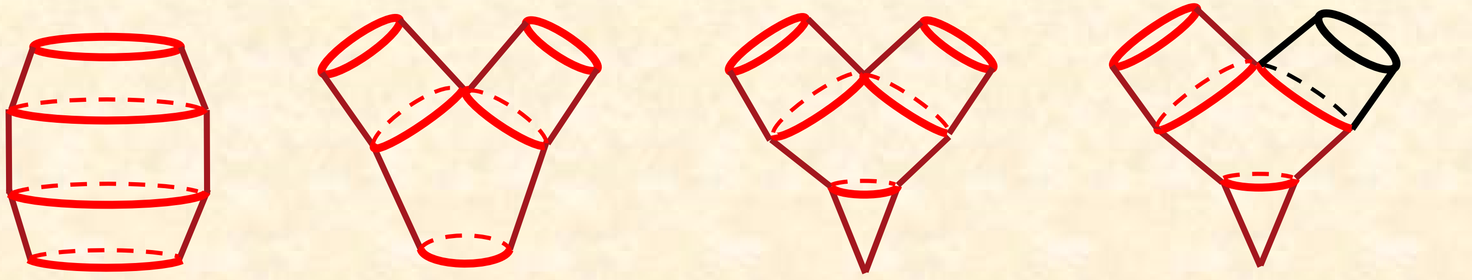
Cf. $w(t) = -1$ (Λ-CDM)

DISCUSSIONS

- **Evolution of CDT**

Fourth gate open!

1st form 2nd form 3rd form 4th form



LDT GCDT CDT w/ W–alg. CDT w/ W&J–alg.

- **Quantization level**

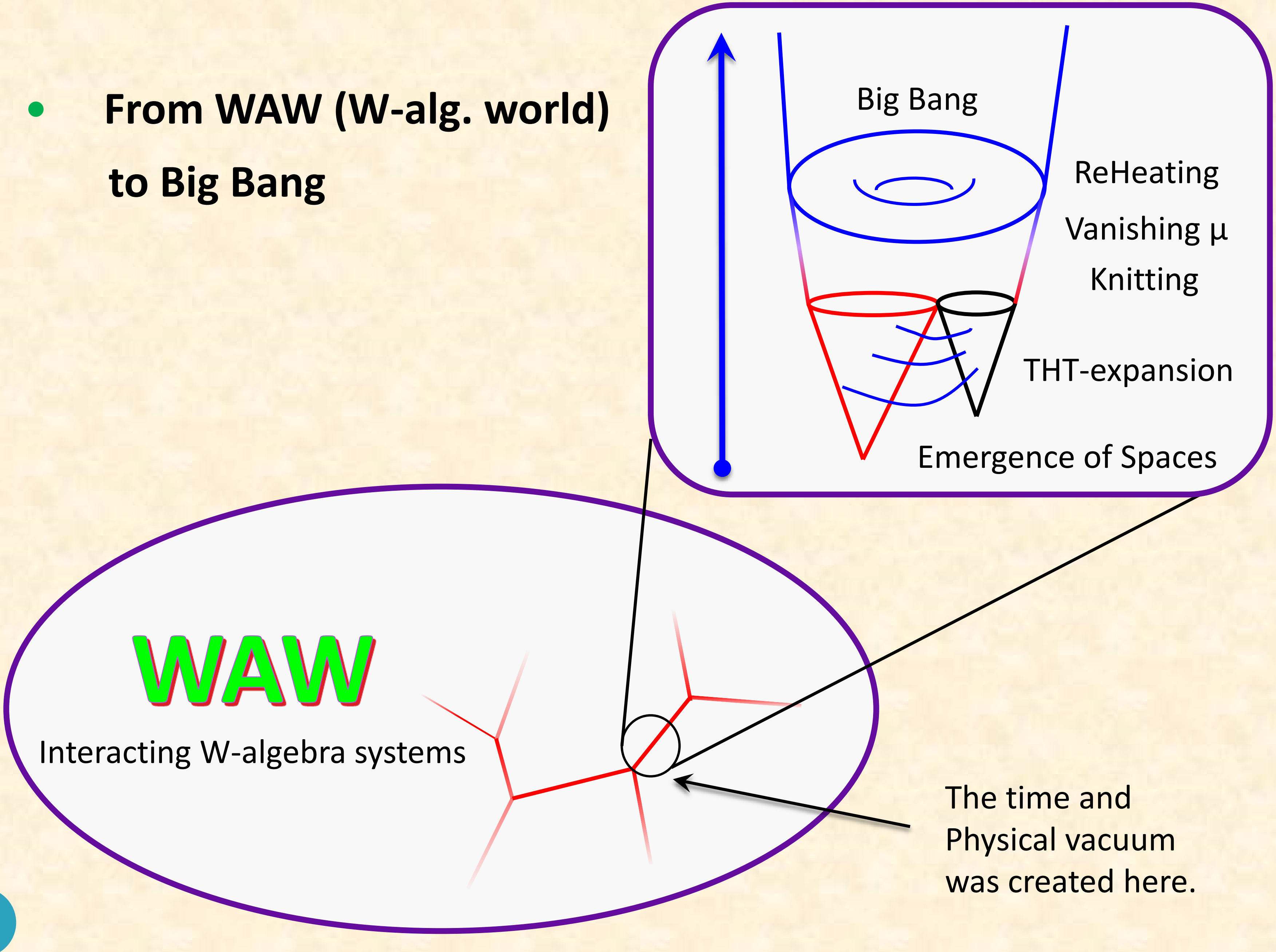
  1st quantization ▪▪▪▪ quantum mechanics

  2nd quantization ▪▪▪▪ intereraction of fields

  3rd quantization ▪▪▪▪ intereraction of spaces

  4th quantization ▪▪▪▪ intereraction of Hilbert spaces of W-alg.

- **From WAW (W-alg. world) to Big Bang**



Big Bang

ReHeating

Vanishing μ

Knitting

THT-expansion

Emergence of Spaces

WAW

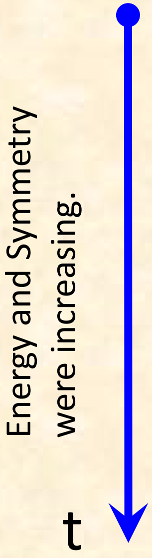
Interacting W-algebra systems

The time and
Physical vacuum
was created here.

SUMMARY

- We constructed **the quantum gravity** from CDT.
- **The simplest model has 2, 3, 4, 6, 10 dimension spacetime.**

WAW



The time and the absolute vacuum was born

The many spaces were born and started by THT-expansion..

When the size of spaces exceeded Planck scale, the dimension enhancement occurred by the Knitting mechanism.

By the Coleman mechanism the cosmological constant vanished and the space was heated, then the Big Bang started.

(Standard Scenario began)

- **We also calculated physical parameters about the late expansion.**