Spherical transverse M5-branes from the plane wave matrix model

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Main topic of my talk

Distribution of eigenvalues in the M(atrix) theory

Matrix models

- Nonperturbative formulation of M/string theories. [BFSS, IKKT, DVV]

$$Z = \int \prod_{\mu=1}^{D} DX^\mu e^{-S}$$

$X^\mu : N \times N$ Hermitian matrices

"quantized version" of embedding function

$$y^\mu : \mathcal{M} \to R^D$$

- BFSS matrix model

$$S = \frac{1}{g^2} \int dt \mathrm{Tr} \left(\frac{1}{2} DX_\mu^2 + \frac{1}{4} [X^\mu, X^\nu]^2 + \mathrm{fermions} \right)$$

Conjecture : This model describes M-theory with light-like compactification

In particular, the model is claimed to be the second quantization describing all fundamental objects such as M2-branes and M5-branes.

Matrix Model = Second Quantization?

Fundamental objects in M-theory : M2-branes (1+2 dim), M5-branes (1+5 dim)

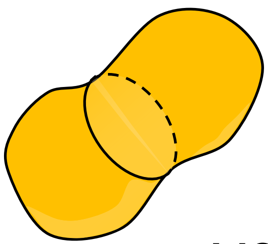
$$\left.\begin{array}{l} |\text{a single M2}\rangle \\ |\text{multiple M2}\rangle \end{array}\right\} \text{Relatively well understood}$$

$$\left.\begin{array}{l} |\text{a single M5}\rangle \\ |\text{multiple M5}\rangle \\ |\text{multiple M2/M5}\rangle \end{array}\right\} \text{Longstanding problem} \Rightarrow \text{This talk}$$

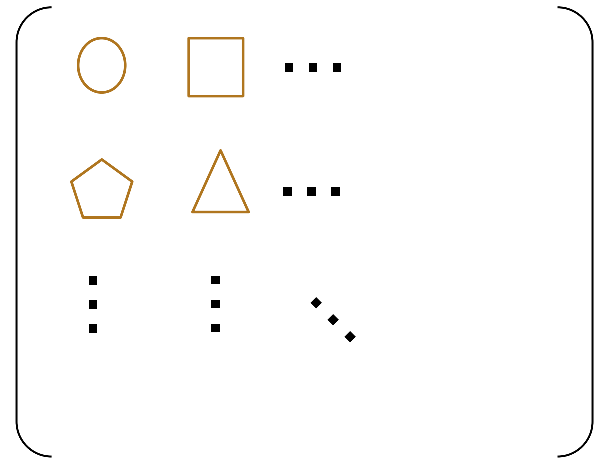
Does the matrix model contain all of these states? If yes, how?

Various matrix configurations

Fundamental objects → Matrix configurations



M2 and M5



- So far, matrix configurations for various objects have been constructed
  - Longitudinal/Transverse M2-brane (F1, D2 in type IIA string)
  - Longitudinal M5-brane (D4 in type IIA string)

[De Wit-Hoppe-Nicolai, Banks-Seiberg-Shenker, Castelino-Lee-Taylor, Imamura …]

However, transverse M5-brane has been missing in the matrix model

Difficulty

- The world volume theory of transverse M5-branes (NS5-branes in IIA string) is not known.

- M5-brane charge is missing in the SUSY algebra of matrix model
  [Banks-Seiberg-Shenker]

No guiding principle to find transverse M5-branes in the matrix model

Today's topic

- Mass deformation of BFSS matrix model is useful to understand this problem

Outline

BFSS matrix model $\Rightarrow$ M-theory on $R^{1,10}$

$\Downarrow$ Mass deformation

Plane wave matrix model (PWMM) $\Rightarrow$ M-theory on pp-wave [BMN]

- M-theory on 11D pp-wave background admits spherical transverse M2 and M5 branes with vanishing light cone energy.

- This M5-brane should be described in PWMM as a vacuum state.

In finding this transverse M5-brane in PWMM,
the target is restricted to the vacuum sector

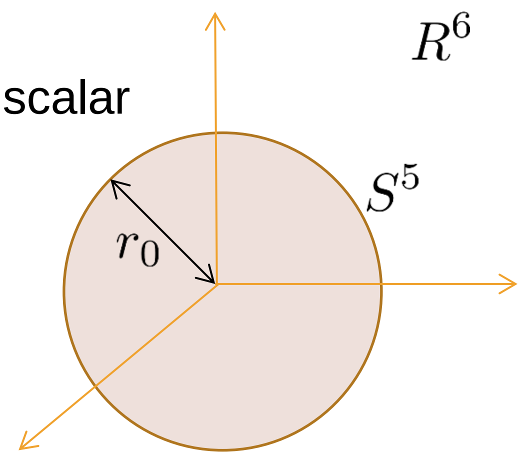
Conjecture [BMN, Maldacena-Sheiki-Jabbari-Raamsdonk] :

" The trivial vacuum (all fields=0) in PWMM corresponds to the spherical M5."

Our result

- To check this conjecture, we applied localization to PWMM.
- We found that in appropriate limit where M5-brane is considered to be realized, the eigenvalue distribution of SO(6) scalars in PWMM forms spherical shells.

Eigenvalue distribution of SO(6) scalar



- For a single M5, the radius of the distribution agrees with the known value of M5.

$$r_0 = \left(\frac{\mu p^+}{6\pi^3}\right)^{1/4} = r_{M5}$$

- We also showed that PWMM has multiple M5-brane states.

Contents

2. M-theory on pp-wave background

PP-wave background

- Maximally supersymmetric pp-wave solution in 11D SUGRA

$$ds^2 = -2dx^+dx^- + dx^M dx^M - \left(\frac{\mu^2}{9}x^i x^i + \frac{\mu^2}{36}x^a x^a\right) dx^+ dx^+$$
$$F_{123+} = \mu$$

$$\begin{cases} M = 1, \cdots, 9 \\ i = 1, 2, 3 \\ a = 4, \cdots, 9 \end{cases}$$

- Let us consider classical M2- and M5-branes on this background

M2-brane

- Dirac-Nambu-Goto action

$$S = -\int d^3\sigma \sqrt{-h} + \int C_3$$

$$\left[\begin{array}{l} h = \det h_{\alpha\beta} \\ h_{\alpha\beta} = g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu \quad \text{: induced metric} \\ X \text{ : world volume} \to R^{1,10} \quad \text{(Embedding function)} \\ dC_3 = F_4 = \mu \, dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^+ \end{array}\right.$$

- Polyakov action

$$S = -\frac{1}{2}\int d^3\sigma \sqrt{-\gamma}\left(\gamma^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu - 1\right) + \int C_3$$

$\gamma_{\alpha\beta}$: auxiliary field

Gauge-fixing

$$\begin{cases} \gamma_{01} = \gamma_{02} = 0 \\ \gamma_{00} = -4\,\det h_{ab} \quad (a, b = 1, 2) \end{cases}$$

$\rightarrow$ Fixing Diffeomorphism

$X^{+} = \sigma^{0}$ $\rightarrow$ EOM for γ_{ab} is trivialized

Light-cone hamiltonian

Poisson bracket

$$H = \frac{2\pi}{p^{+}}\left(P_M^2 + \frac{1}{2}\{X_M, X_N\}^2\right) + \frac{p^{+}}{8\pi}\left(\frac{\mu^2}{9}X_i^2 + \frac{\mu^2}{36}X_a^2\right)$$

$$-\frac{\mu}{6}\left(\epsilon^{ijk}X_i\{X_j, X_k\}\right)$$

p^{+}: Light-cone momentum

$$\begin{cases} M = 1, \cdots, 9 \\ i = 1, 2, 3 \\ a = 4, \cdots, 9 \end{cases}$$

Spherical M2-brane

- Potential for $X_i (i = 1, 2, 3)$ becomes a perfect square

$$V(X_i) = \frac{p^+ \mu^2}{72\pi} \left(X_i + \frac{6\pi}{p^+} \epsilon_{ijk} \{X^j, X^k\} \right)^2$$

- The potential is vanishing when

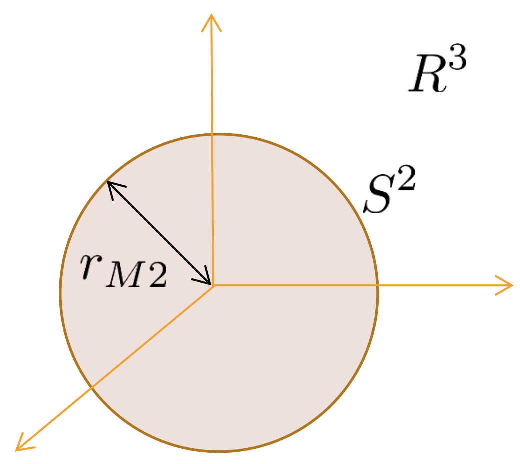
$$X_i + \frac{6\pi}{p^+} \epsilon_{ijk} \{X^j, X^k\} = 0 \qquad X_a = 0 \ (a = 4, \cdots, 9)$$

This is solved by a spherical M2-brane

$$X_1^2 + X_2^2 + X_3^2 = r_{M2}^2$$

$$r_{M2} = \frac{\mu p^+}{12\pi}$$

(in the unit of $T_{M2} = 1$)



Spherical M5-brane

- We can repeat the same computation for an M5-brane
- Action $S = -\int d^6\sigma \sqrt{-h} + \int C_6$
- Hamiltonian

$$H = \frac{\pi^3}{p^+}\left(P_M^2 + \frac{1}{5!}\{X_{M_1}, \cdots, X_{M_5}\}^2\right) + \frac{p^+}{2\pi^3}\left(\frac{\mu^2}{9}X_i^2 + \frac{\mu^2}{36}X_a^2\right)$$

$$-\frac{\mu}{6!}\left(\epsilon^{a_1\cdots a_6} X_{a_1}\{X_{a_2}, \cdots, X_{a_6}\}\right) \qquad \{f_1, \cdots, f_5\} = \epsilon^{a_1\cdots a_5}\partial_{a_1} f_1 \cdots \partial_{a_5} f_5$$

- Potential for $X_a (a = 4, \cdots, 9)$ forms a perfect square

$$V(X_a) = \frac{p^+\mu^2}{72\pi^3}\left(X_{a_1} + \frac{6\pi^3}{5!\mu p^+}\epsilon_{a_1 a_2 \cdots a_6}\{X^{a_2}, \cdots, X^{a_6}\}\right)^2$$

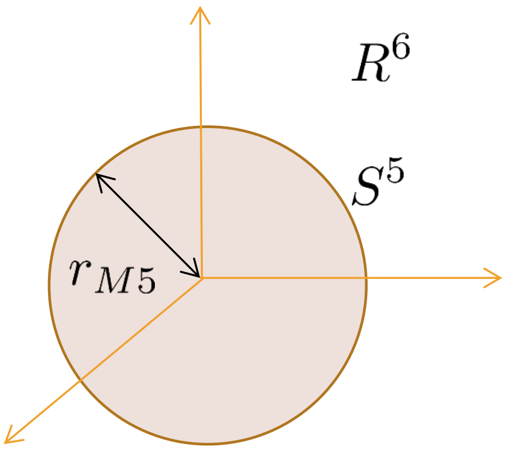
Spherical M5-brane

- Hamiltonian is vanishing for

$$\begin{cases} X_i = 0 \ (i = 1, 2, 3) \\ \sum_{a=4}^{9} X_a^2 = r_{M5}^2 \end{cases}$$

$$r_{M5} = \left(\frac{\mu p^+}{6\pi^3} \right)^{1/4}$$

(in the unit of $T_{M5} = 1$)



Spherical M5-brane

Summary so far

In M-theory on pp-wave, there exist spherical M2- and M5-branes with vanishing light cone energy.

3. The conjecture for M5-branes in PWMM

Matrix model formulation of M-theory

- PWMM is obtained by matrix regularization of M2-brane on pp-wave.

$$X^M(t,\sigma^1,\sigma^2) \to X^M(t) : N\times N \text{ Hermitian matrix}$$

$$\{\ ,\ \} \to iN[\ ,\ ]$$

$$S_{\mathrm{PWMM}} = \frac{1}{g^2}\int dt\, \mathrm{tr}\left[\frac{1}{2}(D_tX_M)^2 - \frac{1}{4}[X_M,X_N]^2 + \frac{1}{2}\Psi^\dagger D_t\Psi - \frac{1}{2}\Psi^\dagger\gamma_M[X_M,\Psi] + \frac{\mu^2}{2}(X_i)^2 + \frac{\mu^2}{8}(X_a)^2 + i\mu\epsilon_{ijk}X_iX_jX_k + i\frac{3\mu}{8}\Psi^\dagger\gamma_{123}\Psi\right],$$

$$\begin{cases} X_i : \text{SO(3) scalar} \\ X_a : \text{SO(6) scalar} \end{cases}$$

- PWMM is conjectured to realize the second quantization of M-theory on pp-wave background [BFSS, BMN].

- It must contain all states in M-theory. In particular, states with zero light-cone energy (spherical M2/M5) must be realized as the vacuum states of PWMM.

Vacua of PWMM

$$S_{\text{PWMM}} = \frac{1}{g^2}\int dt\, \mathrm{tr}\left[\frac{1}{2}(D_t X_M)^2 - \frac{1}{4}[X_M, X_N]^2 + \frac{1}{2}\Psi^\dagger D_t \Psi - \frac{1}{2}\Psi^\dagger \gamma_M [X_M, \Psi] + \frac{\mu^2}{2}(X_i)^2 + \frac{\mu^2}{8}(X_a)^2 + i\mu\epsilon_{ijk}X_i X_j X_k + i\frac{3\mu}{8}\Psi^\dagger \gamma_{123}\Psi\right] ,$$

- Potential for SO(3) scalars

$$V(X_i) = \frac{1}{g^2}\mathrm{tr}\left(\mu X_i + \frac{i}{2}\epsilon_{ijk}[X^j, X^k]\right)^2$$

Any N dimensional representation of SU(2) generator gives a vacuum

$$\begin{cases} X_i = \mu L_i \qquad [L_i, L_j] = i\epsilon_{ijk}L_k & \text{(Fuzzy sphere)} \\ X_a = 0 \end{cases}$$

Fuzzy spheres

$$X^i = \frac{2r_{\rm M2}}{N} L^i$$

Consider N dimensional irreducible rep.

$$\begin{cases} X^i X^i = \dfrac{4r_{\rm M2}^2}{N^2} L^i L^i \simeq r_{\rm M2}^2 \mathbf{1} \\ [X^i, X^j] = \dfrac{2ir_{\rm M2}}{N} \epsilon^{ijk} X^k \simeq 0 \end{cases}$$

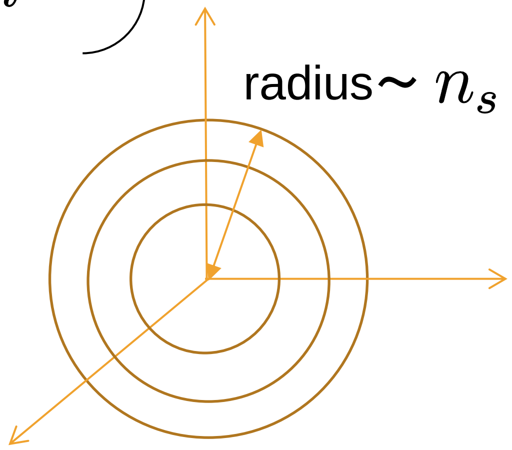
- In the large-N limit, this becomes commutative two-sphere with radius $r_{\rm M2}$

The spherical M2-brane is realized in PWMM as the fuzzy sphere.
But where can we find the M5-brane?

General vacua

In general, representation is reducible

$$X_i = \mu \begin{pmatrix} L_i^{[n_1]} & & & \\ & L_i^{[n_2]} & & \\ & & \ddots & \\ & & & L_i^{[n_\Lambda]} \end{pmatrix}$$



Naively, this will correspond to concentric fuzzy spheres

Still, we cannot see any M5-brane like objects

Conjecture

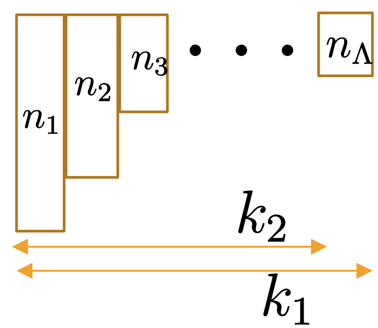
- "Trivial vacuum $(X_i = 0)$ corresponds to a single spherical M5". [BMN]
- More generally, let us make an irreducible decomposition, [Maldacena-Sheiki-Jabbari-Raamsdonk]

$$X_i = \mu L_i = \mu \bigoplus_{s=1}^{\Lambda} L_i^{[n_s]}$$

n_s dim irrep

- Vacua are labelled by partitions of N
  ⇔ Young Tableau

- Let k_a be the length of a-th row



$n_s \to \infty$ is commutative limit of fuzzy sphere (M2-brane limit)

Dual limit $k_a \to \infty$ is conjectured to be the M5-brane limit

For the trivial vacuum ($n_s = 1$), they checked the conjecture by showing that the mass spectra of PWMM and a single M5-brane agree with each other.

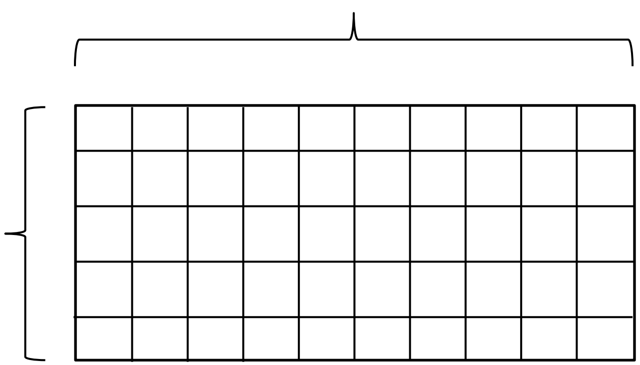
Example: the simplest partition

N_2 copies

$$X_i = \mu \begin{pmatrix} L_i^{[N_5]} & & & \\ & L_i^{[N_5]} & & \\ & & \ddots & \\ & & & L_i^{[N_5]} \end{pmatrix} = L_i^{[N_5]} \otimes 1_{N_2}$$

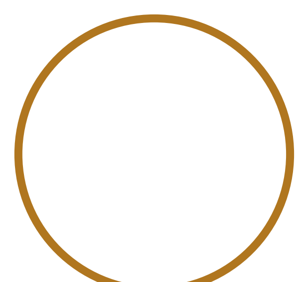
N_2

N_5



$N_2 \gg 1$

N_5 fixed

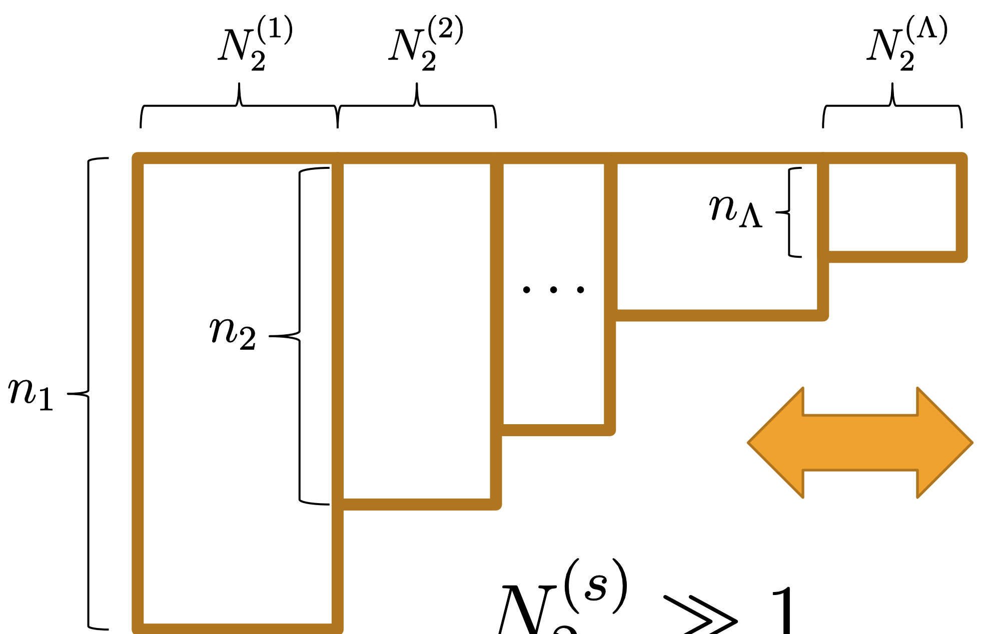


N_5 coincident spherical M5

$p^+ = N_2/R$

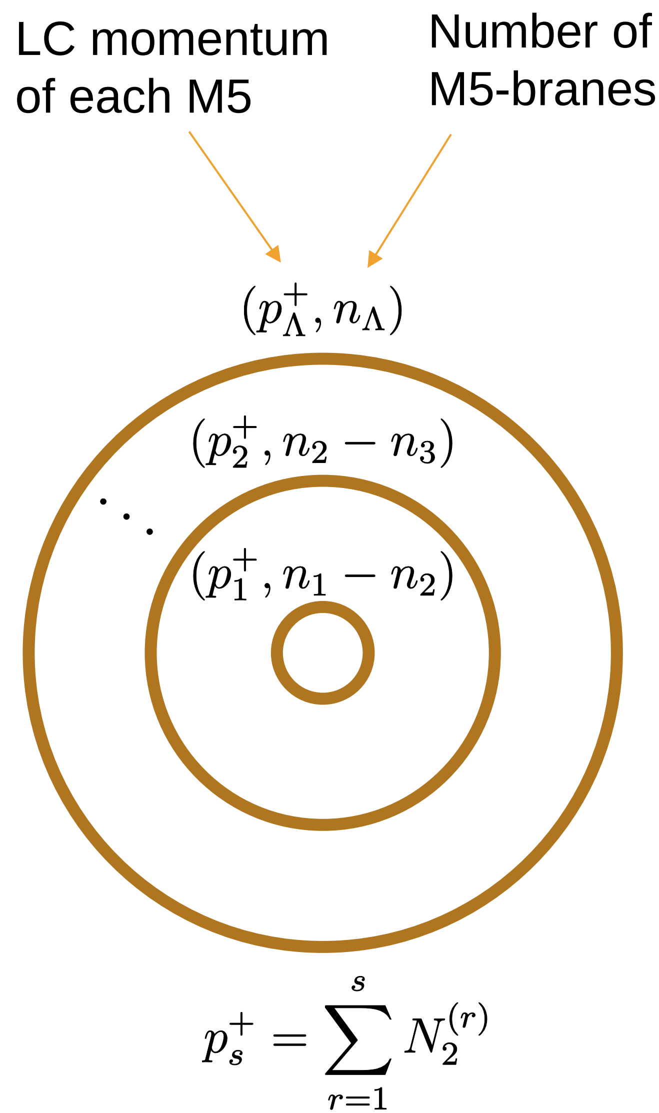
For the most general partition, the conjecture is as follow.

$$X_i = \bigotimes_{s=1}^{\Lambda} L_i^{[n_s]} \otimes 1_{N_2^{(s)}}$$

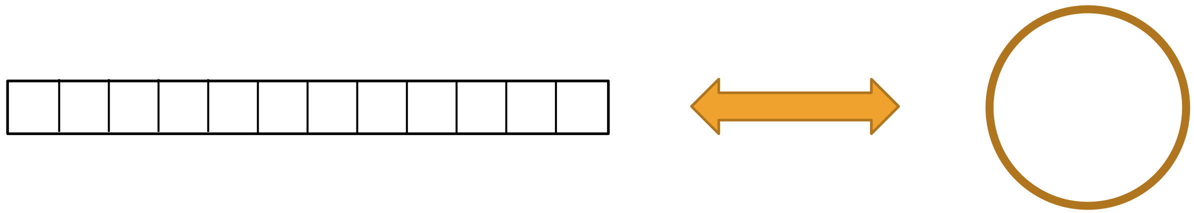


$$N_2^{(s)} \gg 1$$

$$N_2^{(s)}/N_2^{(t)},\ n_s \text{ fixed}$$



Trivial Vacuum = Single M5 ?



Single M5?

- Trivial vacuum does not look like a spherical 1+5 dimensional object.
- However, recall that M-theory is supposed to be realized in the limit,

$$N \to \infty \quad \text{with} \quad p^+ = N/R \quad \text{fixed.}$$

This corresponds to a very strong coupling limit in PWMM

- SO(6) scalars may form the spherical M5-brane at strong coupling.

$$X^m = \underbrace{0 + \text{Large quantum fluctuation}}_{S^5?}$$

We checked this by using the localization

4. M5-branes from PWMM

Decoupling limit

- The system we consider : M5-brane + bulk gravity

- We consider the decoupling limit of M5-brane to focus only on the M5-brane

In terms of the parameters of PWMM, this limit is given by the strong coupling limit in the 't Hooft limit. [Maldacena-Sheiki-Jabbari-Raamsdonk]

Assumption

- We assume that in the strong coupling limit, the low energy modes of the matrices become commuting matrices,

$$[X^A, X^B] \to 0$$

This condition will be needed to describe classical geometric objects in SUGRA.

Localization in PWMM

$$\begin{cases} M, N = 1, \cdots, 9 \\ i, j, k = 1, 2, 3 \\ a, b = 4, \cdots, 9 \end{cases}$$

$$S_{\mathrm{PWMM}} = \frac{1}{g^2}\int dt\, \mathrm{tr}\left[\frac{1}{2}(D_t X_M)^2 - \frac{1}{4}[X_M, X_N]^2 + \frac{1}{2}\Psi^\dagger D_t \Psi - \frac{1}{2}\Psi^\dagger \gamma_M [X_M, \Psi]\right.$$
$$\left. + \frac{\mu^2}{2}(X_i)^2 + \frac{\mu^2}{8}(X_a)^2 + i\mu\epsilon_{ijk}X_i X_j X_k + i\frac{3\mu}{8}\Psi^\dagger \gamma_{123}\Psi\right] ,$$

- For the moment, let us consider the simplest partition,

$$X_i = \mu L_i = \mu L_i^{[N_5]} \otimes \mathbf{1}_{N_2}$$ (N_5=1 ⇔ the trivial vacuum)

- After Wick rotation, we set the boundary condition as

All fields → The vacuum configuration $(\tau \to \pm\infty)$

- We consider a complex scalar field,

$$\phi = X_3 + iX_9 \quad \left(\phi(t) = X_3(t) + i(X_8(t)\sin t + X_9(t)\cos t) \right)$$

There exist supersymmetries which leave ϕ invariant, $Q\phi = 0$ (1/4 BPS)

- Any correlation function made of ϕ can be computed by localization

$$\langle \mathrm{Tr}\phi^{n_1}(t_1)\mathrm{Tr}\phi^{n_2}(t_2)\cdots \rangle$$

- Localization

We add a nice Q-exact term tQV to the action

$\Rightarrow$ The partition function is independent of t

$\Rightarrow$ Taking the large-t limit, the saddle points of the Q-exact term dominate

$\Rightarrow$ Saddle point

$$\phi = X_3 + iX_9 = \mu L_3 + iM$$

M : constant matrix commuting with L_i

◆ Result of the localization

$$\langle \mathrm{Tr}\phi^{n_1}(t_1)\mathrm{Tr}\phi^{n_2}(t_2)\cdots\rangle = \langle \mathrm{Tr}(\mu L_3 + iM)^{n_1}\mathrm{Tr}(\mu L_3 + M)^{n_2}\cdots\rangle_{eff}$$

Effective action

$$S = \frac{1}{g^2}\sum_{i=1}^{N_2} m_i^2 + \sum_{J=0}^{N_5-1}\sum_{i\neq j}\log\left(\frac{((2J)^2+(m_i-m_j)^2)((2J+2)^2+(m_i-m_j)^2)}{((2J+1)^2+(m_i-m_j)^2)^2}\right)$$

Classical part

1-loop determinant

Note : These correlation functions are independent of the time coordinates

(This follows from the SUSY Ward identity)

Since they are invariant under taking the time average,
this implies that they includes only the low energy modes

This result shows that the spectrum of ϕ is determined by the effective action

$$\langle \mathrm{Tr}(z-\phi)^{-1}\rangle = \langle \mathrm{Tr}(z-\mu L_3 + iM)^{-1}\rangle_{eff}$$

Furthermore, under the assumption of the commutativity, eigenvalue distribution of M can be identified with the distribution of the low energy moduli of X^9

◆ In the large-N limit, one can apply the saddle point analysis.

- Weak coupling limit → eigenvalue distribution is trivial $(X_9 = M = 0)$
- Strong coupling limit → There is a non-trivial equilibrium

→ Eigenvalue distribution of X_9 in the decoupling limit of M5-branes

$$\rho(x) = c(a-x^2)^{3/2} \qquad a, c \text{ : constants}$$

Uplift to SO(6) symmetric distribution

- Assuming SO(6) symmetry and the commutativity, we consider the six dimensional eigenvalue distribution $\tilde{\rho}$ defined by

$$\int dx_9 x_9^n \rho(x_9) =: \int dx_4 dx_5 \cdots dx_9 x_9^n \tilde{\rho}(r) \qquad \text{for any } n$$

$$r = \sqrt{x_4^2 + \cdots + x_9^2}$$

- For $\rho(x) = c(a - x^2)^{3/2}$, we find $\tilde{\rho}(r) = \tilde{c}\,\delta(r - r_0)$

SO(6) scalars form a spherical shell in R^6

- Furthermore, the radius of the distribution agrees with the radius of M5,

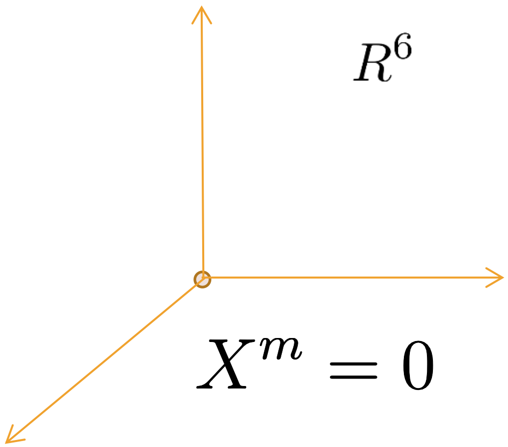
$$r_0 = \left(\frac{\mu p^+}{6\pi^3}\right)^{1/4} = r_{M5}$$

In the M5-brane limit, SO(6) scalars form a spherical M5-brane

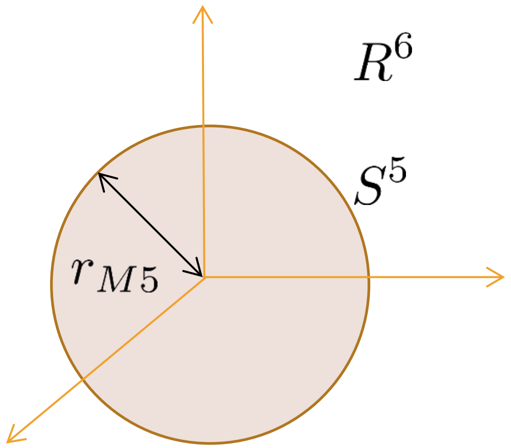
Emergent M5-branes

At weak coupling,
vacuum configuration was trivial

At strong coupling, however,
typical configuration of moduli
is spherical shell



Trivial configuration



Spherical M5-brane

The most general partition

For the most general partition. the eigenvalue integral is given by a multi-matrix model.

$$Z = \int \prod_{s=1}^{\Lambda} \prod_{i=1}^{N_2^{(s)}} dm_{si} Z_{1-loop} e^{-\frac{2n_s}{g^2} m_{si}^2}$$

$$Z_{1-loop} = \prod_{s,t=1}^{\Lambda} \prod_{J=|n_s-n_t|/2}^{(n_s+n_t)/2-1} \prod_{i=1}^{N_2^{(s)}} \prod_{j=1}^{N_2^{(t)}} \left[\frac{\{(2J+2)^2 + (m_{si}-m_{tj})^2\}\{(2J)^2 + (m_{si}-m_{tj})^2\}}{\{(2J+2)^2 + (m_{si}-m_{tj})^2\}^2} \right]^{1/2}$$

Recently, we found an exact solution to this model

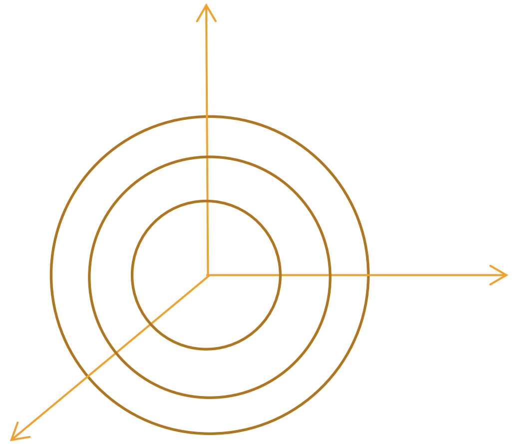
[Asano-Ishiki-Shimasaki-Terashima, to appear in arXiv]

Multiple M5-branes

Solution in the decoupling limit

$$\hat{\rho}_s(x) = \frac{8^{3/4}\sum_{r=1}^{s} N_2^{(r)}}{3\pi\lambda_s^{1/4}}\left[1-\left(\frac{x}{x_s}\right)^2\right]^{3/2}, \quad x_s = (8\lambda_s)^{1/4}, \quad \lambda_s = g^2\sum_{r=1}^{s} N_2^{(r)}$$

The SO(6) symmetric uplift is Λ-stacks of spherical shells



Agrees with the claim of the conjecture!

Summary

- By applying localization, we derived a spherical distribution for low energy modes of SO(6) scalars in the M5-brane limit of PWMM.
- The radius agrees with that of M5-brane
- (multiple) M5-brane states are indeed contained in PWMM

Nice evidence for PWMM to be M-theory !

Outlook

- Excited states? (Rotating M5 etc)
- Theory for multiple M5-branes?